UNIVERSITÀ
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Master Thesis

3rd Update

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Angular Kinetic Energy

- Kinetic energy with the “Industrial Robotics” method (name?):

$$T = \frac{1}{2} Tr\left(WJW^T \right)$$

- Same result with “classic method” without considering inertia tensor:

$$T = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Angular Kinetic Energy

- Two cases analysed:
  - Lumped mass
  - Distributed mass

$$\begin{cases} I_{xx} = \dfrac{-I_x + I_y + I_z}{2} \\ I_{yy} = \dfrac{-I_y + I_x + I_z}{2} \\ I_{zz} = \dfrac{-I_z + I_x + I_y}{2} \end{cases}$$

$$\begin{cases} I_x = I_{yy} + I_{zz} \\ I_x = I_{yy} + I_{zz} \\ I_x = I_{yy} + I_{zz} \end{cases}$$

Lumped mass

- Arms: $\begin{cases} Ix = 0 \\ Iy = \frac{1}{4}mL^2 \\ Iz = \frac{1}{4}mL^2 \end{cases}$ $\begin{cases} Ixx = \frac{1}{4}mL^2 \\ Iyy = 0 \\ Izz = 0 \end{cases}$

- Object: $\begin{cases} Ix = 0 \\ Iy = 0 \\ Iz = 0 \end{cases}$ $\begin{cases} Ixx = 0 \\ Iyy = 0 \\ Izz = 0 \end{cases}$ $\Rightarrow$ Mass matrix not invertible!

Distributed mass

- Arms: $\begin{cases} Ix = 0 \\ Iy = \frac{1}{3}mL^2 \\ Iz = \frac{1}{3}mL^2 \end{cases}$ $\begin{cases} Ixx = \frac{1}{3}mL^2 \\ Iyy = 0 \\ Izz = 0 \end{cases}$

- Object: $\begin{cases} Ix = \frac{1}{4}mr^2 \\ Iy = \frac{1}{4}mr^2 \\ Iz = \frac{1}{2}mr^2 \end{cases}$ $\begin{cases} Ixx = \frac{1}{4}mr^2 \\ Iyy = \frac{1}{4}mr^2 \\ Izz = 0 \end{cases}$

Angular Kinetic Energy

- In both cases, the resulting mass matrices from the two different approaches (“matrix form” and “classic”) are not the same and they differ in the same quantity:

```
M[[1]] == M2[[1]]
M[[2]] == M2[[2]]
FullSimplify[M[[3]] == M2[[3]]]
FullSimplify[M[[4]] == M2[[4]]]
FullSimplify[M[[5]] == M2[[5]]]
```

```
True

True
```

$$\left\{0,\ 0,\ -\frac{l1^2\ m1}{4}-\frac{l2^2\ m2}{4},\ -\frac{l1^2\ m1}{4}-\frac{l2^2\ m2}{4},\ -\frac{l2^2\ m2}{4}\right\} == \{0,\ 0,\ 0,\ 0,\ 0\}$$

$$\left\{0,\ 0,\ -\frac{l1^2\ m1}{4}-\frac{l2^2\ m2}{4},\ -\frac{l1^2\ m1}{4}-\frac{l2^2\ m2}{4},\ -\frac{l2^2\ m2}{4}\right\} == \{0,\ 0,\ 0,\ 0,\ 0\}$$

$$\left\{0,\ 0,\ -\frac{l2^2\ m2}{4},\ -\frac{l2^2\ m2}{4},\ -\frac{l2^2\ m2}{4}\right\} == \{0,\ 0,\ 0,\ 0,\ 0\}$$

Post-Impact dynamics

- Assumption: when the target object is captured, its coordinates are the same as the end-defector ones, and its angular velocity is zero. Its angle remains the same as before the impact.

```
ψp = {EE⟦1⟧, EE⟦2⟧, Ω[t] /. initialConditions1};
ψp' = D[{EE⟦1⟧, EE⟦2⟧, 0}, t];
```

- Since, in plastic impacts $J_o\dot{\psi} = J\dot{p}$, it is possible to write the dynamics as a function of the base+manipulator generalised coordinates only.

$$M'\ddot{p} + C' + Kp = U, \tag{24}$$

where

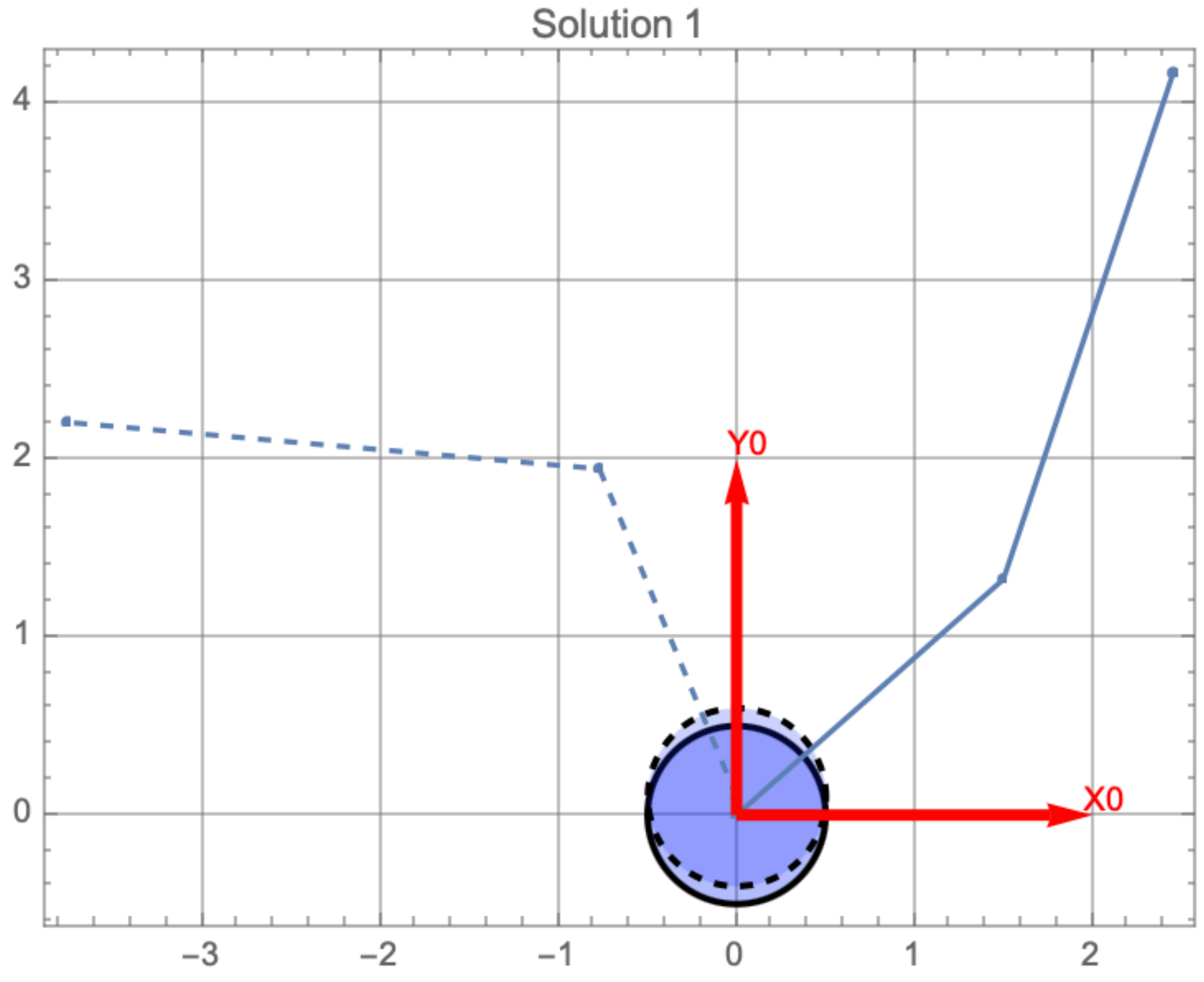
$$M' = M + J^{\mathrm{T}} J_{\mathrm{o}} (J_{\mathrm{o}}^{\mathrm{T}} J_{\mathrm{o}})^{-1} M_{\mathrm{o}} (J_{\mathrm{o}}^{\mathrm{T}} J_{\mathrm{o}})^{-1} J_{\mathrm{o}}^{\mathrm{T}} J \tag{25}$$

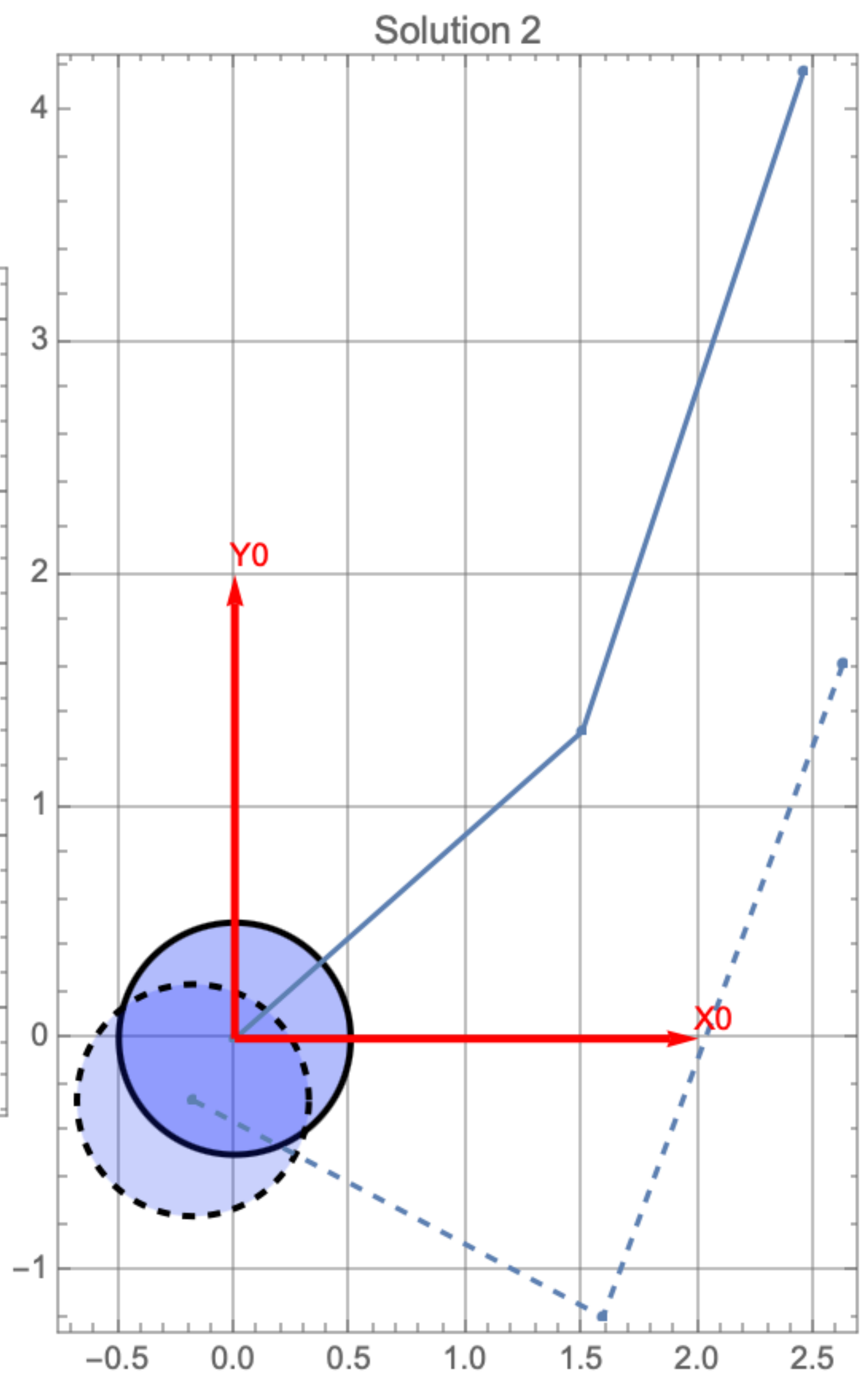
and

$$C' = C + J^{\mathrm{T}} J_{\mathrm{o}} (J_{\mathrm{o}}^{\mathrm{T}} J_{\mathrm{o}})^{-1} M_{\mathrm{o}} (J_{\mathrm{o}}^{\mathrm{T}} J_{\mathrm{o}})^{-1} J_{\mathrm{o}}^{\mathrm{T}} (\dot{J} - \dot{J}_{\mathrm{o}} (J_{\mathrm{o}}^{\mathrm{T}} J_{\mathrm{o}})^{-1} J_{\mathrm{o}}^{\mathrm{T}} J) \dot{p} + J^{\mathrm{T}} (J_{\mathrm{o}}^{\mathrm{T}})^{+} C_{\mathrm{o}}. \tag{26}$$

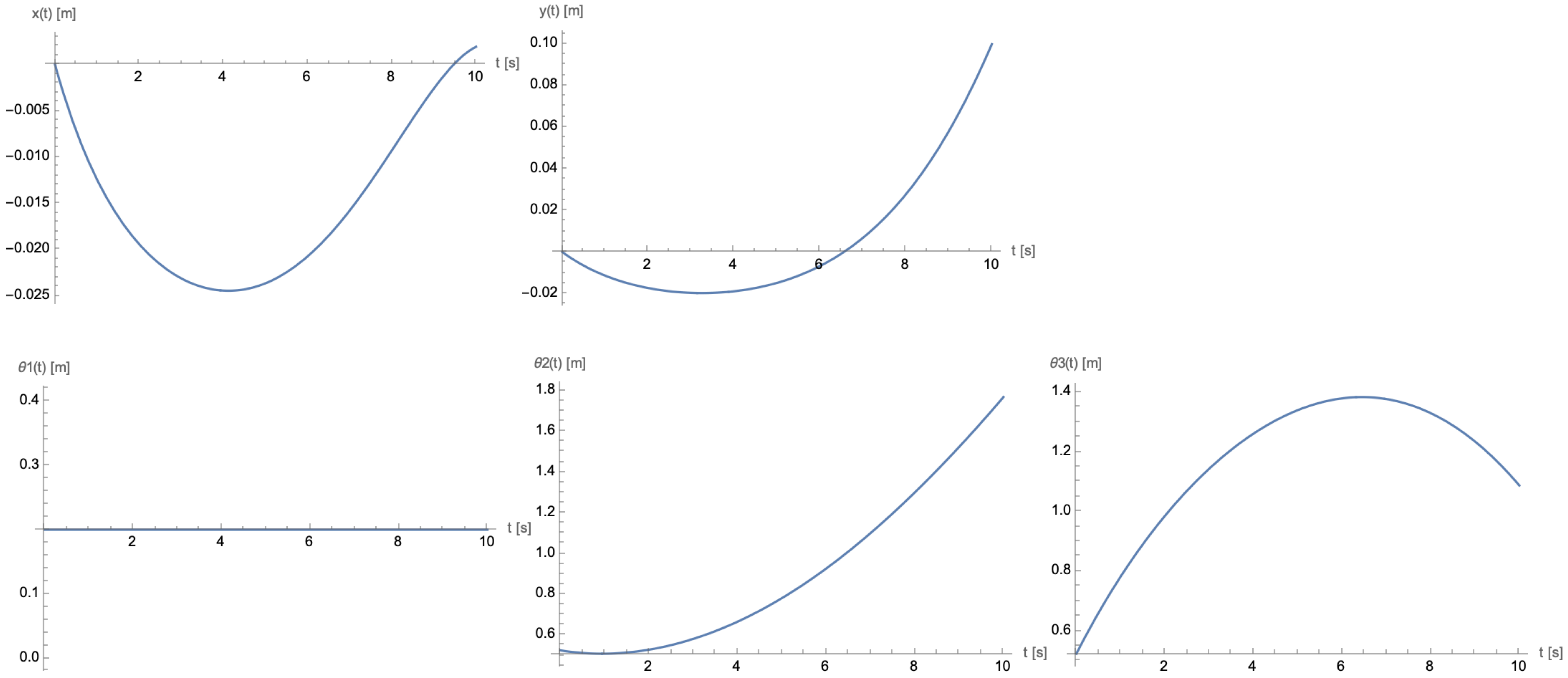
Post-Impact dynamics

- Two simulations performed:
  - $\{\dot{X}_i, \dot{Y}_i\} = \{-1,0\}$
  - $\{\dot{X}_i, \dot{Y}_i\} = \{0, -1\}$



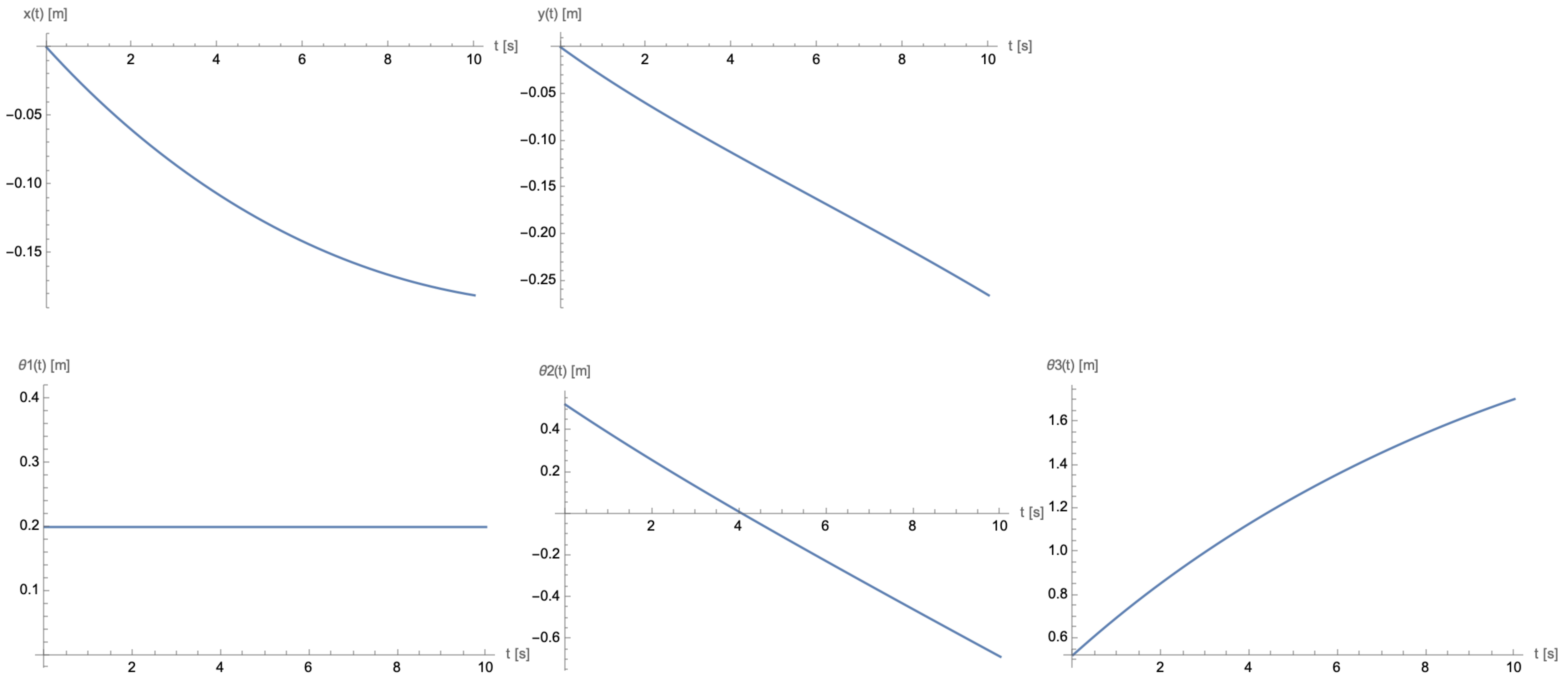


Post-Impact dynamics



First solution

Post-Impact dynamics



Second solution