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Optimal Control of a Space Manipulator for Detumbling of a Target Satellite

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Abstract—**Robotic capture of a tumbling target-satellite means that the space robot's hand and the target grapple point arrive at a rendezvous point with the same velocity and then the chase vehicle mechanically connects into the target using a grapple device. This paper is focused on optimal control of the space manipulator in the postcapture phase so as to bring the tumbling non-cooperative satellite to rest in minimum time while ensuring that the magnitude of the interaction torque between the manipulator and target remains below a prescribed value. First, we seek fast detumbling maneuvers of the target satellite subjected to the torque restriction. The theory of optimal control and Pontryagin's principle are applied to obtain a closed-form solution for the optimal path planning problem, giving a great deal of insight: the vector of the interaction torque is aligned opposite to the direction of the instantaneous angular momentum vector. Second, a coordination control for combined system of the space robot and the target satellite, which acts as the manipulator payload, is developed so that the robot tracks the optimal path while regulating the attitude of the chase vehicle to a desired value. A preliminary illustrative example is appended.**

I. Introduction

In the past decades, there has been growing interest in using space robotic systems for on-orbit servicing of satellites [1]–[6]. Many research works done in this area were motivated by several mission that required the capabilities of automated rendezvous and capture [7]. To verify and to demonstrate the research results and the developments, several missions have already been performed and more are in the horizon [8].

The chaser vehicles are often equipped by an manipulator arm with a grappling device on it [9], [10]. The control system of the space manipulator usually has two mode of operations: capturing and passivation (stabilization). In the capturing phase, the manipulator arm is guided (typically with using vision data) so as its end-effector intercepts the satellite grapple point at a rendezvous point along the trajectory of the tumbling satellite. The capture will be without impact, if the manipulator approaches the target in such a manner that, at the time of capture, the relative velocity between the end-effector and the target grapple point is zero [11]. Otherwise, the effect of impact and on a free-floating space robot has to be taken into account [12]; an optimal trajectory planning to minimize the impulse during contact between two bodies was presented in [13]. After capture of an uncontrolled tumbling satellite by a space manipulator, the satellite should be brought to rest [14], [15]. To accomplish this goal, the space manipulator should gently apply torques to the target satellite for removing any relative velocity, ideally as fast as possible. In this paper, we are dealing particularly with problems occurring in poscapture of tumbling satellite, i.e., the stabilization phase. The main topic presented hereafter is the passivation of a free floating tumbling satellite in minimum time using the space manipulator so that the magnitude of the interaction torque between the manipulator and the target remains below a prescribed value. Form a practical point of view, it is also desirable to maintain the attitude of the robot base undisturbed during the manipulator-satellite interaction.

There are many studies on optimal trajectory planning to guide a robotic manipulator to rendezvous and capture a non-cooperative target satellite [16]–[20]. This problem is closely related to the application of visual servoing [21]–[23] in robotic capture of moving objects with known dynamics. In the case that the target satellite dynamics is uncertain, not only the states but also the target's inertial parameters and its position of center of mass can be estimated from vision data obtained from zero motion of a tumbling satellite [24], [25]. However, there are only few studies on the path planning for passivation of a tumbling satellite and non of which is optimal. In [26], the principle of conservation of momentum was used to damp out the chaser-target relative motion. However, there was no control on the force and moment built up at the connection of the chaser manipulator and the target. Impedance control scheme for a free-floating space robot in grasping of a tumbling target with model uncertainty is presented in [27], however optimal path planning is not addressed in this work.

The magnitude of the interaction torque between the space manipulator and the target must be constrained during the passivation operation for two main reasons: First, too much interaction torque could cause mechanical damage to either the target satellite or to the space manipulator. Second, a large interaction torque may lead to actuation saturation of the space robot's attitude control system. This is because, the reaction of the torque on the space robot base should be eventually compensated for through additional momentum generated by the actuator of its attitude control system, e.g., momentum/reaction wheels, in order to keep the attitude of the base undisturbed. Moreover, it is important to dump

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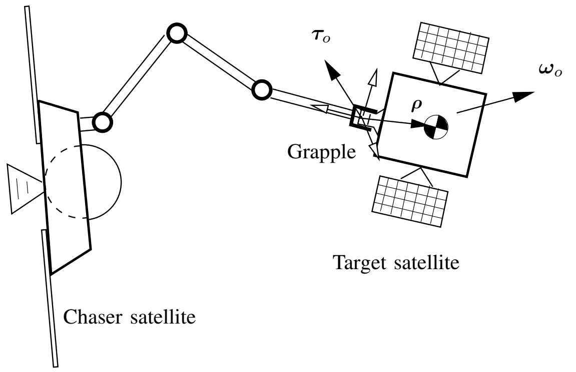

Fig. 1. The free body diagram of chaser and target satellites

the initial velocity of the target as quickly as possible in order to mitigate the risk of collision due to small but nonzero translational drifts of the satellites. Hence, optimal planning of the passivation maneuvers is highly desired. The problem of time-optimal detumbling control of rigid spacecraft is formulated as a nonlinear programming and solved numerically by utilizing an iterative procedure in [28], while non-optimal control approaches have been reported in [29]–[31].

This paper is organized as follow: Section II presents a closed-form solution for time-optimal detumbling control of a rigid spacecraft under the constraint that the Euclidean norm of the braking torques is below a prescribed value. The optimal control theory and Pontryagin's principle are applied to derive the optimal solution. First, it will be shown that for our particular optimal control problem, the system costates and states are related by a nonlinear but static function. Subsequently, the optimal control law is explicitly derived in the form of a nonlinear state feedback. Section III describes a coordination control for the combined system of the space robot and the target satellite so that the manipulator tracks the optimal path while regulating the attitude of its own base to a desired value. Finally, simulation results are shown in section IV.

II. Optimal Detumbling Maneuver

Dynamics of the rotational motion of the target satellite can be expressed by Euler's equation as

$$\dot{\boldsymbol{\omega}}_o = \boldsymbol{\phi}(\boldsymbol{\omega}_o) + \boldsymbol{I}_c^{-1}\boldsymbol{\tau}_o$$

where $\boldsymbol{\omega}_o$ and $\boldsymbol{\tau}_o$ denote the vectors of the angular velocity and input torque, both of which are expressed in the fixed-body frame, $\boldsymbol{I}_c$ is the inertia tensor of the target satellite and

$$\boldsymbol{\phi}(\boldsymbol{\omega}_o) = -\boldsymbol{I}_c^{-1}(\boldsymbol{\omega}_o \times \boldsymbol{I}_c\boldsymbol{\omega}_o). \tag{1}$$

The time-optimal control problem being considered here is how to drive the spacecraft from the given initial angular velocity $\boldsymbol{\omega}_o(0)$ to rest in *minimum time* while the Euclidean norm of the torque input is restricted to be below a prescribed value $\tau_{\max}$. That is the following cost function

$$J = \int_0^{t_f} 1\ dt$$

is minimized subject to terminal condition $\boldsymbol{\omega}_o(t_f) = 0$ while the input torque trajectory should satisfy

$$\|\boldsymbol{\tau}_o\| \leq \tau_{\max}. \tag{2}$$

Denoting vector $\boldsymbol{\lambda} \in \mathbb{R}^3$ as the costates, we can write the system Hamiltonian as

$$H = 1 + \boldsymbol{\lambda}^T\boldsymbol{\phi}(\boldsymbol{\omega}_o) + (\boldsymbol{I}_c^{-1}\boldsymbol{\lambda})^T\boldsymbol{\tau}_o. \tag{3}$$

Then, the theory of optimal control [32], [33] dictates that the time-derivative of the costates must satisfy

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \boldsymbol{\omega}_o} = -\frac{\partial \boldsymbol{\phi}^T}{\partial \boldsymbol{\omega}_o}\boldsymbol{\lambda} \tag{4}$$

where

$$\frac{\partial \boldsymbol{\phi}^T}{\partial \boldsymbol{\omega}_o} = \boldsymbol{I}_c[\boldsymbol{\omega}_o\times]\boldsymbol{I}_c^{-1} - [\boldsymbol{I}_c\boldsymbol{\omega}_o\times]\boldsymbol{I}_c^{-1}, \tag{5}$$

and skew-symmetric matrix $[\boldsymbol{a}\times]$ represents the cross-product, i.e., $[\boldsymbol{a}\times]\boldsymbol{b} = \boldsymbol{a}\times\boldsymbol{b}$. If $\boldsymbol{\tau}_o^*$ is the time-optimal torque history and $\boldsymbol{\omega}_o^*$, $\boldsymbol{\lambda}^*$ represent the solutions of (1) and (4) for $\boldsymbol{\tau}_o = \boldsymbol{\tau}_o^*$ then, according to *Pontryagin's Minimum Principle*, optimal torque $\boldsymbol{\tau}_o^*$ satisfies the equation

$$H(\boldsymbol{\omega}_o^*, \boldsymbol{\lambda}^*, \boldsymbol{\tau}_o^*) \leq H(\boldsymbol{\omega}_o^*, \boldsymbol{\lambda}^*, \boldsymbol{\tau}_o), \quad \forall \boldsymbol{\tau}_o \in \mathbb{R}^3 \ni \|\boldsymbol{\tau}_o\| \leq \tau_{\max} \tag{6}$$

for every $t \in [0,\ t_f)$. Equations (3) and (6) together imply that

$$\boldsymbol{\tau}_o^* = -\frac{\boldsymbol{I}_c^{-1}\boldsymbol{\lambda}^*}{\left\|\boldsymbol{I}_c^{-1}\boldsymbol{\lambda}^*\right\|}\tau_{\max}. \tag{7}$$

Therefore, the dynamics of the closed-loop system becomes

$$\dot{\boldsymbol{\omega}}_o^* = \boldsymbol{\phi}(\boldsymbol{\omega}^*) - \frac{\boldsymbol{I}_c^{-2}\boldsymbol{\lambda}^*}{\left\|\boldsymbol{I}_c^{-1}\boldsymbol{\lambda}^*\right\|}\tau_{\max} \tag{8}$$

The structure of the optimal controller is determined by (4) and (7) together. However, to determine the control input, the initial values of the costates, $\boldsymbol{\lambda}(0)$, should be also obtained. In fact, by choosing different initial values for the costates, we obtain a family of optimal solutions, each of which corresponds to a particular final angular velocity. In general, the two-point boundary value problem for nonlinear systems is challenging. However, as it will be shown in the following, the structure of our particular system (4) and (8) lead to an easy solution when the final velocity is zero. In such a case, it will be shown that the costates and states are related via the following function:

$$\boldsymbol{\lambda}^*(t) = \frac{\boldsymbol{I}_c^2\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}} \qquad \forall t \in [0,\ t_f), \tag{9}$$

despite the fact that the evolutions of the optimal trajectories of the states and costates are governed by two different differential equations (8) and (4). In other words, (9) is a solution to equations (8) and (4). Note that since $\boldsymbol{\omega}_o^*(t) = \boldsymbol{\omega}_o(t) \quad \forall t \in [0,\ t_f)$, $\boldsymbol{\omega}_o^*(t_f)$ is not defined, but is assumed nonzero. In such a case, on substitution of (9) into (8), we arrive at the following autonomous system:

$$\dot{\boldsymbol{\omega}}_o^* = \boldsymbol{\phi}(\boldsymbol{\omega}_o^*) - \frac{\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|}\tau_{\max} \qquad \forall t \in [0,\ t_f). \tag{10}$$

To prove the above claim, we need to show that (9) and (10) satisfy the optimality condition (4). Using (10) in the time-derivative of right-hand side (RHS) of (9) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\lambda}^* = \boldsymbol{I}_c^2 \frac{\boldsymbol{\phi} - \frac{\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|}\tau_{\max}}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}} - \boldsymbol{I}_c^2\boldsymbol{\omega}_o^* \frac{\boldsymbol{\omega}_o^{*T}\boldsymbol{I}_c^2(\boldsymbol{\phi} - \frac{\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|}\tau_{\max})}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|^3\,\tau_{\max}} = \frac{\boldsymbol{I}_c^2\boldsymbol{\phi}}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}}. \tag{11}$$

On the other hand, using (5) and (9) in the RHS of (4) yields

$$-\frac{\partial\boldsymbol{\phi}^T}{\partial\boldsymbol{\omega}_o}\boldsymbol{\lambda}^* = \frac{-\boldsymbol{I}_c[\boldsymbol{\omega}_o^*\times]\boldsymbol{I}_c\boldsymbol{\omega}_o^* + (\boldsymbol{I}_c\boldsymbol{\omega}_o^*)\times\boldsymbol{I}_c\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}} = \frac{\boldsymbol{I}_c^2\boldsymbol{\phi}}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}}. \tag{12}$$

A comparison between (11) and (12) clearly proves that (9) is indeed a solution to the differential equation (4). Furthermore, the Hamiltonian on the optimal trajectory becomes

$$H^* = \boldsymbol{\lambda}^{*T}\boldsymbol{\phi}(\boldsymbol{\omega}_o^*) = -\frac{(\boldsymbol{I}_c\boldsymbol{\omega}_o^*)^T[\boldsymbol{\omega}_o^*\times](\boldsymbol{I}_c\boldsymbol{\omega}_o^*)}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|\,\tau_{\max}} = 0$$

Therefore, the condition for optimality with open end time is also satisfied [33, pp. 213]. The substitution of (9) into (7) gives

$$\boldsymbol{\tau}_o^* = -\frac{\boldsymbol{I}_c\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|}\tau_{\max} \qquad \forall t\in[0,\ t_f) \tag{13}$$

Apparently, the control law (13) constitutes a nonlinear state feedback. The structure of (13) also gives an interesting insight into the optimal control solution: the instantaneous torque vector is aligned opposite to the direction of angular momentum vector.

Clearly, the torque feedback should be turned off right after time t_f because the optimal solution is valid only for the time interval $[0\ t_f)$. However, t_f is not a given variable; rather it is one of the arguments of the optimization process. In order to be able to obtain the terminal time, let us define h as the magnitude of the angular momentum, i.e.,

$$h \triangleq \|\boldsymbol{I}_c\boldsymbol{\omega}_o\|.$$

The time-derivative of h along the optimal trajectory (8) satisfies

$$\dot{h} = \frac{\boldsymbol{\omega}_o^T\boldsymbol{I}_c^2\dot{\boldsymbol{\omega}}_o}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o\|} = -\frac{(\boldsymbol{I}_c\boldsymbol{\omega}_o)^T[\boldsymbol{\omega}_o\times](\boldsymbol{I}_c\boldsymbol{\omega}_o)}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o\|} - \frac{(\boldsymbol{\omega}_o^T\boldsymbol{I}_c^2\boldsymbol{\omega}_o)\tau_{\max}}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o\|^2} = -\tau_{\max}$$

This means that the optimal controller reduces the magnitude of the angular momentum linearly at the constant rate of $\tau_{\max}$. Therefore, given initial angular velocity $\boldsymbol{\omega}_o(0)$, the terminal time can be computed from

$$t_f = \frac{\|\boldsymbol{I}_c\boldsymbol{\omega}_o(0)\|}{\tau_{\max}}. \tag{14}$$

Alternatively, the feedback torque, $\boldsymbol{\tau}_o$, can be turned off when the angular velocity becomes sufficiently small. Assume that the terminal condition is stated by

$$\|\boldsymbol{\omega}_o(t_f)\| \le \epsilon, \tag{15}$$

with ϵ being selected to be arbitrary small. Then, the optimal path can be specified by

$$\dot{\boldsymbol{\omega}}_o^* = \begin{cases} \boldsymbol{\phi}(\boldsymbol{\omega}_o^*) - \frac{\boldsymbol{\omega}_o^*}{\|\boldsymbol{I}_c\boldsymbol{\omega}_o^*\|}\tau_{\max} & \text{if } \|\boldsymbol{\omega}_o^*\| \ge \epsilon \\ \boldsymbol{0}_{3\times1} & \text{otherwise} \end{cases}. \tag{16}$$

III. Control of the Combined System of Manipulator and Target

In the postcapture phase, the space robot and the target satellite constitutes a single free-flying multibody chain. The dynamic equations of the space robot can be expressed in the form [4]

$$\boldsymbol{M}_s\ddot{\boldsymbol{\psi}}_s + \boldsymbol{c}_s(\boldsymbol{\psi}_s, \dot{\boldsymbol{\psi}}_s) = \boldsymbol{u} + \boldsymbol{J}^T\boldsymbol{f}_h, \tag{17}$$

where

$$\dot{\boldsymbol{\psi}}_s = \begin{bmatrix}\boldsymbol{\nu}_b\\ \dot{\boldsymbol{\theta}}\end{bmatrix}, \qquad \boldsymbol{u} = \begin{bmatrix}\boldsymbol{f}_b\\ \boldsymbol{\tau}_m\end{bmatrix}.$$

Here, $\boldsymbol{M}_s$ is the generalized mass matrix of the space manipulator, $\boldsymbol{c}_s$ is generalized Coriolis and centrifugal force, $\boldsymbol{\nu}_b^T = [\boldsymbol{v}_b^T\ \ \boldsymbol{\omega}_b^T]$ is the generalized velocity of the base consisting of the linear and angular velocities, $\boldsymbol{v}_b$ and $\boldsymbol{\omega}_b$, vector $\dot{\boldsymbol{\theta}}$ is the motion rate of the manipulator joint, vector $\boldsymbol{f}_h$ is the force and moment exerted by the manipulator hand, vector $\boldsymbol{f}_b$ is the force and moment exert on the centroid of the base, vector $\boldsymbol{\tau}_m$ is the manipulator joint torque and $\boldsymbol{J}$ is the Jacobian, which takes this form

$$\boldsymbol{J} = \begin{bmatrix}\boldsymbol{J}_b & \boldsymbol{J}_m\end{bmatrix}$$

with $\boldsymbol{J}_b$ and $\boldsymbol{J}_m$ being the Jacobian matrices for the base and for the manipulator arm, respectively. On the other hand, if the target spacecraft is rigid body, then its dynamics motion can be described by

$$\boldsymbol{M}_o\dot{\boldsymbol{\nu}}_o + \boldsymbol{c}_o = -\boldsymbol{A}^T\boldsymbol{f}_h \tag{18}$$

where $\boldsymbol{\nu}_o$ is the six-dimensional generalized velocity vector consisting of the velocity of the center of mass, $\boldsymbol{v}_o$, and the angular velocity, $\boldsymbol{\omega}_o$, components, $\boldsymbol{M}_o$ is the generalized mass matrix that can be written as

$$\begin{bmatrix} m\boldsymbol{1}_3 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}_c\end{bmatrix} \quad \text{and} \quad \boldsymbol{c}_o = \begin{bmatrix} m\boldsymbol{\omega}_o\times\boldsymbol{v}_o\\ \boldsymbol{\omega}_o\times\boldsymbol{I}_c\boldsymbol{\omega}_o\end{bmatrix}$$

with m being the satellite mass, and $\boldsymbol{A}$ can be expressed as

$$\boldsymbol{A} = \begin{bmatrix}\boldsymbol{1}_3 & \boldsymbol{\rho}\times\\ \boldsymbol{0} & \boldsymbol{1}_3\end{bmatrix}$$

with $\boldsymbol{\rho}$ being the position vector of the target-spacecraft contact point with respect to its center of mass. Note that the RHS of (18) is the force and moment exert on the centroid of the target spacecraft. Furthermore, the generalized velocities of the manipulator hand and the target spacecraft are related by the following

$$\boldsymbol{\nu}_o = \boldsymbol{A}\boldsymbol{\nu}_h. \tag{19}$$

The velocity of the manipulator end-effector in the end-effector frame is expressed as

$$\boldsymbol{\nu}_h = \boldsymbol{J}\dot{\boldsymbol{\psi}}_s = \boldsymbol{J}_b\boldsymbol{\nu}_b + \boldsymbol{J}_m\dot{\boldsymbol{\theta}} \tag{20}$$

Using (19) in (20) gives

$$\boldsymbol{\nu}_o = \boldsymbol{A}\boldsymbol{J}_b\boldsymbol{\nu}_b + \boldsymbol{A}\boldsymbol{J}_b\dot{\boldsymbol{\theta}}. \tag{21}$$

The time-derivative of (21) leads to

$$\ddot{\boldsymbol{\theta}} = \boldsymbol{J}_m^{-1}\boldsymbol{A}^{-1}\dot{\boldsymbol{\nu}}_o - \boldsymbol{J}_m^{-1}\boldsymbol{J}_b\dot{\boldsymbol{\nu}}_b + \boldsymbol{c}_d \tag{22}$$

where

$$\boldsymbol{c}_d \triangleq -\boldsymbol{J}_m^{-1}\dot{\boldsymbol{J}}_m\boldsymbol{J}_m^{-1}\boldsymbol{A}^{-1}\boldsymbol{\nu}_o + \boldsymbol{J}_m^{-1}(\dot{\boldsymbol{J}}_m\boldsymbol{J}_m^{-1}\boldsymbol{J}_b - \dot{\boldsymbol{J}}_b)\boldsymbol{\nu}_b$$

is the velocity dependent term.

Now we are interested in writing the equations of motion in terms of the generalized velocities of the bases of the chaser and target satellites, i.e., $\boldsymbol{\nu}_b$ and $\boldsymbol{\nu}_o$. To this end, we define a new velocity vector as

$$\dot{\boldsymbol{\psi}} \triangleq \begin{bmatrix}\boldsymbol{\nu}_b\\ \boldsymbol{\nu}_o\end{bmatrix} \tag{23}$$

The internal force vector $\boldsymbol{f}_h$ can be eliminated from (17) and (18) to yield the following equation

$$\boldsymbol{M}_s\ddot{\boldsymbol{\psi}}_s + \boldsymbol{J}^T\boldsymbol{A}^{-T}\boldsymbol{M}_o\dot{\boldsymbol{\nu}}_o + \boldsymbol{J}^T\boldsymbol{A}^{-T}\boldsymbol{c}_o + \boldsymbol{c}_s = \boldsymbol{u} \tag{24}$$

Upon substitution of $\ddot{\boldsymbol{\theta}}$ from (22) into the corresponding component of $\ddot{\boldsymbol{\psi}}_s$ in (24) the latter equation can be written in this form

$$\boldsymbol{M}\ddot{\boldsymbol{\psi}} + \boldsymbol{c}(\boldsymbol{\psi}, \dot{\boldsymbol{\psi}}) = \boldsymbol{u}, \tag{25}$$

where

$$\boldsymbol{M} \triangleq \boldsymbol{M}_s\begin{bmatrix}\boldsymbol{1} & \boldsymbol{0}\\ -\boldsymbol{J}_m^{-1}\boldsymbol{J}_b & \boldsymbol{J}_m^{-1}\boldsymbol{A}^{-1}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0} & \boldsymbol{J}^T\boldsymbol{A}^{-T}\boldsymbol{M}_o\end{bmatrix}$$

$$\boldsymbol{c} \triangleq \boldsymbol{M}_s\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{c}_d\end{bmatrix} + \boldsymbol{J}^T\boldsymbol{A}^{-T}\boldsymbol{c}_o + \boldsymbol{c}_s,$$

in which we used the the expression of the joint acceleration from (22). Note that (25) describes the dynamic motion of the combined chaser and target satellites in terms of their base variables. The special case of interest is when no force is applied to the base of the chaser satellite. In other words, the joint motion of the manipulator arm is allowed to disturb the base translation but not its attitude. Form a practical point of view, it is important to keep the base attitude unchanged as the spacecraft has to always point its antenna toward the Earth, whereas disturbing the base translation does not pose any significant side effect. Therefore, the generalized force input $\boldsymbol{u}$ consists of a 3×1 zero vector plus the vectors of the chaser base torque and the manipulator joint torque, i.e.,

$$\boldsymbol{u} = \begin{bmatrix}\boldsymbol{0}_{3\times 1}\\ \bar{\boldsymbol{\tau}}\end{bmatrix} \quad \text{where} \quad \bar{\boldsymbol{\tau}} \triangleq \begin{bmatrix}\boldsymbol{\tau}_b\\ \boldsymbol{\tau}_m\end{bmatrix}. \tag{26}$$

In view of the zero components of input vector (26), we will derive the reduced form of the equation of motion (25) in the following. Let us assume that

$$\dot{\boldsymbol{\psi}} \triangleq \begin{bmatrix}\boldsymbol{v}_b\\ \dot{\bar{\boldsymbol{\psi}}}\end{bmatrix} \qquad \text{where} \qquad \dot{\bar{\boldsymbol{\psi}}} \triangleq \begin{bmatrix}\boldsymbol{\omega}_b\\ \boldsymbol{\nu}_o\end{bmatrix}$$

is the velocity components of interest. Also, assume that the mass matrix and the nonlinear vector in (25) are partitioned as

$$\boldsymbol{M} = \begin{bmatrix}\boldsymbol{M}_{11} & \boldsymbol{M}_{12}\\ \boldsymbol{M}_{12}^T & \boldsymbol{M}_{22}\end{bmatrix} \quad \text{and} \quad \boldsymbol{c} = \begin{bmatrix}\boldsymbol{c}_1\\ \boldsymbol{c}_2\end{bmatrix}, \tag{27}$$

so that $\boldsymbol{M}_{11} \in \mathbb{R}^{3\times 3}$, $\boldsymbol{c}_1 \in \mathbb{R}^3$ and the dimensions of rest of submatrices and subvectors are consistent. Then, by eliminating $\dot{\boldsymbol{v}}_b$ from (25), the latter equation can be reduced to

$$\bar{\boldsymbol{M}}\ddot{\bar{\boldsymbol{\psi}}} + \bar{\boldsymbol{c}} = \bar{\boldsymbol{\tau}} \tag{28}$$

where $\bar{\boldsymbol{M}}$ and $\bar{\boldsymbol{c}}$ are constructed from (27) as

$$\bar{\boldsymbol{M}} = \boldsymbol{M}_{22} - \boldsymbol{M}_{12}^T\boldsymbol{M}_{11}^{-1}\boldsymbol{M}_{12}$$
$$\bar{\boldsymbol{c}} = \boldsymbol{c}_2 - \boldsymbol{M}_{12}\boldsymbol{M}_{11}^{-1}\boldsymbol{c}_1$$

Equation (28) shows that through torque control input $\bar{\boldsymbol{\tau}}$, it is possible to simultaneously control the pose of the target satellite and the attitude of the chaser satellite. Therefore, the objective is to develop a coordination controller which sends torque commands to motors of the manipulator joints and to actuators of the attitude control system, e.g., reaction/momentum wheels, in order to not only track the optimal trajectories (16) but also to regulate the base attitude. To achieve this goal, we use a feedback linearization method based on dynamic model (28). Suppose that orientation of the base is represented by quaternion $\boldsymbol{q}^T = [\boldsymbol{q}_v^T \; q_s]$, where $\boldsymbol{q}_v$ and q_s are the vector and scalar parts of the quaternion, respectively. Then adopting a simple PD quaternion feedback [34] for the spacecraft attitude control, an appropriate feedback linearization control torque is given by

$$\bar{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}}_{ff} - \bar{\boldsymbol{M}}\begin{bmatrix}\boldsymbol{K}_p\text{vec}(\delta\boldsymbol{q}) + \boldsymbol{K}_d\boldsymbol{\omega}_b\\ \boldsymbol{K}_v\boldsymbol{v}_o\\ \boldsymbol{K}_\omega(\boldsymbol{\omega}_o - \boldsymbol{\omega}_o^*)\end{bmatrix} \tag{29}$$

where

$$\bar{\boldsymbol{\tau}}_{ff} = \bar{\boldsymbol{c}} + \bar{\boldsymbol{M}}\begin{bmatrix}\boldsymbol{0}_{6\times 1}\\ \dot{\boldsymbol{\omega}}_o^*\end{bmatrix}$$

is the feed forward term. In the above, $\boldsymbol{K}_p > 0$, $\boldsymbol{K}_d > 0$, $\boldsymbol{K}_v > 0$ and $\boldsymbol{K}_\omega > 0$ are the feedback gains,

$$\delta\boldsymbol{q} = \boldsymbol{q} \otimes \boldsymbol{q}^*$$

is the quaternion error, $\boldsymbol{q}^*$ is the desired quaternion, vec$(\cdot)$ returns the vector part or quaternion $(\cdot)$ and the quaternion product operator $\otimes$ is defined as

$$\boldsymbol{q}\otimes \triangleq \begin{bmatrix}q_s\boldsymbol{1}_3 - \boldsymbol{q}_v\times & \boldsymbol{q}_v\\ -\boldsymbol{q}_v^T & q_s\end{bmatrix}.$$

Recall that $\dot{\boldsymbol{\omega}}_o^*$ and $\boldsymbol{\omega}_o^*$ in (29) are obtained from the trajectory generator (16), while $\boldsymbol{\omega}_o$ and $\boldsymbol{v}_o$ can be obtained from the manipulator joint rates and the base velocity by making use of (21). Now substituting the control law (29) into (28) results in a set of three uncoupled differential equations as

$$\dot{\boldsymbol{\omega}}_b + \boldsymbol{K}_d\boldsymbol{\omega}_b + \boldsymbol{K}_p\text{vec}(\delta\boldsymbol{q}) = \boldsymbol{0} \tag{30a}$$
$$\dot{\boldsymbol{v}}_o + \boldsymbol{K}_v\boldsymbol{v}_o = \boldsymbol{0} \tag{30b}$$
$$(\dot{\boldsymbol{\omega}}_o - \dot{\boldsymbol{\omega}}_o^*) + \boldsymbol{K}_\omega(\boldsymbol{\omega}_o - \boldsymbol{\omega}_o^*) = \boldsymbol{0} \tag{30c}$$

The stability proof of system (30a) is given in the Appendix, while exponential stability of the systems (30b) and (30c) is obvious. Therefore, we can say $\boldsymbol{\omega}_o \to \boldsymbol{\omega}_o^*$, $\boldsymbol{v}_o \to \boldsymbol{0}$ and $\boldsymbol{q} \to \boldsymbol{q}_d$ as $t \to \infty$. Note that the role of feedback gains $\boldsymbol{K}_v$ and $\boldsymbol{K}_\omega$ in (29) is to compensate for a possible modelling uncertainty, otherwise a feed forward controller as $\bar{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}}_{ff}$ suffices to achieve the control objective. Ideally, the closed-loop system should not exhibit any transition response

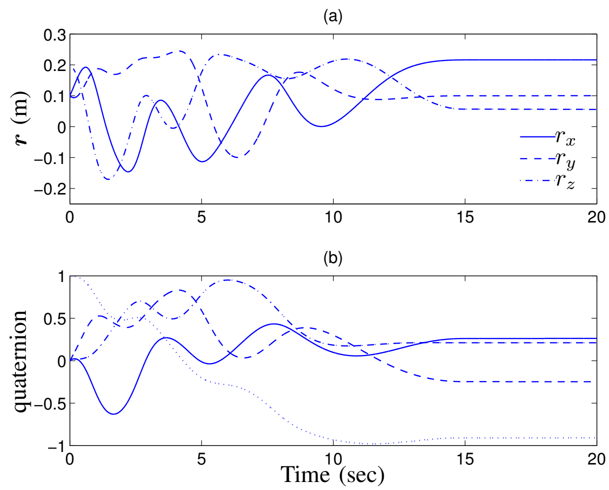

Fig. 2. Trajectories of the manipulator's end-effector position (a) and orientation (b).

because of the initial conditions are set as $\boldsymbol{v}_o(0) = \boldsymbol{0}$ and $\boldsymbol{\omega}_o(0) = \boldsymbol{\omega}_o^*(0)$.

The above development can be summarized in the following

Proposition 1: Suppose that the end-effector of a space manipulator reaches the connecting point of a tumbling satellite at the same velocity and then grasps it rigidly. Also assume that the angular rate of the tumbling satellite at the time of capture is $\boldsymbol{\omega}_o(0)$. Then, if the coordination control law (29) in conjunction with the optimal path (16) is applied to the free-flying space robot, the robot brings the tumbling satellite to state of rest in minimum time, while the magnitude of the torque exerted by the manipulator on the tumbling satellite remains below the prescribed value $\tau_{\max}$.

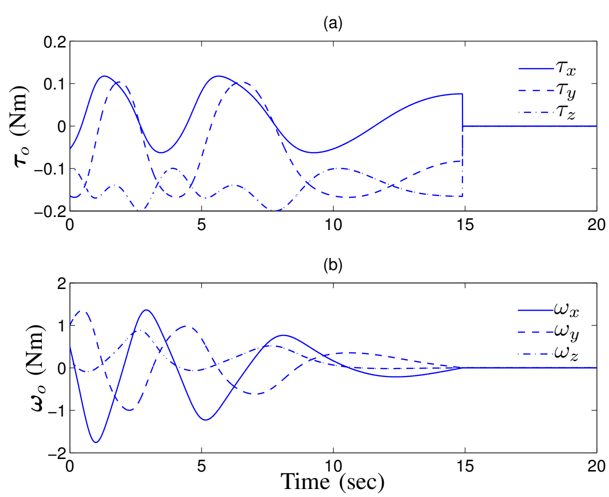

Fig. 3. Trajectories of the angular velocities (a) of the target and the torque interaction (b).

IV. Simulation Results

Consider a target satellite with inertial parameters

$$\boldsymbol{I}_c = \begin{bmatrix} 1 & 0.5 & -1 \\ 0.5 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} \text{ kgm}^2 \quad \text{and} \quad \boldsymbol{\rho} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix} \text{ m},$$

that tumbles with angular rate

$$\boldsymbol{\omega}_o(0) = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.2 \end{bmatrix} \text{ rad/s}$$

at time $t = 0$ sec.. The objective is bring the tumbling rigid-body to rest in minimum time while the magnitude of the braking torque is restricted by

$$\|\boldsymbol{\tau}_o\| \leq 0.2 \text{ Nm}.$$

According to (14), given the initial angular momentum of $h(0) = 2.98$ kgm^2/s the optimal controller is expected to achieve complete passivation within $t_f = 14.9$ sec.. Fig. 2 illustrates trajectories of the position and orientation of the manipulator end-effector during the detumbling maneuvers. Trajectories of the torque exerted by the manipulator on the tumbling satellite are shown in Fig.3a. The subsequent trajectories of the angular velocities are shown in Fig.3b. Apparently, the controller succeeded to reduce the angular velocity of the rigid body to zero within the expected finite time t_f.

V. Conclusions

An optimal control for detumbling maneuvers of a target non-cooperative satellite using a space manipulator has been presented. First, a closed-form solution to the optimal motion planning of how to damp out the initial angular velocity of a rigid body (satellite) in minimum time subject to restriction that the magnitude of the required external torque is maintained below a prescribed value has been derived. Second, a coordination control for the combined system of the space robot and the target satellite has been developed so that the space robot not only provides the optimal maneuvers dictated by the motion planner but also keeps the attitude of its base undisturbed.

Appendix

The quaternion evolves in time according to the following differential equation

$$\dot{\boldsymbol{q}} = \frac{1}{2}\underline{\boldsymbol{\omega}}_b \otimes \boldsymbol{q} \qquad \text{where} \qquad \underline{\boldsymbol{\omega}}_b = \begin{bmatrix} \boldsymbol{\omega}_b \\ 0 \end{bmatrix}. \tag{31}$$

Now, we define the following positive-definite Lyapunov function:

$$V = \frac{1}{2}\delta\boldsymbol{q}_v^T \boldsymbol{K}_p \delta\boldsymbol{q}_v + \frac{1}{2}\|\boldsymbol{\omega}_b\|^2. \tag{32}$$

Then, it can be shown by substitution from the quaternion propagation equation (31) that time derivative of V along trajectory (30a) is

$$\dot{V} = -\boldsymbol{\omega}_b^T \boldsymbol{K}_d \boldsymbol{\omega}_b \tag{33}$$

so that $\dot{V} \leq 0$ for all t. Therefore, according to LaSalle's Global Invariant Set Theorem [35], [36], the equilibrium point reaches where $\dot{V} = 0$, or $\boldsymbol{\omega}_b \equiv \mathbf{0}$. Then, we have from (30a)

$$\boldsymbol{K}_p \mathrm{vec}(\delta \boldsymbol{q}) = 0.$$

On the other hand it is known that two coordinate systems coincides if, and only if, $\delta \boldsymbol{q} = 0$, where the $\delta \boldsymbol{q}$ is the vector component of the quaternion [34]. Therefore, we have global asymptotic convergence of the orientation error.

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