



# DYNAMICS OF A SPACE ROBOT SUBJECTED TO AN IMPACT

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## I.O INTRODUCTION:

- ▶ Introduction
- ▶ Overview
- ▶ Kinematics
- ▶ Dynamics
- ▶ Impact Analysis
- ▶ Mass and Velocity Retrieval
- ▶ Results
- ▶ Q&A

# INTRODUCTION

## I.0 INTRODUCTION:

- Thesis' aim: satellite docking, stabilization and inertia retrieval.
- Docking → *impact*.
- Manipulator: robotic arm:
  - Rigid bodies: *links*
  - Relative motion: *joints*
  - Actuation: *motors*
- Vehicle-Manipulator System: VMS.



Figure 1: ESA's space manipulator [1].



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## 2.0 OVERVIEW:

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# HISTORICAL BACKGROUND

## 2.0 OVERVIEW:

- First space manipulator: Space Remote Manipulator System (SRMS), 11/1981 - 07/2011.
- First space mission: MIR, 1986 - 2001.
- Japanese arm: 2008.
- Canadarm2 (or SSRMS) and MSS: 04/2001
- ESA's manipulator, ERA: 07/2021



Figure 2: Canadarm2 attached to the Mobile Base System [2].

# MAIN CHARACTERISTICS

## 2.0 OVERVIEW:

- Lack of fixed base:
  - Free-flying system;
  - Free-floating system.
- Typical capture sequence:
  1. observation and planning phase;
  2. final approach phase;
  3. impact and grasping/capture phase;
  4. post-capture stabilization phase.
- To avoid impact: zero relative velocity.

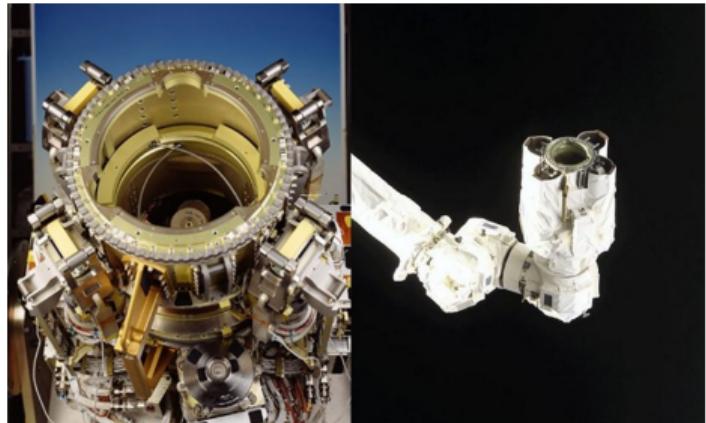


Figure 3: Latching End Effector (LEE) [2].

# MAIN CHARACTERISTICS

## 2.0 OVERVIEW:

- Two 2-links arms attached to a "shoulder".
- Usually redundant kinematics.
- Number of degrees of freedom:  
$$n = 3(m - 1) - 2c_1 - c_2$$

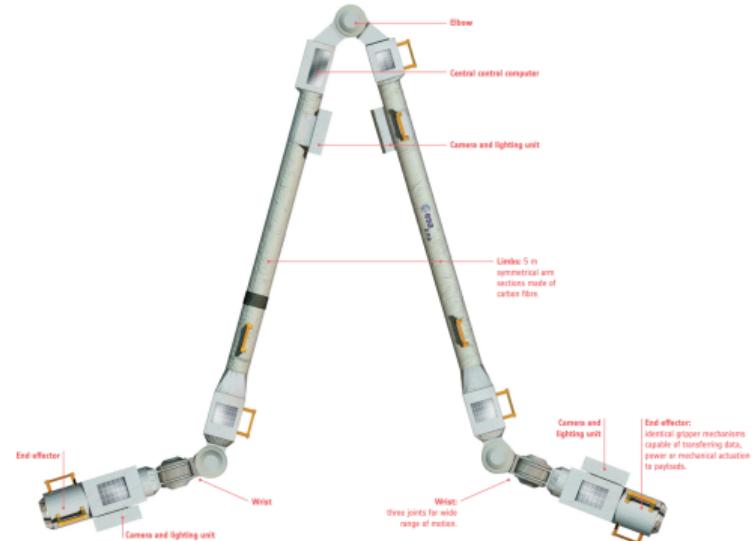


Figure 4: ERA specifications [3].



## STATE OF ART

### 2.0 OVERVIEW:

	SSRMS	ERA	JEMRMS (MA)
Span	14.22 m	11.3 m	9.91 m
Boom Span	7.11 m	7.77 m	3.93 m
DOFs	7	7	6
Joints	Offset	Inline	Offset
Base	Relocatable	Relocatable	Fixed
Positioning Accuracy	65 mm, 0.71°	40 mm, 1°	50 mm, 1.8°
Mass	1336 kg	630 kg	757 kg
Max Handling Capacity	116 Mg	8 Mg	7 Mg
Power Consumption	1360 W (average)	>800 W	2.3 kW

Table 1: SSRMS, ERA and JEMRMS specifications [4].



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## FUNDAMENTALS

### 3.0 KINEMATICS:

- Direct Kinematics.
- Inverse Kinematics.
- Homogeneous Matrix Approach [5].

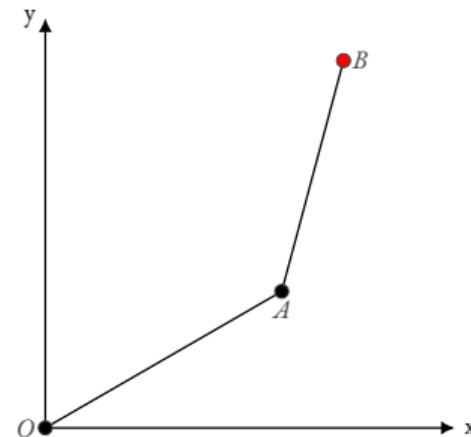


Figure 5: 2D RR (revolute-revolute) manipulator.

## FUNDAMENTALS

### 3.0 KINEMATICS:

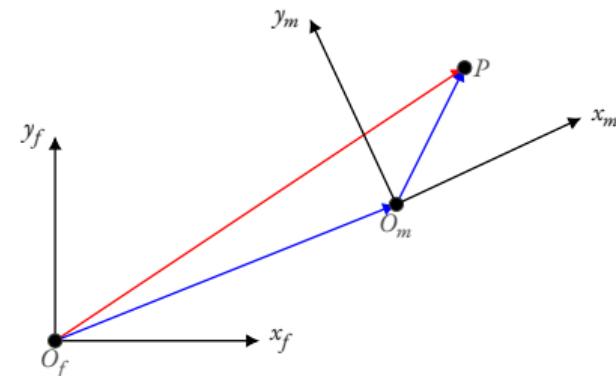
- Roto-Translation Matrix:

$$\begin{bmatrix} {}^f O_f P_x \\ {}^f O_f P_y \\ {}^f O_f P_z \\ 1 \end{bmatrix} = \begin{bmatrix} {}^f R & \begin{bmatrix} {}^f O_f O_m x \\ {}^f O_f O_m y \\ {}^f O_f O_m z \end{bmatrix} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^m O_m P_x \\ {}^m O_m P_y \\ {}^m O_m P_z \\ 1 \end{bmatrix} = {}^f M {}^m M \begin{bmatrix} {}^m O_m P_x \\ {}^m O_m P_y \\ {}^m O_m P_z \\ 1 \end{bmatrix} \quad (1)$$

- Velocity Matrix:  ${}^f W = {}^f M {}^m M \dot{{}^m M}$

$$W = \begin{bmatrix} 0 & -\omega_z & \omega_y & v_x \\ \omega_z & 0 & -\omega_x & v_y \\ -\omega_y & \omega_x & 0 & v_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

- 'Logic Matrix':  $L = \frac{W}{|\omega|}$



**Figure 6:** Definition of a point in a rotated and translated frame with respect to the fixed one.



# DANAVIT-HARTENBERG METHOD

## 3.0 KINEMATICS:

- Used to define reference frames.
- Ordered procedure:
  1.  $z_i$  axis: axis of the revolute joint that connects the link to the following.
  2.  $x_i$  axis: line of minimum distance between  $z_{i-1}$  and  $z_i$ , oriented from  $z_{i-1}$  to  $z_i$ .
  3.  $y_i$  axis: obtained by the vectorial product of the other two axes.

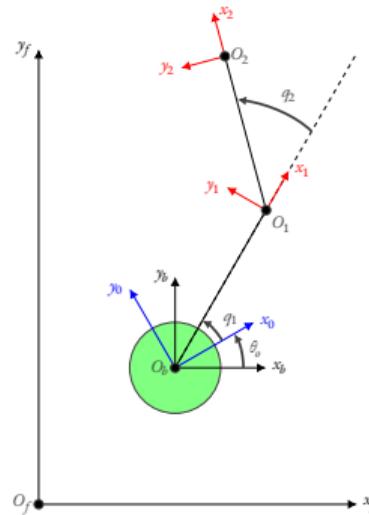


Figure 7: The Danavit-Hartenberg rule has been used to place the local joints' frame (in red) in a planar RR manipulator.



# PLANAR VMS

## 3.0 KINEMATICS:

- VMS' coordinates:  $p = \{x_b, y_b, \theta_b, q_1, q_2\}$
- Satellite coordinates:  $\psi = \{x_O, y_O, \theta_O\}$

$$\begin{aligned} {}^f_b M &= \begin{bmatrix} 1 & 0 & 0 & x_b \\ 0 & 1 & 0 & y_b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^b_0 M = \begin{bmatrix} \cos \theta_0 & -\sin \theta_0 & 0 & 0 \\ \sin \theta_0 & \cos \theta_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &\qquad\qquad\qquad (3) \end{aligned}$$

$${}^0_1 M = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & l_1 \cos q_1 \\ \sin q_1 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2 M = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & l_2 \sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



## PLANAR VMS

### 3.o KINEMATICS:

$$p_{EE} = {}^0_2MO_3 = \begin{bmatrix} l_1 \cos(\theta_0 + q_1) + l_2 \cos(\theta_0 + q_1 + q_2) + x_b \\ l_1 \sin(\theta_0 + q_1) + l_2 \sin(\theta_0 + q_1 + q_2) + y_b \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

with  $O_3 = \{0, 0, 0, 1\}$ .

$${}^fW_{f1_{1:3,1:3}} = \begin{bmatrix} 0 & -\theta_0 - q_1 & 0 \\ \theta_0 + q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad {}^fW_{f2_{1:3,1:3}} = \begin{bmatrix} 0 & -\theta_0 - q_1 - q_2 & 0 \\ \theta_0 + q_1 + q_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

$${}^fL_{f1_{1:3,1:3}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad {}^fL_{f2_{1:3,1:3}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

# SATELLITE KINEMATICS

## 3.o KINEMATICS:

$${}_{o_0}^f M = \begin{bmatrix} 1 & 0 & 0 & x_O \\ 0 & 1 & 0 & y_O \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_{o_1}^{o_0} M = \begin{bmatrix} \cos \theta_O & -\sin \theta_O & 0 & 0 \\ \sin \theta_O & \cos \theta_O & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{o_2}^{o_1} M = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 & r \cos \gamma \\ \sin \gamma & \cos \gamma & 0 & r \sin \gamma \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

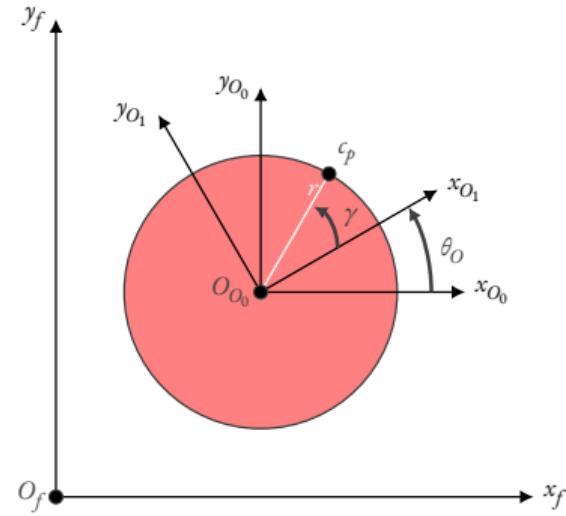


Figure 8: Tumbling object disk approximation.



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## FUNDAMENTALS

### 4.0 DYNAMICS:

- Direct Dynamics.
- Inverse Dynamics.
- Given a *wrench*  $w$  at the EE:

- Power at the joints:  $P_\tau = \tau_w^T \dot{q}$
- Power at the EE:  $P_e = w^T v$
- Conservation of energy:

$$P_\tau = P_e \quad \Rightarrow \quad \tau_w^T \dot{q} = w^T v = w^T J \dot{q} \quad \forall \dot{q} \Rightarrow \quad \tau_w^T = w^T J \quad \Rightarrow \quad \tau_w = J^T w$$

- Equation of motion:  $M(q)\ddot{q} + C(q, \dot{q}) = u + J^T(q)w$



# VMS

## 4.0 DYNAMICS:

- Homogeneous Matrix Approach [5, 6].
- Lagrangian formulation:

$$\mathcal{L}(q, \dot{q}) = \sum_{i=1}^N T_i - U_i, \quad \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = f_i \quad (8)$$

- Pseudo Inertia Tensor:

$${}^i J_k = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} & m_k x_G \\ J_{yx} & J_{yy} & J_{yz} & m_k y_G \\ J_{zx} & J_{zy} & J_{zz} & m_k z_G \\ m_k x_G & m_k y_G & m_k z_G & m \end{bmatrix} \quad (9)$$

with

$$\begin{aligned} J_{xx} &= \int x^2 dm & J_{yy} &= \int y^2 dm & J_{zz} &= \int z^2 dm \\ J_{xy} &= \int xy dm & J_{xz} &= \int xz dm & J_{yz} &= \int yz dm \end{aligned} \quad (10)$$

- Action Matrix:

$${}^i\phi_k = \begin{bmatrix} 0 & -c_z & c_y & f_x \\ c_z & 0 & -c_x & f_y \\ -c_y & c_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{bmatrix} \quad (11)$$

- Non-Lagrangian components:

$$f_{q_i} = \left( \sum_{j=1}^N {}^f\phi_j \right) \otimes {}^fL_{q_i} \quad (12)$$

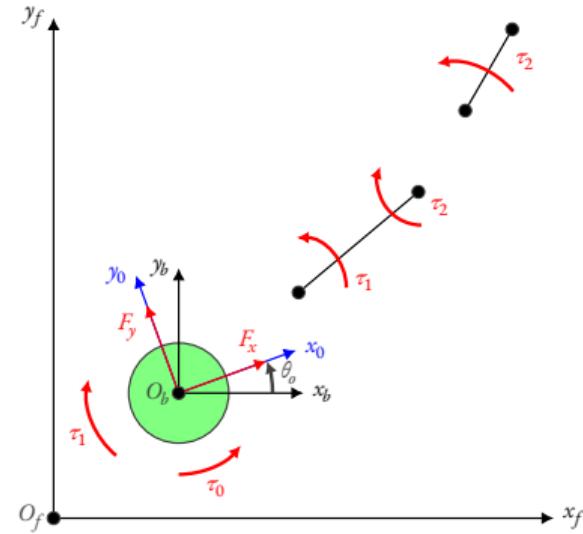


Figure 9: Forces and torques applied to the base and to the links.

# VMS

## 4.0 DYNAMICS:

- Non-Lagrangian components:

$$\begin{aligned}
 f_1 &= \left( {}^f\phi_b + {}^f\phi_1 + {}^f\phi_2 \right) \otimes {}^fL_{fb_x} = F_x \cos \theta_0 - F_y \sin \theta_0 \\
 f_2 &= \left( {}^f\phi_b + {}^f\phi_1 + {}^f\phi_2 \right) \otimes {}^fL_{fb_y} = F_x \sin \theta_0 + F_y \cos \theta_0 \\
 f_3 &= \left( {}^f\phi_b + {}^f\phi_1 + {}^f\phi_2 \right) \otimes {}^fL_{f0} = \tau_0 \\
 f_4 &= \left( {}^f\phi_b + {}^f\phi_1 \right) \otimes {}^fL_{01} = \tau_1 \\
 f_5 &= {}^f\phi_b \otimes {}^fL_{12} = \tau_2
 \end{aligned} \tag{13}$$

- Kinetic Energy

$$T_j = \frac{1}{2} \operatorname{Tr} \left( {}^fW_{fj} {}^fJ_j {}^fW_{fj}^T \right) \tag{14}$$

**Table 2:** Moments of inertia of the planar VMS' bodies, with respect to their centre of mass (in the centre of the base and the arms), with  $i = \{1, 2\}$ .

Body	Tensor of Pseudo Inertia		
Base	$\frac{1}{4}m_b r^2$	0	0
	0	$\frac{1}{4}m_b r^2$	0
	0	0	0
Arms	$\frac{1}{3}m_i l_i^2$	0	0
	0	0	0
	0	0	0



## SATELLITE

### 4.0 DYNAMICS:

- Same procedure or, alternatively:

$$T_O = v_O m_O v_O^T + \frac{1}{2} I_{O_z} \theta_O^2 \quad (15)$$

- Mass matrix:

$$M_O = \begin{bmatrix} m_O & 0 & 0 \\ 0 & m_O & 0 \\ 0 & 0 & \frac{m_O r^2}{2} \end{bmatrix} \quad (16)$$



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## CONSERVATION OF MOMENTUM

### 5.0 IMPACT ANALYSIS:

- Four assumptions [7]:
  - Only the generalized velocities change during the impact, no displacement.
  - At the contact point between the end-effector and the target there are no forces but moments only.
  - We know the satellite's mass and initial velocities.
  - Plastic impact.
- Equations of motion:

$$\begin{cases} M\ddot{p} + C = u + J^T f_I \\ M_O \ddot{\psi} + C_O = -J_O^T f_I \end{cases} \quad (17)$$

and

$$\int_0^\pi M\ddot{p} dt + \int_0^\pi C dt = - \int_0^\pi J^T (J_O^+)^T M_O \ddot{\psi}_O dt + \int_0^\pi (u - J^T (J_O^+)^T C_O) dt \quad (18)$$



## CONSERVATION OF MOMENTUM

### 5.0 IMPACT ANALYSIS:

- Conservation of momentum equation:

$$M(\dot{p}_f - \dot{p}_i) + J^T (J_O^+)^T M_O (\dot{\psi}_f - \dot{\psi}_i) = 0 \quad (19)$$

- For one degree of freedom:

$$m_1(v_{f,1} - v_{i,1}) + m_2(v_{f,2} - v_{i,2}) = 0 \quad (20)$$



# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

- For plastic impacts:

$$J\dot{p}_f = J_O\dot{\psi}_f \quad \Rightarrow \quad \dot{\psi}_f = J_O^+ J \dot{p}_f \quad (21)$$

- Substituting (21) in (19):

$$\dot{p}_f = G^{-1}H \quad (22)$$

where:

$$\begin{aligned} G &= M + J^T(J_O^+)^T M_O J_O^+ J \\ H &= M \dot{p}_i + J^T(J_O^+)^T M_O \dot{\psi}_i \end{aligned} \quad (23)$$



## RIGID BODIES

### 5.0 IMPACT ANALYSIS:

- New equation after impact:

$$\dot{\psi}_f = J_O^+ J \dot{p}_f \quad \Rightarrow \quad \ddot{\psi} = J_O^+ J \ddot{p} + \frac{\partial J_O^+}{\partial t} J \dot{p} + J_O^+ \frac{\partial J}{\partial t} \dot{p} \quad (24)$$

$$\begin{cases} M \ddot{p} + C = u + J^T f_I \\ M_O \ddot{\psi} + C_O = -J_O^T f_I \\ M' \ddot{p} + C' = u \end{cases} \quad (25)$$

where:

$$\begin{aligned} M' &= M + J^T (J_O^T)^+ M_O J_O^+ J \\ C' &= C + J^T (J_O^T)^+ M_O \frac{\partial J_O^+}{\partial t} J \dot{p} + J^T (J_O^T)^+ M_O J^+ \frac{\partial J}{\partial t} \dot{p} + J^T (J_O^T)^+ C_O \end{aligned} \quad (26)$$



# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

Table 3: VMS parameters.

$l_1$	$l_2$	$m_b$	$m_1$	$m_2$	$m_O$	$r$	$\gamma$
5.59 m	5.59 m	419 725 kg	300 kg	300 kg	3000 kg	0.12 m	0.5 rad

Table 4: Simulation's initial positions.

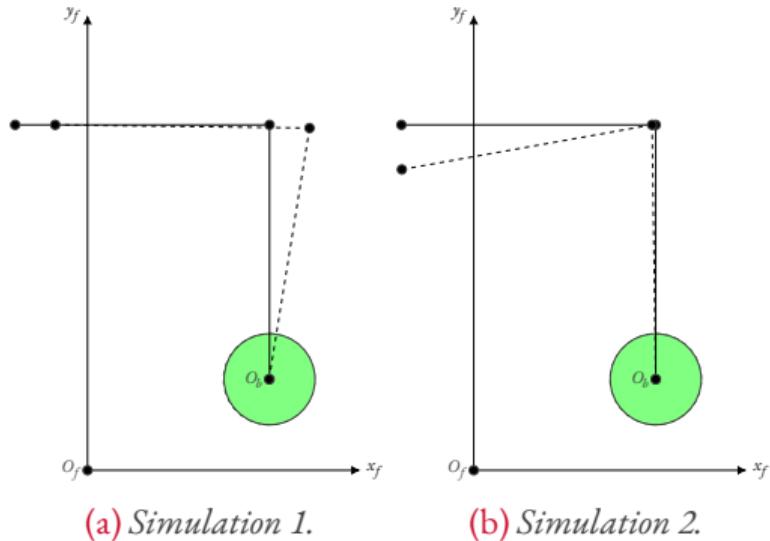
$x_b$	$y_b$	$\theta_0$	$q_1$	$q_2$	$x_O$	$y_O$	$\theta_O$
4 m	2 m	$\pi/2$ rad	0 rad	$\pi/2$ rad	-1.71 m	7.59 m	5.78 rad

# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

**Table 5:** Simulation's initial velocities.

	Simulation 1	Simulation 2
$\dot{\theta}_0$	0 rad s <sup>-1</sup>	0 rad s <sup>-1</sup>
$\dot{x}_b$	0 m s <sup>-1</sup>	0 m s <sup>-1</sup>
$\dot{y}_b$	0 m s <sup>-1</sup>	0 m s <sup>-1</sup>
$\dot{q}_1$	0 rad s <sup>-1</sup>	0 rad s <sup>-1</sup>
$\dot{q}_2$	0 rad s <sup>-1</sup>	0 rad s <sup>-1</sup>
$\dot{x}_O$	1 m s <sup>-1</sup>	0 m s <sup>-1</sup>
$\dot{y}_O$	0 m s <sup>-1</sup>	-1 m s <sup>-1</sup>
$\dot{\theta}_O$	0.01 rad s <sup>-1</sup>	0.01 rad s <sup>-1</sup>



**Figure 10:** Initial and final (dashed) position of the VMS after the impact for the two different simulations, ten seconds long.



# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

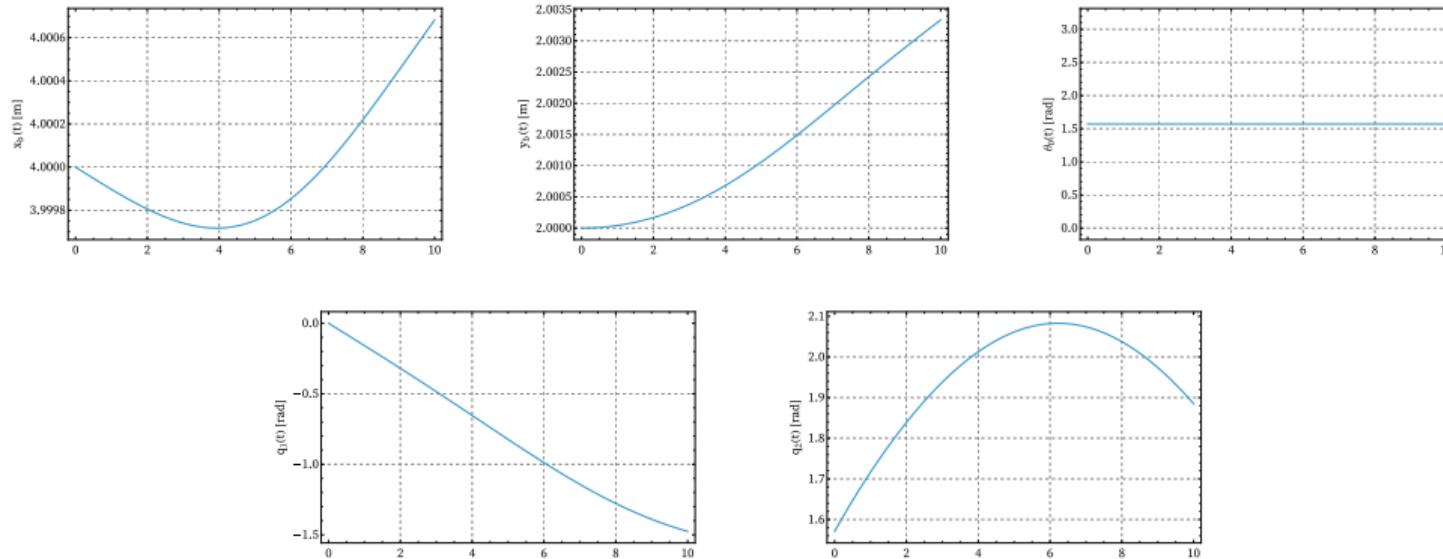


Figure 11: VMS generalized coordinates' displacement after the catching of the payload when no control is performed, Simulation 1.



# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

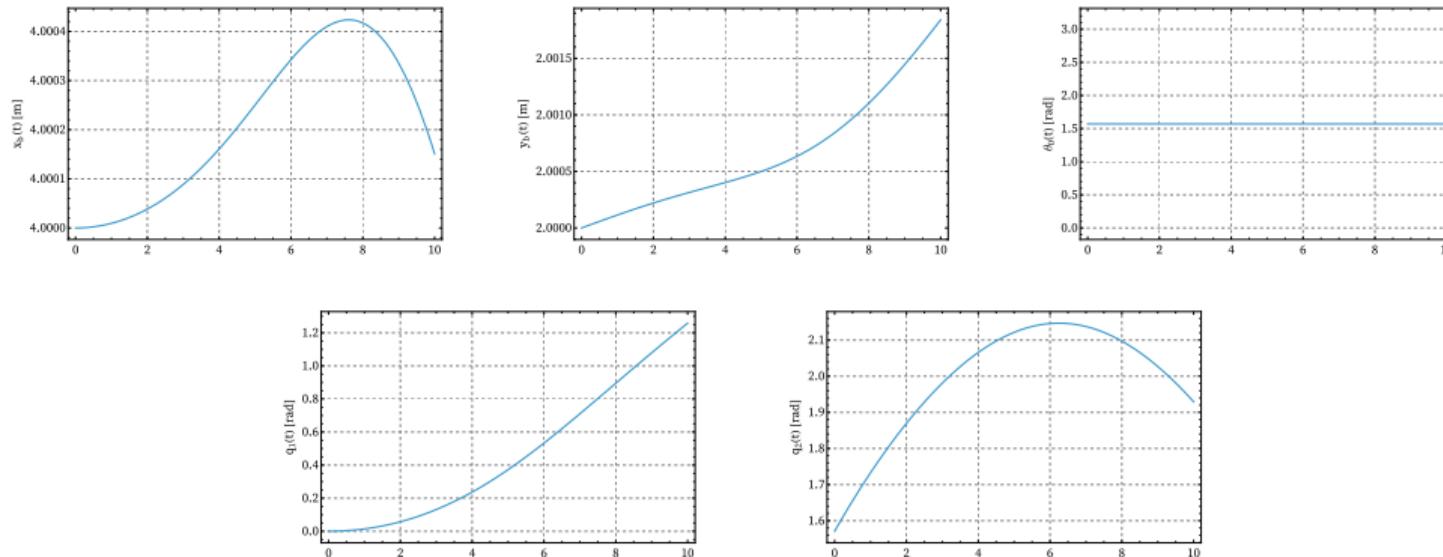


Figure 12: VMS generalized coordinates' displacement after the catching of the payload when no control is performed, Simulation 2.



## RIGID BODIES

### 5.0 IMPACT ANALYSIS:

- Controlled motion:

$$\begin{bmatrix} M'_{tt} & M'_{tr} \\ M'_{rt} & M'_{rr} \end{bmatrix} \begin{bmatrix} \ddot{p}_t \\ \ddot{p}_r \end{bmatrix} + \begin{bmatrix} C'_t \\ C'_r \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (27)$$

- Translational coordinates as a function of rotational ones:

$$\ddot{p}_t = -M'^{-1}_{tt} (M'_{tr} \ddot{p}_r + C'_t) \quad (28)$$

- Final equation:

$$\ddot{p}_r \tilde{M} + \tilde{C} = u \quad (29)$$

- Feedback linearization:

$$u = \hat{M}[\ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q)] + \hat{C} \quad (30)$$



# RIGID BODIES

## 5.0 IMPACT ANALYSIS:

- If  $\hat{M} \approx \tilde{M}$  and  $\hat{C} \approx \tilde{C}$ :

$$\ddot{e} + K_d \dot{e} + K_p e = 0 \quad (31)$$

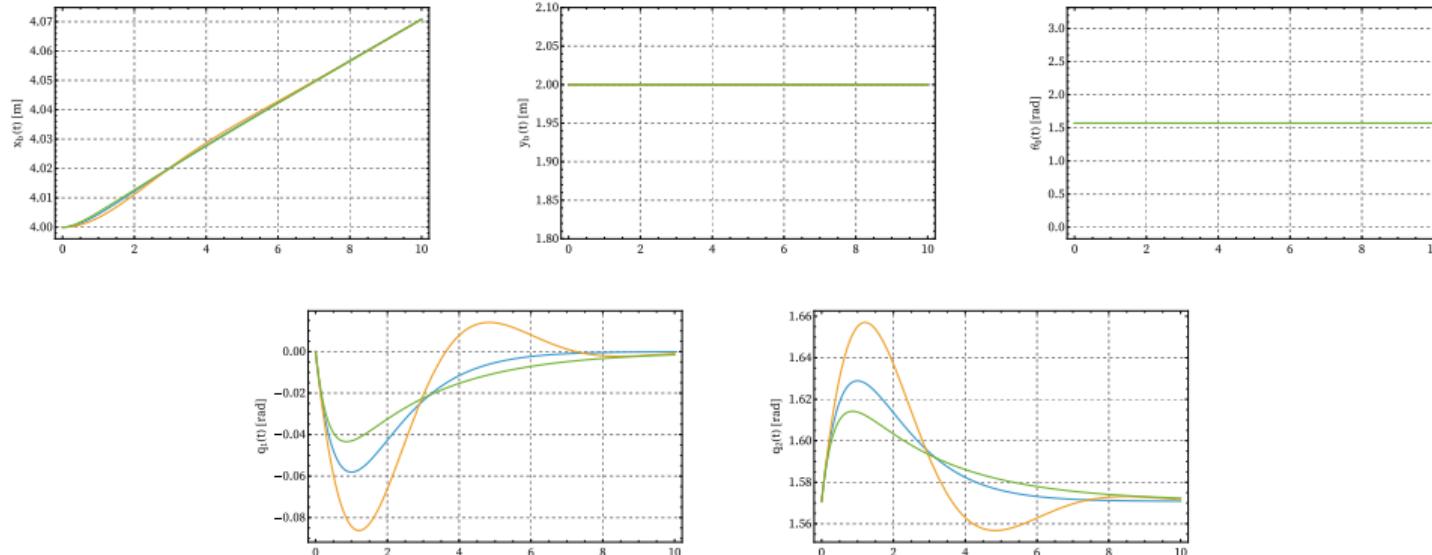
- decoupled;
- linear;
- $\omega_n = \sqrt{K_{p_i}}$  and  $\xi = \frac{K_{d_i}}{2\sqrt{K_{p_i}}}$ .

- Critically damped when

$$\xi = 1 \quad \Rightarrow \quad K_d = 2\sqrt{K_p} \quad (32)$$

# RIGID BODIES

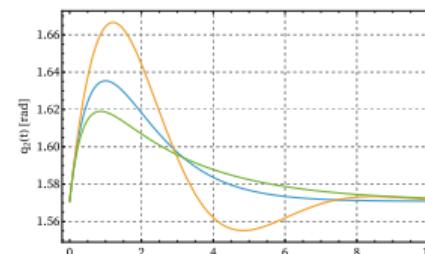
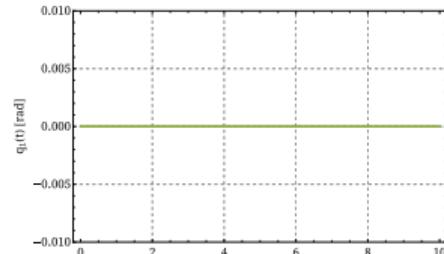
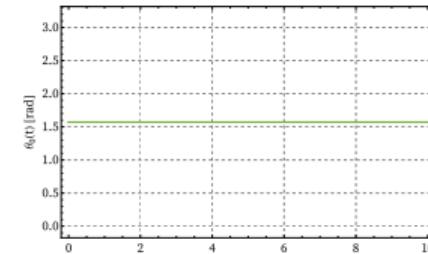
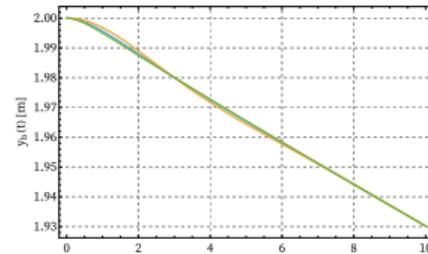
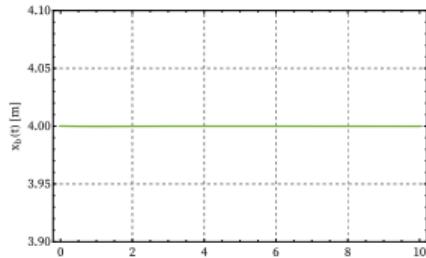
## 5.0 IMPACT ANALYSIS:



**Figure 13:** VMS generalized coordinates' displacement after the catching of the payload when control is performed, Simulation 1. In yellow, an underdamped behaviour ( $\xi = 0.5$ ), in blue a critically damped behaviour ( $\xi = 1$ ), in green an overdamped behaviour ( $\xi = 1.5$ ).

# RIGID BODIES

## 5.0 IMPACT ANALYSIS:



**Figure 14:** VMS generalized coordinates' displacement after the catching of the payload when control is performed, Simulation 2. In yellow, an underdamped behaviour ( $\xi = 0.5$ ), in blue a critically damped behaviour ( $\xi = 1$ ), in green an overdamped behaviour ( $\xi = 1.5$ ).



# ELASTIC BODIES

## 5.0 IMPACT ANALYSIS:

- Arms modeled ad Euler-Bernoulli Beams:

$$\begin{aligned} w(x, t) &= \sum_{n=1}^{\infty} W_n(x) Q_n(t) \\ Q(t) &= A \cos \omega t + B \sin \omega t \\ W(x) &= c_1 \cos(\beta x) + c_2 \sin \beta x + c_3 \cosh \beta x + c_4 \sinh \beta x \end{aligned} \tag{33}$$

- Elasticity Matrix [8]:

$${}_{i+1}^i E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & W_i(x) Q_i(t) \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{34}$$

- The number of coordinates increases:  $p = \{x_b, y_b, \theta_b, q_1, q_2, Q_1, Q_2\}$ .



## ELASTIC BODIES

### 5.0 IMPACT ANALYSIS:

- Elastic potential energy:

$$\begin{aligned} U &= \frac{1}{2} \int_0^{l_i} ES \left( \frac{\partial^2 w(x, t)}{\partial x^2} \right)^2 dx \\ &= \frac{1}{2} \int_0^{l_i} ES \left( \frac{\partial^2 W(x)}{\partial x^2} Q(t) \right)^2 dx \end{aligned} \tag{35}$$

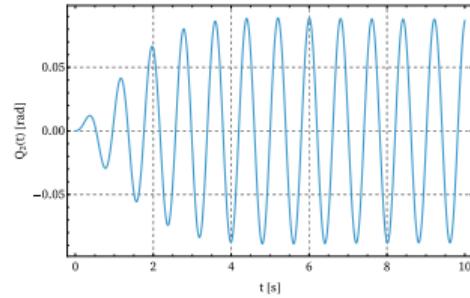
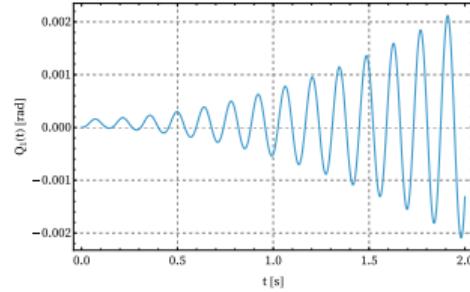
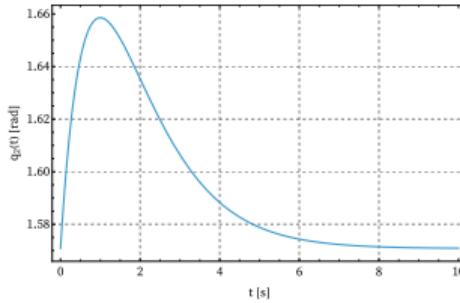
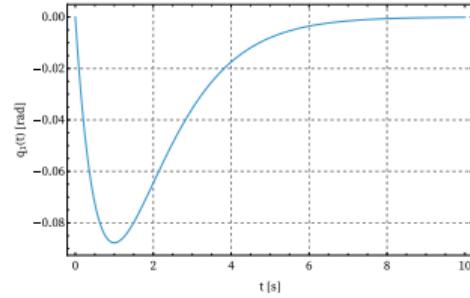
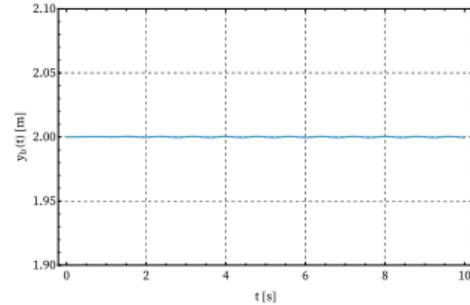
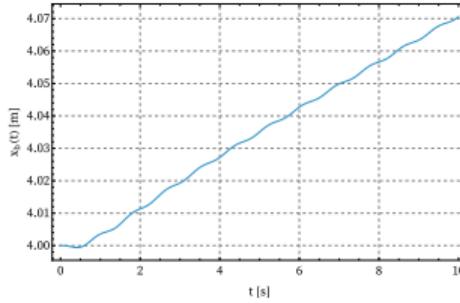
- Included in the stiffness matrix:

$$\begin{aligned} \frac{d}{dt} \frac{\partial U}{\partial \dot{Q}_i} - \frac{\partial U}{\partial Q_i} &= 0 + \int_0^{l_i} ES \frac{\partial^2 W(x)}{\partial x^2} dx \\ M \ddot{p} + C + K p &= u + J^T f_I \\ K_i &= \int_0^{l_i} E_i S_i \frac{\partial^2 W_i(x)}{\partial x^2} dx \end{aligned} \tag{36}$$



# ELASTIC BODIES

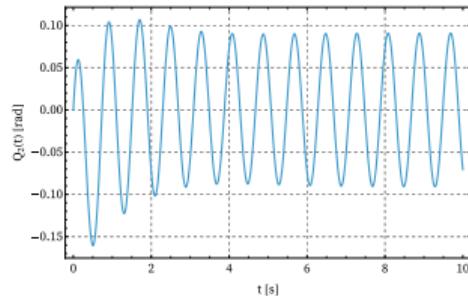
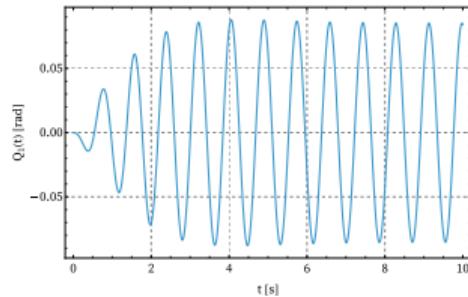
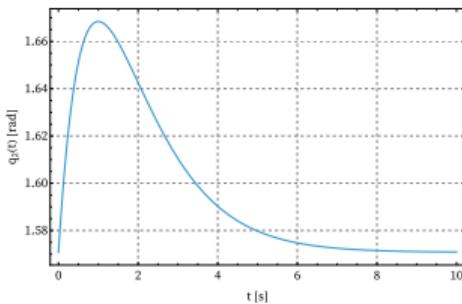
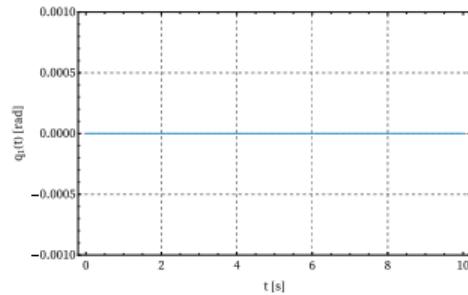
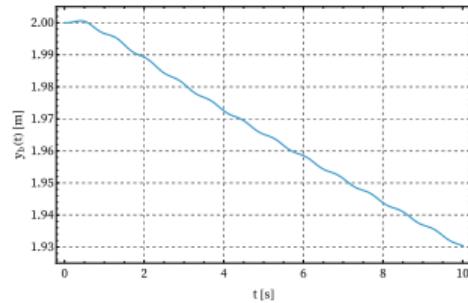
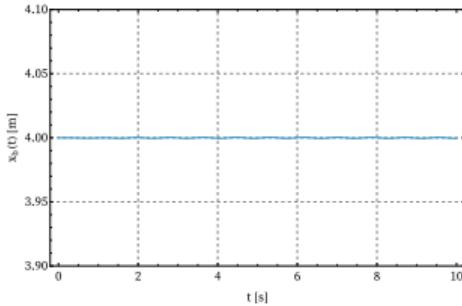
## 5.0 IMPACT ANALYSIS:



**Figure 15:** VMS generalized coordinates' displacement after the catching of the payload when control is performed, Simulation 1. Arms modeled as Euler-Bernoulli beams.

# ELASTIC BODIES

## 5.0 IMPACT ANALYSIS:



**Figure 16:** VMS generalized coordinates' displacement after the catching of the payload when control is performed, Simulation 2. Arms modeled as Euler-Bernoulli beams.



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## 6.0 MASS AND VELOCITY RETRIEVAL:

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- ▶ Results
- ▶ Q&A



## LINEARIZATION

### 6.0 MASS AND VELOCITY RETRIEVAL:

- Approximations:
  - No spacecraft translation;
  - No spacecraft rotation;
  - Linearization around equilibrium position.
- First-order Taylor expansion:

$$D(\mathbf{x}) = D(q_1, q_2, \dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2) = M'(q_1, q_2) \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + C'(q_1, q_2, \dot{q}_1, \dot{q}_2) \quad (37)$$

$$D(\mathbf{x}) \approx D(\bar{\mathbf{x}}) + (\mathbf{x} - \bar{\mathbf{x}})^T \nabla T(\mathbf{x}) \quad (38)$$

- New linearized dynamics:

$$M_{lin} \ddot{q} + C_{lin} \dot{q} + K_{lin} q - \delta = 0 \quad (39)$$



## JOINTS' DECOUPLING

### 6.0 MASS AND VELOCITY RETRIEVAL:

- Displacement of one arm negligible for the displacement of the other one (true when  $\hat{M} = M'$ ).
- New equations:

$$\begin{aligned}\ddot{q}_1 + \frac{\hat{m}_1}{\tilde{m}_1} k_d \dot{e}_1 + \frac{\hat{m}_1}{\tilde{m}_1} k_p e_1 &= 0 \\ \ddot{q}_2 + \frac{\hat{m}_2}{\tilde{m}_2} k_d \dot{e}_2 + \frac{\hat{m}_2}{\tilde{m}_2} k_p e_2 &= 0\end{aligned}\tag{40}$$

- Damping coefficient:

$$\xi' = \frac{\hat{m} k_v}{2\omega_n \tilde{m}} = \frac{\hat{m} 2\sqrt{k_p}}{2\sqrt{\frac{\hat{m}}{\tilde{m}}} k_p \tilde{m}} = \sqrt{\frac{\hat{m}}{\tilde{m}}}\tag{41}$$

# JOINTS' DECOUPLING

## 6.0 MASS AND VELOCITY RETRIEVAL:

- Position Roots: find overshoot, find parametrix solution of equation and solve:  
 $q(t^*, \tilde{m}) = q^*$ .
- Mass Fit: minimize error with non-linear regression, parameter is the mass. We can add noise:

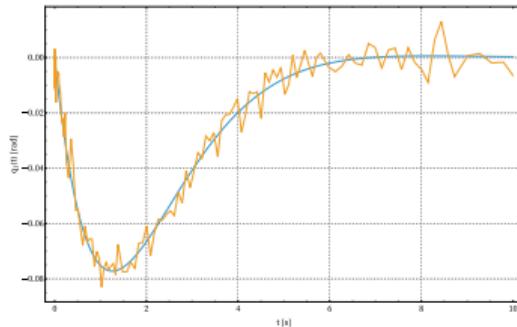


Figure 17: Noisy data to simulate a real scenario in orange.

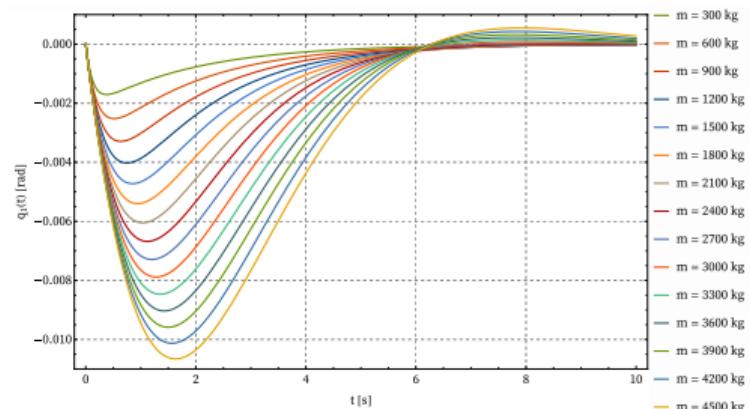


Figure 18: Parametric plot of the first joint's time evolution, first simulation. The real mass is 3000 kg.



# JOINTS' DECOUPLING

## 6.0 MASS AND VELOCITY RETRIEVAL:

- Derivative Roots:

$$\begin{aligned} \tilde{m}\ddot{q} + \hat{m}k_d\dot{q} + \hat{m}k_p(q - q_d) &= 0 \\ \tilde{m}\ddot{q} + \hat{m}k_d\dot{q} + \hat{m}k_pq &= \hat{m}k_pq_d \\ q(t) = q_d &\left[ 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \right] + \left[ q_0 \cos(\omega_d)t + \frac{\dot{q}_0 + \xi\omega_n q_0}{\omega_d} \sin(\omega_d t) \right] e^{-\xi\omega_n t} \quad (42) \\ \tan(\sqrt{1-\xi^2}\omega_n t^*) &= \beta(\xi) \end{aligned}$$

- Damping coefficient and natural frequency fit: non-linear regression of closed-form solution, parameters are  $\xi$  and  $\omega_n$ .

# COUPLED SOLUTION

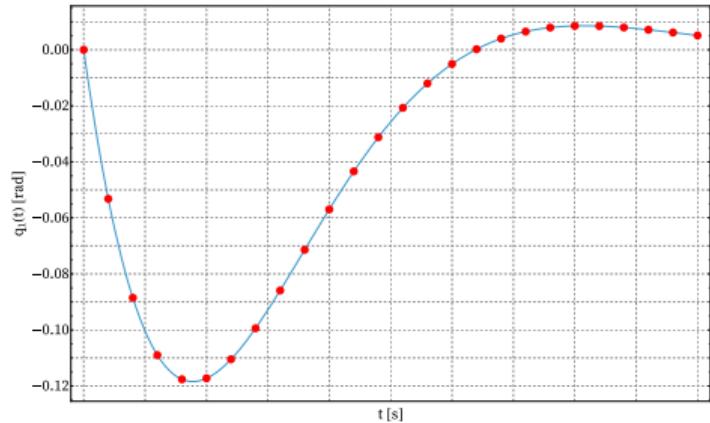
## 6.0 MASS AND VELOCITY RETRIEVAL:

- Continous Domain: solve coupled equations with Wolfram's ParametricNDSolve →  $q(m, t)$ :

$$\begin{aligned}\Gamma(m) &= \int_0^T \left[ q(m, t) - q_{data}(t) \right]^2 dt \\ m^* &= \min_m \Gamma(m)\end{aligned}\quad (43)$$

- Discrete Domain: equations solved with Maple, closed-form solution:

$$\Gamma(m) = \sum_{k=0}^{N_p} \left[ q(m, \frac{T}{N_p} k) - q_{data}(\frac{T}{N_p} k) \right]^2 \quad (44)$$



**Figure 19:** In blue is the continous solution, in red is the discretized one.



## VELOCITY EXTRACTION

### 6.0 MASS AND VELOCITY RETRIEVAL:

- From final velocities relation:

$$\dot{p}_f = G^{-1}H \quad (45)$$

we can write:

$$\begin{aligned} H &= G\dot{p}_f \\ &\Rightarrow M\dot{p}_i + J^T(J_O^+)^T M_O \dot{\psi}_i = G\dot{p}_f \\ &\Rightarrow \dot{\psi}_i = M_O^{-1} J_O^T (J^+)^T (G\dot{p}_f - M\dot{p}_i) \end{aligned} \quad (46)$$

- Good results for translational velocities, big numerical errors for  $\dot{\theta}_O$ .
- Previous assumption: docking ( $\dot{\theta}_O \approx 0$ ).



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# MASS EXTRACTION

## 7.0 RESULTS:

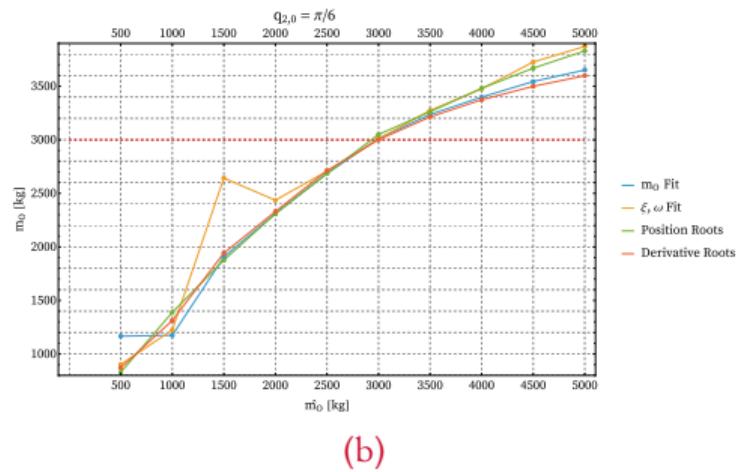
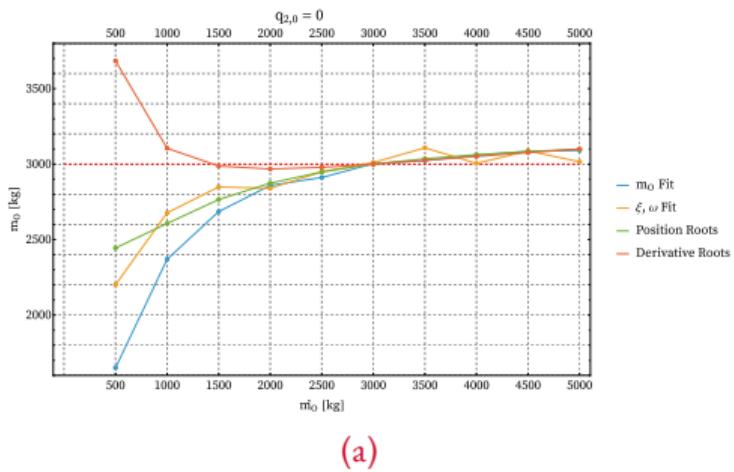
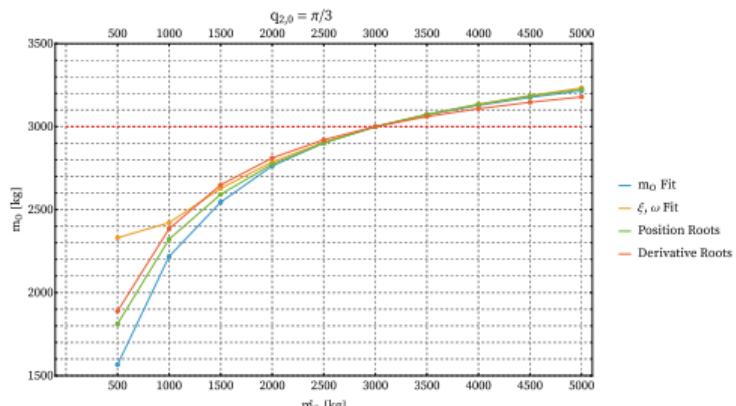


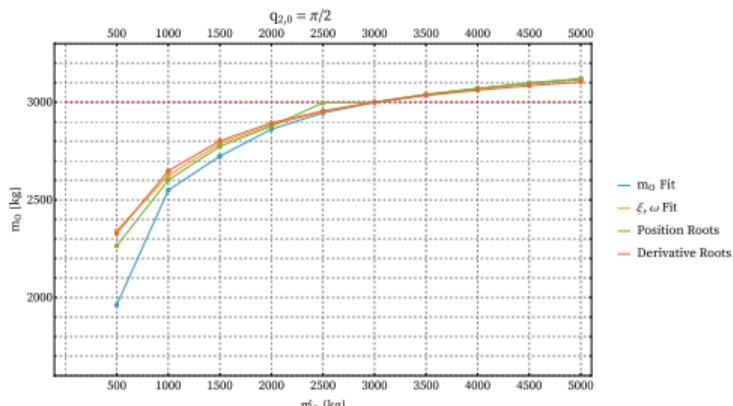
Figure 20: Joints' Decoupling.

# MASS EXTRACTION

## 7.0 RESULTS:



(a)

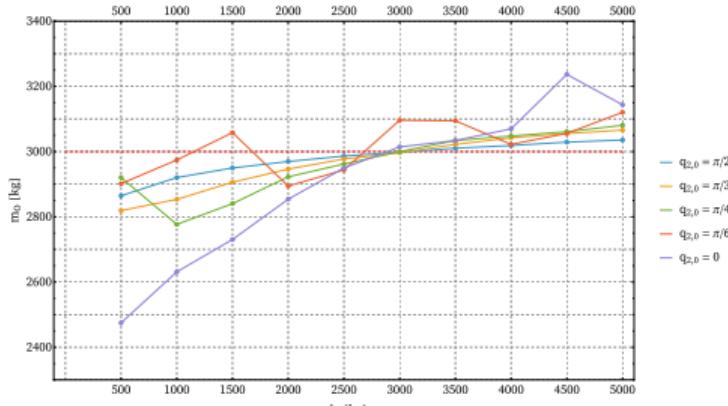


(b)

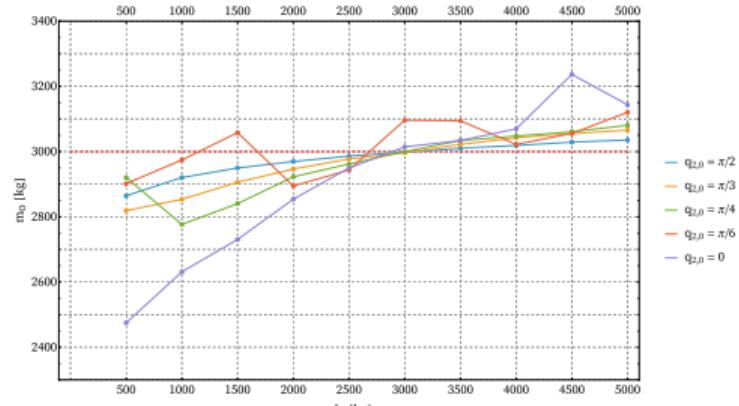
Figure 21: Joints' Decoupling.

# MASS EXTRACTION

## 7.0 RESULTS:



(a)

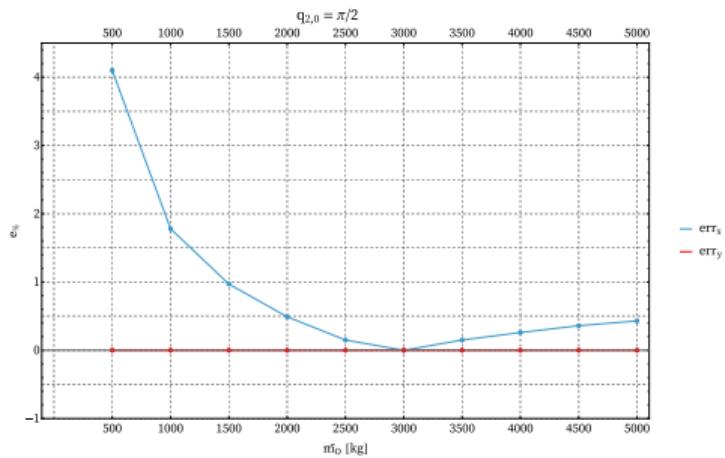


(b)

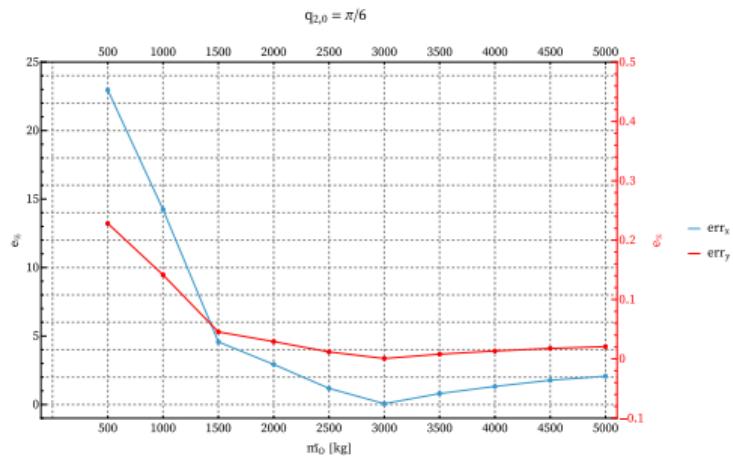
Figure 22: Coupled Solution.

# VELOCITY EXTRACTION

## 7.0 RESULTS:



(a)

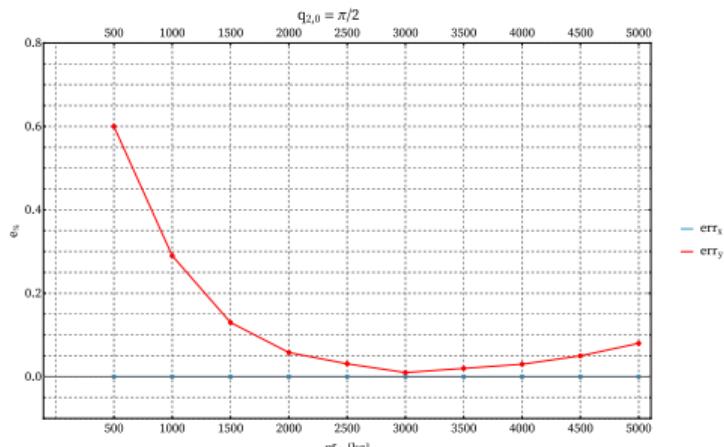


(b)

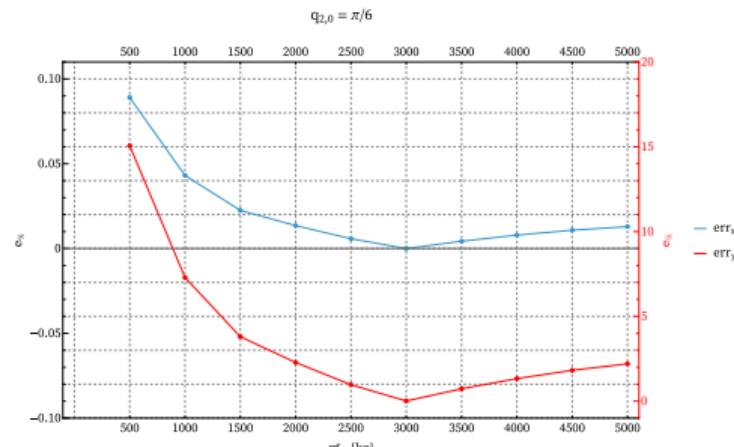
Figure 23: Velocity error for Simulation 1.

# VELOCITY EXTRACTION

## 7.0 RESULTS:



(a)



(b)

Figure 24: Velocity error for Simulation 2.



# CONCLUSION

## 7.0 RESULTS:

- Joints' Decoupling:
  - Pro: simple closed-form equation
  - Cons: depend significantly on VMS configuration.
  - $\xi$  and  $\omega_n$  fit best one.
  - When  $\hat{m} > m$  better results: displacement is smaller, linearization error reduced.
- Coupled Solution:
  - Pro: less susceptible to VMS configuration.
  - Cons: require more computational power (continuous method).
  - Discrete method is faster ( $\approx 0.03$  s vs 1.5 s)



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Q & A  
*Thank you for listening!*



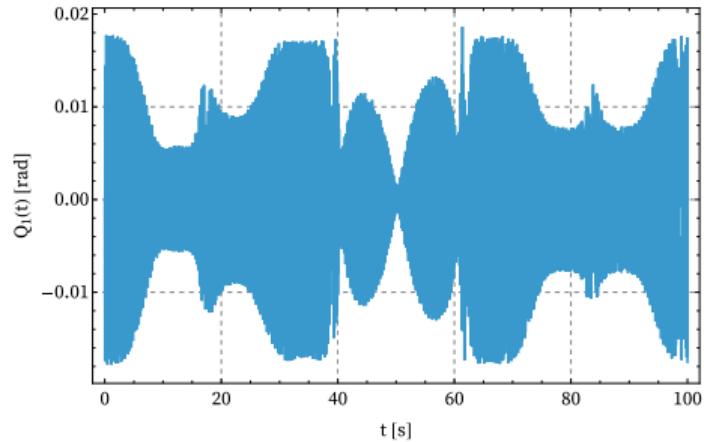
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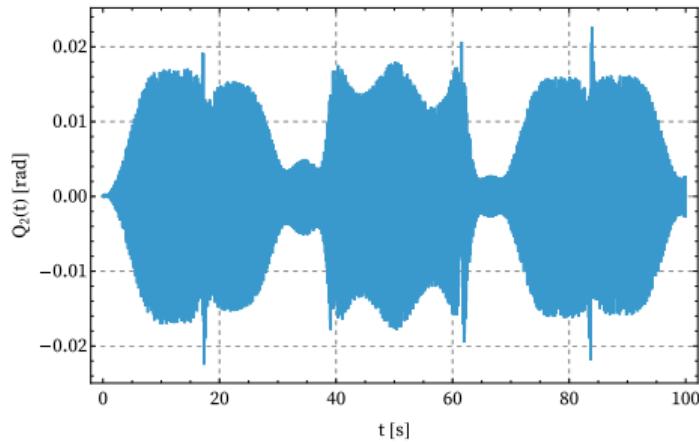
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# ELASTIC VIBRATIONS

## 9.o Q&A:



(a) First joint.

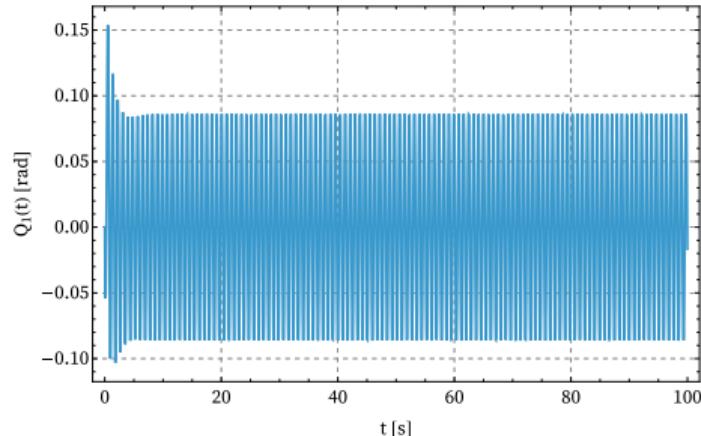


(b) Second joint.

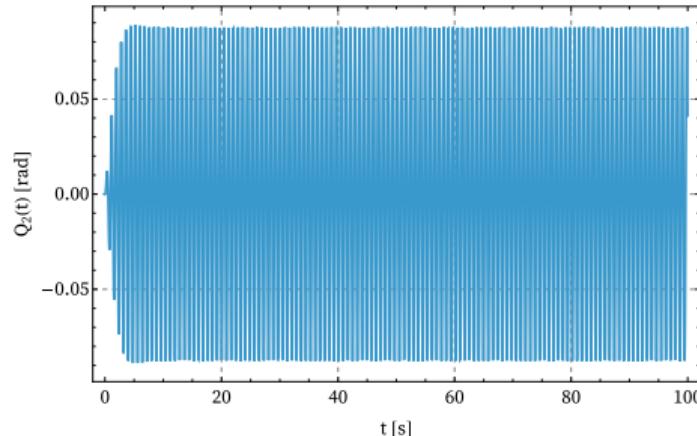
Figure 25: Extended temporal window of joints' elastic coordinates for Simulation 1. The oscillations change in time due to the movement of the arms.

# ELASTIC VIBRATIONS

## 9.o Q&A:



(a) First joint.



(b) Second joint.

**Figure 26:** Extended temporal window of joints' elastic coordinates for Simulation 1. The oscillations are constant since the arms do not move after the impact.