NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN SALZBURG 2025

Workshop on Nonlinear Partial Differential Equations at the Paris-Lodron University
Salzburg, November 05 – 07 2025

Book of Abstracts

Timetable

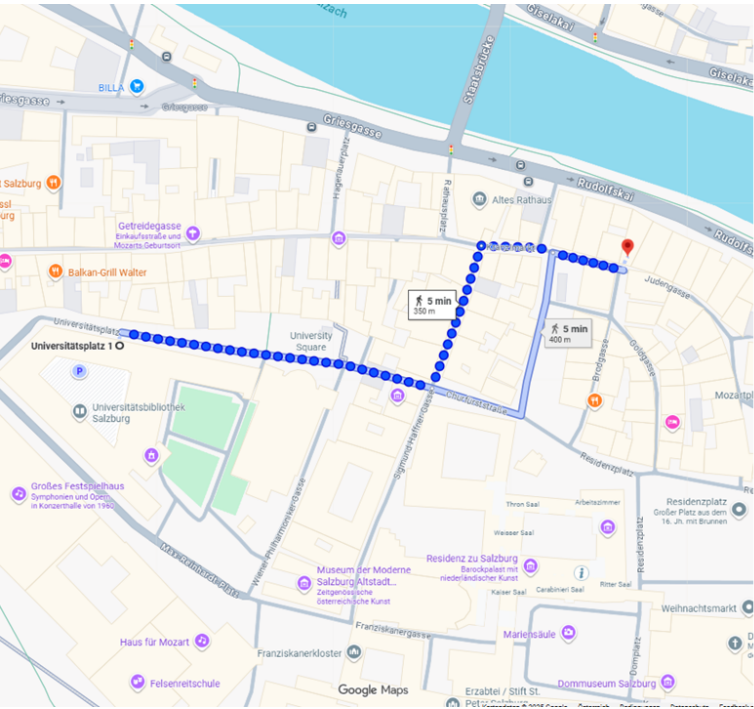## Wednesday, November 5th

9:50 –10:00	Opening	
10:00 –10:45	**Nicola Fusco** Università di Napoli, Italy	Consistency for the surface diffusion flow in three dimensions
10:50 –11:20	Coffee	
11:20 –12:20	**Contributed Talks**	Pedra Andrade, Julian Blawid, Michał Borowski, Filippo Cassanello
12:25 –14:00	Lunch	
14:00 –14:45	**Raffaella Servadei** Università degli Studi di Urbino Carlo Bo, Italy	Nonlocal fractional problems with lack of compactness
14:50 –15:30	Coffee	
15:20 –16:30	**Contributed Talks**	Filomena De Filippis, Theo Elenius, Guillermo García-Sáez, Max Lipton
16:35 – 17:20	**Giovanni Molica Bisci** Università San Raffaele Roma, Italy	Variational Methods in Finsler Geometry

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Thursday, November 6th

9:00 – 9:45	**Manuel Friedrich** University of Linz, Austria	Regularity for minimizers of the Griffith energy
9:50 –10:20	Coffee	
10:20 –11:20	**Contributed Talks**	Marta Macrì, Leah Schätzler, Kristian Moring, Cintia Pacchiano
11:25 –12:10	**Carolin Kreisbeck** Katholische Universität Eichstätt-Ingolstadt, Germany	Nonlocal gradients in variational problems: Heterogeneous horizons and local boundary conditions
12:15 – 14:00	Lunch	
14:00 –14:45	**Ulisse Stefanelli** University of Vienna, Austria	A free boundary problem in accretive growth
14:50 –15:40	Photo and Coffee	
15:40 – 16:55	**Contributed Talks**	Fabian Rupp, Alessandro Scagliotti, Manuel Schlierf, Lovelesh Sharma, Yawen Feng
18:30	Conference Dinner	

Friday, November 7th

09:00–09:45	**Ljangjun Weng** Scuola Normale Superiore, Italy	The Gauss curvature flow and its variation
09:50 –10:20	**Coffee**	
10:20 – 11:20	**Contributed Talks**	Lubomira Softova, Eduard Stefanescu, Michael Strunk, Andreas Vikelis
11:25 – 12:10	**Antonio Iannizotto** Università degli Studi di Cagliari, Italy	Fine boundary regularity for the fractional p-Laplacian

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The conference dinner takes place at the restaurant

„Goldene Kugel"

Address:
Judengasse 3, 5020 Salzburg

Our reservation for the department of mathematics is set for **6.30 pm** in the „Bürgerstube"

Invited Talks

Variational Methods in Finsler Geometry

Giovanni Molica Bisci

Università San Raffaele Roma, Italy

The theory of Sobolev spaces on complete Riemannian manifolds is well-established and has been extensively employed in the analysis of various elliptic problems. Although Finsler geometry constitutes a natural generalization of Riemannian geometry, the corresponding theory of Sobolev spaces on non-compact Finsler manifolds remains largely undeveloped. Motivated by the growing interest in this area within the mathematical literature, the primary aim of this talk is to present some recent results concerning non-compact Randers spaces and their applications to quasilinear elliptic equations. The approach is principally based on new abstract Sobolev embedding theorems, along with variational and topological methods developed in the recent monograph *Nonlinear Problems with Lack of Compactness*, De Gruyter Series in Nonlinear Analysis and Applications, Volume 36 (2021), co-authored with P. Pucci.

Regularity for minimizers of the Griffith energy

Manuel Friedrich

University of Linz, Austria

We discuss regularity for the crack set of a minimizer for the Griffith fracture energy, arising in the variational modeling of brittle materials. In the planar setting, we prove an epsilon-regularity result showing that the crack is locally a $C^{1,1/2}$ curve outside of a singular set of zero Hausdorff measure. The main novelty is that, in contrast to previous results, no topological constraints on the crack are required. Joint work with Camille Labourie and Kerrek Stinson.

Consistency for the surface diffusion flow in three dimensions

Nicola Fusco

Università di Napoli, Italy

We discuss the existence the flat flow solutions for the surface diffusion equation via the discrete minimizing movements scheme proposed by Cahn and Taylor in 1994. We prove that in 3D the scheme converges to the unique smooth solution of the equation, provided that the initial set is sufficiently regular.

Fine boundary regularity for the fractional p-Laplacian

Antonio Iannizzotto

Università degli Studi di Cagliari, Italy

We consider the following nonlinear, nonlocal elliptic equation with Dirichlet-type conditions:

$$\begin{cases} (-\Delta)_p^s u = f(x) & \text{in } \Omega \\ u = 0 & \text{in } \Omega^c, \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p > 1$, $s \in (0,1)$, and the leading operator is the fractional p-Laplacian defined by

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \to 0^+} \int_{B_\varepsilon^c(x)} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))}{|x - y|^{N+ps}} \, dy.$$

The reaction is a function $f \in L^q(\Omega)$ for some $q \in (N/s, \infty]$. We focus on a fine boundary regularity property of weak solutions of (0.1), namely, $u/d^s \in C^\alpha(\overline{\Omega})$ for some $\alpha \in (0,1)$ depending on the data, where $d(\cdot)$ denotes the distance from Ω^c. Such qualitative property comes with a quantitative counterpart, i.e., there exists $C > 0$ depending on the data s.t.

$$\left\| \frac{u}{d^s} \right\|_{C^\alpha(\overline{\Omega})} \le C \|f\|_{L^q(\Omega)}^{\frac{1}{p-1}}.$$

We will first prove the result above for $q = \infty$, then we will tackle the case $q < \infty$ by means of a nonlocal perturbative method. The motivation for our study lies in several applications from nonlinear analysis, such as Hopf's lemma, Sobolev vs. Hölder minima, and the study of problems with singular reactions.

Work in collaboration with S. Mosconi, M. Squassina [1,2,3].

References

[1] A. Iannizzotto, S. Mosconi, Fine boundary regularity for the singular fractional p-Laplacian, *J. Differential Equations* **412** (2024) 322–379.

[2] A. Iannizzotto, S. Mosconi, Fine boundary regularity for the fractional p-Laplacian with unbounded reactions (in preparation).

[3] A. Iannizzotto, S. Mosconi, M. Squassina, Fine boundary regularity for the degenerate fractional p-Laplacian, *J. Funct. Anal.* **279** (2020) art. 108659.

Nonlocal gradients in variational problems: Heterogeneous horizons and local boundary conditions

Carolin Kreisbeck

Katholische Universität Eichstätt-Ingolstadt, Germany

Building on recent advances in nonlocal hyperelasticity, we discuss a class of variational problems involving integral functionals with nonlocal gradients. Specific to our set-up is a space-dependent interaction range that vanishes at the boundary of the reference domain. This ensures that the operator depends only on values within the domain and localizes to the classical gradient at the boundary, which allows for a seamless integration of nonlocal modeling with local boundary values. Our main contribution is a comprehensive study of the associated Sobolev spaces, including the analysis of a trace operator and the proof of a Poincaré inequality. A central aspect of our technical approach lies in exploiting connections with pseudo-differential operator theory. As an application, we establish the existence of minimizers for functionals with quasiconvex or polyconvex integrands depending on heterogeneous nonlocal gradients, subject to local Dirichlet-, Neumann- or mixed-type boundary conditions. This is joint work with Hidde Schönberger (UCLouvain).

On the problem of uniqueness in the Calculus of Variations

Jan Kristensen

Oxford University, UK

It is well-known that even under favourable conditions that ensure both existence and (partial) regularity of minimizers their uniqueness is not guaranteed. In this talk we show how exactly uniqueness is connected to convexity of the variational integral. We also give some results showing how uniqueness (and regularity) of minimizers can be ensured using smallness conditions on the data. It is important to emphasize that these smallness conditions are too weak to allow for a direct application of any known Implicit Function Theorem.

The talk is based on joint work with Judith Campos Cordero (Mexico City), Bernd Kirchheim (Leipzig) and Jan Kolar (Prague).

Nonlocal fractional problems with lack of compactness

Raffaella Servadei

Università degli Studi di Urbino Carlo Bo, Italy

Several important problems arising in many research fields, such as physics and differential geometry, lead to consider elliptic equations when a lack of compactness occurs. From the mathematical point of view, the main interest relies on the fact that often the tools of nonlinear functional analysis, based on compactness arguments, cannot be used, at least in a straightforward way, and some new techniques have to be developed.

Aim of the talk is to present some of these techniques, together with their applications to nonlocal and nonlinear PDEs driven by the fractional Laplacian. The results obtained generalize to the nonlocal setting what is known in the classical setting of elliptic PDEs.

A free boundary problem in accretive growth

Ulisse Stefanelli

University of Vienna, Austria

Accretive growth, in which material is added at the boundary of a system, is a central phenomenon in biology, natural processes, and engineering applications. Mathematically, it can be described by a stationary Hamilton-Jacobi equation governing the moving boundary, coupled with a PDE for an activation field (such as nutrients, temperature, or stress) defined on the evolving domain. This leads to a free boundary problem with a highly nonlinear, coupled structure.

In this talk, I will present an existence analysis of such a problem. I will begin with the growth subproblem, establishing regularity of the sublevel sets of the solution. In particular, I will show that the growing domains admit a uniform Poincaré inequality, which provides the necessary framework to prove an existence result for the fully coupled free boundary system.

The Gauss curvature flow and its variation

Liangjun Weng

Scuola Normale Superiore, Italy

The Gauss curvature flow is a fully nonlinear geometric evolution equation in which a strictly convex, closed hypersurface evolves with normal velocity equal to its Gauss curvature. Originating from Firey's work in the 1970s, it was introduced as an idealized model for the wearing process of a convex stone on a beach and has since developed deep connections with PDE and geometry. In this talk, we will discuss the classical and recent progress on the Gauss curvature flow, highlighting contributions by Andrews, Guan–Ni, Brendle–Choi–Daskalopoulos, and many others. In particular, we will discuss the convergence of a convex body to a round point in finite time and emphasize the central roles played by Firey's entropy, the Blaschke–Santaló inequality, and non-collapsing estimates in the asymptotic analysis of the flow. If time permits, we will discuss natural variants of the Gauss curvature flow in the capillary setting, which form part of a joint program with Xinqun Mei (Peking) and Guofang Wang (Freiburg).

Contributed Talks

Sharp Regularity Estimates for Degenerate Fully Nonlinear Equations

Pedra Andrade

Paris Lodron Universität Salzburg, Austria

In this talk, I will discuss sharp Hölder regularity results for the gradient of solutions to a class of degenerate fully nonlinear elliptic equations. These equations lose ellipticity when the gradient vanishes and involve nonlinear first-order Hamiltonian terms. Such features create significant analytical challenges, particularly in regimes where the gradient is very small or very large. I will present a strategy that combines perturbation techniques with a maximum principle argument, which allows us to overcome these obstacles and obtain optimal interior regularity estimates.

Morphoelastic Growth at Large Strains

Julian Blawid

University of Regensburg, Germany

We discuss a model for morphoelastic growth of second gradient viscoelastic materials. Morphoelastic growth refers to the growth of tissue under the influence of elastic stress. Our model features a multiplicative decomposition of the deformation gradient into a part related to growth and an elastic contribution. While the volumetric change due to growth of the tissue is determined by an ordinary differential equation on a Banach space, the total deformation solves a quasistatic momentum balance equation. The growth process is driven by the absorption of nutrients, the concentration of nutrients is modeled by the solution of a parabolic reaction-diffusion equation. To prove existence of solutions we use a minimizing movement scheme for the momentum balance equation and exploit the regularity theory of De Giorgi, Nash and Moser for the nutrient equation. Moreover, we discuss matters concerning injectivity and self-contact.

Smoothness of weight sharply discards Lavrentiev's gap for double phase functionals

Michał Borowski

University of Warsaw, Poland

I will discuss how increasing the smoothness of the weight in double phase functionals affects the range of exponents admissible for discarding Lavrentiev's gap. In detail, I consider functionals of the type

$$u \mapsto \int_\Omega \left(|\nabla u(x)|^p + a(x)|\nabla u(x)|^q\right) dx, \quad 1 \le p < q, a(\cdot) \ge 0.$$

It is well-known that without additional assumptions imposed on a and p, q, it may happen that minimizing the functional over Lipschitz functions results in a strictly greater value than over Sobolev functions. This produces Lavrentiev's phenomenon, and further, the lack of regularity of minimizers. To overcome Lavrentiev's gap, I consider $a \in C^{k,\alpha}$, and show that it generates the sharp admissible range of exponents to be $q \le p + (k+\alpha)\max(1, p/N)$. In particular, this result generalizes prior results considering Hölder continuous a, and is applicable for every given exponents p and q. In the talk, I will further elaborate on the double phase functionals, the result, and its proof.

The talk is based on the following preprint:

M. Borowski, Smoothness of weight sharply discards Lavrentiev's gap for double phase functionals, arXiv:2509.06567. (2025).

Integral Harnack estimates and the rate of extinction of singular fractional diffusion

Filippo Cassanello

University of Cagliari, Italy

In this talk we discuss several integral Harnack-type inequalities for local weak solutions of parabolic equations with measurable and bounded coefficients, describing singular s-fractional p-Laplacian diffusion. Then we apply the aforementioned estimates to evaluate the decay rate of the local mass and supremum of the solutions as they approach a possible extinction time. Yet we show consistency of our general decay estimates by studying the extinction phenomenon for weak solutions of the Cauchy-Dirichlet problem, by means of an approximation procedure that carefully avoids the use of an integrable time derivative.

μ-ellipticity and nonautonomous integrals

Filomena De Filippis

Università degli studi di Parma, Italy

μ-ellipticity describes certain degenerate forms of ellipticity typical of convex integrals at linear or nearly linear growth, such as the area integral or the iterated logarithmic model. The regularity of solutions to autonomous or totally differentiable problems is classical after Bombieri, De Giorgi and Miranda, Ladyzhenskaya and Ural'tseva and Frehse and Seregin. The anisotropic case is a later achievement of Bildhauer, Fuchs and Mingione, Beck and Schmidt and Gmeineder and Kristensen. However, all the approaches developed so far break down in presence of nondifferentiable ingredients. In particular, Schauder theory for certain significant anisotropic, nonautonomous functionals with Hölder continuous coefficients was only recently obtained by C. De Filippis and Mingione. We will see the validity of Schauder theory for anisotropic problems whose growth is arbitrarily close to linear within the maximal nonuniformity range, and discuss sharp results and insights on more general nonautonomous area type integrals. From recent, joint work with Cristiana De Filippis (Parma) and Mirco Piccinini (Pisa).

Characterizations and properties of solutions to parabolic problems of linear growth

Theo Elenius

Aalto University, Finland

We consider notions of weak solutions to a general class of parabolic problems of linear growth, formulated independently of time regularity. Equivalence with variational solutions is established using a stability result for weak solutions. A key tool in our arguments is approximation of parabolic BV functions using time mollification and Sobolev approximations. We also prove a comparison principle and a local boundedness result for solutions. When the time derivative of the solution is in L^2 our definitions are equivalent with the definition based on the Anzellotti pairing.

Second-order Sobolev regularity for a class of generalized nonlinear p-parabolic equations

Yawen Feng

University of Jyväskylä, Finland

In this talk, we consider a general form of a degenerate or singular parabolic equation

$$u_t - |Du|^\gamma(\Delta u + (p-2)\Delta_\infty^N u) = 0$$

with $-1 < \gamma < \infty$ and $1 < p < \infty$, which includes the standard p-parabolic equation and the normalized (or game theoretic) p-parabolic equation. We prove the local $W^{2,2}$-regularity of its planar viscosity solutions in certain ranges. Here we establish a systematic approach where the question about the second-order regularity boils down to verifying that a certain coefficient matrix is positive definite. This is joint work with M. Parviainen(University of Jyväskylä) and S. Sarsa (University of Jyväskylä)

A general theory of nonlocal elasticity based on nonlocal gradients

Guillermo García-Sáez

Universidad de Castilla-La Mancha

Bessel potential spaces have gained renewed interest due to their robust structural properties and applications in fractional partial differential equations (PDEs). These spaces, derived through complex interpolation between Lebesgue and Sobolev spaces, are closely related to the Riesz fractional gradient introduced by Shieh and Spector in [4,5]. In [6], the equations of nonlocal nonlinear elasticity based on those gradients are studied and related with the well-known Eringen's model. Recently, a broader class of nonlocal gradients have been introduced based on general kernels in [7,8] that include the particular case of the Riesz fractional gradient. In this talk we present the results obtained in [9, 10] in which we derive the equations of nonlinear elasticity based on the nonlocal gradients for general kernels. Furthermore, we perform a formal linearization of the equations to obtain the linear equations based on those nonlocal gradients. We prove existence and uniqueness of solutions providing a general nonlocal Poincaré and Korn's inequality using a traslation operator from the nonlocal Bessel potential spaces to the classical Sobolev spaces. We also study the connection with the Eringen's model in the most general setting and the localization of the equations for varying horizons.

References
[4] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations I. *Advances in Calculus of Variations*, 8(4):321–336, 2015.

[5] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations II. *Advances in Calculus of Variations*, 11(3):289–307, 2018.

[6] J. C. Bellido, J. Cueto, and C. Mora-Corral. Eringen's model via linearization of nonlocal hyperelasticity. *Mathematics and Mechanics of Solids*, 29(4):686–703, 2023.

[7] J. C. Bellido, C. Mora-Corral, and H. Schönberger. Nonlocal gradients: Fundamental theorem of calculus, Poincaré inequalities and embeddings. Preprint, arXiv:2402.16487, 2024.

[8] J. Cueto, C. Kreisbeck, and H. Schönberger. Γ-convergence involving nonlocal gradients with varying horizon: Recovery of local and fractional models. *Nonlinear Analysis: Real World Applications*, 85:104371, 2025.

[9] J. C. Bellido, J. Cueto, and G. García-Sáez. Compact embeddings of Bessel potential spaces. Preprint, arXiv:2506.01677, 2025.

[10] J.C. Bellido and G. García-Sáez *A general theory of nonlocal elasticity based on nonlocal gradients and connections with Eringen's model*, On preparation.

Complex methods in the Möbius energy integrals of helix curves

Max Lipton

Massachusetts Institute of Technology, USA

The Möbius knot energy is a topic of interest in analysis due to its ability to serve as a topological barrier between knot types. The first variation of the Möbius energy is large when the curve's difference vectors are in close alignment with its binormal. This indicates helices are fruitful line of inquiry in analyzing the Möbius energy. In my recent paper, I set up the pointwise energy integral of a helix with given pitch parameter $\rho > 0$ and meromorphically extend the integrand to $\mathbb{C}$. By summing up all of the residues, we can bound the pointwise energy by $\frac{C}{\rho^2}$. Estimating the locations of the poles and their residues required a careful analysis of the solutions to $\frac{\sin(z)}{z} = i\rho$, involving the use of uncommon transcendental constants, satisfying differrential inequalities on small intervals, and the reproducing property of a Bergman kernel. The calculations are done with first principles and do not rely on powerful theorems from Nevanlinna Theory or RKHS.

Maximum principle for some nonlinear elliptic systems

Marta Macrì

University of L'Aquila, Italy

We prove a maximum principle for a class of nonlinear elliptic systems in divergence form.

The result is obtained by assuming a componentwise sign condition.

Global higher integrability for systems with p-growth structure in noncylindrical domains

Kristian Moring

Paris Lodron Universität Salzburg, Austria

In this talk, we address higher integrability for Cauchy-Dirichlet problems with p-growth structure on time-dependent (noncylindrical) domains. We discuss the assumptions and techniques ensuring that the gradient of any weak solution is integrable up to the lateral and initial boundaries beyond the natural exponent p. The talk is based on joint work with Leah Schätzler and Christoph Scheven.

Unified A-priori Estimates for Minimizers under *p,q*-Growth and Exponential Growth

Cintia Pacchiano Camacho

Universidad Nacional Autónoma de México, Mexico

In this talk, we present a unified regularity framework for variational integrals with non-uniformly elliptic integrands, including those exhibiting p, q-growth or exponential-type growth. We consider general energy functionals of the form

$$\int_\Omega f(x, Du)\, dx,$$

where the integrand $f(x, \xi)$ may satisfy natural growth, (p, q)-growth, or exponential growth conditions. We establish that under suitable structural assumptions on the second derivatives of f with respect to the gradient variable, any local minimizer is locally Lipschitz continuous. This key result allows us to reduce complex non-uniformly elliptic problems to a standard growth setting, where classical regularity theory can be applied.

Our analysis includes models beyond the uniformly elliptic case, such as anisotropic energies, the double phase functional, the $p(x)$-Laplacian, and exponential growth integrals. We show that a-priori estimates on the gradient and second derivatives of minimizers can be derived under general conditions involving functions g_1, g_2, and g_3 controlling the ellipticity and the regularity of f. These estimates serve as a crucial step in proving higher regularity results.

The results presented extend and unify various existing regularity theories and provide new insights, particularly for variational problems where the integrand grows faster than any polynomial at infinity. We conclude with examples demonstrating the applicability of our theory in multiple settings, including degenerate, anisotropic, and exponential-type energies.

Balanced quasistatic evolutions of critical points in metric spaces

Alessandro Scagliotti

Technische Universität München, Germany

Quasistatic evolutions of critical points of time-dependent energies exhibit piecewise smooth behavior, making them useful for modeling continuum mechanics phenomena like elastic-plasticity and fracture. Traditionally, such evolutions have been derived as vanishing viscosity and inertia limits, leading to balanced viscosity solutions. However, for nonconvex energies, these constructions have been realized in Euclidean spaces and assume non-degenerate critical points.
In this talk, we take a different approach by decoupling the time scales of the energy evolution and of the transition to equilibria. Namely, starting from an equilibrium configuration, we let the energy evolve, while keeping frozen the system state; then, we update the state by freezing the energy, while letting the system transit via gradient flow or an approximation of it (e.g., minimizing movement or backward differentiation schemes). This approach has several advantages. It aligns with the physical principle that systems transit through energy-minimizing steady states. It is also fully constructive and computationally implementable, with physical and computational costs governed by appropriate action functionals. Additionally, our analysis is simpler and more general than previous formulations in the literature, as it does not require non-degenerate critical points. Finally, this approach extends to evolutions in locally compact metric path spaces, and our axiomatic presentation allows for various realizations.

Existence of variational solutions for doubly nonlinear equations in noncylindrical domains

Leah Schätzler

Aalto University, Finland

In this talk, we discuss existence and basic regularity results for doubly nonlinear systems of the type
$$\partial_t\big(|u|^{q-1}u\big) - \operatorname{div}\big(|Du|^{p-2}Du\big) = 0$$
with parameters $q \in (0,\infty)$ and $p \in (1,\infty)$ in a bounded noncylindrical domain $E \subset \mathbb{R}^n \times [0,T)$. Merely assuming that $\mathcal{L}^{n+1}(\partial E) = 0$, we have the existence of variational solutions $u \in L^\infty\big(0,T; L^{q+1}(E,\mathbb{R}^N)\big)$. If E does not shrink too fast, the power $|u|^{q-1}u$ of the solution u constructed in the first step admits a distributional time derivative. Moreover, under suitable conditions on E, u is continuous with respect to time. The talk is based on joint work with Christoph Scheven, Jarkko Siltakoski, and Calvin Stanko.

On gradient flow approaches to Willmore boundary problems

Manuel Schlierf

Paris Lodron Universität Salzburg, Austria

For immersed spheres and axi-symmetric tori, the asymptotics of the Willmore flow are well-understood below a critical initial-energy-threshold. This talk treats the case of compact surfaces with boundary. First, I'll present results for the Willmore flow with Dirichlet boundary data if the initial surface is an axi-symmetric annulus. Again, the asymptotics can be characterized below a critical initial-energy-threshold depending on the boundary data. Finally, I'll comment on future work for different boundary conditions in the more general, non-axi-symmetric case.

On an eigenvalue problem associated with mixed operators under mixed boundary conditions

Lovelesh Sharma

Indian Institute of Technology Jodhpur, India

In this talk, I am inquisitive about a class of eigenvalue problems involving both local as well as nonlocal operators, precisely the classical Laplace operator and the fractional Laplace operator in the presence of mixed boundary conditions, that is

$$\begin{cases} \mathcal{L}u = \lambda u, \ u > 0 \text{ in } \Omega, \\ u = 0 \text{ in } U^c, \\ \mathcal{N}_s(u) = 0 \text{ in } \mathcal{N}, \\ \dfrac{\partial u}{\partial \nu} = 0 \text{ in } \partial\Omega \cap \overline{\mathcal{N}}, \end{cases} \qquad (P_\lambda)$$

where $U = (\Omega \cup \mathcal{N} \cup (\partial\Omega \cap \overline{\mathcal{N}}))$, $\Omega \subseteq \mathbb{R}^n$ is a non empty open set, $\mathcal{D}, \mathcal{N}$ are open subsets of $\mathbb{R}^n \setminus \bar{\Omega}$ such that $\overline{\mathcal{D} \cup \mathcal{N}} = \mathbb{R}^n \setminus \Omega$, $\mathcal{D} \cap \mathcal{N} = \emptyset$ and $\Omega \cup \mathcal{N}$ is a bounded set with smooth boundary, $\lambda > 0$ is a real parameter and

$$\mathcal{L} = -\Delta + (-\Delta)^s, \text{ for } s \in (0, 1).$$

I will discuss establishing the existence and some characteristics of the first eigenvalue and associated eigenfunctions to the mixed local and nonlocal problem with mixed boundary conditions, based on the topology of the disjoint sets $\mathcal{D}$ and $\mathcal{N}$. Next, I will discuss how we apply these results to establish bifurcation type results, both from zero and infinity for the an asymptotically linear problem inclined with the Original problem. Lastly, I shall motivate other standard open problems related to singular and critical exponents, which are very delicate topics of study in PDEs. Lastly, I shall motivate to other standard open problems related to singular and critical exponents, which are very delicate topics of study in PDEs.

Boundedness of Solutions to the Co-normal Problem for a Class of Nonlinear Elliptic Equations

Lubomira Softova

University of Salerno, Italy

We consider the following co-normal problem for nonlinear elliptic equations:

$$\begin{cases} \operatorname{div} \boldsymbol{a}(x, u, Du) = b(x, u, Du), & x \in \Omega, \\ \boldsymbol{a}(x, u, Du) \cdot \boldsymbol{\nu}(x) = \psi(x, u), & x \in \partial\Omega, \end{cases} \tag{0.2}$$

where $\Omega \subset \mathbb{R}^n$, $n \geq 2$, is a bounded domain with Lipschitz boundary, and $\boldsymbol{\nu}$ denotes the outward unit normal vector at $x \in \partial\Omega$.

The nonlinear terms $\boldsymbol{a}(x, z, \xi)$ and $b(x, z, \xi)$ are *Carathéodory functions* satisfying natural growth conditions. The divergence-form operator is modeled on the Laplacian and is assumed to satisfy the following *coercivity condition*: there exist constants $\gamma, \Lambda > 0$ such that

$$\boldsymbol{a}(x, z, \xi) \cdot \xi \geq \gamma |\xi|^2 - \Lambda |z|^{2^*} - \Lambda \varphi_1^2(x),$$

while the boundary term satisfies

$$|\psi(x, z)| \leq \psi_1(x) + \psi_2(x)|z|^\beta$$

under suitable Lebesgue and Morrey regularity assumptions on the data φ_1, ψ_1, and ψ_2.

Our main result asserts that any weak solution in $W^{1,2}(\Omega)$ of the co-normal problem (0.2) is globally essentially bounded.

Random Schrödinger operators on compact manifolds

Eduard Stefanescu

Technische Universität Graz, Austria

We investigate the eigenvalues of a random Schrödinger operator of Anderson type, given by $-\Delta_g + V_\omega$, on compact manifolds with complex-valued potential. We show that, with high probability, the eigenvalues can be bounded in terms of the L^q-norm of the potential for all $q \geq 1$. Through the use of randomization techniques, we obtain improved bounds that surpass the sharp deterministic estimates established by Sogge. This advancement draws on ideas introduced by Bourgain in the context of almost-sure scattering for lattice Schrödinger operators. This is joint work with Jean-Claude Cuenin and Konstantin Merz.

Boundary regularity for parabolic systems with nonstandard (p,q)-growth conditions in convex domains

Michael Strunk

Paris Lodron Universität Salzburg, Austria

In this talk, we consider parabolic systems

$$\partial_t u^i - \operatorname{div}\big(a(|Du|)Du^i\big) = f^i \qquad \text{in } \Omega_T, \quad \text{for } i = 1, \ldots, N \in \mathbb{N}$$

in a space-time cylinder $\Omega_T = \Omega \times (0,T)$ taken over a bounded and open convex set $\Omega \subset \mathbb{R}^n$ ($n \geq 2$) and $T > 0$. The inhomogeneity $f = (f^1, \ldots, f^N) \colon \Omega_T \to \mathbb{R}^N$ is assumed to be of class $L^{n+2+\sigma}(\Omega_T, \mathbb{R}^N)$ for some $\sigma > 0$. The coefficients $a \colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ are of Uhlenbeck-type structure and subject to nonstandard p, q-growth conditions

$$C_1(\mu^2 + s^2)^{\frac{p-2}{2}} \leq a(s) + a'(s)s \leq C_2(\mu^2 + s^2)^{\frac{q-2}{2}} \qquad \text{for any } s > 0$$

for some positive constants $0 < C_1 \leq C_2$ and a parameter $\mu \in (0,1]$, where we assume the relation

$$2 \leq p \leq q < p + \frac{4}{n+2}$$

to hold true. As our main result, we establish a local Lipschitz estimate up to the lateral boundary of the space-time domain for any weak solution that takes homogeneous boundary values on the considered part of the domain.

Measure-valued solutions for non-associative finite plasticity

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The variational treatment of evolutionary nonassociative elasto-plasticity at finite strains remains unexplored. In this direction, following the concept of energetic solutions, we present an existence result for measure-valued solutions of the quasistatic evolution problem which are stable and balance the energy. In particular, we apply a modification of the standard time-discretization scheme, considering Young measures generated by piecewise constant interpolants of time-discrete solutions of a properly defined minimization problem. A key point in our analysis is the limit passage in the dissipation. The later calls for time-continuity properties of the stresses which are not expected in the quasistatic framework. To overcome this obstacle we introduce a regularization of the generalized stress in the definition of our energetic solutions.