UNIVERSITÀ DI TRENTO
Department of Industrial Engineering

Master Thesis

7th Update

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Impact Model

- Assumptions:
  - although the generalized velocities change substantially, the generalized coordinates of the system remain the same over the impact duration
  - at the contact point between the end-effector and the target there are forces but not moments. Impact occurs at a single point, which is unable to transmit a local moment

Impact Model

$$\begin{cases} M\ddot{p} + C = u + J^T f_I \\ M_O \ddot{\psi} + C_O = -J_O^T f_I \end{cases}$$

where $J^T f_I$ and $J_O^T f_I$ are the torques and forces applied to the VMS and object's generalized coordinates respectively

$$f_I = -(J_O^T)^+ (M_O \ddot{\psi}_O + C_O)$$

$$M(\dot{p}_f - \dot{p}_i) + J^T (J_O^+)^T M_O (\dot{\psi}_f - \dot{\psi}_i) = 0$$

Impact Model

$$J\dot{p}_f = J_O\dot{\psi}_f \Rightarrow \dot{\psi}_f = J_O^+ J\dot{p}_f$$

$$\dot{p}_f = G^{-1}H$$

$$\text{where} \begin{cases} G = M + J^T(J_O^+)^T M_O J_O^+ J \\ H = M\dot{p}_i + J^T(J_O^+)^T M_O \dot{\psi}_i \end{cases}$$

Differentiate $\dot{\psi}_f = J_O^+ J\dot{p}_f$ and substitute in $M_O\ddot{\psi} + C_O = -J_O^T f_I$

Impact Model

Final VMS dynamics with payload at the EE position:

$$M'\ddot{p} + C' = u$$

where $$\begin{cases} M' = M + J^T (J_O^T)^+ M_O J_O^+ J \\ C' = C + J^T (J_O^T)^+ M_O \dfrac{\partial J_O^+}{\partial t} J\dot{p} + J^T (J_O^T)^+ M_O J^+ \dfrac{\partial J}{\partial t} \dot{p} + J^T (J_O^T)^+ C_O \end{cases}$$

Controlled based motion

Equation of motion can be rewritten:

$$\begin{bmatrix} M'_{tt} & M'_{tr} \\ M'_{rt} & M'_{rr} \end{bmatrix} \begin{bmatrix} \ddot{p}_t \\ \ddot{p}_r \end{bmatrix} + \begin{bmatrix} C'_t \\ C'_r \end{bmatrix} = \begin{bmatrix} \overline{0} \\ u \end{bmatrix}$$

$$\ddot{p}_t = -M_{tt}^{-1'}(M'_{tr}\ddot{p}_r + C'_t)$$

$$\ddot{p}_r\tilde{M} + \tilde{C} = u$$

Controlled based motion

$$\ddot{p}_r\tilde{M} + \tilde{C} = u$$

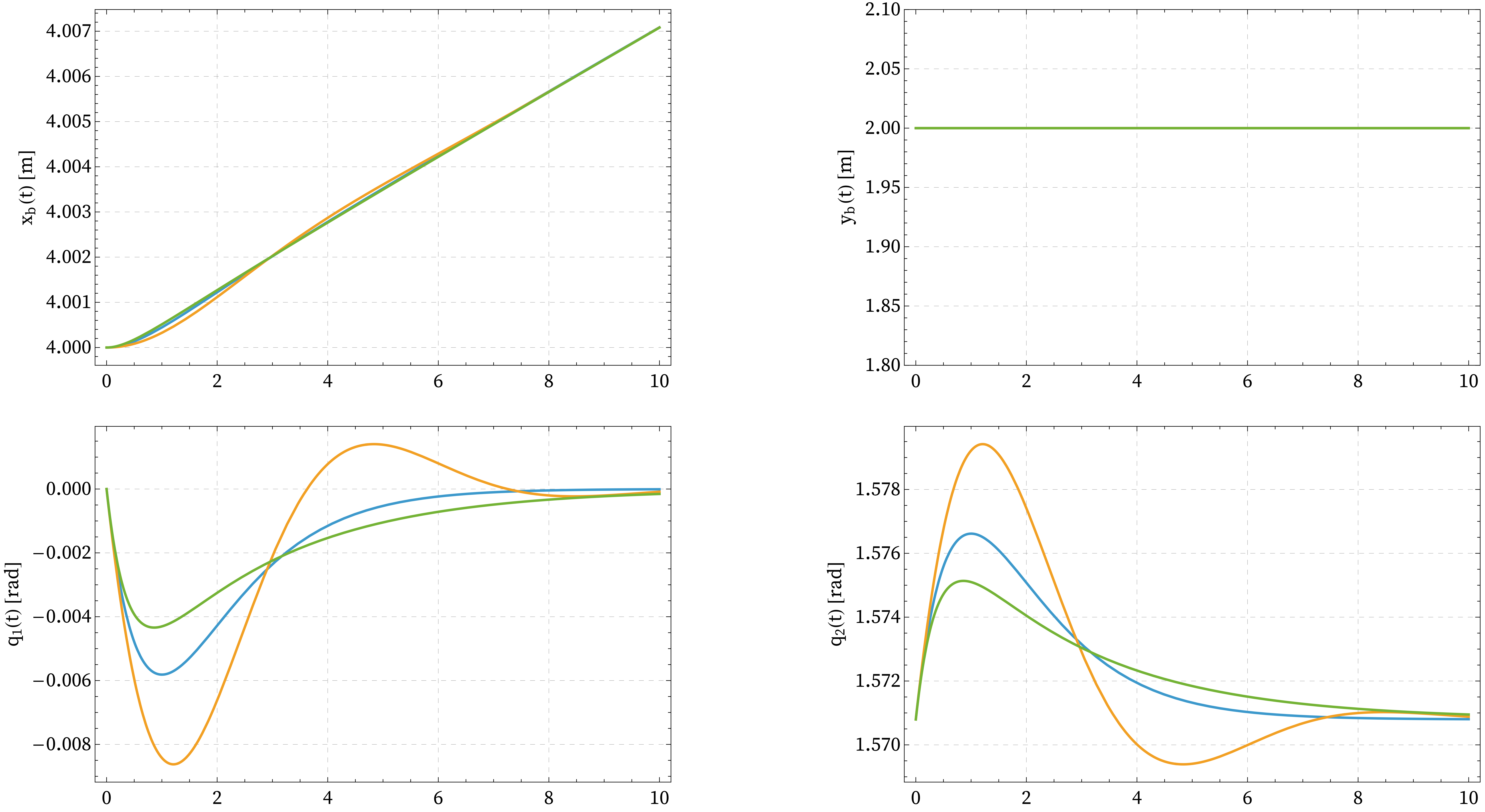
$$u = \tilde{M}[K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q)] + \tilde{C}$$

$$\ddot{e} + K_d\dot{e} + K_p e = 0$$

with $e = q - q_d$, $\omega_n = \sqrt{K_{p_i}}$ and $\xi = \frac{K_{d_i}}{2\sqrt{K_{p_i}}}$

$$\xi = 1 \quad \Rightarrow \quad K_d = 2\sqrt{K_p}$$

Controlled based motion



Linearized equations

Initial equation: $M'\ddot{p} + C' = u$

Linearize equations:

$M_{lin}\ddot{q} + C_{lin}\dot{q} + K_{lin}q = cost$, constant given by desired position

Eigenvalue problem:

$$\{K_{lin} - \omega^2 M_{lin}\}\{u_1, u_2\}^T$$

M_{lin} and K_{lin} don't depend on $q_{1,0}$:

$$M_{lin}(q_{2,0}), K_{lin}(q_{2,0}, m_O)$$

Matrices

- When we keep the same initial conditions for M_{lin} and K_{lin}, if $m_O = m \Rightarrow M_{lin} = K_{lin} \forall q_{2,0}$:

```
Chop@FullSimplify[Mcoupled] /. q200 → π / 3 // MatrixForm
```

//MatrixForm=

$$\begin{pmatrix} 295620. & 142135. \\ 142135. & 94235.9 \end{pmatrix}$$

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//MatrixForm=

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//MatrixForm=

$$\begin{pmatrix} 295620. & 142135. \\ 142135. & 94235.9 \end{pmatrix}$$

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Chop@FullSimplify[Kcoupled /. m0 → 3000 /. q200 → π / 3 /. q100 → 0] // MatrixForm
```

//MatrixForm=

$$\begin{pmatrix} 295620. & 142135. \\ 142135. & 94235.9 \end{pmatrix}$$

- When we set $q_{2,0} = \pi/2$, the second row has the same values:

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Chop@FullSimplify[Mcoupled] /. q200 → q20 // MatrixForm
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MatrixForm=

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MatrixForm=

$$\begin{pmatrix} 200479. & 94235.9 \\ 94235.9 & 94235.9 \end{pmatrix}$$

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Chop@FullSimplify[Kcoupled /. m0 → 3000 /. q200 → π / 2 /. q100 → 0] // MatrixForm
```

MatrixForm=

$$\begin{pmatrix} 200479. & 94235.9 \\ 94235.9 & 94235.9 \end{pmatrix}$$

Matrices

- When the mass is correct ($m_O = m \Rightarrow M_{lin} = K_{lin}$):

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \text{ with } m_{i,j} = k_{i,j} = x_{i,j}$$

$$\Longrightarrow \det\left((1 - \omega^2) \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \right) = 0$$

$$\omega_{1,2} = 1$$

$$U = \begin{bmatrix} ind & ind \\ ind & ind \end{bmatrix}$$

Matrices

- When the mass is incorrect ($m_O \neq m \Rightarrow M_{lin} \neq K_{lin}$) and $q_{2,0} = \pi/2$:

$$\begin{bmatrix} k_1 & k_2 \\ k_2 & k_2 \end{bmatrix} - \omega^2 \begin{bmatrix} m_1 & m_2 \\ m_2 & m_2 \end{bmatrix} \text{ with } m_i \neq k_i$$

$$\Longrightarrow \det\left(\begin{bmatrix} k_1 - \omega^2 m_1 & k_2 - \omega^2 m_2 \\ k_2 - \omega^2 m_2 & k_2 - \omega^2 m_2 \end{bmatrix} \right) = 0$$

$$\begin{cases} \omega_1 = \sqrt{\dfrac{k_2}{m_2}} \\ \omega_2 = \sqrt{\dfrac{k_2 - k_1}{m_2 - m_1}} \end{cases} \Longrightarrow \begin{bmatrix} k_1 - \dfrac{k_2}{m_2} m_1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} \dfrac{k_1 m_2 - k_2 m_1}{m_2 - m_1} & \dfrac{k_1 m_2 - k_2 m_1}{m_2 - m_1} \\ \dfrac{k_1 m_2 - k_2 m_1}{m_2 - m_1} & \dfrac{k_1 m_2 - k_2 m_1}{m_2 - m_1} \end{bmatrix}$$

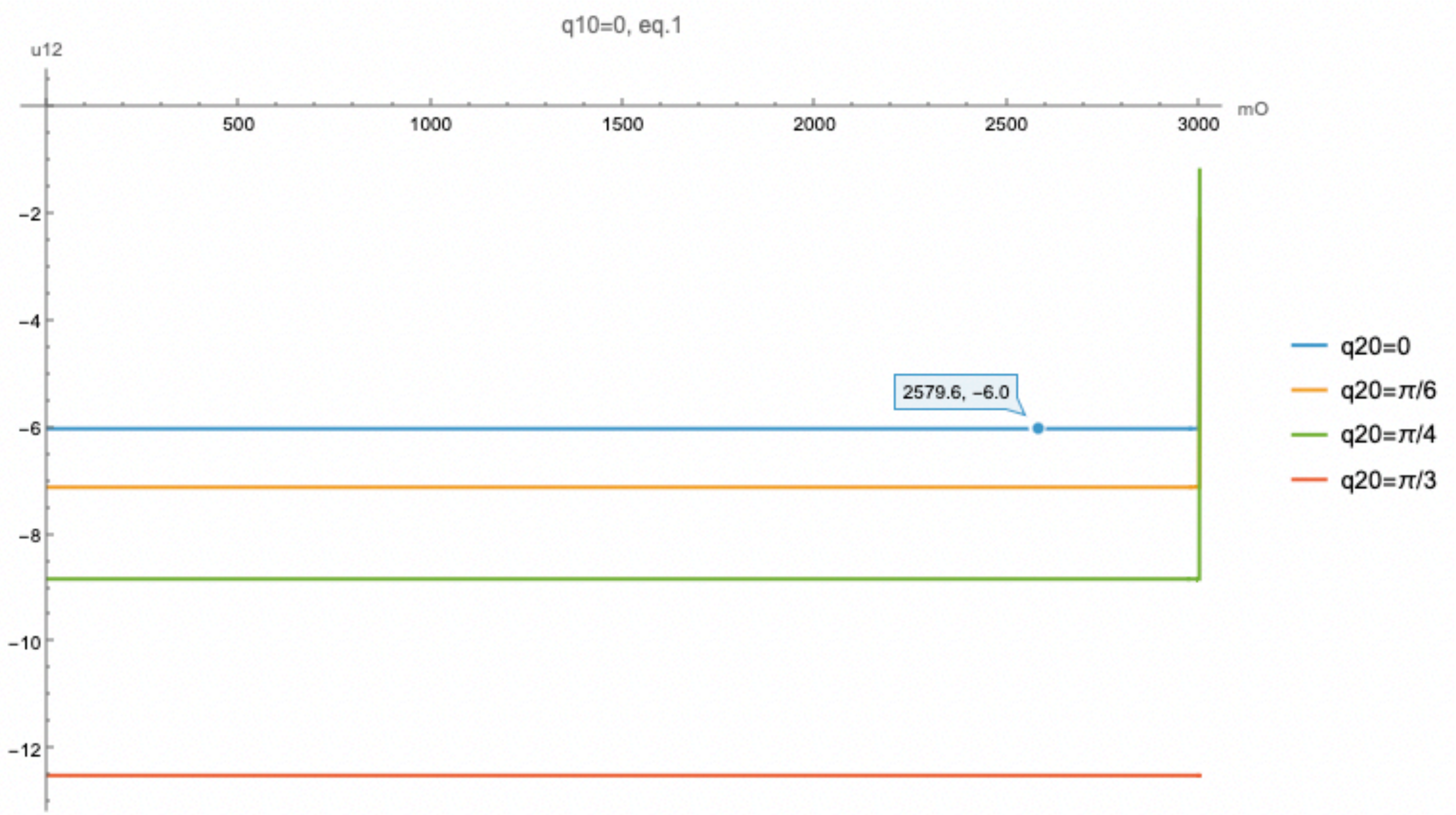
Matrices

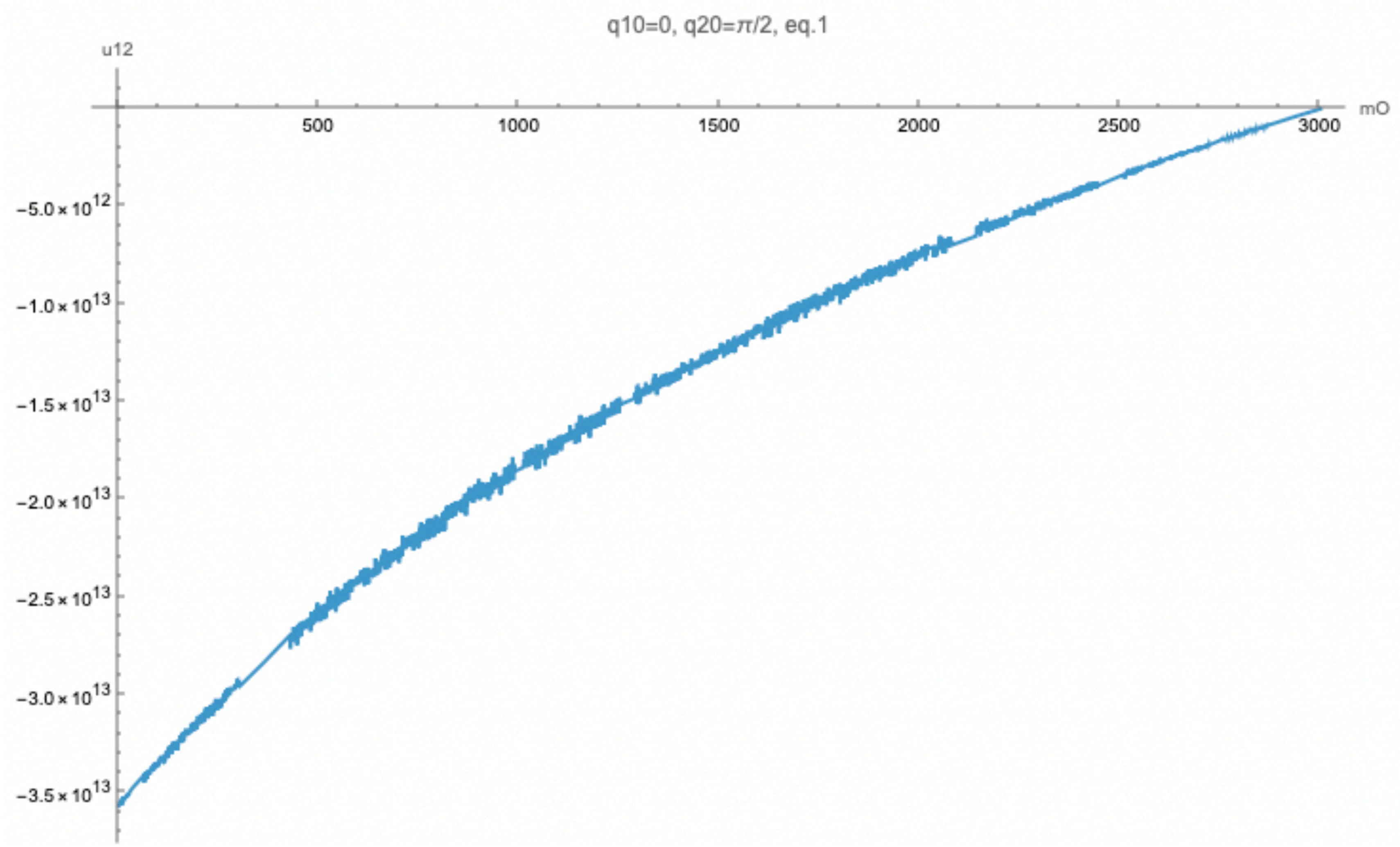
- When the mass is incorrect ($m_O \neq m \Rightarrow M_{lin} \neq K_{lin}$) and $q_{2,0} = \pi/2$:

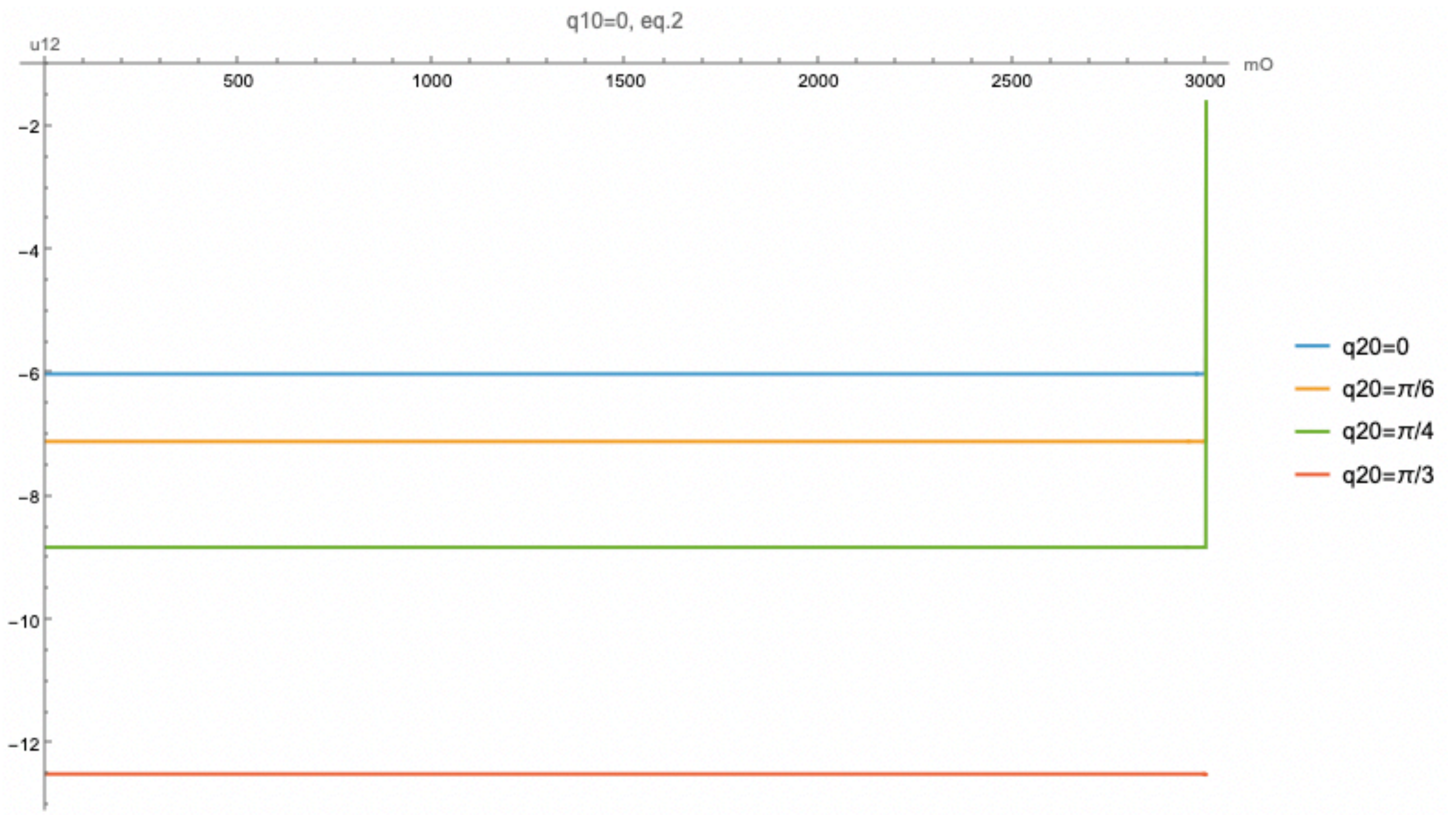
$$\begin{bmatrix} k_1 - \frac{k_2}{m_2} m_1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} \frac{k_1 m_2 - k_2 m_1}{m_2 - m_1} & \frac{k_1 m_2 - k_2 m_1}{m_2 - m_1} \\ \frac{k_1 m_2 - k_2 m_1}{m_2 - m_1} & \frac{k_1 m_2 - k_2 m_1}{m_2 - m_1} \end{bmatrix}$$

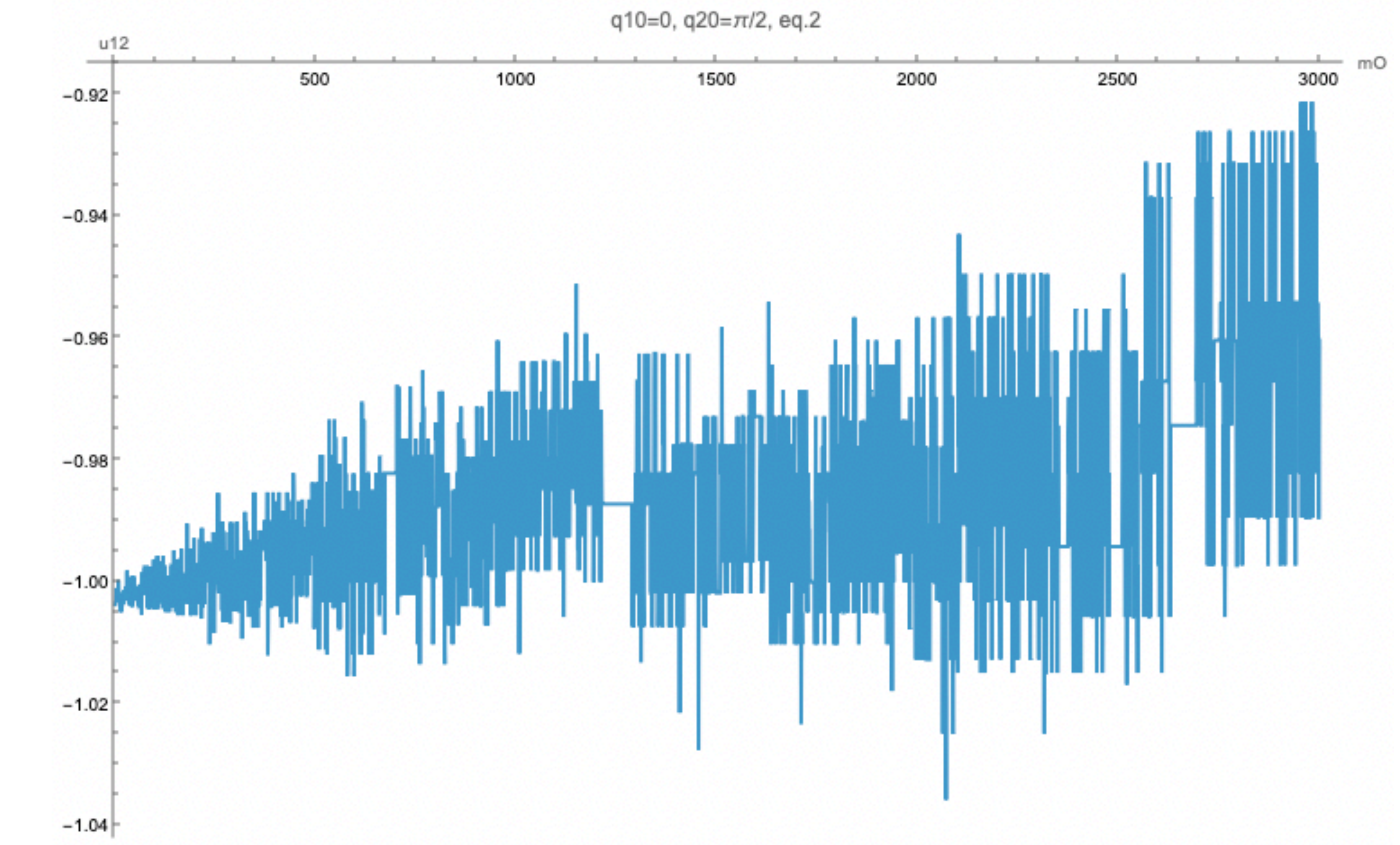
$$U = \begin{bmatrix} 0 & 1 \\ ind & -1 \end{bmatrix} \text{ or } U = \begin{bmatrix} ind & 1 \\ ind & -1 \end{bmatrix}$$

First frequency

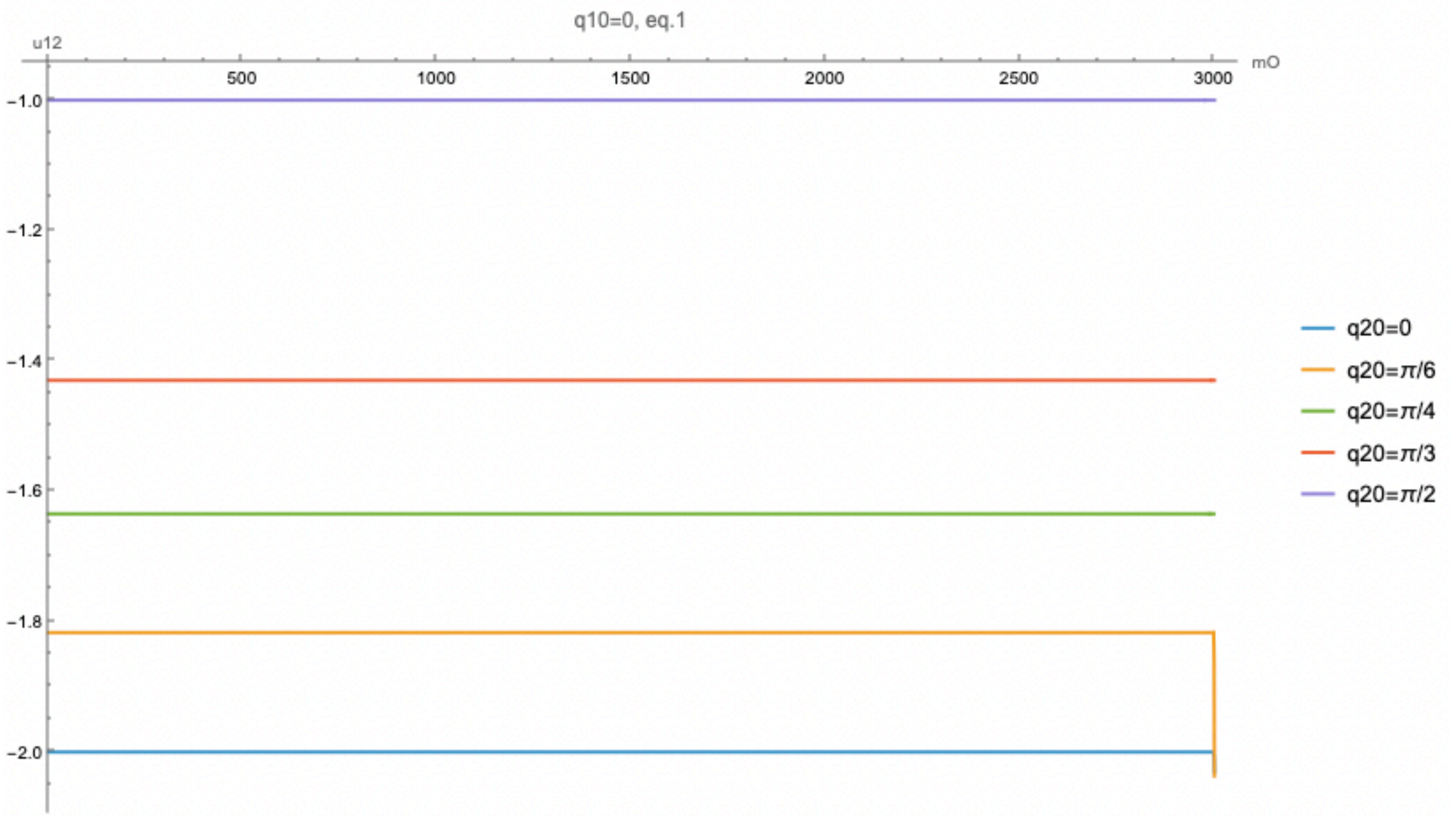


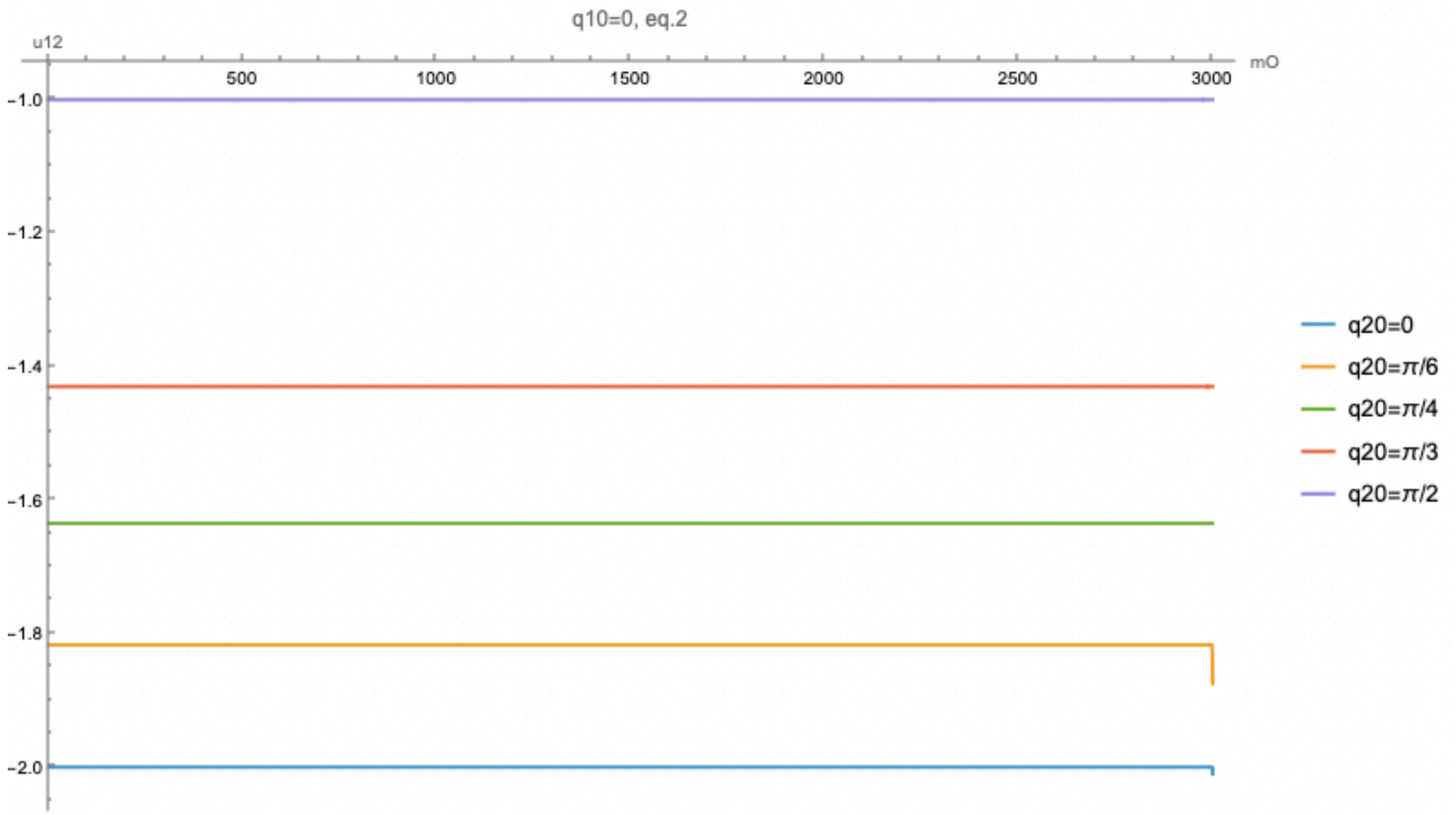






Second frequency



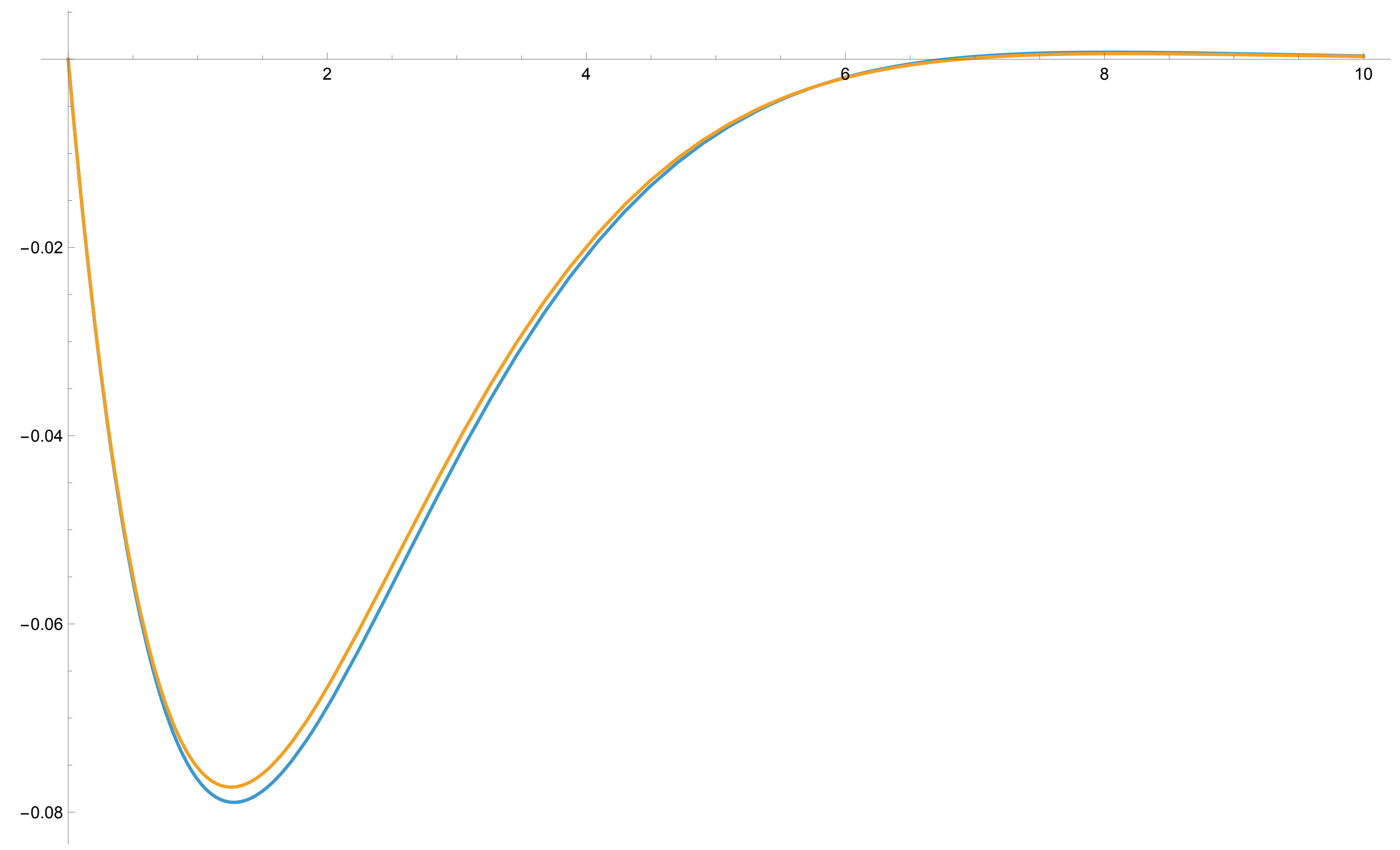


Real dynamics vs Approximated One

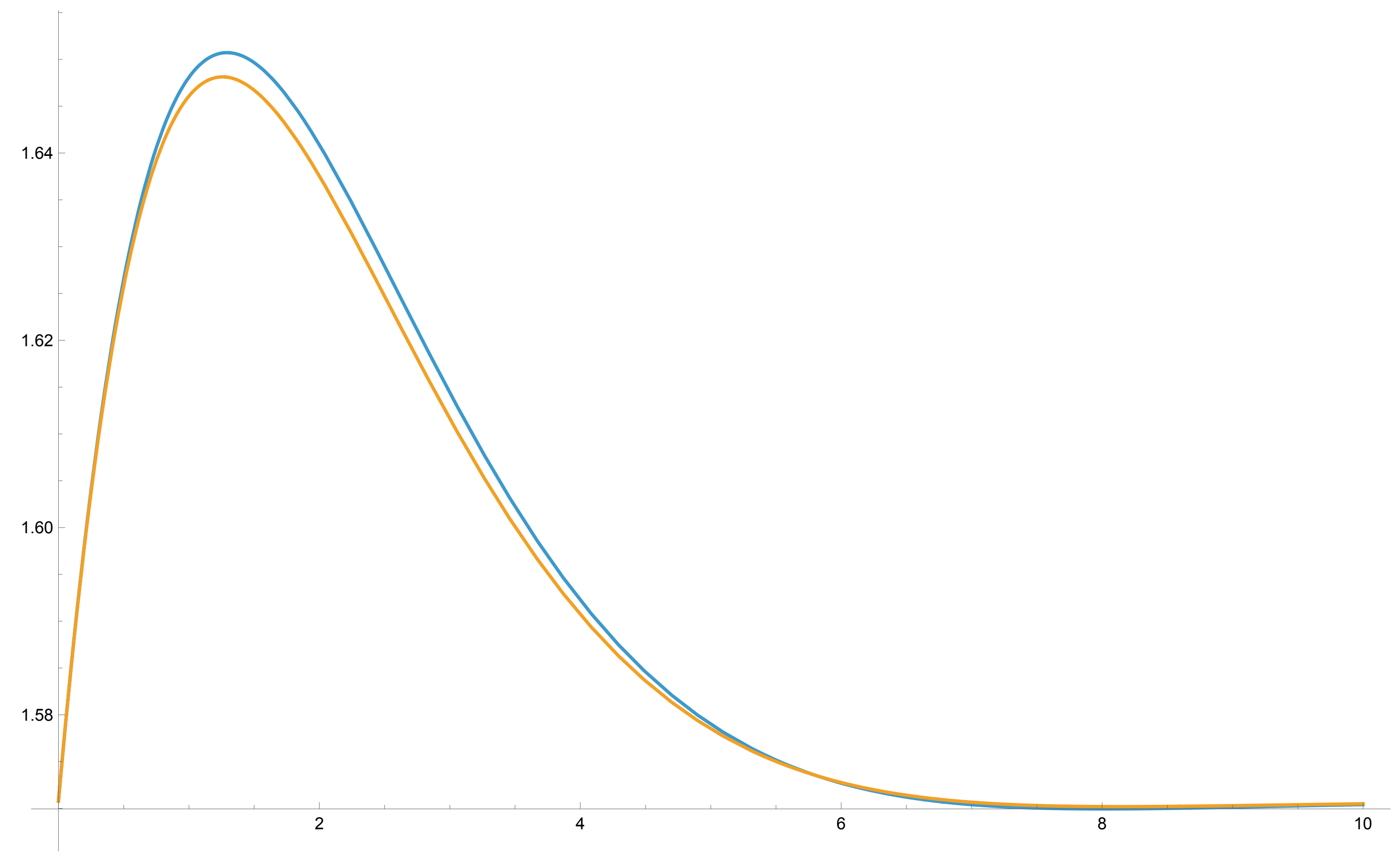
Assumption: we can neglect the other arm's contribution in the linearized equation:

$$\begin{cases} q_2(t) = q_{2,0},\ \dot{q}_2(t) = 0,\ \ddot{q}_2(t) = 0 \quad for \quad q_1 \\ q_1(t) = q_{1,0},\ \dot{q}_1(t) = 0,\ \ddot{q}_1(t) = 0 \quad for \quad q_2 \end{cases}$$

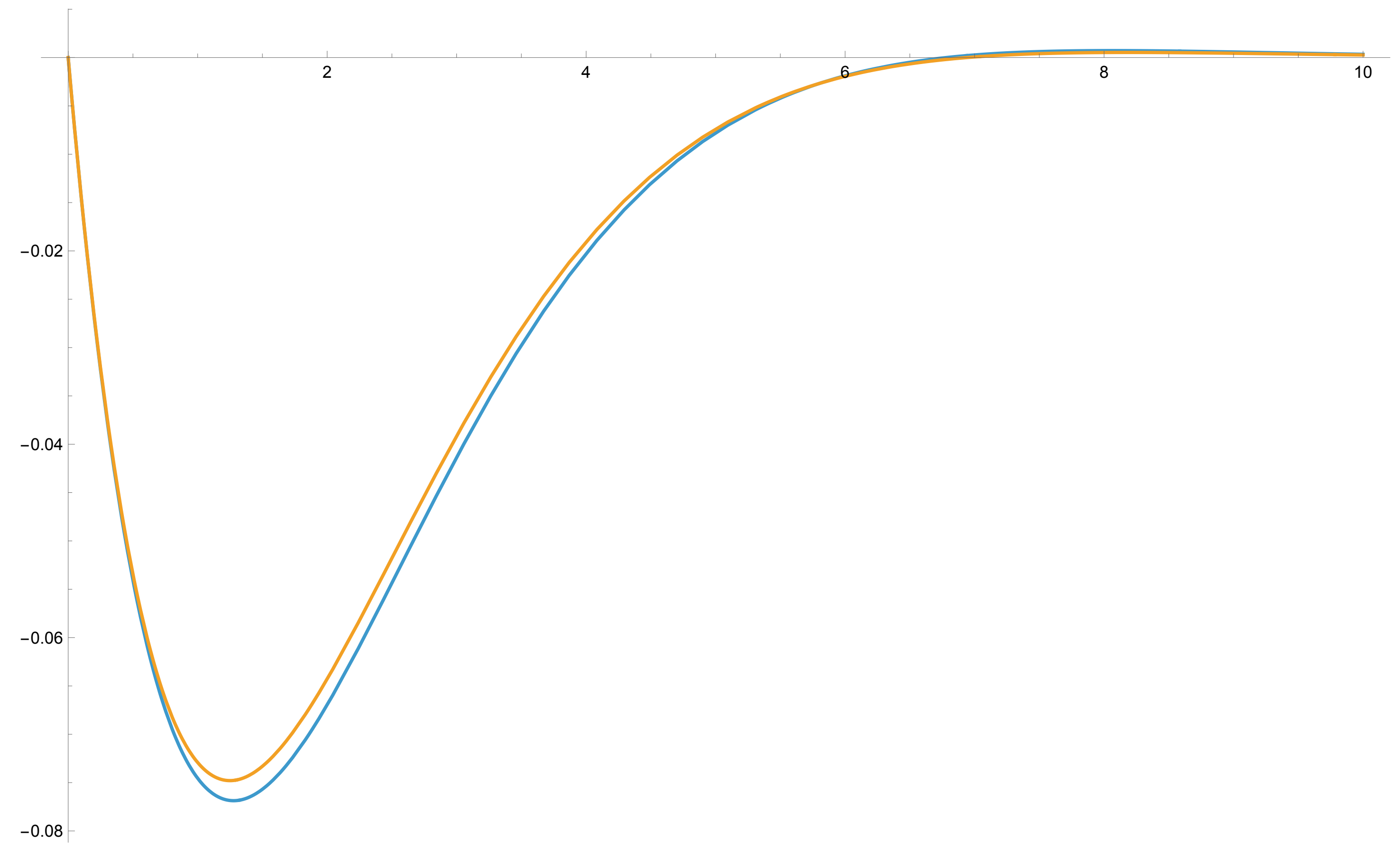
Real dynamics vs Approximated One



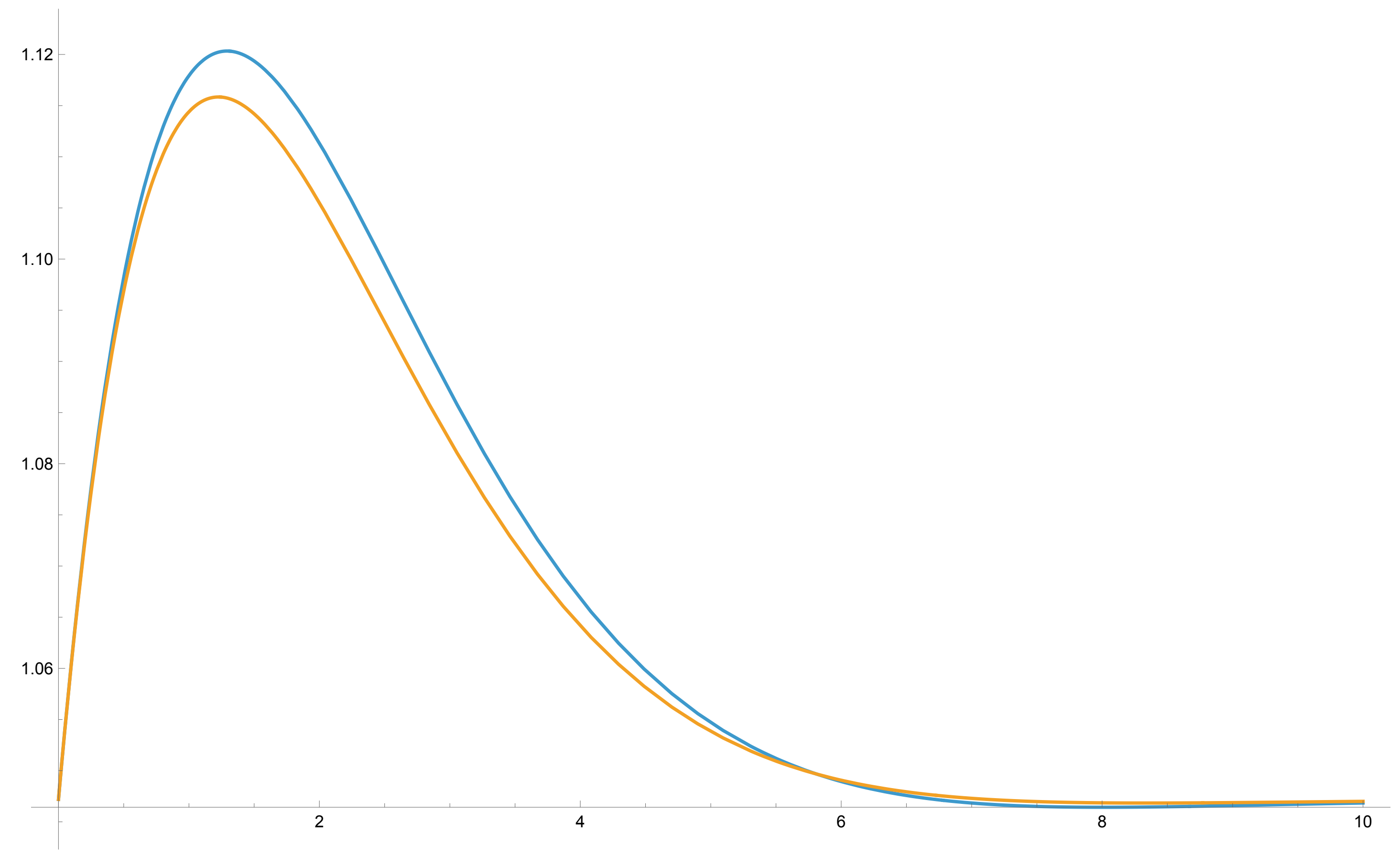
q1(t), q20 = π/2



q2(t), q20 = π/2

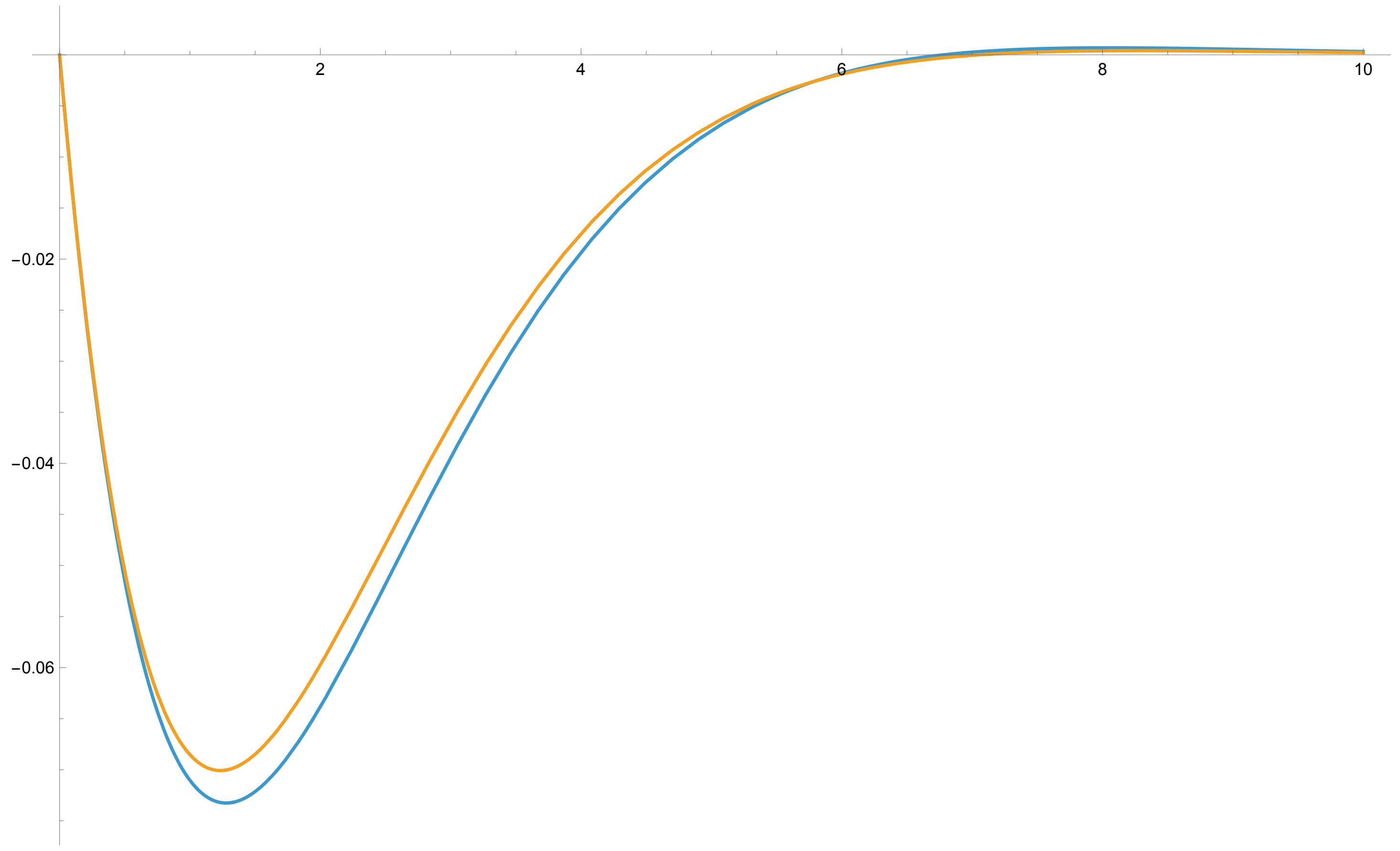


q1(t), q20 = π/3

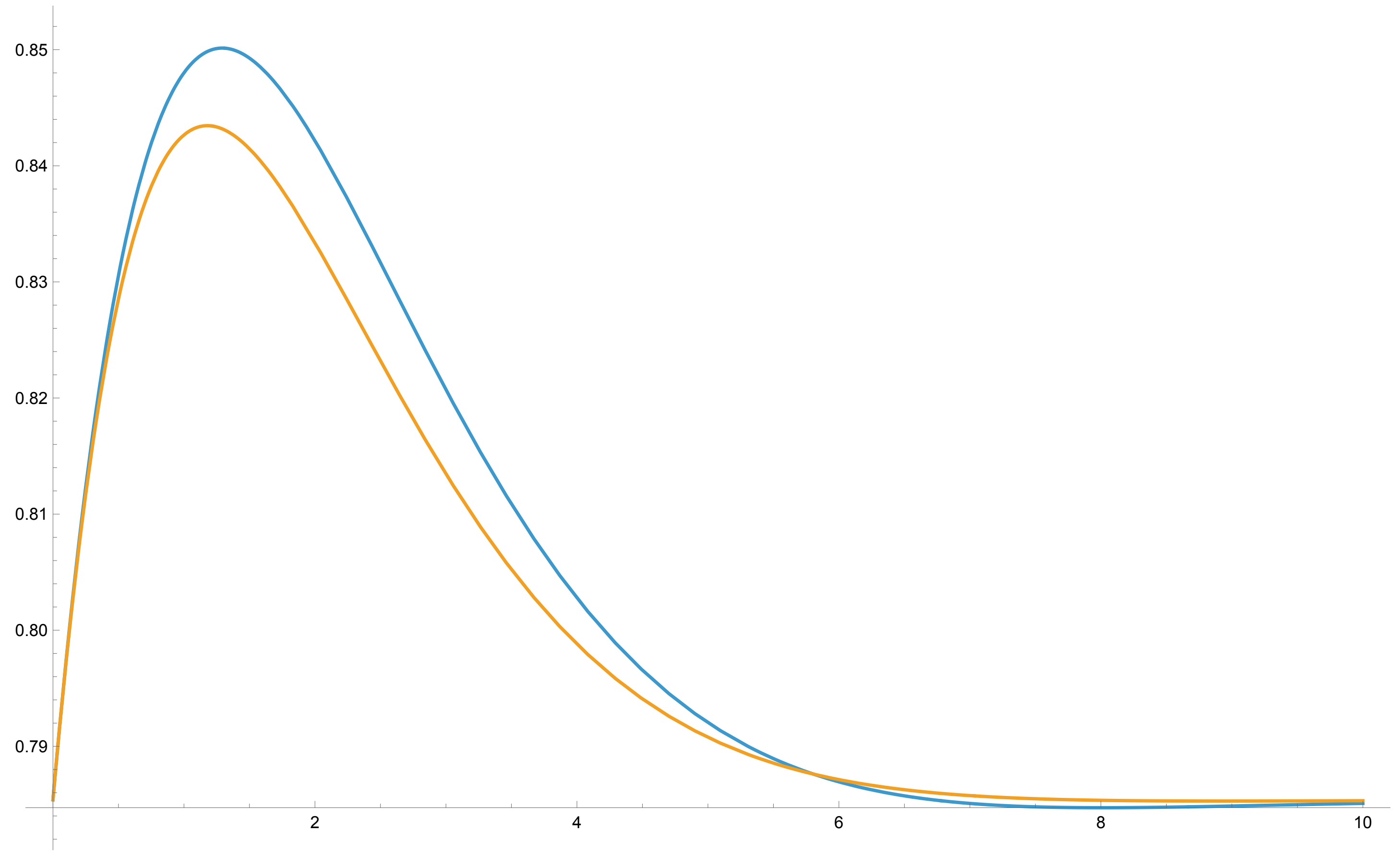


q2(t), q20 = π/3

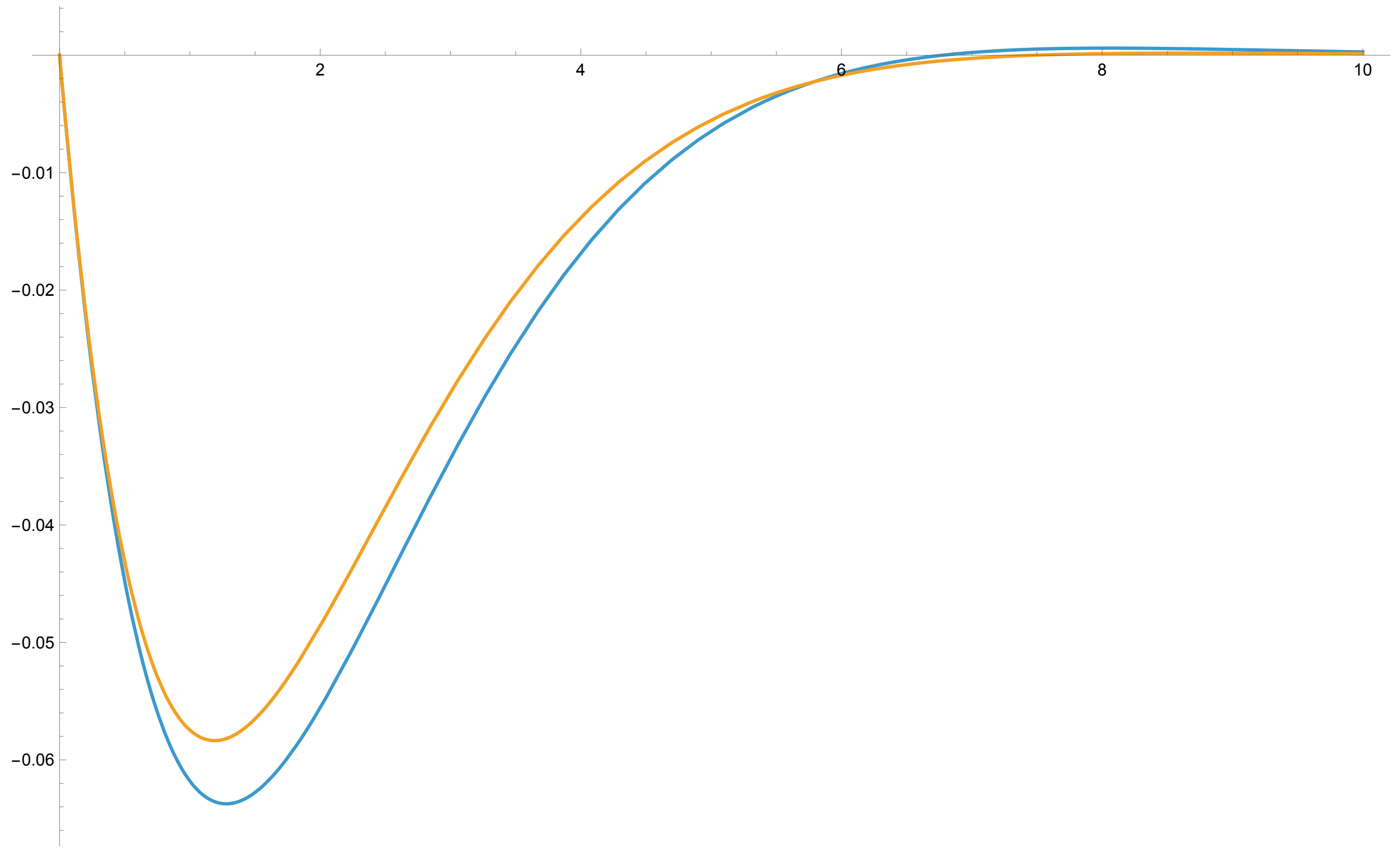
Real dynamics vs Approximated One



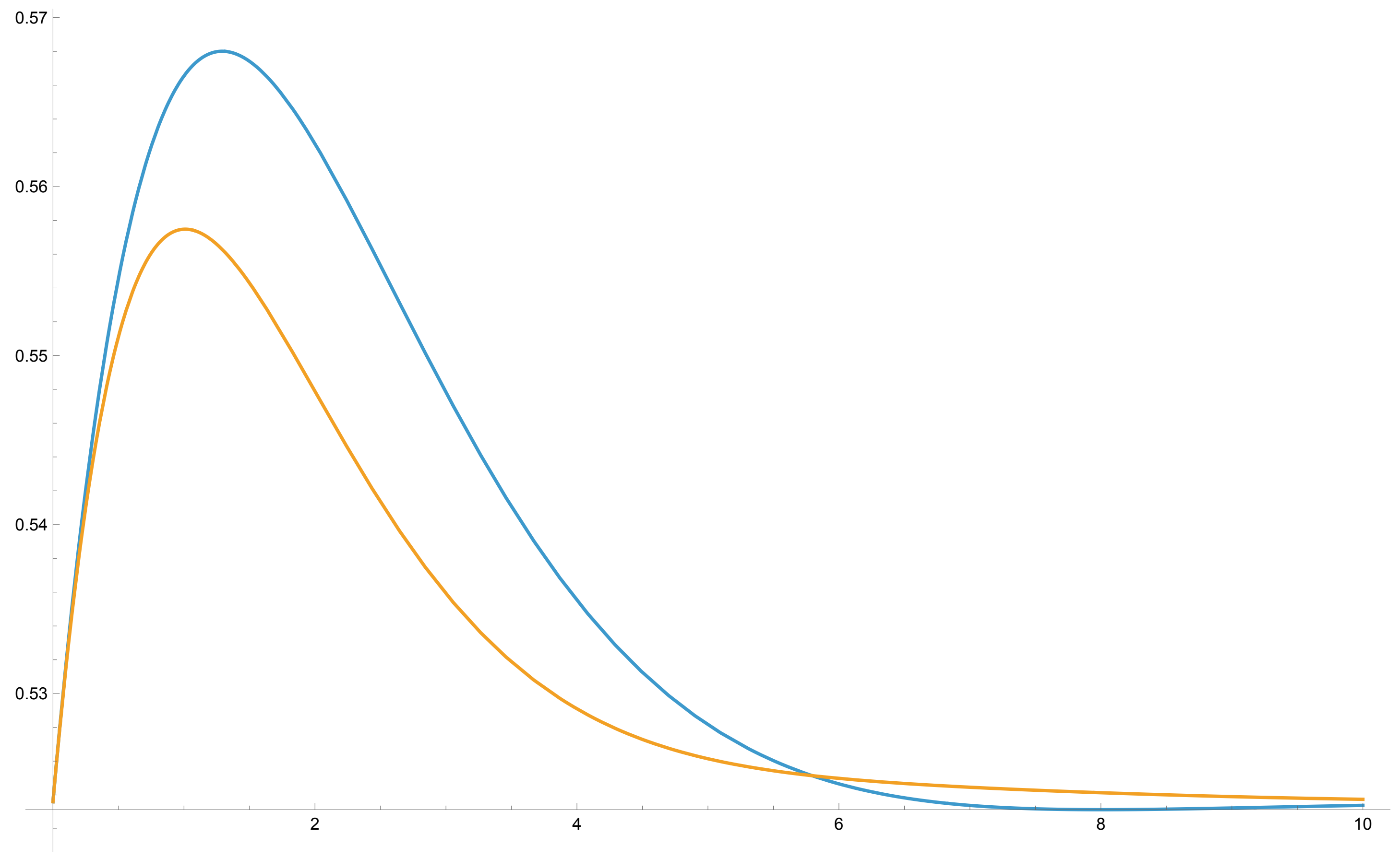
q1(t), q20 = π/4



q2(t), q20 = π/4

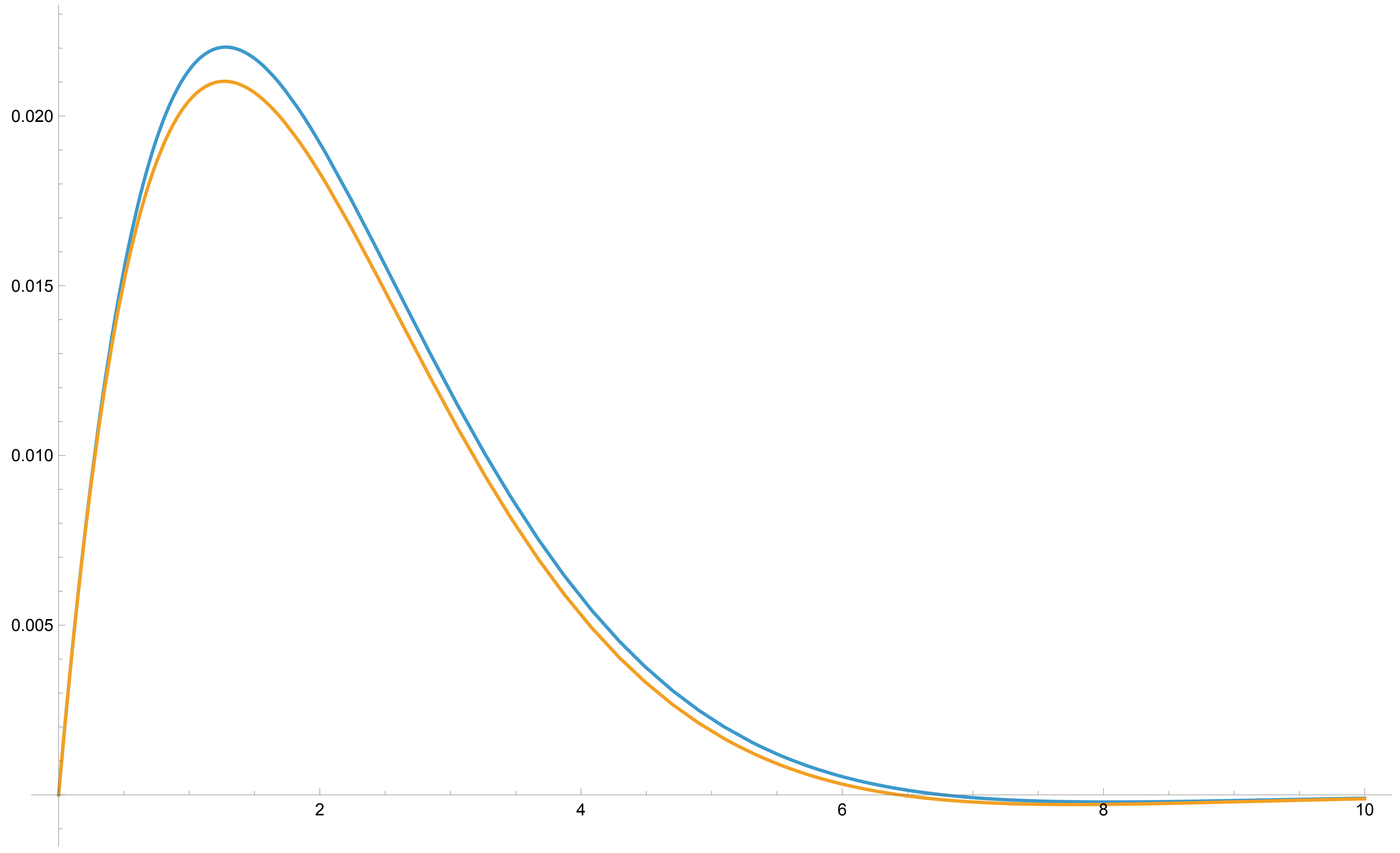


q1(t), q20 = π/6

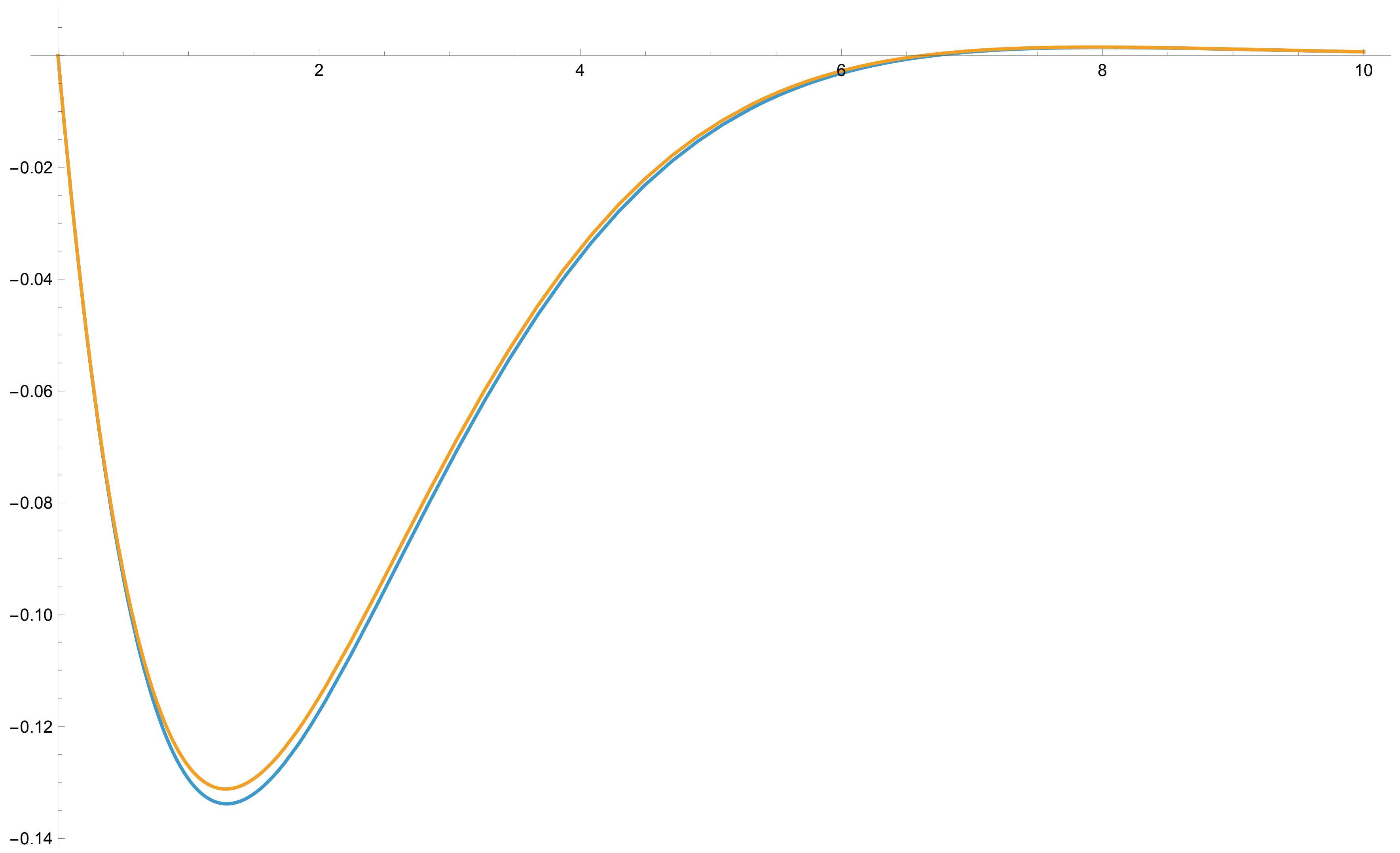


q2(t), q20 = π/6

Real dynamics vs Approximated One



q1(t), q20 = 0



q2(t), q20 = 0

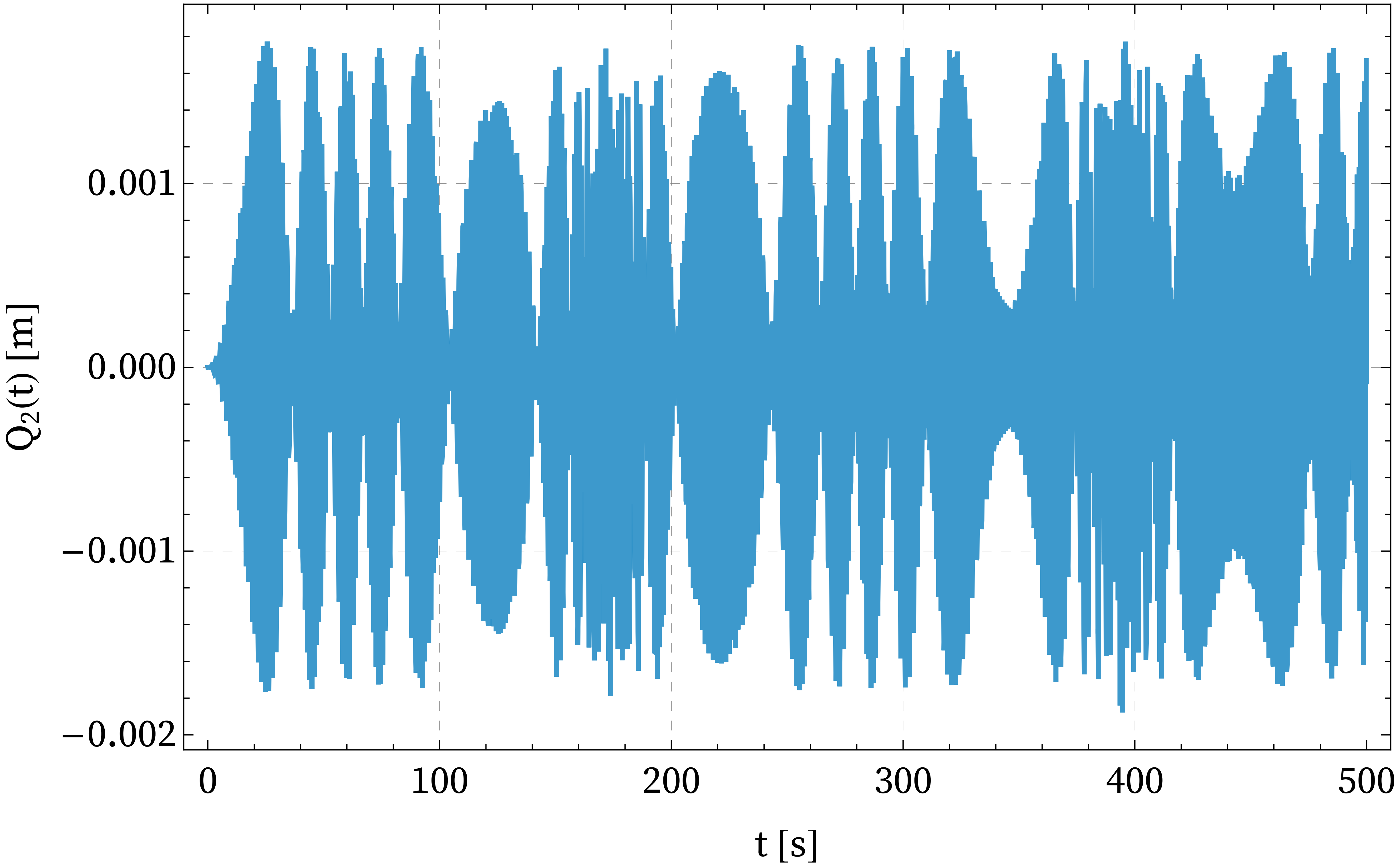
If $m_O = m$, the plots are the same

Elastic deformation

- Problems of wrong initial velocities addressed

- Change in amplitude due change in arm's configuration: when the arms are 90º, the beat stops

- Computed closed-loop control on arms



Controlled Simulation

The arms are always in 90º configuration