Inflationary Redistribution vs. Trading Opportunities

with Competitive Search Incomplete Markets

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Paper: github.com/phantomachine/csim

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Motivation

Yellen (2016) in *Macroeconomic Research After the Crisis*:

> *[O]nce heterogeneity is taken into account, other important channels emerge. For example, spending by many households and firms appears to be quite sensitive to changes in labor income, business sales, or the value of collateral that in turn affects their access to credit ...*

> *[S]tudying monetary models with heterogeneous agents **more closely** could help us shed new light on these aspects of the monetary transmission mechanism.*

Welfare and Inequality costs ...

- of ***inflation***?
- (Corrollary) and of "***financial innovation***"?
  - access to credit, or
  - lower cost of financial-risk-insurance participation

Towards a *quantitative* Hetero-Agent New Monetarist view with (some) explicit trading frictions:

- Endogenous Baumol-Tobin (liquid asset/money demand)
- Segmented Asset Markets (access to financial insurance)
- Role of market participation friction and search externality (trading opportunities)
  - Equilibrium trade-off in *intensive* vs. *extensive* margins w.r.t. inflation policy

CSIM

Competitive **S**earch **I**ncomplete **M**arkets

a Heterogeneous Agent New Monetarist model

Big Picture

1. Each period, agent market participation:
   - *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
   - *worker/consumer* in **CM:** work; precautionary asset (money); neoclassical Walrasian market
2. **Matching friction** in **DM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings
   - competitive firms create trading posts (some cost); post terms of trades
   - buyers direct their search decisions to these trading posts
   - Equilibrium spending cycle
     - liquid wealth accumulation at different rates (Baumol-Tobin)
     - monetary policy has real effects via *uneven equilibrium liquidity demands*
3. **Costly participation** in the Walrasian **CM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
4. Equilibrium trade-offs:
   - **Intensive margins**: quantities in both markets
   - **Extensive margins**: participation rates in **CM**; matching prob. **DM**

Big Picture

Special Classes

- Agents must go to CM after DM (no ex-ante participation choice) and $\chi = 0$

$\Rightarrow$ **Lagos-Wright** (JPE2005) with competitive search, or **Rocheteau-Wright** (ECTA2008)

- CM Participation cost $\chi = 0$, no CM utility of consumption, and strictly convex disutility of labor

$\Rightarrow$ **Menzio-Shi-Sun** (JET2013)

Giants

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*

2. `CSIM` `Python` **Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium

3. **Economic insight** (baseline calibrated model):
   - *Redistributive inflation*:
     - Inflation tends to redistribute from rich to poor, but reduces average welfare.
     - Standard mechanism in most HA models.
   - Search externality through *trading opportunities*:
     - With sufficiently high inflation, *extensive margins* (DM **trading opportunities***; CM participation) dominate redistribution effect
     - and result in possible reduction of average welfare
     - a rise in inequality of real money balances
   - "*Financial innovation*" experiments: raise average welfare, but increase liquid wealth inequality

Model

Fiat money supply, measurement units

- Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- Labor is the numeraire good.
  - Denote ωM as the nominal wage rate.
  - A dollar's worth of money is equivalent to $1/\omega M$.
- $1/\omega$: per-capita real money balance (in labor units)
  - If M: initial aggregate stock of money
  - Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*
- Nominal wage growth rate γ:

$$\gamma(\tau) \equiv \omega_{+} M_{+} / \omega M$$

Model

Firms, matching, submarket location in DM

Matching technology:

- Given buyer type: $\mathbf{s} := (m; \omega)$
- Sellers' matching probability and buyers': $s(\mathbf{s}) = \mu[b(\mathbf{s})]$
  - $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

Expected profit-max. decision rule:

- Create trading posts $(x, q)(\mathbf{s})$, and labor hiring from competitive CM market:
-

$$\mu[b(\mathbf{s})][x(\mathbf{s}) - q(\mathbf{s})] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

- Implies quantity traded at a submarket with positive market tightness:

$$q(\mathbf{s}) = Q[(x, b)(\mathbf{s})] \equiv x(\mathbf{s}) - \frac{k}{\mu[b(\mathbf{s})]}$$

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m; \omega)$:

$$W(\mathbf{s}) = \max_{(C,l,y)\in\mathbb{R}^3_+} \left\{ U(C) - A \cdot l + \beta \bar{V}\left(\mathbf{s}_{+1}\right) : pC + y \leq m + l \right\},$$

- takes ω as given
- $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1+\tau)}$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m;\omega)$:

$$B(\mathbf{s}) = \max_{x\in[0,m],q\in\mathbb{R}_+} \left\{\beta\left[1-b\left(x,q\right)\right]\left[\bar{V}\left(\frac{\omega m+\tau}{\omega_{+1}\left(1+\tau\right)},\omega_{+1}\right)\right] + b\left(x,q\right)\left[u(q)+\beta\bar{V}\left(\frac{\omega\left(m-x\right)+\tau}{\omega_{+1}\left(1+\tau\right)},\omega_{+1}\right)\right]\right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets $(\tilde{x},\tilde{q})$ where $\tilde{x}-\tilde{q} \leq m \leq k$
  - For *this* submarket, optimal to choose $b^{\star}(\mathbf{s}) = x^{\star}(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta\bar{V}\left(\frac{\omega m+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)$

Model

Ex ante **market participation decisions**

Ex-ante value of fair lottery $(\pi_1, 1-\pi_1)$ over prizes $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1-\pi_1)\tilde{V}(z_2, \omega) : \right. \\ \left. \pi_1 z_1 + (1-\pi_1) z_2 = m \right\},$$

where *ex-post* value is

$$\tilde{V}(z, \omega) = \begin{cases} \max\{W(z-\chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega; \tau) := \begin{cases} \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} & \text{if pay } \chi \text{ in CM} \\ A \in \left[0, \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\}\right) & \text{if pay } \chi \text{ before entering CM} \end{cases}$$

Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ, is a

- list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- a list of policies $\mathbf{s} \mapsto (l^\star, y^\star, b^\star, x^\star, q^\star, z^\star, \pi^\star)(\mathbf{s})$ supporting the value functions;
- market tightness $\mathbf{s} \mapsto \mu \circ b^\star(\mathbf{s})$ given matching technology μ: firms max. profits;
- an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$G(E) = \int P(\mathbf{s}, E) \mathrm{d}G(\mathbf{s}) \qquad \forall E \in \mathcal{E},$$

  $\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^\star, x^\star, q^\star, z^\star, \pi^\star)$ and $\mu \circ b^\star$under τ; and,
- a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G.

SME: Computation

Incomplete Information Estimation

Method of Simulated Moments

- CM: $U\left(C\right)=\bar{U}_{CM}\frac{C^{1-\sigma_{CM}}}{1-\sigma_{CM}}$ and $-A\cdot l$
- DM: $u\left(q\right)=\frac{(q+0.001)^{1-\sigma_{DM}}-(0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- DM matching technology: $\mu(b)=1-b$.

Table: Benchmark estimates

Parameter	Value	Empirical Targets	Description (MSM*)
β	0.99	-	$1/\beta-1\approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized
A	1	-	Normalized
$\bar{m}$	$0.95\times U^{-1}(A)$	-	Sufficient condition (2.3)
χ	$0.132\times\bar{m}=0.125$	Aux reg. $(\pi, M/PY)$	Fixed cost, CM participation *
k	0.05103	Aux reg. $(\pi, M/PY)$	Fixed cost, trading post *
$\bar{U}_{CM}$	0.002710	Mean Hours $(\frac{1}{3})$	CM preference scale *

Incomplete Information Estimation

Method of Simulated Moments: Result

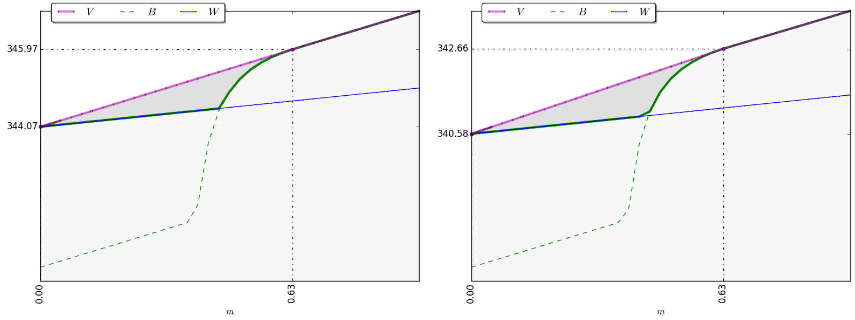
Table: Implied Target Estimators

	Auxiliary Stats	Model Stats*	Description (MSM*)
χ	0.60	0.70	Aux reg. (π,M/PY) mean
k	-0.31	-0.21	Aux reg. (π,M/PY) slope at mean
$\bar{U}_{CM}$	0.33	0.34	Mean Hours ($\frac{1}{3}$)

SME: Results

SME

Value Functions (Theorems 1 and 2)



(a) $\tau = 0$ (0% inflation) (b) $\tau = 0.1$ (10% inflation)

Figure: Value functions from two economies.

Steady State

Markov Policy Functions: Theorems 1 and 2

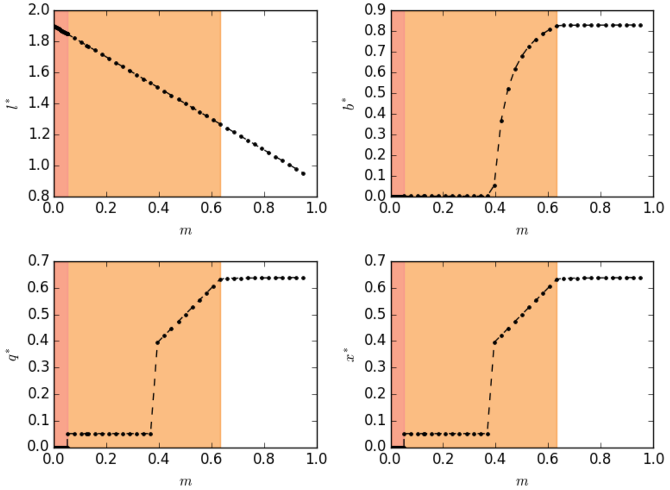
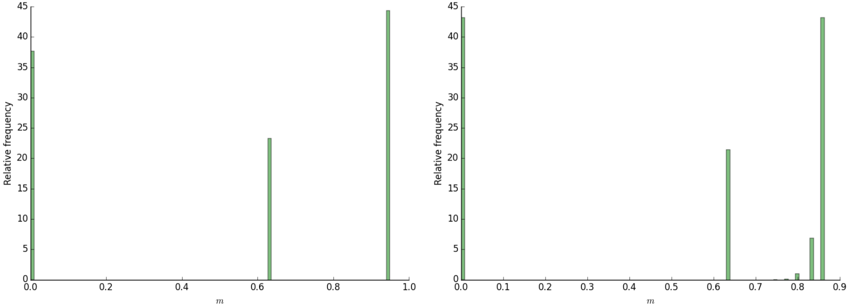


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Money Distribution



(a) $\tau = 0$ (0% inflation)

(b) $\tau = 0.1$ (10% inflation)

Figure: Money distribution under different equilibrium inflation rates

Comparative SME: Costs of Inflation

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$

- Near-zero limited **CM** asset market participation
- **DM** poses "virtually only" trading friction ...
- **DM Search externality:** intensive-extensive trade-off
  - given b, if buyer pays more x, then more q consumable
  - given x, buyer wanting greater trading opportunity b, eats less q

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$ (cont'd)

- With anticipated higher inflation and V concave, *lower* (higher) *marginal valuation* of money to *high* (low)-balance agents
- One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
  - standard insight in HA-CIA (İmrohoroğlu-Prescott, 1991) or HANK (Kaplan-Moll-Violante, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
  - *feature of competitive search*

General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ($\chi > 0$):

- more incentive to trade off q and l for higher DM-match (**DM extensive**) *and CM-participation rates* (**CM extensive**)

Comparative Steady States

Economic Mechanism - numerical result

- So higher anticipated inflation means:
  - ex ante agents participate less in CM, but work more intensively in CM, spend more x in DM to maintain (C, q) (**intensive margins**) ...
  - **vs.** more likely to match b each round of trade with less q; and participate in CM more, but work less intensively (opposite force: **extensive margin**s)
- With sufficiently high inflation, extensive margins dominates intensive margins
  - Gini inequality may rise with inflation rate (or τ), if paying χ during CM (baseline economy)
  - Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Comparative steady states

Intensive vs Extensive Margins

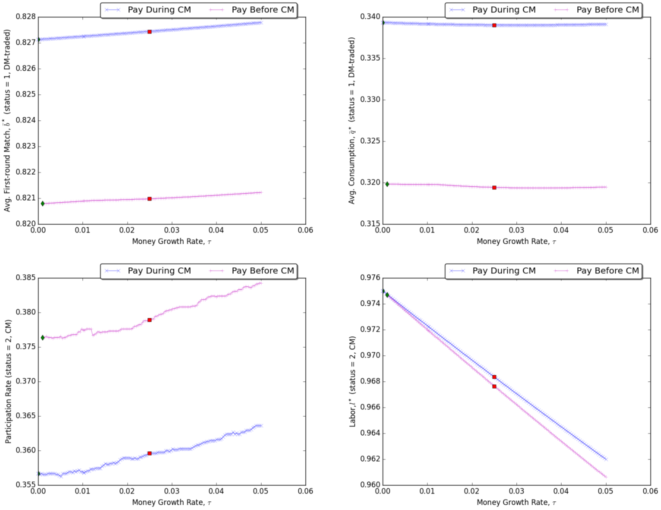
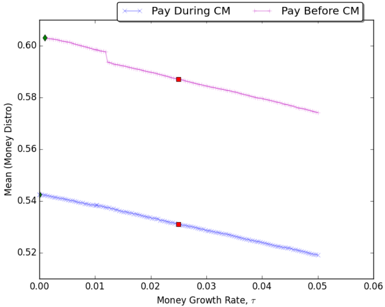


Figure: Comparative steady states

Effect on Distribution

Comparative steady states



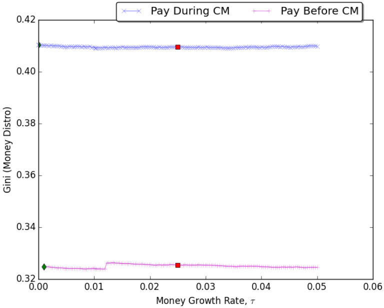


Figure: Comparative steady states — distribution

Effect on Welfare

Comparative steady states: Utilitarian Welfare

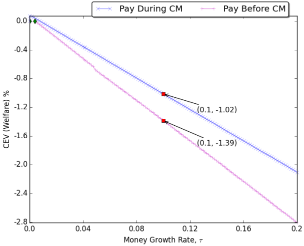


Table: Size matters

Economy	**Welfare Cost***	**Remarks**
CSIM (This Model)	1.02 or 1.39	CS-HA
İmrohoroğlu-Prescott (1991, JMCB)	0.9	Bewley-CIA-HA
Chiu-Molico (2010, JME)	0.41	RM-HA
Lagos-Wright (2005, JPE)	1.32	RM-TIOLI-RA
Lucas (2000, ECTA)	0.87	CIA-RA

* *Note*: % CEV, $0\% - 10\%$ inflation pa

Comparative SME: "Financial Innovation"

Comparative Steady States

Financial Innovation

Interpretation 1:

- From Pay χ *Before CM* ($\Longrightarrow$ ZBL) to *During CM* ($\Longrightarrow$ NBL)

Table: FI #1

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) **ZBL**	0.33	0.22
(After) **NBL**	0.41	0.20

Interpretation 2:

- From $\chi > 0$ to $\chi = 0$, fix: Pay χ before CM ($\Longrightarrow$ ZBL)

Table: FI #2

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) **ZBL**	0.33	0.22
(After) **ZBL** but $\chi = 0$	0.41	0.22

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*

2. `CSIM` `Python` **Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium

3. With high enough inflation, *extensive margins* (DM **trading opportunities***; CM participation) can dominate redistribution effect; and result in
   - a reduction of average welfare
   - a rise in inequality of money balances (if some can "borrow" to overcome CM participation cost)

4. *"Financial innovation"* raises average welfare, but raises liquid wealth inequality

Back Pocket

On the Shoulders of Giants

Go Back Literature

Baumol-Tobin liquidity channel of monetary transmission:

- Alvarez, Atkeson and Edmond (2009, QJE)
- Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - Exogenous measures of *active* households who can "go to the bank"
- c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- Rocheteau, Weill and Wong (2015, NBER):
  - Undirected search: random matching and bargaining.
  - Exogenous upper bound on labor supply—incomplete insurance in CM.
  - Money is non-neutral and generates redistribution (akin to our intensive margin).
  - c.f., no "extensive margin" trade-off.
- Head, Liu, Menzio, Wright (2012, JEEA):
  - Inflation affects price dispersion only in shifting support of price distribution
  - Money is neutral

Existence of SME with unique agent distribution:

- Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- $\Rightarrow$ Hopenhayn-Prescott (1992, ECTA)

SME properties

Bounded labor implies NBL

Go Back Equilibrium NBL

- Sufficient condition (primitives):

$$\exists \bar{m} \in \left(0, U_1^{-1}(A)\right) \iff A < U_1(\bar{m}) < U_1(0)$$

- Implies **bounded** and **interior** *optimal labor choice*:

$$l^\star(m, \omega) = \bar{y}^\star(\omega) - m + \bar{C}^\star \in (0, l_{\max}(\omega, \tau))$$

where

$$\begin{aligned} l_{\max}(\omega, \tau) &:= y_{\max}\ (\omega; \tau) - 0 + \bar{C}^\star \\ &= \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} + U^{-1}(A) < 2U^{-1}(A) \end{aligned}$$

- Equivalent interpretation: $y_{\max}\ (\omega; \tau)$ is a **N**atural **B**orrowing **L**imit.

SME properties

Theorem 1 (CM policies and value)

1. $W(\cdot,\omega) \in \mathcal{V}[0,\bar{m}]$, i.e., it is continuous, increasing and concave on $[0,\bar{m}]$. Moreover, it is linear on $[0,\bar{m}]$.
2. The partial derivative functions $W_1(\cdot,\omega)$ and $\bar{V}_1(\cdot,\omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left(\frac{\omega y^\star(m,\omega)+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)\begin{cases}\leq A, & y^\star(m,\omega)\geq 0\\ \geq A, & y^\star(m,\omega)\leq y_{\max}(\omega;\tau)\end{cases},$$

   and the envelop condition:

$$W_1(m,\omega) = A,$$

3. The stationary Markovian policy rules $y^\star(\cdot,\omega)$ and $l^\star(\cdot,\omega)$ are scalar-valued and continuous functions on $[0,\bar{m}]$.
   - The function $y^\star(\cdot,\omega)$, is constant valued on $[0,\bar{m}]$.
   - The optimizer $l^\star(\cdot,\omega)$ is an affine and decreasing function on $[0,\bar{m}]$.
   - Moreover, for every (m,ω), the optimal choice $l^\star(m,\omega)$ is interior.

SME properties

Theorem 2 (DM policies and value)

1. Fo any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m: $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^{\star}(m, \omega) = x^{\star}(m, \omega) = q^{\star}(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω), where $m \in [k, \bar{m}]$ and $b^{\star}(m, \omega) > 0$:
   - $(x^{\star}, b^{\star}, q^{\star})(m, \omega)$ are unique, continuous, and increasing in m.
   - $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
   - $B(m, \omega)$ is strictly increasing in m.
   - $b^{\star}$and $x^{\star}$, respectively, satisfy the FoCs

$$
\begin{aligned}
u \circ Q\left[x^{\star}(m, \omega), b^{\star}(m, \omega)\right] + b^{\star}(m, \omega)\left(u \circ Q\right)_2\left[x^{\star}(m, \omega), b^{\star}(m, \omega)\right] \\
= \beta\left[\bar{V}\left(\phi\left(m, \omega\right), \omega_{+1}\right) - \bar{V}\left(\phi^{\star}\left(m, \omega\right), \omega_{+1}\right)\right],
\end{aligned}
$$

$$
\left(u \circ Q\right)_1\left[x^{\star}(m, \omega), b^{\star}(m, \omega)\right] = \frac{\beta}{1 + \tau}\left(\frac{\omega}{\omega_{+1}}\right) \bar{V}_1\left[\phi^{\star}(m, \omega), \omega_{+1}\right].
$$

Steady State

Agent sample path: 10% money growth

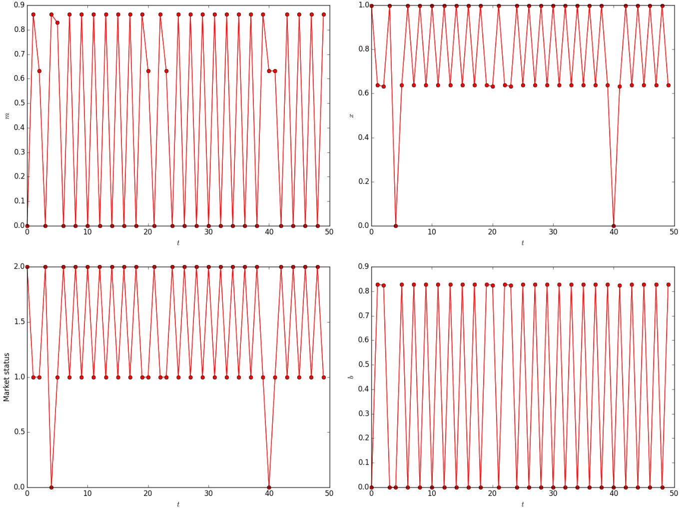


Figure: Agent sample path ($\tau = 0.1$)