

1 Surface Densities Prewet a Near Critical Membrane  
2 Supplementary Material

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18 **Overview**

19 In section 1 we provide details on the Landau Theory introduced in the main text. We first derive  
20 the Landau coefficients used in our expression of the tether free energy,  $f_{tether}$ . Next we detail the  
21 derivation of the surface free energy,  $f_{surf}$  expressed as a function of three ‘surface’ variables  $\phi_0, \psi, \rho$ .  
22 We then examine the behavior of  $f_{surf}$  near the membrane critical point, showing enhancement  
23 of prewetting through the diverging effective surface enhancement and surface chemical potential  
24 as criticality is approached. We then derive a relationship between membrane’s susceptibility and  
25 the enhancement of prewetting. In Section 2 we provide the supplemental figures referenced in the  
26 main text. Here we discuss the effect of finite tether-length used in our simulations, properties  
27 of our bulk lattice polymers system and alternative polymer mixtures, expand on the three-phase  
28 coexistence region, and discuss membrane- and bulk-dominated surface denstites

<sup>29</sup> **1 Landau Theory**

Our mean-field theory starts with the Landau Functional shown in Equation 1 of the main text:

$$\begin{aligned}
 \mathcal{L} &= \mathcal{L}_{2D} + \mathcal{L}_{3D} \\
 \mathcal{L}_{2D} &= \int d^2\vec{x} f_{2D}(\psi, \rho, \phi_0) \\
 \mathcal{L}_{3D} &= \int d^2\vec{x} dz \frac{1}{2} \nabla \phi^2 + f_{3D}(\phi) \\
 f_{2D}(\psi, \rho, \phi_0) &= \underbrace{\frac{t_{mem}}{2} \psi^2 + \frac{u_{mem}}{4!} \psi^4 - \lambda_\psi \psi}_{f_{membrane}} \\
 &\quad - \underbrace{\frac{5}{6}\rho + \frac{3}{2}\rho^2 - \frac{3}{3!}\rho^3 + \frac{2}{4!}\rho^4 - \frac{1}{4} - \lambda_\rho \rho}_{f_{tether}} \\
 &\quad - \underbrace{h_\psi \rho \psi - h_\phi \rho \phi_0}_{f_{int}}
 \end{aligned} \tag{1}$$

where  $f_{3D}$  is the free energy of a system displaying a conventional condensation transition:

$$f_{3D} = \frac{t_{bulk}}{2} \phi^2 + \frac{u_{bulk}}{4!} \phi^4 - \mu_{bulk} \phi \tag{2}$$

- <sup>30</sup> Our goal is to minimize  $\mathcal{L}$  over order parameters  $\rho, \psi$  and order parameter profile  $\phi(z)$ . Two  
<sup>31</sup> derivatives  $\frac{\partial \mathcal{L}}{\partial \psi} = \frac{\partial \mathcal{L}}{\partial \rho} = 0$  minimize the free energy over purely ‘surface’ terms. A functional  
<sup>32</sup> derivative  $\frac{\delta \mathcal{L}}{\delta \phi(z)} = 0$  minimizes the energy over forms of the density profile  $\phi(z)$ .

<sup>33</sup> **1.1 Derivation of  $f_{tether}$**

The free energy of the tethers in our model is expressed as a Landau free energy, a function of powers of the order parameter  $\rho$  and Landau coefficients  $a, b, c, d, e$

$$f_{tether}(\rho) = a\rho + \frac{b}{2}\rho^2 + \frac{c}{3!}\rho^3 + \frac{d}{4!}\rho^4 + e - \lambda_\rho \rho$$

The tethers are a non-interacting 2-dimensional lattice gas. The free energy of a 2D gas may be written as  $f_{tether} = \rho \log \rho$ . We approximate this by expanding  $f_{tether}$  in powers of  $\rho$  at  $\rho_\star$

$$\rho \log \rho|_{\rho_\star} = (\rho - \rho_\star)(1 + \log \rho_\star) + \frac{(\rho - \rho_\star)^2}{2\rho_\star} - \frac{(\rho - \rho_\star)^3}{6\rho_\star^2} + \frac{(\rho - \rho_\star)^4}{12\rho_\star^3}$$

$$f_{tether}(\rho, \rho_\star) = \frac{-3\rho_\star^4 - 12\rho_\star^4 \log \rho_\star}{12\rho_\star^3} + \frac{\rho(-10\rho_\star^3 + 12\rho_\star^3 \log \rho_\star)}{12\rho_\star^3} + \frac{3\rho^2}{2\rho_\star} - \frac{\rho^3}{2\rho_\star^2} + \frac{\rho^4}{12\rho_\star^3} - \lambda_\rho \rho$$

Setting  $\rho_\star = 1$ , as used in all calculations here, we obtain the free energy of the tethers and values of all Landau coefficients:

$$f_{tether}(\rho, \rho_\star = 1) = -\frac{1}{4} - \frac{5}{6}\rho + \frac{3}{2}\rho^2 - \frac{3}{3!}\rho^3 + \frac{2}{4!}\rho^4 - \lambda_\rho \rho$$

- <sup>34</sup> This Taylor series provides an analytically tractable approximation to the ideal gas free energy of  
<sup>35</sup> the tethers near  $\rho = 1$ . However, because its first derivative is finite at  $\rho = 0$ , some of our physical  
<sup>36</sup> phases spill over into negative values of  $\rho$ . As such we interpret  $\rho$  more as an order parameter than  
<sup>37</sup> as a more rigorously defined tether density.

## 38 1.2 Derivation of surface free-energy

39 Contributions from the bulk are a function of  $\phi(z)$ . To simplify calculations we express the bulk  
40 energy as a function of initial and final values of  $\phi$ ,  $\phi(z = 0) = \phi_0$  and  $\phi(z = \infty) = \phi_\infty$ . these  
41 solely determine the behaviour of the density profile. We write  $f_{3D}(\phi) - f_{3D}(\phi_\infty)$  as  $f_{3D}^{\phi-\phi_\infty}$ .

First we perform the functional derivative,  $\frac{\delta \mathcal{L}_{bulk}}{\delta \phi(z)}$

$$\begin{aligned}\mathcal{L}_{bulk}(\phi(z)) &= \int_0^\infty dz \frac{1}{2} (\nabla \phi)^2 + f_{3D}^{\phi-\phi_\infty} \\ \delta \mathcal{L}_{bulk}[\phi(z)] &= \int_0^\infty dz \nabla \phi \nabla \delta \phi + \frac{\partial f_{3D}^{\phi-\phi_\infty}}{\partial \phi} \delta \phi \\ \frac{\delta \mathcal{L}_{bulk}[\phi(z)]}{\delta \phi(z)} &= \frac{\partial f_{3D}^{\phi-\phi_\infty}}{\partial \phi} - \nabla^2 \phi = 0\end{aligned}\tag{3}$$

The last equality serves as a second order ODE for  $\phi(z)$  which we solve given boundary conditions at  $z = 0, \infty$ , making use of mathematical tools developed in [1].

$$\begin{aligned}\frac{\delta \mathcal{L}_{bulk}}{\delta \phi(z)} &= 0 \\ \frac{\partial^2 \phi}{\partial z^2} &= \frac{\partial f_{3D}^{\phi-\phi_\infty}}{\partial \phi} \\ \frac{\partial \phi}{\partial z} \frac{\partial^2 \phi}{\partial z^2} &= \frac{\partial \phi}{\partial z} \frac{\partial f_{3D}^{\phi-\phi_\infty}}{\partial \phi} \\ \frac{1}{2} \frac{\partial}{\partial z} \left( \frac{\partial \phi}{\partial z} \right)^2 &= \frac{\partial}{\partial z} f_{3D}^{\phi-\phi_\infty} \\ \frac{1}{2} \int_{z=0}^{z=\infty} dz \frac{\partial}{\partial z} \left( \frac{\partial \phi}{\partial z} \right)^2 &= \int_{z=0}^{z=\infty} dz f_{3D}^{\phi-\phi_\infty} \\ \frac{1}{2} \left( \left( \frac{\partial \phi}{\partial z} \right)_{z=\infty}^2 - \left( \frac{\partial \phi}{\partial z} \right)_{z=0}^2 \right) &= f_{3D}^{\phi-\phi_\infty}(\phi_\infty) - f_{3D}^{\phi-\phi_\infty}(\phi_0) \\ \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)_{z=0}^2 &= f_{3D}^{\phi-\phi_\infty}(\phi_0) \\ \frac{\partial \phi}{\partial z} &= \pm \sqrt{2 f_{3D}^{\phi-\phi_\infty}(\phi_0)}\end{aligned}\tag{4}$$

Finally we substitute this into our original functional and rewrite as an integration over values

of  $\phi$  instead of  $z$  to define the bulk contribution,  $\Delta f_{bulk}$

$$\begin{aligned}
\Delta f_{bulk}(\phi(z)) &= \int_0^\infty dz \frac{1}{2} (\nabla\phi)^2 + f_{3D}^{\phi-\phi_\infty} \\
&= \int_0^\infty dz (\nabla\phi)^2 \\
&= \int_{\phi_0}^{\phi_\infty} dz \frac{d\phi}{dz} \frac{dz}{d\phi} (\nabla\phi)^2 \\
&= \int_{\phi_0}^{\phi_\infty} d\phi \nabla\phi \\
\Delta f_{bulk}(\phi_0, \phi_\infty) &= \int_{\phi_0}^{\phi_\infty} d\phi \sqrt{(2(f_{\phi_0} - f_{\phi_\infty}))}
\end{aligned} \tag{5}$$

The sign of  $\frac{\partial\phi}{\partial z}$  depends on whether  $\phi_0$  is above or below  $\phi_\infty$ : polymer density always moves towards the bulk density  $\phi_\infty$ . An expression for the gradient may be obtained by substituting in the expression for  $f_{3D}^{\phi-\phi_\infty}(\phi)$  and  $u_{bulk} = \frac{1}{6}(6t_{bulk}\phi_\infty + \phi_\infty^3)$ :

$$\frac{\partial\phi}{\partial z} = (\phi - \phi_\infty) \frac{\sqrt{(12t_{bulk} + \phi^2 + 2\phi\phi_\infty + 3\phi_\infty^2)}}{2\sqrt{6}} \tag{6}$$

Writing  $\Delta f_{bulk}$  as a function of  $\phi_0$  and  $\phi_\infty$  now allows minimization of the systems Landau free energy over values of  $\phi_0, \rho$ , and  $\psi$ . After solutions are obtained we compute the energy of each of these solutions. This can be expressed as

$$\begin{aligned}
f_{surf} &= \Delta f_{bulk}(\phi_0, \phi_\infty) + f_{2D}(\phi_0, \psi, \rho) \\
&= \int_{\phi_0}^{\phi_\infty} d\phi \sqrt{2(f_{\phi_0} - f_{\phi_\infty})} + f_{2D} \\
f_{surf} &= \int_{\phi_0}^{\phi_\infty} d\phi \left\{ \nabla\phi - \frac{\partial f_{2D}}{\partial\phi} \right\} + \frac{\partial f_{2D}}{\partial\phi}|_{\phi_\infty}
\end{aligned} \tag{7}$$

where the final equation is the integration between the curves of  $\nabla\phi$  and  $-\frac{\partial f_{2D}}{\partial\phi_0}$  shown in Figure 4B of the main text.

### 1.3 Enhancement of prewetting regime near membrane critical point

In our Monte-Carlo simulations and numerical calculations, we find a large expansion of the prewetting region as the membrane is brought near to its critical point. We now aim to show using a simplified Landau theory how this enhancement arises. First we simplify the free energies of membrane and tether components, and obtain expressions for minimal values of  $\psi$  and  $\rho$  to quadratic order:

$$f_{2D} = \frac{t_{mem}}{2}\psi^2 - \lambda_\psi\psi - \frac{1}{4} - \frac{5}{6}\rho + \frac{3}{2}\rho^2 - \lambda_\rho\rho - h_\psi\psi\rho - h_\phi\phi\rho$$

$$\begin{aligned}
\frac{\partial f_{2D}}{\partial \psi} &= \frac{\partial f_{2D}}{\partial \rho} = 0 \\
\psi_{min} &= -\frac{-5h_\psi - 6(h_\psi \lambda_\rho - 3\lambda_\psi - h_\phi h_\psi \phi_0)}{6(3t_{mem} - h_\psi^2)} \\
\rho_{min} &= -\frac{5t_{mem} - 6(t_{mem} \lambda_\rho + h_\phi \lambda_\psi + t_{mem} h_\phi \phi_0)}{6(h_\psi^2 - 3t_{mem})} \tag{8}
\end{aligned}$$

This simplification of  $f_{2D}$  excludes fourth order terms in  $\psi$  and  $\rho$  is only reasonable for  $t_{mem} > t_{c,mem}$ . Substituting  $\psi_{min}, \rho_{min}$  into the original  $f_{surf}$  we obtain the surface free energy in terms of  $\phi_0$ :

$$\begin{aligned}
f_{surf}(\phi_0, \psi, \rho) &= \Delta f_{bulk}(\phi_0, \phi_\infty) - \frac{t_{mem} h_\phi^2}{2(3t_{mem} - h_\psi^2)} \phi_0^2 - \frac{h_\phi (t_{mem} (5 + 6\lambda_\rho) + 6h_\psi \lambda_\psi)}{6(3t_{mem} - h_\psi^2)} \phi_0 \\
&\quad - \frac{t_{mem} (79 + 60\lambda_\rho + 36\lambda_\rho^2) + 6(-3h_\psi^2 + 2h_\psi (5 + 6\lambda_\rho) \lambda_\psi + 18\lambda_\psi^2)}{72(3t_{mem} - h_\psi^2)} \tag{9}
\end{aligned}$$

50 Here second order couplings in  $\phi_0$  emerge after the minimizing over the membrane and tether  
51 degrees of freedom. Each of these terms has a denominator,  $3t_{mem} - h_\psi^2$  which defines the membrane  
52 critical point as  $t_{mem} = h_\psi^2/3$ . The surface enhancement (prefactor to the  $\phi_0^2$  term) is proportional  
53 to  $(J_{c,mem} - J_{mem})^{-1}$  which is consistent with the scaling of the Ising model susceptibility exponent  
54  $\gamma = 1$  in mean-field.

#### 55 1.4 Membrane susceptibility and prewetting

The susceptibility-mediated enhancement of prewetting can be seen directly by examining higher-order interactions between membrane and tether components. We split  $\mathcal{L}_{2D}$  into non-interacting membrane and tether contributions and interaction terms mixing all components:

$$\begin{aligned}
\mathcal{L}_{2D} &= \int d^2 \vec{x} \{ f_{mem}(\psi) + f_{tether}(\rho) + f_{int}(\psi, \rho, \phi_0) \} \\
f_{int}(\psi, \rho, \phi_0) &= -h_\psi \rho \psi - h_\phi \rho \phi_0 \tag{10}
\end{aligned}$$

We focus on the interactions and define  $\mathcal{L}_{int} = \int d^2 \vec{x} f_{int}(\psi, \rho, \phi_0)$ . We calculate  $\mathcal{L}_{int}$  as a sum of cumulants:

$$\begin{aligned}
\mathcal{L}_{int} &\approx \mathcal{L}_{int}^0 - \langle \mathcal{L}_{int} \rangle - \frac{1}{2} (\langle \mathcal{L}_{int}^2 \rangle - \langle \mathcal{L}_{int} \rangle^2) - \dots \\
\langle \mathcal{L}_{int} \rangle &= \int d^2 \vec{x} h_\psi \psi \rho + h_\phi \phi_0 \rho \\
\langle \mathcal{L}_{int}^2 \rangle &= \int d^2 \vec{x} \rho (h_\psi \psi + h_\phi \phi_0) \int d^2 \vec{x}' \rho' (h_\psi \psi' + h_\phi \phi_0) \\
\langle \mathcal{L}_{int} \rangle^2 &= \left( \int d^2 \vec{x} h_\psi \psi \rho + h_\phi \phi_0 \rho \right)^2 \tag{11}
\end{aligned}$$

We assume  $t_{mem} > t_{c,mem}$  and the membrane is at its critical composition,  $\psi = 0$ . We assume we are far from the prewetting critical point and neglect second order terms in  $\phi_0$

$$\begin{aligned}
\langle \mathcal{L}_{int} \rangle &= \langle \rho \rangle (h_\phi \langle \phi_0 \rangle + h_\psi \langle \psi \rangle) \\
\langle \mathcal{L}_{int}^2 \rangle - \langle \mathcal{L}_{int} \rangle^2 &= h_\psi^2 \langle \rho \rangle^2 \int d^2 \vec{x} \psi_x \int d^2 \vec{x}' \psi_{x'} - \left( \int d^2 \vec{x} h_\psi \psi_x \rho_x \right)^2 \\
&= h_\psi^2 \langle \rho \rangle^2 \int d^2 \vec{x} \int d^2 \vec{x}' \langle \psi_x \psi_{x-x'} \rangle - h_\psi^2 \langle \rho \rangle^2 \langle \psi_x \rangle^2 \\
&= h_\psi^2 \langle \rho \rangle^2 \int d^2 \vec{x} \langle \psi_x^2 \rangle - \langle \psi_x \rangle^2 \\
&= h_\psi^2 \langle \rho \rangle^2 \int d^2 \vec{x} G(\vec{x}) \\
&= h_\psi^2 \langle \rho \rangle^2 \int dr G(r) \\
&= h_\psi^2 \langle \rho \rangle^2 \chi_\psi
\end{aligned} \tag{12}$$

<sup>56</sup> Here  $\chi_\psi$  is the susceptibility of the membrane and  $G(r)$  is the two-point correlation function. The  
<sup>57</sup> integral over all space of  $G(r)$  is  $\chi_\psi$ . Here we have assumed that the membrane order parameter is  
<sup>58</sup> translationally invariant and that  $G(r)$  is spherically symmetric.

$$\mathcal{L}_{int} \approx \mathcal{L}_{int}^0 - \langle \rho \rangle (h_\phi \langle \phi_0 \rangle + h_\psi \langle \psi \rangle) - \frac{h_\psi^2 \langle \rho \rangle^2}{2} \chi_\psi \tag{13}$$

At the critical composition,  $\psi = 0$  and critical temperature,  $\chi_\psi \rightarrow \infty$ , implying that interactions become very favorable even with negligible contributions from other terms. Substituting  $\mathcal{L}_{int}$  into our original Landau functional, we obtain a new free energy function, neglecting fourth order terms:

$$f_{2D} = -\frac{5}{6}\rho + \frac{3}{2}\rho^2 - \frac{1}{4} - \lambda_\rho \rho - h_\phi \phi_0 \rho - \frac{h_\psi^2 \rho^2}{2} \chi_\psi \tag{14}$$

Minimizing over  $\psi$  and  $\rho$  and substituting back in the free energy, as in equation 9 above, we express  $f_{2d}$  just in terms of surface polymer density  $\phi_0$

$$f_{2D} = -\frac{h_\phi^2}{2(h_\psi^2 \chi_\psi - 3)} \phi_0^2 + \frac{h_\phi(5 + 6\lambda_\rho)}{6(h_\psi^2 \chi_\psi - 3)} \phi_0 + \frac{-18h_\psi^2 \chi_\psi + 36\lambda_\rho^2 + 60\lambda_\rho + 79}{72(h_\psi^2 \chi_\psi - 3)} \tag{15}$$

<sup>59</sup> This is similar to Equation 9, where the denominator vanishes at the ‘shifted’ membrane critical  
<sup>60</sup> point  $\chi_\psi = \frac{3}{h_\psi^2}$ . This differs from the calculation above where the divergence near the critical point  
<sup>61</sup> is stronger, as the Ising susceptibility exponent  $\gamma = 7/4$  instead of 1. A full treatment of our model  
<sup>62</sup> beyond mean-field would likely expand prewetting boundaries beyond the values we present here.

63 **2 Supplementary Figures**

64 **2.1 Effect of tether length in monte-carlo simulations**

65 Our Monte Carlo Simulations implement tethers with a finite three-dimensional length while the  
66 tethers in our landau theory lack a three-dimensional component. We examined the effect of tether  
67 length with simulations varying tether length from 1 – 10 monomers 1. As expected, prewet phases  
68 are stable at higher bulk temperatures as tether length is increased. The form of the polymer density  
69 profile is influenced by tether length. The density profile has a maximum at short but non-zero  
70 distances from the wall (see Main text Figure 2D,Figure 3C, Supplemental Figure 1). The location  
71 of this maximum extends further from the wall as tether length is increased. This is in contrast to  
72 conventional wetting and prewetting where the density profile is monotonic with distance from the  
73 membrane.

74 The non-standard behavior of the density profile is explained by the entropic repulsion expe-  
75 rienced by polymers near a surface. Along the length of the tether, the membrane- and tether-  
76 mediated interactions between polymers are the same. However there are more interactions available  
77 to polymers at finite distance from the surface. With a non-zero tether length, polymers prefer to  
78 concentrate their density away from the surface while still within the tether length.

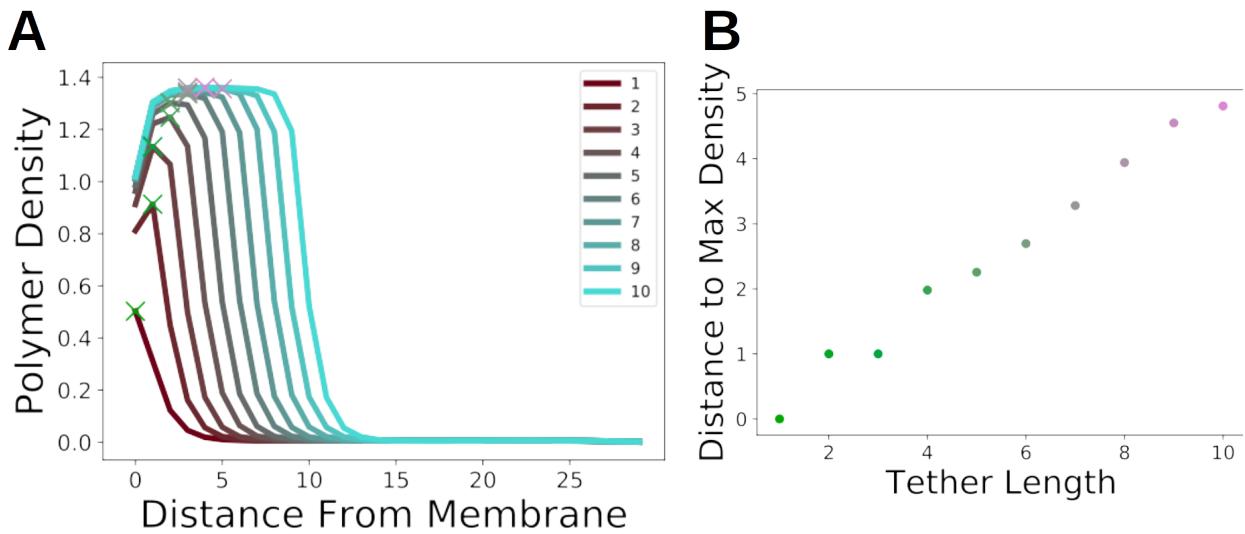


Figure 1: **Effect of Tether Length** **A)** Density profiles of simulations at various tether lengths. Simulations with longer tethers have a broader maximum further from the membrane. **B)** Distance of maximum polymer Density plotted against tether length

79 **2.2 Simulations of bulk polymer mixtures**

80 Our 3D lattice polymer simulations phase-separate into coexisting dense and dilute phases  
81 in lieu of membrane and tethers. Here, as in the main text, red and blue polymers of equal length  
82 ( $n=20$  monomer units) interact attractively with one another and like polymers repel each other.  
83 Polymers phase-separate through increasing  $J_{bulk}$  (See Figure 2). Higher polymer concentrations  
84 phase-separate at lower  $J_{bulk}$ .

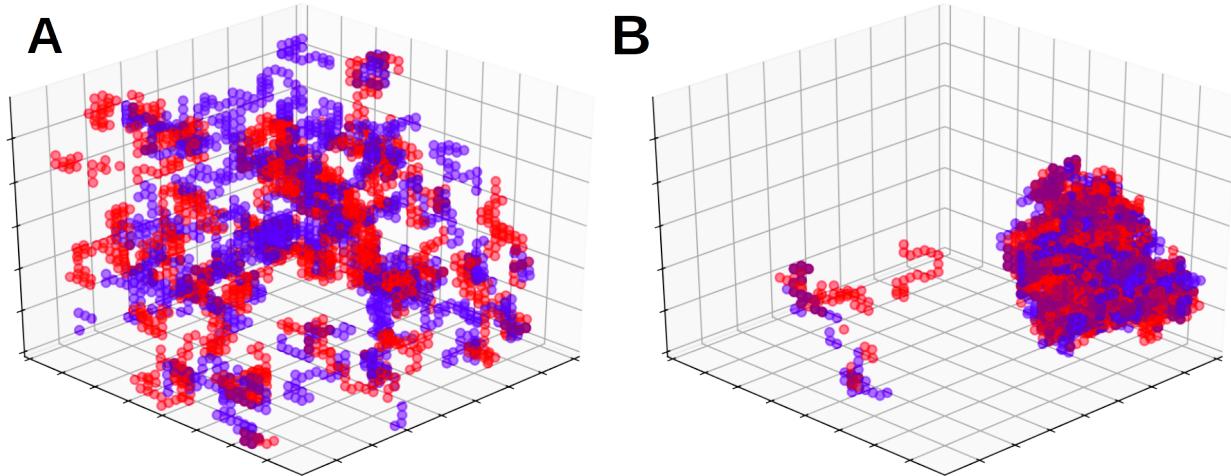


Figure 2: **Monte-Carlo Simulations of Bulk Lattice Polymers** **A)** Lattice polymer simulations show phase coexistence. Simulations at  $0.25k_B T$  have a homogeneous polymer density. **B**) Simulations at strong polymer coupling  $1.0k_B T$ , bottom, see coexisting dense and dilute phases.

85 **2.3 Alternative bulk polymer mixtures**

86 We also ran simulations with different bulk polymer mixtures and observed similar phases to those  
87 in our symmetric mix of red and blue polymers (See Figure 3). We simulated a unary polymer  
88 mixture with strong nearest-neighbor interactions, an asymmetric mixture of polymers ( $n=5$  and  
89  $n=20$ ), and a mixture of two short ( $n=5$ ) polymers. We observe one, two, and three surface phase-  
90 coexistence in all of these mixtures. The classical theories of wetting and prewetting only require a  
91 bulk systems that phase separates, and a surface with affinity for one of these phases. Diverse bulk  
92 systems ought display these phases, so long as they can phase-separate in the absence of a surface.

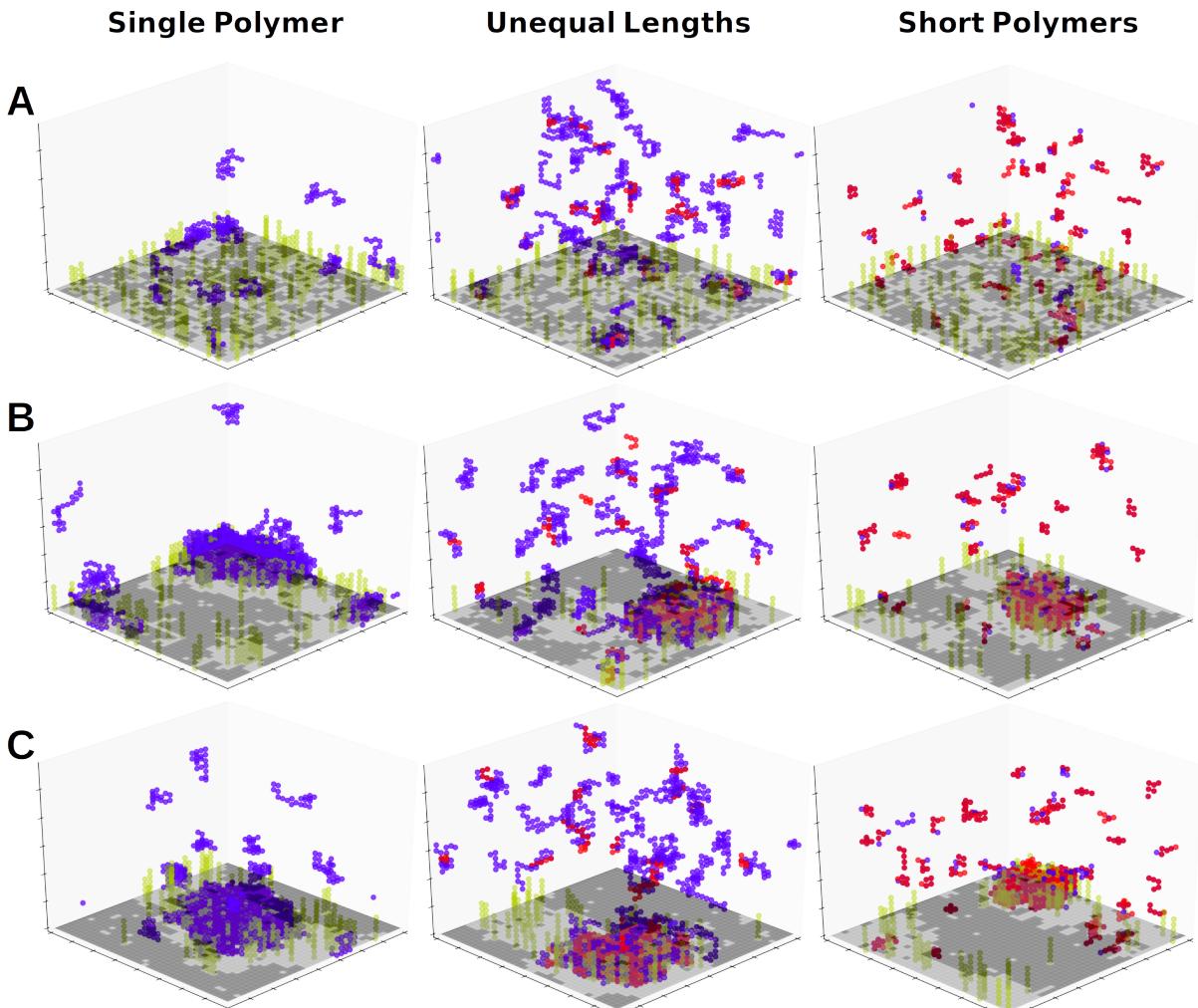


Figure 3: **Several Bulk Polymer Mixtures Produce Similar Results.** A) Single surface-phase system with a single polymer species (left), two polymers of unequal length (middle), and short polymers (right) B) Surface phase coexistence in mixtures as above C) Three surface phase coexistence

93 **2.4 Two phase coexistence regions bound three-phase coexistence regions on**  
 94 **the surface phase diagram**

95 Our landau theory predicts three two-phase coexistence regions bounding the three-phase coexis-  
 96 tence region (main text Figure 7A). We examined our Landau theory in these regions 4A. Shifting  
 97 the surface composition eliminates one of the three stable solutions. After finding a parameter  
 98 regime in simulations demonstrating three-phase coexistence we shifted membrane and tether com-  
 99 positions to access the two phase coexistence regions 4. Decreasing membrane order at constant  
 100 tether density eliminates the prewet dry phase ( $l_d$  Dry- $l_o$  Prewet). Likewise increasing membrane  
 101 order with a slight increase in tether density shows  $l_o$ -Dry  $l_o$ -Prewet Coexistence. Finally, removing  
 102 tethers eliminates the polymer rich phase, displaying a  $l_o$ -Dry  $l_d$ -Dry membrane.

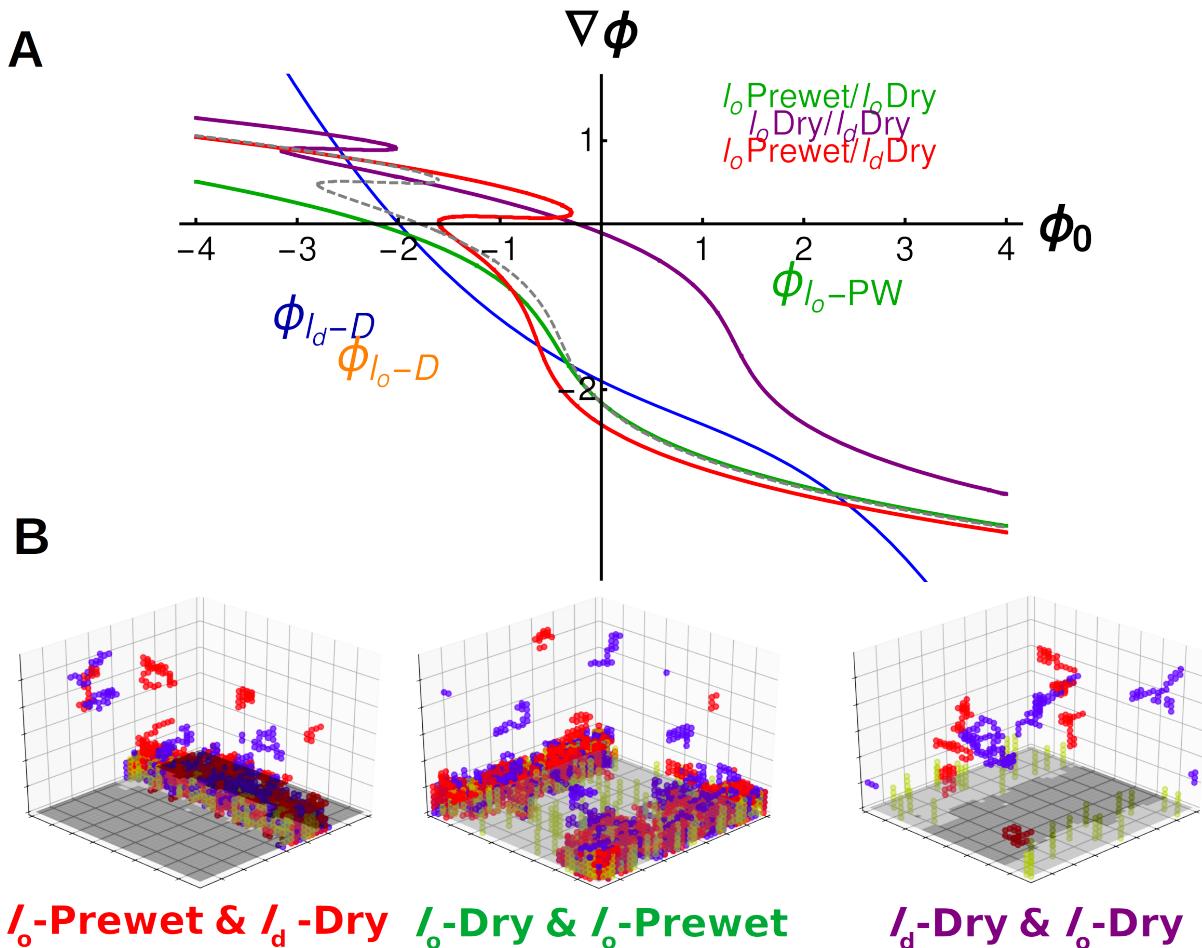


Figure 4: **Two-phase coexistence near three-phase coexistence** **A)** Gradient construction within each two phase region depicted in Figure 6A. **B)** Two-phase coexistence regions observed in simulation

103 **2.5 Dependence of three-phase coexistence region on bulk and membrane tem-**  
 104 **peratures**

105 The three-phase coexistence region requires a phase-separated membrane and polymers that prewet  
 106 in the absence of membrane interactions. We calculated the phase diagram in our Landau theory  
 107 over membrane and bulk couplings in Figure 5A. Three-phase coexistence occurs at low membrane  
 108 and bulk temperatures, and the parameter regime shrinks as either temperature is increased. We  
 109 plotted the surface phase-diagram at several points within this phase diagrams in Figure 5B. De-  
 110 creasing  $J_{mem}$  shrinks the  $l_o$  Dry -  $l_d$ -Dry coexistence region and respective edge of the three phase  
 111 region. Decreasing  $J_{bulk}$  shrinks the  $l_o$  Dry- $l_o$  Prewet coexistence region. Outside of the parameter  
 112 regime where three-phases coexist, two-phases coexist. The character of these phases depends on  
 113 whether the dominant force is membrane or bulk.

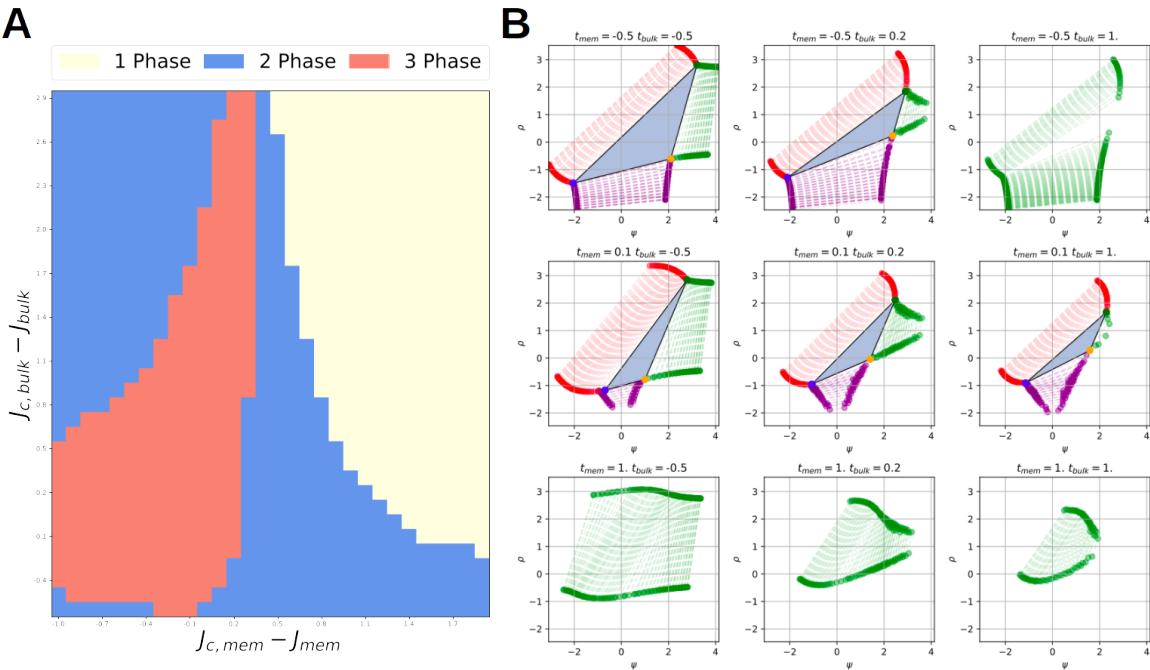


Figure 5: **Bulk and Membrane Coupling govern 3-phase coexistence** **A)** Phase diagram over membrane and bulk couplings showing one, two, and three phase regions (yellow, blue, and red). Three phase coexistence generally requires strong membrane and bulk interactions, but extends to weaker  $J_{bulk}$  near the membrane critical point. **B)** Surface phase diagrams at various membrane and bulk couplings. Weak membrane couplings collapse the purple ( $l_0$ -dry,  $l_d$ -dry) region, top to bottom. Weak bulk couplings shrink the green ( $l_o$ -prewet,  $l_o$ -dry) edge, left to right.

<sup>114</sup> **2.6 Surface densities may be dominated by bulk or membrane forces**

<sup>115</sup> Near  $J_{c,mem}$ , two surface phases coexist at very low  $J_{bulk}$ ; at high  $J_{bulk}$  surface phases may also  
<sup>116</sup> coexist at low  $J_{mem}$  (main text Figure 6A). We explored our Landau theory in these two limits  
<sup>117</sup> to determine the character of these phases Figure 6. Order parameter profiles show that while  
<sup>118</sup> the bulk-dominated phase is similar to that of classical prewetting [1] the membrane dominated  
<sup>119</sup> phases only have marginal enrichment relative to the bulk equilibrium density  $\phi_\infty$  as with a classical  
<sup>120</sup> ‘surface transition’ [2]. This is similar to Nakanishi and Fisher’s description of wetting and surface  
<sup>121</sup> transitions [3], where surface transitions and prewetting are revealed to be the same transition  
<sup>122</sup> viewed from different regions of a higher-dimensional phase diagram. Biological systems likely  
utilize these forces in a variety ways where membrane and bulk systems both have a role.

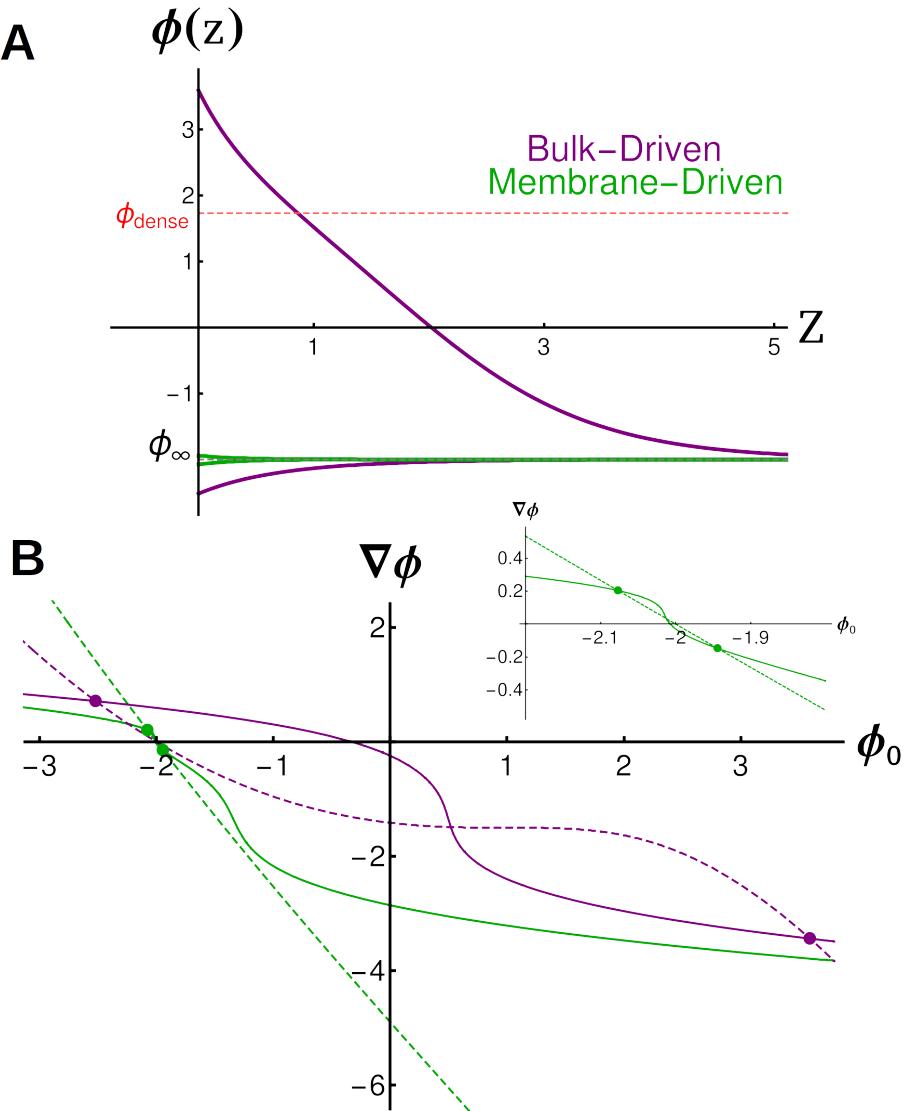


Figure 6: **Bulk and Membrane Dominated Surface densities** **A)** Density profiles for surface densities dominated by bulk (purple) and membrane (green) forces. **B)** Gradient construction for bulk and membrane dominated systems. A near-critical membrane allows surface coexistence when the gradient term is steep, but only over a narrow range of  $\phi_0$  values

123      **Simulation Parameters For Supplemental Figures**

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Figure	Tether Density	Membrane Order	$J_{bulk}, k_B T$	$J_{mem}, T_c$	$J_{tether}$	$\mu_{bulk}$
1A/B	0.25	0.8	0.4	1.1	1.0	-4.5
2A, One Phase	N/A	N/A	0.25	N/A	N/A	0.06
2A, Coexistence	N/A	N/A	1.0	N/A	N/A	0.06
3A Single	0.094	0.5	0.55	2.0	1.0	-4.5
3A, Unequal	0.0625	0.5	1.62	2.0	1.0	-3.0
3A, Short	0.0625	0.5	3.31	2.0	1.0	-3.5
3B, Single	0.094	0.5	0.55	1.0	1.0	-4.5
3B, Unequal	0.0625	0.5	1.62	1.0	1.0	-3.0
3B, Short	0.0625	0.5	3.31	1.05	1.0	-3.5
3C, Single	0.094	0.5	0.55	0.9	1.0	-4.5
3C, Unequal	0.0625	0.5	1.64	0.0	1.0	-3.0
3C, Short	0.0625	0.5	3.31	0.9	1.0	-3.5
4B, $l_o - Prewet/l_d Dry$	0.08	0.2	0.5	0.5	1.0	-4.5
4B, $l_o - Dry/l_o - Prewet$	0.14	1	0.5	0.5	1.0	-4.5
4B, $l_d - Dry/l_o - Dry$	0.02	0.5	0.5	0.5	1.0	-4.5

126      **References**

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