Computing Signal-to-Noise Ratio for Astronomical Point Sources

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1. Computing SNR Given the Exposure Time and Stellar Magnitude

1.1. Calculating the Signal

The signal-to-noise ratio, signal/noise, is a key parameter in assessing the quality of an astronomical observation. The signal, S, is the total number of photons received by the telescope's detector during a specified total exposure time and is defined as

$$S = (\mathrm{QE_{tot}}\ \tau\ t_{exp}\ f_{STAR}) \tag{1}$$

where:

$$\begin{aligned} \mathrm{QE}_{tot} &= \text{total system quantum efficiency} \\ \tau &= \text{additional efficiency factor for source photon detection (usually just set to 1)} \\ t_{exp} &= \text{total exposure time (in seconds)} \\ f_{STAR} &= \text{photon flux per second from a source of magnitude, m, entering the telescope} \end{aligned}$$

where the total exposure time, t_{exp}, is the sum of the individual integration times (e.g., if $N_{exp} = 10$ and each individual exposure is 300 seconds then $t_{exp} = N_{exp}\ \times 300\mathrm{s} = 3000$ s) and where the stellar flux, f_{STAR}, is defined as

$$f_{STAR} = \mathrm{F_0\ C_{Ap}\ T_{atm}}\ \frac{\pi}{4}\mathrm{D}^2\ \Delta\lambda\ 10^{-0.4\mathrm{m}}\ \ \text{in units of photons s}^{-1} \tag{2}$$

In equation 2, the variables are:

$$\begin{aligned} \mathrm{F_0} &= \text{flux zeropoint (photons s}^{-1}\text{ cm}^{-2}\text{ nm}^{-1}\text{ for a }0^{th}\text{ magnitude star)} \\ \mathrm{C_{Ap}} &= \text{aperture correction } (\leq 1)\text{. Set to } < 1\text{, as appropriate, if there is light} \\ &\ \text{from the source that lies outside default SNR calculation box size.} \\ \mathrm{T_{atm}} &= \text{transmission from top of atmosphere to the telescope (= 1 for space telescope)} \\ \mathrm{D} &= \text{telescope aperture diameter (in cm)} \\ \Delta\lambda &= \text{bandwidth (in nm)} \\ \mathrm{m} &= \text{stellar magnitude} \end{aligned}$$

Note that $\mathrm{F_0}$ is calibrated in several different magnitude systems so one must make sure that $\mathrm{F_0}$ and the stellar magnitude value are in the same photometric system. The $\mathrm{F_0}$ is a function of wavelength, λ. For the AB magnitude system, the $\mathrm{F_0}(\lambda)$ is

$$\mathrm{F_0}(\lambda)_{\mathrm{AB}} = \frac{5.5099\ \times\ 10^6}{\lambda(\text{in nm})}\ \text{photons s}^{-1}\text{ cm}^{-2}\text{ nm}^{-1} \tag{3}$$

1.2. Calculating the Noise

We consider 5 noise terms: photon noise from the source, photon noise from the sky, dark current, read noise, and thermal emission from the optical telescope assembly. The specific equations for these noise terms are given below.

$$\sigma_{Source} = \sqrt{\mathrm{QE_{tot}}\ \mathrm{t_{exp}}\ \tau\ f_{\mathrm{STAR}}} \tag{4}$$

$$\sigma_{Sky} = \sqrt{\mathrm{QE_{tot}}\ \mathrm{t_{exp}}\ f_{\mathrm{SKY}}} \tag{5}$$

where

$$f_{SKY} = \mathrm{F_0}\ \frac{\pi}{4}\mathrm{D}^2\ \Delta\lambda\ 10^{-0.4\Sigma}\ (\Theta^2\ \mathrm{N_{PIX}}) \tag{6}$$

and where

Σ	=	sky brightness (in mag/square arcsecond)
Θ	=	pixel scale (in arcseconds/pixel)
$\mathrm{N_{PIX}}$	=	number of pixels in the SNR computation box

The dark current noise per pixel is

$$\sigma_{dark\ current} = \sqrt{\mathrm{DC}\ \mathrm{t_{exp}}\ \mathrm{N_{PIX}}} \tag{7}$$

where DC is the dark current in electrons s^{-1} pixel^{-1}.

By definition, the read noise value per pixel per exposure is

$$\sigma_{read\ noise} = RN \tag{8}$$

where RN is the read noise level in electrons pixel^{-1} per individual readout (*i.e.*, per individual exposure).

1.2.1. Thermal Emission

Thermal emission from the optical telescope assembly (OTA) is largely limited to wavelengths longer than 2 microns. The precise point at which the thermal emission competes with or exceeds the sky background level is dependent primarily on the OTA temperature. The Planck law describes how thermal emission varies with temperature and wavelength. The Planck equation is:

$$\mathrm{B}(\lambda, \mathrm{T}) = \frac{2hc^2}{\lambda^5}\ \left[\frac{1}{e^{(hc/\lambda k_B \mathrm{T})} - 1}\right] \tag{9}$$

where $\mathrm{B}(\lambda, \mathrm{T})$ is the Planck blackbody emission function in units of erg s^{-1} cm^{-3} $\mathrm{steradian}^{-1}$ and where h is Planck's constant (6.6261×10^{-27} erg s), c is the speed of light (2.99792×10^{10} cm s^{-1}), and k_B is Boltzmann's constant (1.3807×10^{-16} erg degK^{-1}). The OTA temperature, T, is given in degrees Kelvin and the central wavelength, λ, is specified in cm.

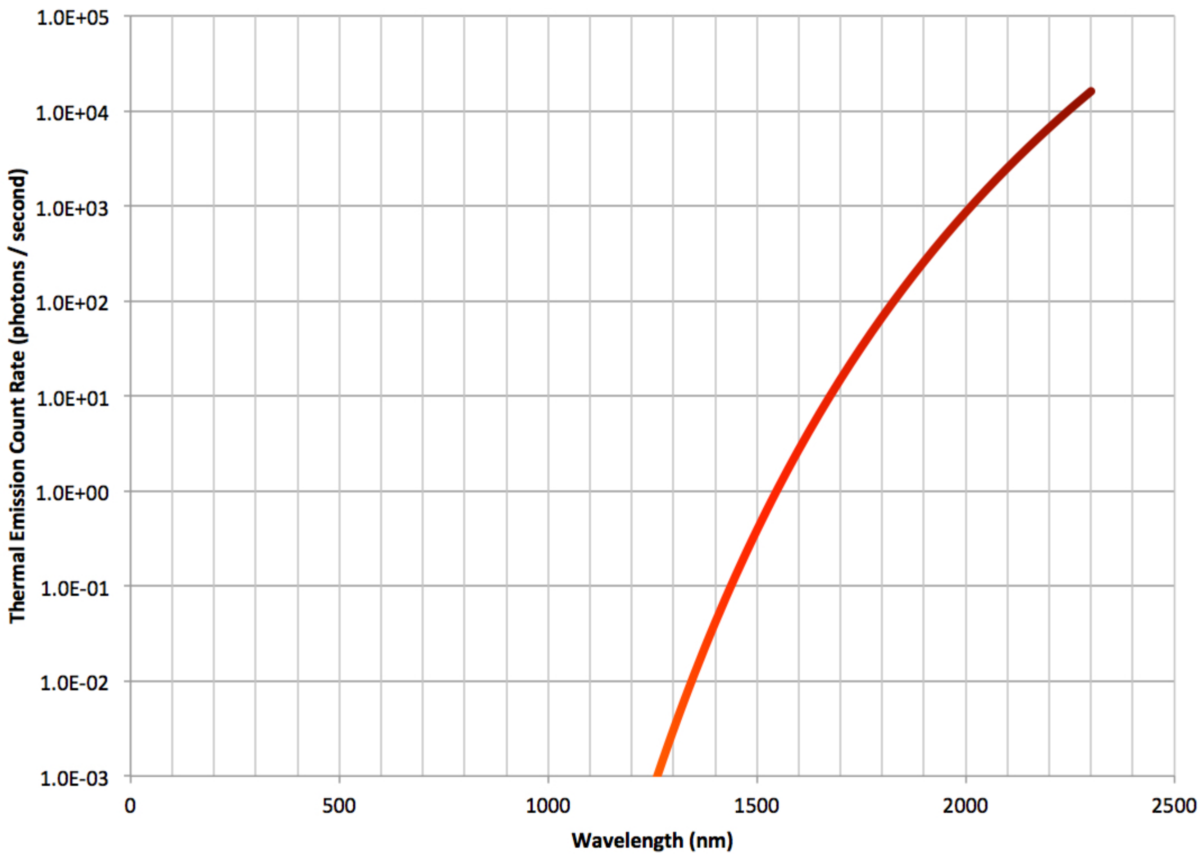

Fig. 1.— The photon count rate due to thermal emission from a 10-meter space telescope operating at a temperature of 280^o K. The thermal emissivity adopted here is $\epsilon_T = 0.03$ and the total QE is 0.55.

The thermal background level will be the sum of emission from each mirror. Assuming all mirrors in the OTA are at the same temperature, the photon count rate from thermal emission is:

$$\mathrm{C_{Thermal}} = \epsilon_T \ \mathrm{B}(\lambda, \mathrm{T}) \left(\frac{hc}{\lambda}\right)^{-1} \left(\frac{\pi}{4} D^2\right) \mathrm{QE_{tot}} \ \Omega \ \Delta\lambda_{cm} \tag{10}$$

where $\mathrm{C_{Thermal}}$ is in units of photons s^{-1} and where ϵ_T is the combined total thermal emissivity (e.g., for a 3-bounce system consisting of glass optics coated with Al a value of $\epsilon_T = 0.1$ is reasonable), Ω is the solid angle of the photometric aperture (in steradians) which is just $\Omega = 2.3504 \times 10^{-11}$ $(\Theta^2 \ \mathrm{N_{PIX}})$ (cf. sec. 1.2), and $\Delta\lambda_{cm}$ is the bandwidth in cm. Figure 1 shows the thermal background photon rate for a 10m telescope operating at 280 K. The noise from thermal emission is then just:

$$\sigma_{\mathrm{Thermal}} = \sqrt{\mathrm{t_{exp}} \ \mathrm{C_{Thermal}}} \tag{11}$$

1.3. Total Noise Estimate

The total noise level, N, is the quadrature sum of these five independent noise terms with relevant multipliers to include the multiple pixels in the SNR computation region and the total number of individual exposures. Specifically:

$$N = \sqrt{\sigma^2_{\rm Source} \ + \ \sigma^2_{\rm Sky} \ + \ {\rm N_{PIX}} \ \sigma^2_{\rm dark\ current} \ + \ {\rm N_{PIX}\ N_{exp}} \ \sigma^2_{\rm read\ noise} \ + \ \sigma^2_{\rm Thermal}} \quad (12)$$

which becomes

$$N = \sqrt{{\rm QE_{tot}\ t_{exp}}\ (\tau\ f_{\rm STAR} + f_{\rm SKY}) + ({\rm DC\ t_{exp}\ N_{PIX}}) + ({\rm RN^2\ N_{PIX}\ N_{exp}}) + ({\rm t_{exp} C_{Thermal}})} \quad (13)$$

1.4. SNR Equation

Hence, the SNR, or S/N, as a function of exposure time and stellar magnitude is

$$S/N = \frac{{\rm QE_{tot}}\ \tau\ t_{exp}\ f_{STAR}}{\sqrt{{\rm QE_{tot}\ t_{exp}}\ (\tau\ f_{\rm STAR} + f_{\rm SKY}) + ({\rm DC\ t_{exp}\ N_{PIX}}) + ({\rm RN^2\ N_{PIX}\ N_{exp}}) + ({\rm t_{exp} C_{Thermal}})}} \quad (14)$$

2. Computing the Exposure Time Given the SNR and Stellar Magnitude

One can manipulate Equation 14 to yield companion quadratic equations that allow you to compute either the exposure time or the point source magnitude given the other two variables. The sections here give the resulting formulae for these other related computations.

If one instead wishes to compute the total exposure time needed to achieve a given SNR for a star of magnitude, m, then the relevant equation is:

$${\rm t_{exp}} = \frac{-b \ + \ \sqrt{b^2 \ - \ 4ac}}{2a} \quad (15)$$

where

$$\begin{aligned} a &= \left\{{\rm QE_{tot}}\ \tau\ f_{STAR}\right\}^2 \\ b &= \left\{-(SNR^2)\ \left[{\rm QE_{tot}}\ (\tau\ f_{STAR} \ + \ f_{SKY}) \ + \ {\rm C_{Thermal}} \ + \ DC\ N_{PIX}\right]\right\} \\ c &= \left\{-(SNR^2)\ \left[RN^2\ N_{PIX}\ N_{exp}\right]\right\} \end{aligned}$$

3. Computing the Stellar Magnitude Given the SNR and the Exposure Time

If one wishes to compute the limiting stellar magnitude, m, given a desired SNR and a desired total exposure time, $\rm t_{exp}$, then the relevant equation is:

$$m = -2.5{\rm log}_{10}\left[\frac{\kappa}{({\rm F_0\ C_{Ap}\ T_{atm}}\ \frac{\pi}{4}{\rm D^2}\ \Delta\lambda)}\right] \quad (16)$$

where:

$$\kappa = \frac{-b_0 \ + \ \sqrt{b_0^2 \ - \ 4a_0c_0}}{2a_0} \tag{17}$$

and where

$$\begin{aligned}
a_0 &= \left\{\mathrm{QE_{tot}} \ \tau \ t_{exp}\right\}^2 \\
b_0 &= \left\{-(SNR^2) \ \mathrm{QE_{tot}} \ t_{exp}\right\} \\
c_0 &= \left\{-(SNR^2) \ \left[(\mathrm{QE_{tot}} \ f_{SKY} \ + \ \mathrm{C_{Thermal}}) \ t_{exp} \ + \ (N_{PIX} \ DC \ t_{exp}) \ + \ (RN^2 \ N_{PIX} \ N_{exp})\right]\right\}
\end{aligned}$$