

# Inflationary Redistribution vs. Trading Opportunities

## Costs of Inflation and Financial Innovation

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Paper: [github.com/phantomachine/csim](https://github.com/phantomachine/csim)

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# Motivation

Yellen (2016) in *Macroeconomic Research After the Crisis*:

*[O]nce heterogeneity is taken into account, other important channels emerge. For example, spending by many households ... quite sensitive to changes in ... income [ ... and etc. (sic) ... ] that in turn affects their access to credit ...*

*[S]tudying monetary models with heterogeneous agents **more closely** could help us shed new light on these aspects of the monetary transmission mechanism.*

# Cost of Inflation

## Some literature

**Taxonomy.** Welfare Costs/benefits of inflation

1. Intertemporal distortionary tax, distorting consumption (all models): *raises cost*
2. Precautionary liquidity management activities: *raises cost*
3. Redistributes money wealth from “rich” to “poor”: *lowers cost*

Models:

- ▶ İmhoroglu-Prescott (1991); Kaplan *et al* (2017):
  - ▶ #3 reduced heterogeneity, small inflation cost (unless other *exogenous frictions*)
- ▶ Lagos-Wright (2005) (quasilinearity + bargaining):
  - ▶ kill #3 (RA model) and accentuate #2, large inflation cost
- ▶ Molico (2006) (relaxed LW quasilinearity):
  - ▶ brings back #3, reduced heterogeneity, smaller inflation cost
- ▶ Molico-Chiu (2010) (Costly CM participation): accentuate #2
  - ▶ but #3 still strong, reduced heterogeneity, smaller inflation cost

# Question

What happens to welfare cost

- ▶ of *inflation*?
- ▶ (Corollary) and of “*financial innovation*”?

when agents

1. do not choose to always participate in complete-insurance markets (CM), and,
2. face *new trade-off*: extensive (trading probability) vs. intensive (terms of trade) in frictional trades (DM)?

# Key Takeaways

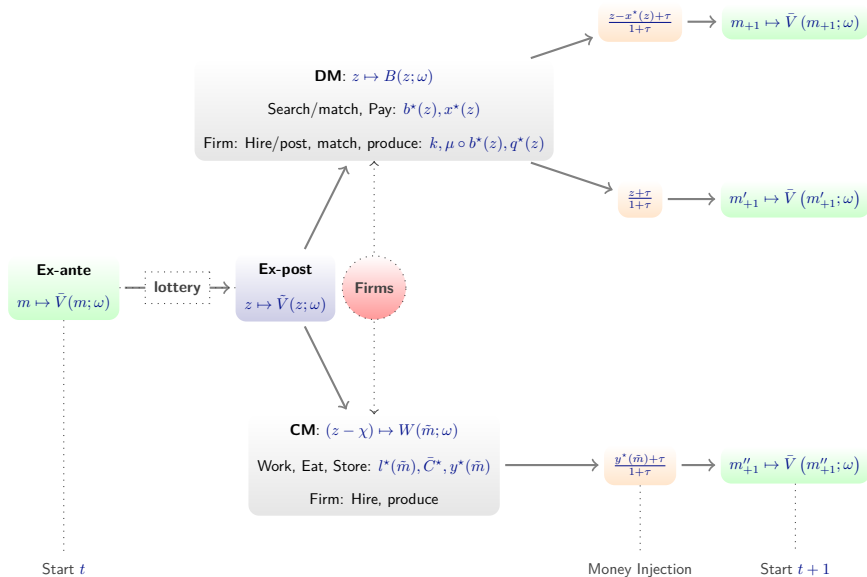
1. **Exists** *stationary monetary equilibrium* with *unique* non-degenerate distribution of agents
2. **Economic insight:**
  - ▶ **What we already know:** *Redistributive inflation*
    - ▶ Rich to poor, but reduces average welfare.
    - ▶ Mechanism: *intensive margin* (all HA models)
  - ▶ **What's new:** Search externality affects *trading probabilities*
    - ▶ Sufficiently high inflation: *extensive margins* (DM **trading opportunities\***; CM participation) dominate redistribution effect
    - ▶ and result in possible reduction of average welfare
    - ▶ a *rise in inequality* of real liquid asset holdings (*contra*. Molico and Chiu, 2010)
  - ▶ **“Financial innovation”**: raise average welfare, but increase liquid wealth inequality
3. **CSIM Python Library:** *Open-source*, exploits computational geometry for convexification, fast solution: highly parallelized and JIT-able (for structural estimation)

# CSIM

Competitive Search Incomplete Markets

a Heterogeneous Agent New Monetarist model

# Model



# Model

## Special Classes

- ▶ Agents must go to CM after DM (no ex-ante participation choice) and  $\chi = 0$

⇒ **Lagos-Wright** (JPE2005) with competitive search, or  
**Rocheteau-Wright** (ECTA2008)

- ▶ CM Participation cost  $\chi = 0$ , no CM utility of consumption, and strictly convex disutility of labor

⇒ **Menzio-Shi-Sun** (JET2013)



# Model

*Ex-post* CM worker/consumer

Worker named  $\mathbf{s} := (m, \omega)$ :

$$W(\mathbf{s}) = \max_{(C, l, y) \in \mathbb{R}_+^3} \left\{ U(C) - A \cdot l + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- ▶ takes  $\omega$  as given
- ▶  $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1 + \tau)}$

# Model

## Ex-post DM buyer

Buyer named  $\mathbf{s} := (m, \omega)$ :

$$B(\mathbf{s}) = \max_{x \in [0, m], q \in \mathbb{R}_+} \left\{ \beta [1 - b(x, q)] \left[ \bar{V} \left( \frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right. \\ \left. + b(x, q) \left[ u(q) + \beta \bar{V} \left( \frac{\omega (m - x) + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right\}.$$

► If  $m \leq k$ , buyer can only afford to visit submarkets  $(\tilde{x}, \tilde{q})$  where  $\tilde{x} - \tilde{q} \leq m \leq k$

► For *this* submarket, optimal to choose

$$b^*(\mathbf{s}) = x^*(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left( \frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right)$$

# Model

## Ex-ante market participation decisions

*Ex-ante* value of fair lottery  $(\pi_1, 1 - \pi_1)$  over prizes  $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1 - \pi_1) \tilde{V}(z_2, \omega) : \right. \\ \left. \pi_1 z_1 + (1 - \pi_1) z_2 = m \right\},$$

where *ex-post* value is

$$\tilde{V}(z, \omega) = \begin{cases} \max \{W(z - \chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega; \tau) := \begin{cases} \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\} & \text{if pay } \chi \text{ in CM} \\ A \in [0, \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\}) & \text{if pay } \chi \text{ before entering CM} \end{cases}$$

# Model

## Stationary Monetary Equilibrium

### Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy  $\tau$ , is a

- ▶ list of value functions  $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$ , satisfying the Bellman functionals ...
- ▶ a list of policies  $\mathbf{s} \mapsto (l^*, y^*, b^*, x^*, q^*, z^*, \pi^*)(\mathbf{s})$  supporting the value functions;
- ▶ market tightness  $\mathbf{s} \mapsto \mu \circ b^*(\mathbf{s})$  given matching technology  $\mu$ : firms max. profits;
- ▶ an ergodic distribution of wealth  $G(\mathbf{s})$  s.t.

$$G(E) = \int P(\mathbf{s}, E) dG(\mathbf{s}) \quad \forall E \in \mathcal{E},$$

$\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$  is a Markov kernel induced by  $(l^*, x^*, q^*, z^*, \pi^*)$  and  $\mu \circ b^*$  under  $\tau$ ; and,

- ▶ a wage rate function  $\mathbf{s} \mapsto \omega(\mathbf{s})$  satisfying labor market clearing

# SME

## Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution  $G$ .

# SME: Computation

# Computation

- ▶ Program runs in Python 2.7/3.6
  - ▶ with optional OpenMPI parallelization capabilities
- ▶ Finding a steady state equilibrium
  - ▶ ~90 seconds on workstation running on 18 parallel cores

# Incomplete Information Estimation

## Method of Simulated Moments

- ▶ CM:  $U(C) = \bar{U}_{CM} \frac{C^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$  and  $-A \cdot l$
- ▶ DM:  $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- ▶ DM matching technology:  $\mu(b) = 1 - b$ .

**Table:** Benchmark estimates

Parameter	Value	Empirical Targets	Description ( <b>MSM*</b> )
$\beta$	0.99	-	$1/\beta - 1 \approx 1\%$
$\sigma_{CM}$	2	-	CM risk aversion
$\sigma_{DM}$	1.01	-	Normalized
$A$	1	-	Normalized
$\bar{m}$	$0.95 \times U^{-1}(A)$	-	Sufficient condition (2.3)
$\chi$	$0.132 \times \bar{m} = 0.125$	Aux reg. $(\pi, M/PY)$	Fixed cost, CM participation *
$k$	0.05103	Aux reg. $(\pi, M/PY)$	Fixed cost, trading post *
$\bar{U}_{CM}$	0.002710	Mean Hours $(\frac{1}{3})$	CM preference scale *



# Incomplete Information Estimation

Method of Simulated Moments: Result

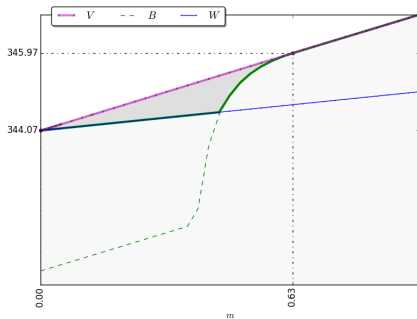
**Table:** Implied Target Estimators

	Auxiliary Stats	Model Stats*	Description ( <b>MSM*</b> )
$(\chi, k)$	0.60	0.70	Mean $M/PY$
	-0.31	-0.21	Aux reg $(\pi, M/PY)$ slope at mean
$\bar{U}_{CM}$	0.33	0.34	Mean Hours ( $\frac{1}{3}$ )

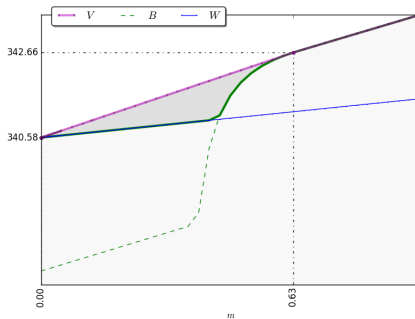
# SME: Results

# SME

## Value Functions (Theorems 1 and 2)



(a)  $\tau = 0$  (0% inflation)

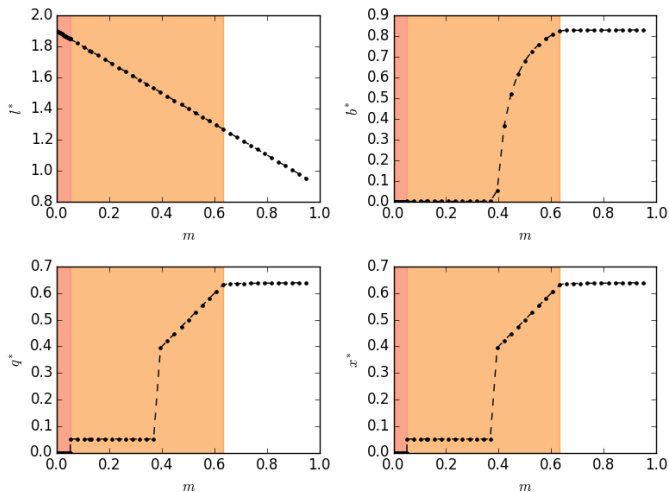


(b)  $\tau = 0.1$  (10% inflation)

**Figure:** Value functions from two economies.

# Steady State

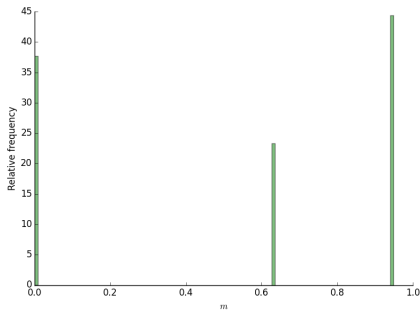
## Markov Policy Functions: Theorems 1 and 2



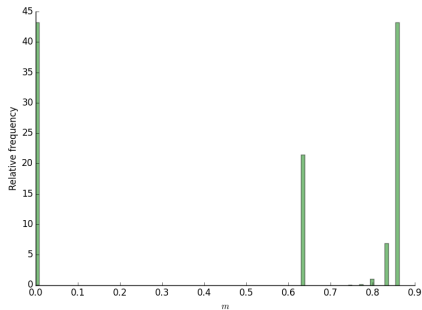
**Figure:** Markov policy functions with  $\tau = 0.1$

# Steady State

## Money Distribution



(a)  $\tau = 0$  (0% inflation)



(b)  $\tau = 0.1$  (10% inflation)

**Figure:** Money distribution under different equilibrium inflation rates

# Comparative SME: Costs of Inflation

# Comparative Steady States

Economic Mechanism (a.k.a intuition)

**Limiting case,  $\chi \searrow 0$**

- ▶ Near-zero limited **CM** asset market participation
- ▶ **DM** poses “virtually only” trading friction ...
- ▶ **DM Search externality:** intensive-extensive trade-off
  - ▶ given  $b$ , if buyer pays more  $x$ , then more  $q$  consumable
  - ▶ given  $x$ , buyer wanting greater trading opportunity  $b$ , eats less  $q$

# Comparative Steady States

Economic Mechanism (a.k.a intuition)

## Limiting case, $\chi \searrow 0$ (cont'd)

- ▶ With anticipated higher inflation and  $V$  concave, *higher* (lower)-balance agents have *lower* (higher) *marginal valuation* of liquid asset
- ▶ One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
  - ▶ standard insight in HA-CIA (İmrohoroğlu-Prescott, 1991) or HANK (Kaplan-Moll-Violante, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- ▶ Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
  - ▶ *feature of competitive search*

## General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ( $\chi > 0$ ):

- ▶ more incentive to trade off  $q$  and  $l$  for higher DM-match (**DM extensive**) and CM-participation rates (**CM extensive**)



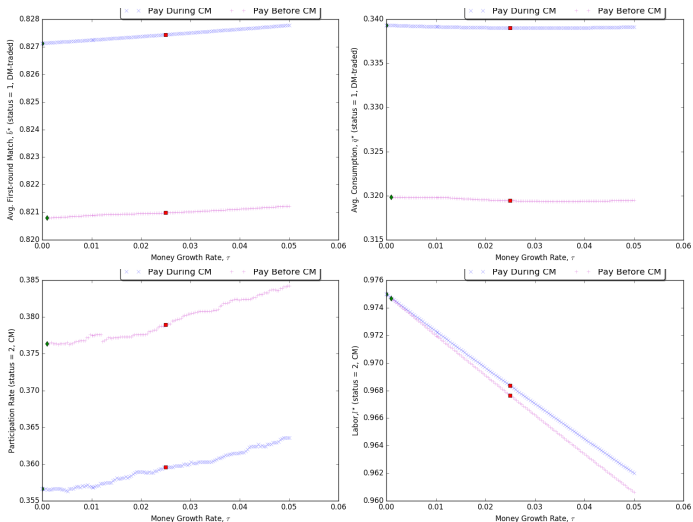
# Comparative Steady States

## Economic Mechanism - numerical result

- ▶ So higher anticipated inflation means:
  - ▶ ex ante agents participate less in CM, but work more intensively in CM, spend more  $x$  in DM to maintain  $(C, q)$  (**intensive margins**) ...
  - ▶ **vs.** more likely to match  $b$  each round of trade with less  $q$ ; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- ▶ With sufficiently high inflation, extensive margins dominates intensive margins
  - ▶ Gini inequality may rise with inflation rate (or  $\tau$ ), if paying  $\chi$  during CM (baseline economy)
  - ▶ Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

# Comparative steady states

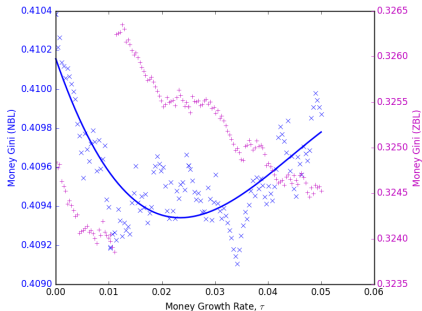
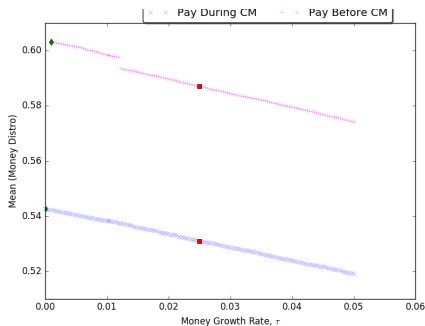
## Intensive vs Extensive Margins



**Figure:** Extensive margins (left) vs Intensive margins (right)

# Effect on Distribution

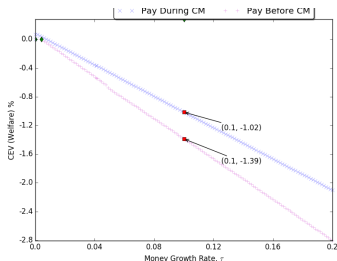
## Comparative steady states



**Figure:** Distributional consequences

# Effect on Welfare

## Comparative steady states: Utilitarian Welfare



**Table: Size matters**

Economy	Welfare Cost*	Remarks
CSIM (This Model)	-0.28	$\chi = 0.001$
	1.02	$\chi = 0.125$
	1.39	$\chi = 0.125$ , ZBL
İmrohoroglu-Prescott (1991, JMCB)	0.90	Bewley-CIA-HA
Chiu-Molico (2010, JME)	0.41	RM-HA
Lagos-Wright (2005, JPE)	1.32	RM-TIOLI-RA
Lucas (2000, ECTA)	0.87	CIA-RA

\* Note: % CEV, 0% – 10% inflation p.a.

# **Comparative SME: “Financial Innovation”**

# Comparative Steady States

## Financial Innovation

**Interpretation 1:** *more access to CM borrowing*

- From Pay  $\chi$  *Before CM* ( $\Rightarrow$  ZBL) to *During CM* ( $\Rightarrow$  NBL)

**Table:** FI #1

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) ZBL	0.33	0.22
(After) NBL	0.41	0.20

# Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*
2. **CSIM Python Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium
3. With high enough inflation, *extensive margins* (DM **trading opportunities\***; CM participation) can dominate redistribution effect; and result in
  - ▶ a reduction of average welfare
  - ▶ a rise in inequality of money balances (if some can “borrow” to overcome CM participation cost)
4. “*Financial innovation*” raises average welfare, but raises liquid wealth inequality

# Quantitative Next Steps

- ▶ Not all trades use “money”—proportion of money exchanges small
  - ▶ some exchanges supported by unsecured credit and banking (with A. Head, J. Lee, I. Ng)
  - ▶ claims on capital may further exacerbate inequality effects (with J. Lee, S. Ong, L. Wang)
- ▶ Re-consider welfare cost of inflation, by accounting for welfare along:
  - ▶ deterministic transition (with J. Lee, L. Wang and A. Sun)
  - ▶ stochastic transitions (business cycles) (with J. Lee, L. Wang, and A. Sun)

Quantitative answer may differ from comparative steady state analysis.



**Back Pocket**

# On the Shoulders of Giants

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[Literature](#)

Baumol-Tobin liquidity channel of monetary transmission:

- ▶ Alvarez, Atkeson and Edmond (2009, QJE)
- ▶ Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - ▶ Exogenous measures of *active* households who can “go to the bank”
- ▶ c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- ▶ Rocheteau, Weill and Wong (2015, NBER):
  - ▶ Undirected search: random matching and bargaining.
  - ▶ Exogenous upper bound on labor supply—incomplete insurance in CM.
  - ▶ Money is non-neutral and generates redistribution (akin to our intensive margin).
  - ▶ c.f., no “extensive margin” trade-off.
- ▶ Head, Liu, Menzio, Wright (2012, JEEA):
  - ▶ Inflation affects price dispersion only in shifting support of price distribution
  - ▶ Money is neutral

Existence of SME with unique agent distribution:

- ▶ Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- ▶  $\Rightarrow$  Hopenhayn-Prescott (1992, ECTA)

# Big Picture

1. Each period, agent market participation:
  - ▶ *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
  - ▶ *worker/consumer* in **CM**: work; precautionary asset (money); neoclassical Walrasian market
2. **Matching friction** in **DM**  $\Rightarrow$  *ex-post* agent **heterogeneity** of money holdings
  - ▶ competitive firms create trading posts (some cost); post terms of trades
  - ▶ buyers direct their search decisions to these trading posts
  - ▶ Equilibrium spending cycle
    - ▶ liquid wealth accumulation at different rates (Baumol-Tobin)
    - ▶ monetary policy has real effects via *uneven equilibrium liquidity demands*
3. **Costly participation** in the Walrasian **CM**  $\Rightarrow$  *ex-post* agent **heterogeneity** of money holdings.
4. Equilibrium trade-offs:
  - ▶ **Intensive margins**: quantities in both markets
  - ▶ **Extensive margins**: participation rates in **CM**; matching prob. **DM**

# Model

## Fiat money supply, measurement units

- ▶ Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- ▶ Labor is the numeraire good.

- ▶ Denote  $\omega M$  as the nominal wage rate.
  - ▶ A dollar's worth of money is equivalent to  $1/\omega M$ .

- ▶  $1/\omega$ : per-capita real money balance (in labor units)

- ▶ If  $M$ : initial aggregate stock of money
  - ▶ Then:  $1/\omega = M \times 1/\omega M$  initial (per-capita) stock of money, *measured in units of labor*

- ▶ Nominal wage growth rate  $\gamma$ :

$$\gamma(\tau) \equiv \omega_+ M_+ / \omega M$$

# Model

## Firms, matching, submarket location in DM

### Matching technology:

- ▶ Sellers and buyers:  $s(\mathbf{s}) = \mu [b(\mathbf{s})]$ , given buyer type:  $\mathbf{s} := (m; \omega)$ 
  - ▶  $\mu(b)$ :  $\mu' < 0$  and  $\mu'' < 0$

### Expected profit-max. decision rule:

- ▶ Create trading posts  $(x, q)(\mathbf{s})$ , and labor hiring from competitive CM market:
- ▶

$$\mu [b(\mathbf{s})] [x(\mathbf{s}) - q(\mathbf{s})] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

# SME properties

## Bounded labor implies NBL

- Sufficient condition (primitives):

$$\exists \bar{m} \in (0, U_1^{-1}(A)) \iff A < U_1(\bar{m}) < U_1(0)$$

- Implies **bounded** and **interior optimal labor choice**:

$$l^*(m, \omega) = \bar{y}^*(\omega) - m + \bar{C}^* \in (0, l_{\max}(\omega, \tau))$$

where

$$\begin{aligned} l_{\max}(\omega, \tau) &:= y_{\max}(\omega; \tau) - 0 + \bar{C}^* \\ &= \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\} + U^{-1}(A) < 2U^{-1}(A) \end{aligned}$$

- Equivalent interpretation:  $y_{\max}(\omega; \tau)$  is a **Natural Borrowing Limit**.

# SME properties

## Theorem 1 (CM policies and value)

1.  $W(\cdot, \omega) \in \mathcal{V}[0, \bar{m}]$ , i.e., it is continuous, increasing and concave on  $[0, \bar{m}]$ . Moreover, it is linear on  $[0, \bar{m}]$ .
2. The partial derivative functions  $W_1(\cdot, \omega)$  and  $\bar{V}_1(\cdot, \omega_{+1})$  exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau} \left( \frac{\omega}{\omega_{+1}} \right) \bar{V}_1 \left( \frac{\omega y^*(m, \omega) + \tau}{\omega_{+1} (1+\tau)}, \omega_{+1} \right) \begin{cases} \leq A, & y^*(m, \omega) \geq 0 \\ \geq A, & y^*(m, \omega) \leq y_{\max}(\omega; \tau) \end{cases},$$

and the envelop condition:

$$W_1(m, \omega) = A,$$

3. The stationary Markovian policy rules  $y^*(\cdot, \omega)$  and  $l^*(\cdot, \omega)$  are scalar-valued and continuous functions on  $[0, \bar{m}]$ .
  - ▶ The function  $y^*(\cdot, \omega)$ , is constant valued on  $[0, \bar{m}]$ .
  - ▶ The optimizer  $l^*(\cdot, \omega)$  is an affine and decreasing function on  $[0, \bar{m}]$ .
  - ▶ Moreover, for every  $(m, \omega)$ , the optimal choice  $l^*(m, \omega)$  is interior.

# SME properties

## Theorem 2 (DM policies and value)

1. For any  $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$ , the DM buyer's value function is increasing and continuous in  $m$ :  $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$ .
2. For any  $m \leq k$ , optimal decisions are  $b^*(m, \omega) = x^*(m, \omega) = q^*(m, \omega) = 0$ , and  $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$ , where  $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$ .
3. At any  $(m, \omega)$ , where  $m \in [k, \bar{m}]$  and  $b^*(m, \omega) > 0$ :
  - ▶  $(x^*, b^*, q^*)(m, \omega)$  are unique, continuous, and increasing in  $m$ .
  - ▶  $B_1(m, \omega)$  exists if and only if  $\bar{V}_1[\phi(m, \omega), \omega]$  exists.
  - ▶  $B(m, \omega)$  is strictly increasing in  $m$ .
  - ▶  $b^*$  and  $x^*$ , respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q[x^*(m, \omega), b^*(m, \omega)] + b^*(m, \omega) (u \circ Q)_2[x^*(m, \omega), b^*(m, \omega)] \\ = \beta [\bar{V}(\phi(m, \omega), \omega_{+1}) - \bar{V}(\phi^*(m, \omega), \omega_{+1})], \end{aligned}$$

$$(u \circ Q)_1[x^*(m, \omega), b^*(m, \omega)] = \frac{\beta}{1 + \tau} \left( \frac{\omega}{\omega_{+1}} \right) \bar{V}_1[\phi^*(m, \omega), \omega_{+1}].$$



# Steady State

Agent sample path: 10% money growth

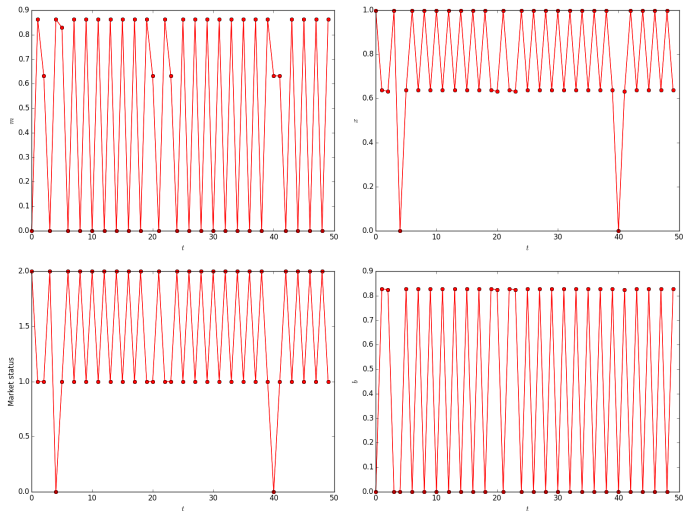


Figure: Agent sample path ( $\tau = 0.1$ )