

Problem. (Multi-hop Neighbors in ER networks.) For an Erdős-Rényi network $G = \mathcal{G}(n, p)$, each edge in network G has a fixed probability p of being present or $(1 - p)$ of being absent, independently of the other edges. We further independently sample on network G for two times, where each present edge has a fixed probability $(1 - s)$ of being absent, and obtain two sampled networks $G_1 = \mathcal{G}(n, ps)$ and $G_2 = \mathcal{G}(n, ps)$. Therefore, a present edge in G is present in both sampled networks with probability s^2 .

We define that two different nodes u and v in network G are 2-hop neighbors if and only if their shortest distance in G is exactly 2. Provided that $0 < s \leq 1$ and $p \ll 1 \ll np$, please prove the following statements:

- (1) the summation of expected number of 2-hop neighbors for all nodes in network G can be approximated by $n^3 p^2$;
- (2) the probability of nodes u, v being 2-hop neighbors in both sampled networks G_1 and G_2 if they are 2-hop neighbors in network G can be approximated by s^4 .