# On Orbit Servicing Flexible Space Robots Dynamics and Control During Capturing Target

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Abstract—Dynamics and control of a flexible space robot capturing a static target was presented in this paper. The dynamics model of the robot system is derived with Lagrangian formulation. The control method of flexible space during capturing target was discussed. Finally, some simulation tests have conducted that the proposed method can assurance the stability of capturing the object and reach a good effectiveness for capturing planning.

Keywords dynamics, control, space robots, flexible multibody, capture target

# I. INTRODUCTION

Future space missions will use flexible, lightweight, multibody space systems. Flexible space robots, as well as large flexible space structures in general, have unveiled a new and challenging field of dynamics and control. So understanding the dynamics and control of large flexible structures, capable of varying their geometric configuration, is also receiving more attention.

Dynamics of multibody systems is a field that has been studied by many researchers. A variety of approaches which differ in complexity and level of efficiency are available. Main focus of most of them is on Newton-Euler, Lagrangian or Kane's formulations. Notes on kinematic and dynamics modeling for multibody systems in space are made in [2, 3]. In [4], the generalized Jacobian matrix is introduced. It is utilized by the authors in [5], where solution algorithms for the inverse kinematics of space robotr systems are presented.

In recent years, the capture of a free-floating target in orbit by a space robot has become an important mission. In most cases capturing operation should precede missions like servicing, inspection and repairing which are critical for the survival of existing satellites in orbit, can be seen in fig 1. In general, a whole capturing operation consists of four specific phases: approach phase, contact and impact phase, capturing phase, and stabilization phase. The path planning problem to a free-floating target for space robots with angular momentum is addressed in [6]. The capture of a spinning object using a dual flexible arm manipulator is studied in [7]. Control schemes for the capture of targets within a free-floating system's workspace were presented in [8, 9]. Notes on dynamical modeling and control of space robots during a capturing operation are made in [10].

In this paper, the kinematic, dynamics and control of flexible space robots during capturing a static object are developed. This paper is organized as follows. In section 2 we present kinematic and dynamic model of the flexible space robots during capturing a static object. In section 3 we propose a methodology for controlling the flexible space robots. In section 4 the simulation and experimental system is presented. In section 5 some experimental results are discussed and the conclusions are demonstrated.

#### II. MODEL AND OPERATOR DEFINITION

# A. Coordinate system

In this section, the equations of motion of a flexible space robotic system are obtained. Fig. 1 shows a model of a space robot consisting of 3 flexible arms connected with a rigid base satellite via revolute joints.  $\Sigma$  is Global coordinate system and  $\Sigma_0 \Sigma_1 \dots \Sigma_n$  denote moving coordinate. Physical parameters of 3 flexible arms are assumed to be exactly equals each other. The links are numbered in the order of their connection to each other, assuming the first one to be pivoted to the base. Let consider the local system of coordinates  $O_n x_n y_n z_n$  with its origin  $O_n$  coincides with this link initial position, the axis  $O_n x_n$  is coincident with its axial line, and the axis  $O_n z_n$  is coincident with the joint  $n.O_n x_n, O_n z_n$  and  $O_n y_n$  are orthogonal one another.

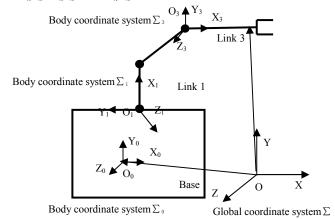


Fig. 1 Mechanic model of a space flexible robot



# B. Dynamics of flexible space robots

Flexible links are assumed as Euler–Bernoulli beams, whose elastic deflection is described by finite element discrete method as

$$\boldsymbol{u}_{if} = \boldsymbol{S}_{i} \boldsymbol{q}_{if} \tag{1}$$

Where  $S_i$  is the shape function matrix and  $q_{if}$  is time dependent vector of the generalized nodal coordinates. It was assumed that every flexible manipulator has three orders coordinate to describe the deformation of flexible body. Defined q as generalize coordinate of the rigid base, can be written as:

$$\boldsymbol{q} = \begin{bmatrix} x_0, y_0, z_0, \theta_{0x}, \theta_{0y}, \theta_{0z} \end{bmatrix}^T$$

In which, the first three part, describe the position of rigid base, while the other three part describe the orientation of base.

Defining  $\theta$  as generalized rotation coordinate of the three flexible manipulators, which represents the relative angular displacement across that joint, can be written as:

$$\boldsymbol{\theta} = \left[\theta_1, \theta_2, \theta_3\right]^T$$

 $q_f$  was defined as nodal coordinate of the flexible body. In this system  $q_f$  was:

$$\boldsymbol{q}_{f} = [q_{f11}, q_{f12}, q_{f13}, q_{f21}, q_{f22}, q_{f23}, q_{f31}, q_{f32}, q_{f33}]^{T}$$

Define  $\mathbf{x} = \begin{bmatrix} q & \theta & q_f \end{bmatrix}^T$  as the generalized coordinate of system. The dynamics equation of motion of a free-flying flexible space robot as a multibody system is, in general, expressed in the following form:

$$\boldsymbol{M}(q,\theta,q_f) \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \ddot{\boldsymbol{\theta}} \\ \ddot{\boldsymbol{q}}_f \end{bmatrix} + \boldsymbol{H}(q,\theta,q_f) \begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{q}_f \end{bmatrix} + \boldsymbol{K} \begin{bmatrix} q \\ \theta \\ q_f \end{bmatrix} = \begin{bmatrix} F_b \\ \tau_{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} J_b^T \\ J_{\theta}^T \\ 0 \end{bmatrix} F_h (2)$$

In which,  $M(q,\theta,q_f)$  is mass matrix;  $H(q,\theta,q_f)$  are vectors of centrifugal and Coriolis forces; K is stiffness matrix;  $F_b$  is force on the centroid of the base;  $\tau_\theta$  torque on the manipulator joints;  $F_h$  impact force exert on the manipulator hand;  $J_b$  and  $J_\theta$  is Jacobian Matrix of the satellite and manipulator part respectively.

### C. System momentum

The manipulator motion induces the base satellite to rotate and translate, and momentum conservation law of the whole system gives "non-holonomic" constraint. In the absence of external forces and moments, the linear momentums and angular momentums of the robot are conserved and assumed to be initially zero. The angular momentum equation and the linear momentum equation of the space robot system can be written as:

$$\mathbf{P}_{0} = \sum_{i=1}^{3} \int_{0}^{L_{i}} \rho_{i} \dot{\mathbf{r}}_{i} dx_{i} + \sum_{i=1}^{3} m_{i} \dot{\mathbf{r}}_{i} \Big|_{x_{i} = L_{i}} + m_{0} \dot{\mathbf{r}}_{0}$$
(3)

$$\mathbf{L}_{0} = \sum_{i=1}^{3} \int_{0}^{L_{i}} \rho_{i} \mathbf{r}_{i} \times \dot{\mathbf{r}}_{i} dx_{i} + \sum_{i=1}^{3} m_{i} \mathbf{r}_{i} \times \dot{\mathbf{r}}_{i} \Big|_{x_{i} = L_{i}}$$

$$+ \sum_{i=1}^{3} \int_{0}^{L_{i}} \mathbf{H}_{i}^{r} \omega_{i} dx + \mathbf{H}_{v}^{b} \omega_{0} + m_{0} \mathbf{r}_{0} \times \dot{\mathbf{r}}_{0}$$

$$(4)$$

In which,  $L_i$  is the length of link i;  $\rho_i$  is density along the distance of the length of link i;  $m_i$  is the mass of link i, which includes masses of actuator, transmission device and other rigid parts of joint i+1;  $m_0$  is the mass of the base;  $\mathbf{H}_i^r$  is the inertial tensor of a unit length elemental link i around mass center of the element;  $\mathbf{H}_{\nu}^b$  is the inertial tensor of the base around its mass center;  $\mathbf{P}_0$ ,  $\mathbf{L}_0$  are initial linear momentums and angular momentums of this system respectively, in this paper,  $\mathbf{P}_0$  and  $\mathbf{L}_0$  are constant zero without external force.

# D. Zero disturbance to base

In this system, it assumed that there is not the external force applied to the base ( $F_b$ =0). At the initial condition, the angular momentum and the linear momentum of flexible space robots are zero. Hence equation (3) and (4) can be written as flowing forms:

$$J_b \dot{\boldsymbol{q}} + J_m \dot{\boldsymbol{\theta}} + J_f \dot{\boldsymbol{q}}_f = 0 \tag{5}$$

Hence the flexible manipulator zero attitude disturbance to rigid base can be written as:

$$\dot{\boldsymbol{q}} = -J_b^-(J_m \dot{\boldsymbol{\theta}} + J_f \dot{\boldsymbol{q}}_f) = 0 \tag{6}$$

The joint motion and the deformation motion given by this equation are guaranteed to make zero disturbance on the base attitude.

# E. Capture phases

The phases of flexible space robot capture the static target can be divided into six sections as follows:

- 1) Observation. Space robot observe the initial position of the static target by telescope mounted on endeffector. In this paper, it is assumed that the target is within the space robot's work space.
- 2) Confirm the trajectory path from initial position of end- effector to the static target. In this paper, the trigonometric spline function was used to plan the trajectory path in Cartesian space.
- 3) Inverse kinematic solution in terms of choosing the key knot points in this trajectory to solve the joint space.
- 4) Capturing target. Adjusting the orientation of endeffector and applying the force  $F_h$  to capture the static

target.

### 5) Orientation stabilization of the system.

In this paper, when flexible space robots capture object, impact force  $F_h$  will disturb the base, so  $J_b^T$  should be zero.

The equation (2) of motion of flexible space robots, and can be written as

$$\boldsymbol{\tau}_{\theta} = \mathbf{M}_{\theta\theta} \ddot{\boldsymbol{\theta}} + \mathbf{M}_{\theta f} \ddot{\boldsymbol{q}}_{f} + \boldsymbol{H}_{1}(\boldsymbol{\theta}, \boldsymbol{q}_{f}) - \boldsymbol{J}_{\theta}^{T} \boldsymbol{F}_{h}$$
 (7)

$$0 = \mathbf{M}_{\theta f} \ddot{\boldsymbol{\theta}} + \mathbf{M}_{ff} \ddot{\boldsymbol{q}}_f + \boldsymbol{H}_2(\boldsymbol{\theta}, \boldsymbol{q}_f) + \mathbf{K}_{ff} \boldsymbol{q}_f$$
(8)

Then equation (8) can be written as:

$$\ddot{\boldsymbol{q}}_{f} = -\mathbf{M}_{ff}^{-1}(\mathbf{M}_{\theta f}\ddot{\boldsymbol{\theta}} + \boldsymbol{H}_{2}(\boldsymbol{\theta}, \boldsymbol{q}_{f}) + \mathbf{K}_{ff}\boldsymbol{q}_{f})$$
(9)

Substitute equation (9) into equation(7), can be gained as flowing forms:

$$\tau_{\theta} = \mathbf{M}_{\theta\theta} \ddot{\theta} - \mathbf{M}_{\theta f} [\mathbf{M}_{ff}^{-1} (\mathbf{M}_{\theta f} \ddot{\theta} + \mathbf{H}_{2}(\theta, q_{f}) + \mathbf{K}_{ff} q_{f})] + \mathbf{H}_{1}(\theta, q_{f}) - J_{\theta}^{T} F_{h}$$
(10)

#### III. CONTROL SCHEME

#### A. Capture describe

To make a flexible space manipulator capture the static object efficiently, it is necessary to control its end-effector's trajectory. However, due to elastic deflections of the links, the kinematics of the flexible space manipulator is different from the usual rigid manipulators and fixed robots, and thus it is very difficult to solve it analytically. Hence, their control systems must handle the difficult problems of accommodating and compensating for low frequency resonances and nonlinear actuator saturation.

Defining x as position and orientation coordinate of the static target, which is captured by space robots. x can be written as:

$$\mathbf{x} = \begin{bmatrix} x_t & y_t & z_t & \theta_{tx} & \theta_{ty} & \theta_{tz} \end{bmatrix}^T$$

At the beginning, using forward kinematics calculates the position and orientation of end-effector at initial joint, and the flexible manipulator under un-deformation state. So:

$$\boldsymbol{\theta}_{init} = \left[\theta_1, \theta_2, \theta_3\right]^T$$

$$\mathbf{q}_{finit} = [0, 0, 0, 0, 0, 0, 0, 0, 0]^T$$

Second the trajectory should be certain to make the endeffector reach the target position. In order to implement a typical motion controller in the flexible space robots joint space, the trajectory in the Cartesian space has to map into the joint space by applying the inverse kinematics, the inverse kinematics solution is multi-solutions for multi-DOF manipulator, hence, this mapping relationship is not simple inverse kinematics problem. Also it is necessary to ensure that the trajectory of the manipulator end-effector is zero disturbance to base.

Here, in order to simplify trajectory planning problem the authors keep the orientation of the manipulator end-effector constant. At the same time, assuming this orientation can satisfy capture requirement, i.e. only consider the position and velocity of end-effector during trajectory planning.

#### B. Trajectory control scheme

This section briefly explains Computed Torque Control Law for flexible space robots hands capturing static target. In recent years the control of space robot system has received increased. Because of the high dynamic coupling between the arms and its floating base, the dynamic and control of space robot system become extremely complicated [11].

From dynamic equation(2), (6) and equation (10), considering the control law:

$$\tau_{t} = \mathbf{M}_{\theta\theta} \boldsymbol{u} + \boldsymbol{H}_{1}(\boldsymbol{\theta}, q_{f}) \dot{\boldsymbol{x}} - \mathbf{M}_{\theta f} [\mathbf{M}_{ff}^{-1} (\mathbf{M}_{\theta f} \boldsymbol{u} + \boldsymbol{H}_{2}(\boldsymbol{\theta}, q_{f}) \dot{\boldsymbol{x}} + \mathbf{K}_{ff} \boldsymbol{x})]$$
(11)

Define  $\mathbf{x}_d$  as the desired trajectory of end-effector. Hence the  $\dot{\mathbf{x}}_d$  and  $\ddot{\mathbf{x}}_d$  will be gained. Consider the PD control law:

$$u = \ddot{x}_d + K_d(\dot{x}_d - \dot{x}) + K_p(x_d - x) = \ddot{x}_d + K_d\dot{e} + K_pe$$
 (12)

In which,  $\mathbf{K}_d$ ,  $\mathbf{K}_p$  are gain matrix, and this matrix is always invertible.

Then the closed loop system may then be expressed by the state equation:

$$\ddot{\boldsymbol{e}} + \boldsymbol{K}_{d} \dot{\boldsymbol{e}} + \boldsymbol{K}_{p} \boldsymbol{e} = 0 \tag{13}$$

Hence  $(\dot{\boldsymbol{e}}, \boldsymbol{e}) = (0,0)$  due to the invertible matrix  $\boldsymbol{K}_d$ ,  $\boldsymbol{K}_p$ , is the equilibrium point of the global stabilization.

Hence in terms of the initial condition  $(\dot{x}_0, x_0)$ , we can gain  $(\dot{x}, x) \rightarrow (\dot{x}_d, x_d)$ . Hence substitute (12) into equation (11), we can gain the control law:

$$\tau_{t} = \hat{\mathbf{M}}(q, q)(\ddot{\mathbf{x}}_{d} + \mathbf{K}_{d}\dot{\mathbf{e}} + \mathbf{K}_{p}\mathbf{e}) + \hat{\mathbf{H}}(\theta, q_{f})\dot{\mathbf{x}} + \hat{\mathbf{K}}\mathbf{x}$$
(14)

# C. Capture control scheme

During the moment of impact between the manipulator and target, an impulse force is generated, the reaction of this force will badly effect the space robot system and target, and it maybe damage either the manipulator, or the target, especially, this force will make the attitude of the space base unstable. Hence it is very important to control the endeffector of space robot to capture the target.

From the above mentioned, the acceleration of the endeffector expressed as:

$$\ddot{\boldsymbol{q}}_{E} = J_{m} \ddot{\boldsymbol{\theta}} + \dot{J}_{m} \dot{\boldsymbol{\theta}} + J_{f} \ddot{\boldsymbol{q}}_{f} + \dot{J}_{f} \dot{\boldsymbol{q}}_{f}$$
(15)

Therefore, in this study, the generalized angle accelerations can be expressed as:

$$\ddot{\boldsymbol{\theta}} = \mathbf{M}_{\theta\theta}^{-1} J_m^T (J_m \mathbf{M}_{\theta\theta}^{-1} J_m^T)^{-1} (\ddot{\boldsymbol{q}}_E - \dot{J}_m \dot{\boldsymbol{\theta}} - J_f \ddot{\boldsymbol{q}}_f - \dot{J}_f \dot{\boldsymbol{q}}_f)$$
(16)

Considered the second-order system as follows:

$$\mathbf{D}_{pc}\ddot{\mathbf{e}} + \mathbf{K}_{dc}\dot{\mathbf{e}} + \mathbf{K}_{pc}\mathbf{e} = -F_h \tag{17}$$

In which,  $\mathbf{D}_c$ ,  $\mathbf{K}_{dc}$ ,  $\mathbf{K}_{pc}$  are gain matrix, and this matrix is always invertible.

So equation (15) (16) (17) can be used to obtain the control torques:

$$\tau_{c} = \mathbf{M}_{\theta\theta}^{-1} J_{m}^{T} (J_{m} \mathbf{M}_{\theta\theta}^{-1} J_{m}^{T})^{-1} \{ \ddot{\boldsymbol{q}}_{E} - \boldsymbol{D}_{pc}^{-1} (\boldsymbol{K}_{dc} \dot{\boldsymbol{e}} + \boldsymbol{K}_{pc} \boldsymbol{e} + \boldsymbol{F}_{h})$$

$$- \dot{J}_{m} \dot{\boldsymbol{\theta}} - J_{f} \ddot{\boldsymbol{q}}_{f} - \dot{J}_{f} \dot{\boldsymbol{q}}_{f} \} - J_{\theta}^{T} \boldsymbol{F}_{h}$$

$$(18)$$

#### IV. SIMULATION

In this section, simulation is conducted to verify the proposed the dynamics model and the control approach of flexible space robots during capturing target. In the simulation model, we consider that 2 body's problem shown as Figure 1, the one body is the rigid base, and the others are flexible manipulator. The table 1 shows s the parameters of flexible space robots.

Table 1: Parameters of flexible space robots

	Length	Mass	Jxx,Jyy,Jzz	Density	Young's modulus
Base	1,1,1	10	{0.015, 0.015, 0.015}	3.6 kg/m	_
Link 1	0.2	1	{0.015, 0.015, 0.015}	1.5 kg/m	210
Link2	0.2	1	{0.015, 0.015, 0.015}	1.5 kg/m	210

In the simulation study, the trigonometric spline function was used to plan the desired tracking trajectory in Cartesian space, then choosing the key knot points in this trajectory as the interval reference point using interval algorithm, then the inverse kinematic solutions was calculate and selected. According to desired position, the displacement of base is studied as fig 2, the torque of two joint as Fig 3.

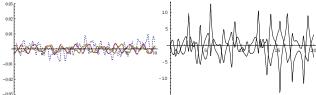


Fig 2 The displacement of base

Fig 3 The torque of the two joint

#### V. CONCLUSION

The dynamics and control of flexible space robots during capturing target are discussed in this paper. These flexible robots are built very light in order to save expensive launch energy. Furthermore, for some tasks in space, payloads grasped by flexible space robots are presented. Thus, development of modeling and control techniques for these flexible space robots to capture target would be an extremely useful. For these intentions above, Computed Torque Control Law with zero disturbance has been presented. The control scheme has been studied for a 2-link 2-joint flexible manipulator to capture a static target. Experimental results show that the system responses are in good agreement with simulation results. Investigating these results, it can be concluded that our control scheme is effective with some assumptions.

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