Project:

Project Working Title: Differential Scattering Cross-section of Neutrinos about a Massive Object

Project End Date: 7 May 2022 (The end of week 6 of spring term)

Student:

Name: Thomas Knudson
Affiliation: Department of Physics, Oregon State University

Statement: I will work regularly and diligently on this project throughout the year and initiate meetings with my advisor to seek feedback and guidance on the research. I understand that a significant portion of the research should be completed by the end of winter term to enable me focus on the writing process in the PH403 class.

Student Signature: ___________________________

Advisor:

Name Dr. Kathryn Hadley
Affiliation Department of Physics, Oregon State University

I have read this thesis proposal. I agree that the scope is reasonable for completion by May 7, 2022 and that sufficient progress can be made by early winter term 2022 to allow significant revision of the thesis during the winter and spring terms of 2022.

Advisor Signature*:* ___________________________

Project Summary

The analysis of the kinematics objects traversing any geometry of spacetime is a fundamentally complicated and abstract process. Not only does the Physicist have to reset their Classical expectations and intuitions about the perceived order of events, duration of time, and length of distances measured in the context of spacetime, but they must also adopt a whole new suite of complex and compact notation. As a result, the representation for the equations of motion take on an extremely foreign and difficult to interpret form. By creating a computational framework to wrangle these geodesic equations, we can provide a more accessible and intuitive interpretation for the trajectories of both massive and massless particles moving about a curved spacetime.

The curvature of this spacetime will be generated by a single massive object with initial parameters matching the description of a Schwarzschild black hole, but the toolset will not be constrained to just the local regions about the black hole. The framework will be able to calculate and display the differential scattering cross sections of particles with variable parameters as they attempt to orbit the black hole and are either captured or flung free. In addition, velocity and acceleration vector field representations for each specified particle will be generated to map out local and global regions of the spacetime to assist in normalizing the Physicist's intuition in these extreme environments.

Project Description

Introduction

When considering new systems and the interactions allowed by them, we tend to gravitate towards two main features: energy and kinematics. Classically, kinematics tends to be the root focus when being introduced to new and more complex systems: e.g., the equations of motion (EOM) for a harmonic oscillator are the cornerstone in understanding its behaviour and evolution through time. While this system is versatile as an approximation for other more complicated systems, we quickly disregard the vectors and trajectories in favor of a discussion about its energy. As we move away from Newtonian Mechanics, each system or formalism strongly suggests, if not outright requires, we abandon our familiar and direct kinematic equations for a more enigmatic exploration through energy conservation.

One such example of this shift is from Classical Thermodynamics into Statistical Mechanics. The student, eager to discuss small finite amounts of particles, immediately recognizes the futility attempting to use kinematics to describe the evolution of the system. On one hand, this can be expected of Quantum Mechanical (QM) systems, but these complications arise in Classical Mechanics as velocities approach significant fractions of the speed of light. While Special Relativity (SR) does not "break" the discussion of kinematics in the same fashion as QM, SR plays havoc with the student's everyday intuition molded from Galilean Relativity.

Luckily, we are still able to discuss trajectories in this more complicated formalism, but they require different terminology. The most immediate consequence of the relativistic formalism comes from the disagreement from observers on lengths and times measured and even the order in which events occur. To rectify this, we transition from a discussion of time and space as separate entities to a single entity: spacetime. The method in which we measure distances also changes from the very familiar Pythagorean (or Euclidean) distance formula to a hyperbolic distance formula. This is evident in the comparison of each space's line element:

$$\underbrace{ds^2 = dx^2 + dy^2}_{\text{2-D } Euclidean} \quad \text{v.s.} \quad \underbrace{ds^2 = dx^2 - c^2dt^2}_{\text{2-D } Minkowski}$$

Equation 1. Line elements describing distance in $\mathbb{E}^2$ and $\mathbb{M}^2$

In the exploration of flat spacetimes (or Minkowski space), we are obliged to refresh our understanding of and the importance of an inertial reference frame. This tool is largely disregarded in undergraduate physics courses due to the explicit requirement of the Newtonian mechanics' model. From this reevaluation of what different inertial observers see, we ultimately arrive at two seemingly inconsequential invariant measurements: proper time and proper distance.

$$d\tau^2 = \; g(\sigma^t, \sigma^t), \qquad ds^2 = \; g\left(\sigma^i, \sigma^j\right) \quad i, j \neq t$$

*Equation **2**. Generalized proper time and proper distance.*

The next complication comes from Einstein considering the presence of massive objects on spacetime. Simply put: mass *distorts* or *curves* spacetime. The simplest and most prominent example of this is the Schwarzschild solution to Einstein's field equations, in which an isolated, non-rotating, and spherically symmetric massive object is placed at the origin. It is easy to see from the line element of this space why this particular geometry for spacetime is used as the first non-flat introduction to General Relativity (GR):

$$ds^2 = -\left(1 - \frac{2GM}{c^2r}\right)(c^2dt^2) + \frac{dr^2}{\left(1 - \frac{2GM}{c^2r}\right)} + r^2(\sin^2\theta \, d\varphi^2 + d\theta^2)$$

*Equation **3**. The line element of the Schwarzschild geometry.*

The next hurdle comes from length contraction and time dilation are also manifestations from curved spacetime, even when objects are moving at velocities slow enough to avoid the framework of Special Relativity. The subtle problem here is that now our grid that describes the *fabric* of spacetime is now also affected by the presence of massive objects. The short answer is that we can adopt a coordinate system that is immune to these effects and is either referred colloquially as the Schwarzschild or Bookkeeper's coordinates: r, θ, φ, t. This type of coordinate system is a geometric representation (unaffected by the curvature of spacetime).

The path a free-falling object takes is always correspond to a straight worldline from the perspective of its rest frame. These paths adopt a special name: *geodesic*, with a sub-classification that *null geodesics* are paths that can only be traveled by massless particles. As an example of the perplexing nature of geodesic equations (GR's EOM analog), below is the generalized geodesic equation for motion about the Schwarzschild geometry:

$$\frac{1}{2}\dot{r}^2 = \frac{E^2 - kc^4}{2c^2} + k\frac{G_N M}{r} - \frac{{L_z}^2}{2r^2} + \frac{G_N M L_z^2}{c^2 r^3}$$

*Equation **4**. The geodesic equation for generalized motion in Schwarzschild spacetime.*

While this can be cleaned up by suppressing constants (typically G_N and c), the functional form still conceals information about this object's trajectory: Where is it at some time t? How fast is it moving? What's the acceleration? Where did it start? How do I build an intuitive understanding? This is the focus of the project: to help translate these compact and foreign equations from General Relativity into visualizations and other representations that tie back into and build off already existing intuitions and reasoning skillsets.

Plan of work

We will study the equations of motion for massive and massless particles in the presence of curved spacetime. A Schwarzschild black hole will serve as the source of curvature for analysis of limiting cases that will be used as the foundation for sense making and error checking in computational runtimes. The project itself has been subdivided into three major phases: Scaffolding, Curving, and Refinement.

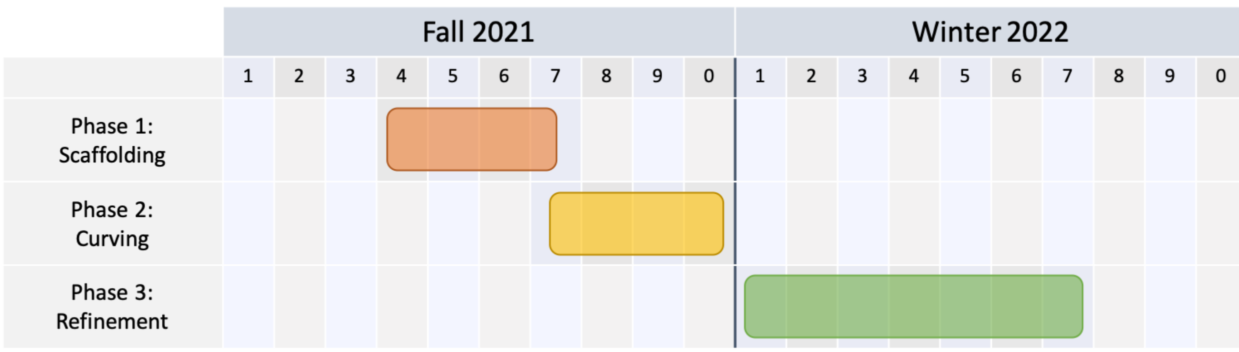

Figure 1. Waterfall breakdown of each phase of the project with expected time estimations.

The core of the toolset is going to depend on a series of key design and implementation decisions about the underlying framework. One of the key features is to have an intuitive code base for Physicists to use, which implies a latticework of grid points mapping out the space. Ideally, this latticework will be agnostic with respect to the active geometry and type of curvature. This stage will focus on the creation of the latticework, its documentation and the manifestation of key mathematical tools such as differential scattering and the representation and parameterization of a geodesic.

The differential scattering cross-section has a general form that can be parameterized based off the impact parameter, $\sigma(b) = \int \frac{d\sigma(b)}{d\Omega}(\theta, \phi) d\Omega$ [2], but we will need to still solve the EOM, Equation **4**, and mesh the two together. Care will need to be taken to avoid any implicit assumptions about the object traversing the spacetime to avoid unphysical geodesics and cross-sections: e.g., the impact parameter has a different form for massive versus massless particles [1,3]. Below, in Figure 2, the generalize user story has been mapped out to provide insight into the conceptual process of the toolset.

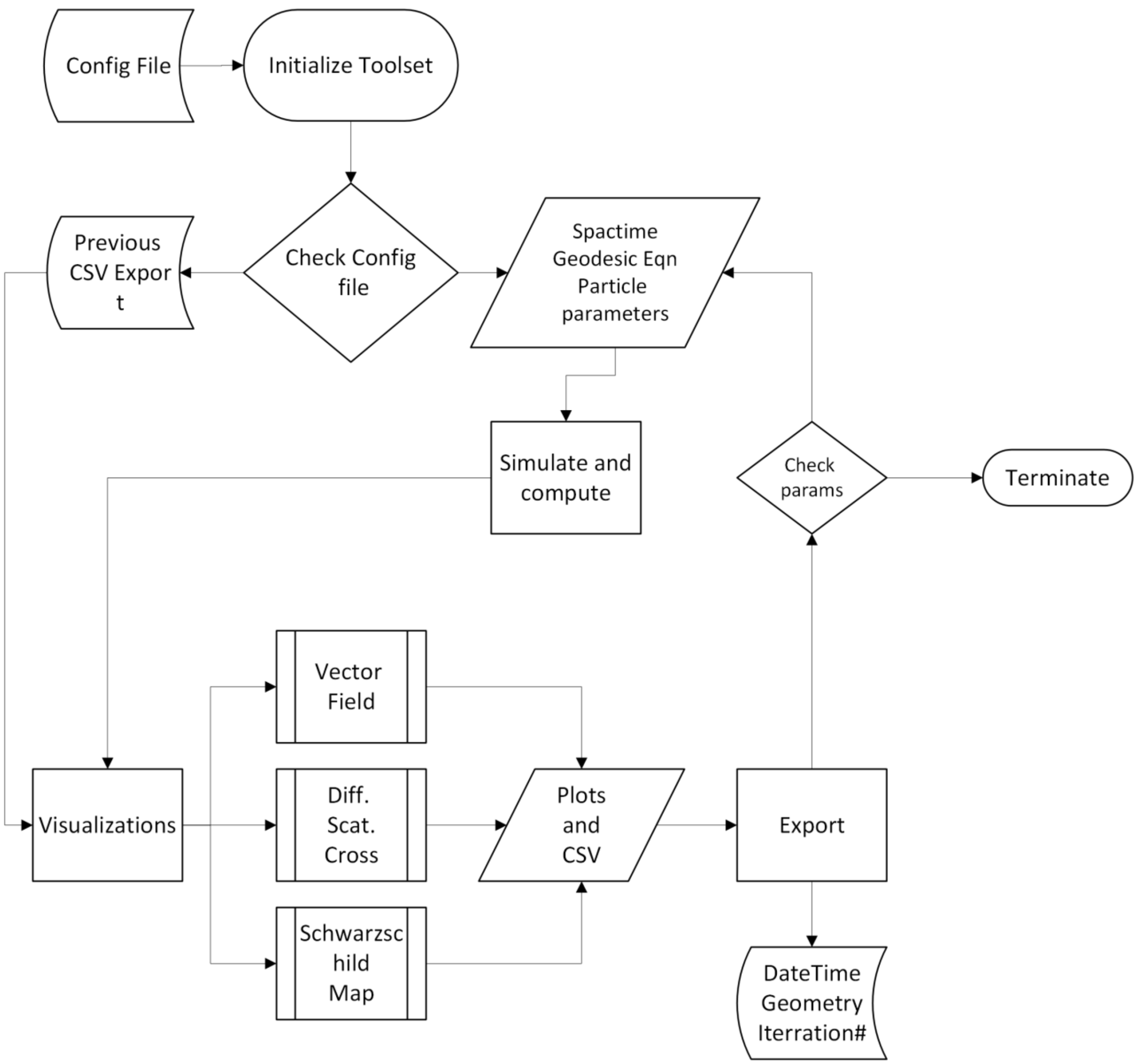

Figure 2. Flow chart detailing the typical user story for interacting with the toolset. ***TODO: denote with color or something else which parts of this will be completed in each phase – should end up highlighting the minimal viable product available by conclusion of Phase 1****.*

- *Phase 2*
- *Phase 3*

Timeline

This project has an ultimate deadline of: Week 6 in Spring Term 2022. In order to meet this deadline, the project has been subdivided into three major phases that correspond directly to prerequisite partitions of the thesis. The estimates of each portion are show below in Figure 3.

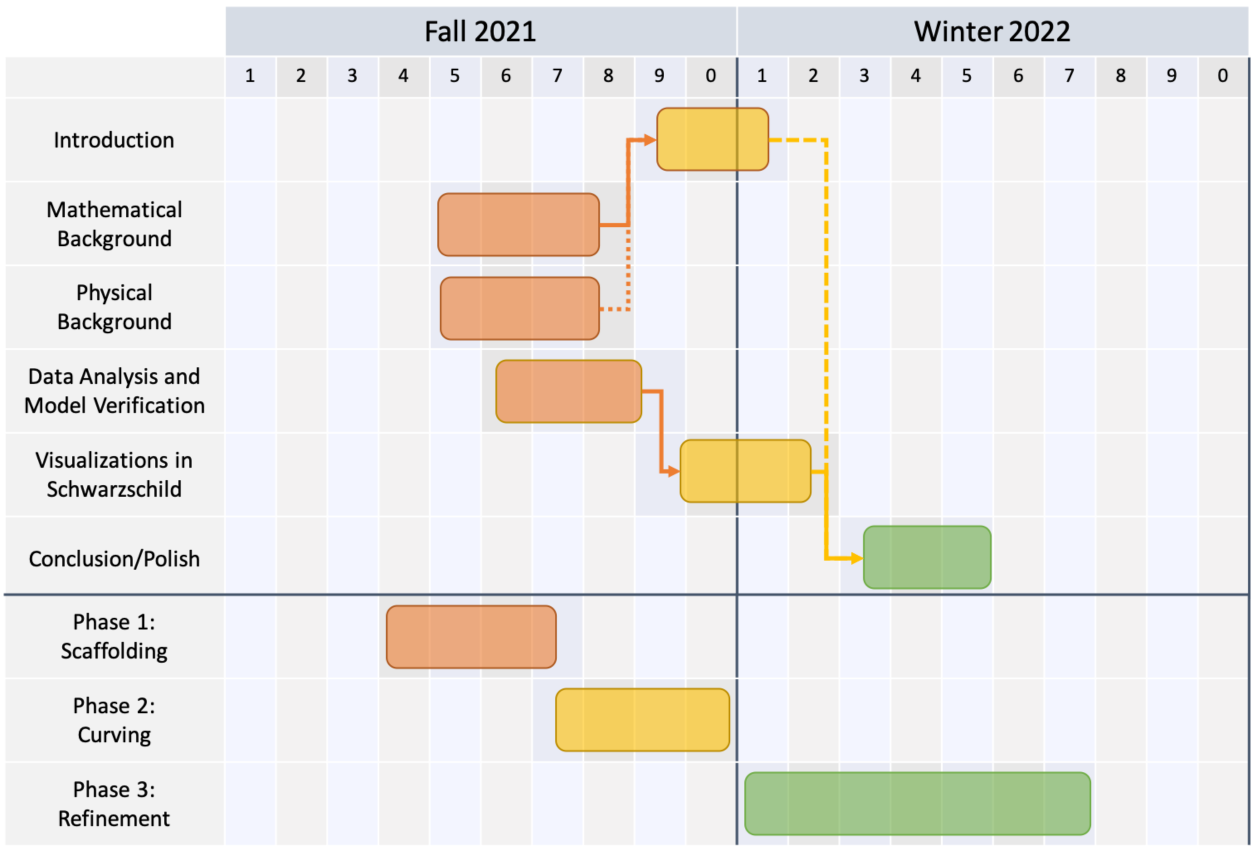

Figure 3. A waterfall diagram for both portions of the project, displaying sections of work with respect to their prerequisite parts. The top half is specifically dedicated to synthesis of sections of the Thesis, while the bottom half denotes each major phase of the implementation and research aspect.

Phase 1: Scaffolding begins during Week 4 of the Fall 2021 term and involves the mocking out the framework. Currently the intent is to have a multidimensional grid (*Latticework*) that is spacetime agnostic and can transform between geometric and physical representations. Alongside this, a few helper functions will be implemented to allow changing of coordinate systems and calculation of geodesics with the corresponding differential scattering cross-sections. Concurrently, work will begin on the synthesis of the thesis itself with respect to communicating and refining key mathematical and physical concepts.

Phase 2: Curving will begin, at the latest, during Week 7 of the Fall term. This is where the Latticework will be instructed how to respond to the presence of different spacetime geometries. The ability to export and import data will also be added during this step, as all required functionality to create the core set of visualizations will be in place. The phase also coincides naturally with the synthesis of the visualizations and data analysis portions of the thesis.

The transition into the final phase, **Phase 3: Refinement**, occurs as the output of the toolset is verified to work correctly. At this point, the codebase will freeze and only permit refactorization focused on bug fixing, increased readability, and/or improved flow of logic. Documentation of the codebase should occur concurrently with each major feature addition, but this will receive special attention at this stage, as the well documented codebase will go hand in hand with the final synthesis and refinement of the Thesis.

Time permitting, the codebase will be refined to be modular and well documented, with a particular addition focus on extendibility. This framework is intended to be integrated into similar codebases, such as either the Stardisk or Chymera projects. The underlying latticework of grid points will be implemented without assumptions of the underlying properties for the geometry of spacetime to facilitate straightforward modular extension for more complicated spacetimes such as the Reissner-Nordström or Kerr geometries.

Data management

Local copies of files will be saved on personal computer and back-ups created at regular intervals. A GitHub repository will also serve as an informal back-up but its main purpose will be for version control capabilities. Data retention for this project will include drafts of the Thesis, notes/supplemental documents discussing equations and how to utilize them, and files/code used to generate plots.

Facilities, Equipment and Other Resources

To facilitate fast prototyping and quick iterations, Python 3 will be used as the base language for creation of the minimal viable product. The language is lightweight enough to allow for design, debugging, and testing to occur on a home computer. Ultimately this code base will be optimized to run on the HPC to allow for higher resolution plots (more expensive in terms of computational run time). A GitHub repository will serve as a version control source for the codebase throughout the lifetime of the project and as an off-site backup of work as the project continues. The repository can be expanded upon to document code functionality to both aide in the writing of the thesis and in the future usability of the framework.

References Cited

[1] E.F Taylor and J. A. Wheeler, "Exploring Black Holes: Introduction to General Relativity”.

[2] J. R. Taylor, “Classical Mechanics”.

[3] C. W. Misner, K. S. Thorne, and J. A. Wheeler, "Gravitation”.