Nonlinear Partial Differential Equations in Salzburg 2024

Book of Abstracts

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Timetable

Wednesday, October 23rd

9:50–10:00	**Opening**	
10:00–10:45	**Xavier Cabre** ICREA & Universitat Politècnica de Cataluny	Stable solutions to semilinear elliptic equations are smooth up to dimension 9
10:50–11:20	**Coffee**	
11:20–12:20	**Contributed Talks**	Nuno Alves, Manuel Schlierf, Berikbol Torebek, Kristian Moring
12:20–14:00	**Lunch**	
14:00–14:45	**Aris Daniilidis** TU Wien, Austria	Many functions are completely determined by their slope and their critical values
14:50–15:20	**Coffee**	
15:20–16:05	**Contributed Talks**	Elias Döhrer, Michael Strunk, Fabian Bäuerlein
16:10 – 16:55	**Anna dall Acqua** Universität Ulm, Germany	Elastic Graphs with clamped boundary

Thursday, October 24th

9:00 – 9:45	**Gulia Treu** Università degli Studi di Padova, Italy	Recent results on nonoccurrence of the Lavrentiev gap for Dirichlet problems in quite general domains
9:50–10:20	**Coffee**	
10:20–11:20	**Contributed Talks**	Antonia Diana, Teresa Rauscher, Daniela Di Donato, Anass Ouannasser
11:25–12:10	**Heiko von der Mosel** Aachen University, Germany	Knot equivalence at low regularity
12:10 – 14:00	**Lunch**	
14:00–14:45	**Thomas Körber** University of Vienna, Austria	The Penrose inequality in extrinsic geometry
14:50–15:45	**Coffee and group foto**	
15:45 – 16:45	**Contributed Talks**	Simone Carano, Alberto Maione, Calvin Stanko, Claudio Afeltra
16:50-17:30	**Salvator Moll** Universitat de València, Spain	Existence and regularity of solutions to manifold-constrained ROF elliptic systems
19:00	**Conference Dinner**	

Friday, October 25th

10:00–10:45	**Zoltan Balogh** Universität Bern	Logarithmic-Sobolev inequalities on non-compact Euclidean submanifolds: sharpness and rigidity
10:50–11:20	**Coffee**	
11:20–12:05	**Jose Mazon** Universitat de Valencia, Spain	Gelfand-Type problems in Random Walk Spaces

List of Abstracts

Wednesday, October 23rd

Stable solutions to semilinear elliptic equations are smooth up to dimension 9

Xavier Cabre

ICREA & Universitat Politècnica de Catalunya. Barcelona, Spain

The regularity of stable solutions to semilinear elliptic PDEs has been studied since the 1970's. It was initiated by a work of Crandall and Rabinowitz, motivated by the Gelfand problem in combustion theory. The theory experienced a revival in the mid-nineties after new progress made by Brezis and collaborators. I will present these developments, as well as a recent work, in collaboration with Figalli, Ros-Oton, and Serra, which finally establishes the regularity of stable solutions up to the optimal dimension 9. I will also describe a more recent paper of mine which provides full quantitative proofs of the regularity results.

Many functions are completely determined by their slope and their critical values

Aris Daniilidis

Technical University Vienna, Austria

A convex continuous function can be determined, up to a constant, by its metric slope (distance of the subdifferential to zero). In this talk, I will present extensions of this result in a general framework and discuss possible applications.

Elastic Graphs with clamped boundary

Anna Dall Acqua

University Ulm, Germany

In the Bernoulli model of an elastic rod described by a curve, the elastic energy is given by integral of the curvature squared with respect to arc-length. We study the minimization of this energy on curves given by the graph of a sufficiently smooth function satisfying Dirichlet boundary conditions. Using invariances of the problem, we are able to integrate the Euler-Lagrange equation once in two different ways. To illustrate the idea and the power of the method, we give also another application to unstable Willmore surfaces of revolution.

Thursday, October 24th

Recent results on nonoccurrence of the Lavrentiev gap for Dirichlet problems in quite general domains

Gulia Treu

Università degli Studi di Padova, Italy

We will present some recent results on the non occurrence of the Lavrentiev gap for multidimensional scalar functionals of the type

$$\int_\Omega f(x, u(x), \nabla u(x))\, dx \qquad u \in \varphi + W_0^{1,p}(\Omega)$$

where Ω is an open bounded domain in $\mathbb{R}^N$, $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ and φ is a Lipschitz function. In particular we will discuss conditions on the Lagrangian f and on the domain Ω that ensure that for every $u \in \varphi + W_0^{1,p}(\Omega)$ there exists a sequence $\{u_k\}_{k\in\mathbb{N}}$, $u_k \in \varphi + W_0^{1,\infty}(\Omega)$, such that u_k strongly converges in $W^{1,p}(\Omega)$ to u and

$$\lim_{k\to\infty} \int_\Omega f(x, u_k(x), \nabla u_k(x))\, dx = \int_\Omega f(x, u(x), \nabla u(x))\, dx.$$

All the results are contained in joint papers with Pierre Bousquet (Université de Toulouse) and Carlo Mariconda (Università di Padova).

Knot equivalence at low regularity

Heiko von der Mosel

Aachen University, Germany

Motivated by the study of scale-invariant knot energies whose energy spaces do not embed into C^1 we provide a new criterion for knot equivalence at low regularity in terms of local distortion. For two general path-connected closed subsets of $\mathbb{R}^n$ whose Hausdorff-distance is smaller than the scale at which their local distortion is controlled, we prove by means of a pseudo-gradient flow that the fundamental groups of the sets' complements are isomorphic. Applied to knot theory this yields new stability results for knot equivalence, as well as the existence of symmetric critical prime knots for scale-invariant knot energies such as the Möbius energy.

The Penrose inequality in extrinsic geometry

Thomas Körber

University of Vienna, Austria

The Riemannian Penrose inequality is a fundamental result in mathematical relativity. It has been a long-standing conjecture of G. Huisken that an analogous result should hold in the context of extrinsic geometry. In this talk, I present recent joint work with M. Eichmair that resolves this conjecture: The exterior mass of an asymptotically flat support surface S with nonnegative mean curvature is bounded in terms of the area of the outermost free boundary minimal surface supported on S. If equality holds, then the complement of a compact subset of S is a half-catenoid. To prove this result, we study minimal capillary surfaces supported on S that minimize the free energy and discover a quantity associated with these surfaces that is nondecreasing as the contact angle increases.

Existence and regularity of solutions to manifold-constrained ROF elliptic systems

Salvador Moll

Universitat de València, Spain

In this talk, I will present some recent results in collaboration with Esther Cabezas-Rivas and Vicent Pallardó-Julià on existence, uniqueness and regularity of solutions to some manifold constrained elliptic systems. These systems arise as the Euler-Lagrange equations for minimizers of generalized Rudin-Osher-Fatemi energy functionals. In the scalar (unconstrained) valued case, the ROF functional is the following one:

$$\int_\Omega |Du| + \frac{\lambda}{2}\int_\Omega |u-f|^2\,dx, \tag{0.1}$$

and the ROF model for denoising consists in finding the minimizer of (0.1) in the space of Bounded Variation functions for a given datum f.

We consider the case when Ω is a compact surface with smooth boundary $\partial\Omega \neq \emptyset$ and functions are constrained to the setting $u(\Omega) \subset \mathcal{N}$, with $(\mathcal{N}, h)$ being a complete, connected and smooth Riemannian manifold.

I will show existence of minimizers of the corresponding generalization of (0.1) and Lipschitz regularity under curvature restrictions on $\mathcal{N}$ and topological ones on the range of f.

Friday, October 24th

Logarithmic-Sobolev inequalities on non-compact Euclidean submanifolds: sharpness and rigidity

Zoltan Balogh

Universität Bern, Switzerland

We provide Michael–Simon-type L^p-logarithmic-Sobolev in- equalities on complete, not necessarily compact n-dimensional submanifolds Σ of the Euclidean space $\mathbb{R}^{n+m}$. Our estimate is sharp, and it involves the mean curvature of Σ. Equality can only occur if and only if Σ is isometric to the Euclidean space $\mathbb{R}^n$ and the extremizer is a Gaussian. Applications are provided to sharp hypercontractivity estimates of Hopf–Lax semigroups on submanifolds.

Gelfand-Type problems in Random Walk Spaces

Jose Mazon

Universitat de Valencia, Spain

This lecture deals with Gelfand-type problems

$$\begin{cases} -\Delta_m u = \lambda f(u) & \text{in } \Omega,\ \lambda > 0, \\ u = 0 & \text{on } \partial_m \Omega, \end{cases} \tag{0.2}$$

being Δ_m the Laplacian in the framework of Random Walk Spaces, which includes as particular cases: Gelfand-type problems on locally finite weighted connected graphs and driven by convolution integrable kernels. As the same assumption on the nonlinearity f as in the local case, we show there exists an extremal parameter $\lambda^* \in (0, \infty)$ such that, for $0 \leq \lambda < \lambda^*$, problem (0.2) admits a minimal bounded solution u_λ and there are not solution for $\lambda > \lambda^*$. Moreover, assuming f is convex, problem (0.2) admit a minimal bounded solution for $\lambda = \lambda^*$. The minimal bounded solution u_λ are stable, being, in the case that f is strictly convex, the only stable solutions. We give examples of simples weighted graphs illustrative of the many situations that can occur for the solutions of Gelfand-type problems on weighted graphs.

Joint work with A. Molino and J. Toledo