

Inflationary Redistribution vs. Trading Opportunities

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Paper: github.com/phantomachine/csim

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Question

How big is the welfare cost and inequality effects of inflation?

... in a heterogenous agent monetarist model?

- ▶ A competitive (directed) search approach to incomplete markets—endogenous Baumol-Tobin model
- ▶ Role of market frictions and search externality
- ▶ Equilibrium trade-off in *intensive* vs. *extensive* margins w.r.t. inflation policy
- ▶ Consequence for average welfare and inequality of wealth (money holdings)
- ▶ A more flexible quantitative model with endogenous limited financial markets participation
 - ▶ can be taken to data

What We Do

- ▶ A directed search model of money and *ex-post* heterogeneous agents:
 - ▶ Endogenous Baumol-Tobin liquidity channel
 - ▶ Menzio, Shi, Sun (MSS2013, JET): deterministic steady state;
 - ▶ no aggregate dynamics, characterize zero inflation monetary equilibrium
 - ▶ pointers to more work ... inflation, dynamics
- ▶ Characterize *stationary monetary equilibrium* (SME)
 - ▶ existence of SME with unique non-degenerate distribution of agents
- ▶ A computational geometry solution to nonconvexities arising from matching externalities
- ▶ Compute various non-stochastic steady state equilibrium regimes over different monetary policies

Shoulders of Giants

On the Shoulders of Giants

Baumol-Tobin liquidity channel of monetary transmission:

- ▶ Alvarez, Atkeson and Edmond (2009, QJE)
 - ▶ Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
 - ▶ Exogenous measures of *active* households who can “go to the bank”
- ▶ c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- ▶ Rocheteau, Weill and Wong (2015, NBER):
 - ▶ Undirected search with random matching and bargaining.
 - ▶ Exogenous upper bound on labor supply—incomplete insurance in CM.
 - ▶ Money is non-neutral and generates redistribution (akin to our intensive margin).
 - ▶ c.f., no “extensive margin” trade-off.
- ▶ Head, Liu, Menzio, Wright (2012, JEEA):
 - ▶ Inflation affects price dispersion only in shifting support of price distribution
 - ▶ Money is neutral

Existence of SME with unique agent distribution:

- ▶ Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- ▶ ⇒ Hopenhayn-Prescott (1992, ECTA)

Model: Big Picture

1. Ex-ante agents identical. Ex post, agents realize different wealth outcomes
2. Each period, each agent decides who to be:
 - ▶ An *anonymous buyer* who – search in **DM**: decentralised; competitive search; matching frictions, or,
 - ▶ a *worker/consumer* – **CM**: work; asset; neoclassical Walrasian market.
3. **Matching friction in DM** \Rightarrow *ex-post* agent **heterogeneity** of money holdings.
 - ▶ competitive firms create trading posts (some cost); post terms of trades
 - ▶ buyers direct their search decisions to these trading posts.
4. **Equilibrium spending cycle** — wealth accumulation at different rates (Baumol-Tobin)
 - ▶ monetary policy affect the real economy, through affecting agents' *uneven equilibrium liquidity demands*
5. **Costly participation** in the Walrasian **CM** \Rightarrow *ex-post* agent **heterogeneity** of money holdings.
6. Equilibrium trade-offs:
 - ▶ **Intensive margins**: quantities in both markets
 - ▶ **Extensive margins**: participation rates in **CM**; matching prob. **DM**

Model

Fiat money supply, measurement units

- ▶ Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- ▶ $1/\omega$: per-capita real money balance (in labor units)
 - ▶ If M : initial aggregate stock of money
 - ▶ Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*

Model

Agents and Goods in DM

- ▶ Finite *set of types* of individuals and firms/goods, I
- ▶ Measure-one continuum of individuals of type $i \in I$
- ▶ An individual consumes good i and produces good $(i + 1) \bmod |I|$

Model

Firms, matching, submarket location in DM

- ▶ Firm chooses:
 - ▶ how many trading posts in submarket $(x, q)(\mathbf{s})$ and labor hiring, given wage 1, as function of who visits post.
 - ▶ “who” indexed by current state of a buyer’s money balance (units of equiv. labor) and aggregate money supply growth rate: $\mathbf{s} := (m; \omega)$
 - ▶ each trading post costs k units of labor to create
 - ▶ zero expected profit

Model

Firms, matching, submarket location in DM

► Matching technology:

- Exists bijective relation between sellers' matching probability and buyers':
 $s(\mathbf{s}) = \mu[b(\mathbf{s})]$
- Trading post's matching probability (market tightness), $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

► Decision rule (equilibrium), at each fixed \mathbf{s} :

$$\mu(b)(x - q) - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

► Implies quantity traded at a submarket with positive market tightness:

$$q(\mathbf{s}) = Q[(x, b)(\mathbf{s})] \equiv x(\mathbf{s}) - \frac{k}{\mu(b(\mathbf{s}))}$$

Model

Ex-ante Individuals

- ▶ *ex-ante* decide to be buyer/worker, depending on current state of individual money balance (units of equiv. labor) and aggregate money supply growth rate: $\mathbf{s} := (m; \omega)$
- ▶ *ex-post*, if *buyer*, then:
 - ▶ decides on *where to shop* \equiv *how much to pay and eat* $(x, q)(\mathbf{s})$
- ▶ *ex-post*, if *worker*, then:
 - ▶ decides how much to work $l^*(\mathbf{s})$

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m; \omega)$:

$$W(\mathbf{s}) = \max_{(C, l, y) \in \mathbb{R}_+^3} \left\{ U(C) - h(l) + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- ▶ takes ω as parametric
- ▶ $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1 + \tau)}$

Model

Change of decision variables in DM (equilibrium)

- ▶ Trading post's matching probability a convex and decreasing function of buyer's matching probability: $\mu(b)$
- ▶ Implies in any equilibrium, each active trading post trades in:

$$q(\mathbf{s}) = Q[(x, b)(\mathbf{s})] \equiv x(\mathbf{s}) - \frac{k}{\mu(b(\mathbf{s}))}$$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m; \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0, m], q \in \mathbb{R}_+} \left\{ \beta [1 - b(x, q)] \left[\bar{V} \left(\frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right. \\ \left. + b(x, q) \left[u(q) + \beta \bar{V} \left(\frac{\omega (m - x) + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets (\tilde{x}, \tilde{q}) where $\tilde{x} - \tilde{q} \leq m \leq k$
 - For *this* submarket, optimal to choose
$$b^*(\mathbf{s}) = x^*(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left(\frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right)$$

Model

Individuals: ex-ante individual

At start of date:

- ▶ Let $\tilde{V}(s) = \max \left\{ B(s), W(m - \chi, \omega) \right\}$
- ▶ **Lemma:** \tilde{V}_m exists for all $m > 0$
- ▶ But \tilde{V} not concave ...
 - ▶ Exists profitable solution where ex-ante agent can randomize between pure strategies $\{buyer, seller\}$
 - ▶ Next we describe fair lotteries

Model

Individuals: convexifying ex-ante preferences

Recall ex-ante value:

$$\bar{V}(\mathbf{s}) = \alpha W(\mathbf{s}) + (1 - \alpha) V(\mathbf{s}),$$

where $V(\mathbf{s})$ is the value of playing a fair lottery $(\pi_1, 1 - \pi_1)$ over the prizes $\{z_1, z_2\}$, i.e.,

$$V(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1 - \pi_1) \tilde{V}(z_2, \omega) : \pi_1 z_1 + (1 - \pi_1) z_2 = m \right\};$$

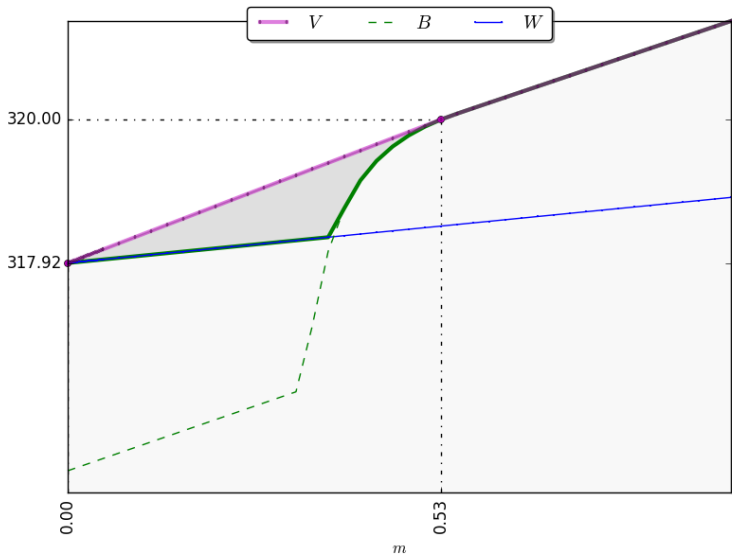
and, given a lottery outcome z , the individual's value becomes

$$\tilde{V}(z, \omega) = \begin{cases} \max \{W(z - \chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

where $y_{\max}(\omega; \tau) := \min \{\bar{m} - \frac{\tau}{\omega}, \bar{m}\}$

Model

Individuals: ex-ante individual



Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ , is a

- ▶ list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- ▶ a list of policies $\mathbf{s} \mapsto (l^*, y^*, b^*, x^*, q^*, z^*, \pi^*)(\mathbf{s})$ supporting the value functions;
- ▶ market tightness $\mathbf{s} \mapsto \mu \circ b^*(\mathbf{s})$ given matching technology μ : firms max. profits;
- ▶ an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$T(G)(E) = \int P(\mathbf{s}, \mathcal{E}) dG(\mathbf{s}) \quad \forall E \in \mathcal{E},$$

$\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^*, x^*, q^*, z^*, \pi^*)$ and $\mu \circ b^*$ under τ ; and,

- ▶ a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME: Existence w/ unique distro

Theorem 1 (CM policies and value)

1. $W(\cdot, \omega) \in \mathcal{V}[0, \bar{m}]$, i.e., it is continuous, increasing and concave on $[0, \bar{m}]$. Moreover, it is linear on $[0, \bar{m}]$.
2. The partial derivative functions $W_1(\cdot, \omega)$ and $\bar{V}_1(\cdot, \omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau} \left(\frac{\omega}{\omega_{+1}} \right) \bar{V}_1 \left(\frac{\omega y^*(m, \omega) + \tau}{\omega_{+1} (1+\tau)}, \omega_{+1} \right) \begin{cases} \leq A, & y^*(m, \omega) \geq 0 \\ \geq A, & y^*(m, \omega) \leq y_{\max}(\omega; \tau) \end{cases},$$

and the envelop condition:

$$W_1(m, \omega) = A,$$

3. The stationary Markovian policy rules $y^*(\cdot, \omega)$ and $l^*(\cdot, \omega)$ are scalar-valued and continuous functions on $[0, \bar{m}]$.
 - ▶ The function $y^*(\cdot, \omega)$, is constant valued on $[0, \bar{m}]$.
 - ▶ The optimizer $l^*(\cdot, \omega)$ is an affine and decreasing function on $[0, \bar{m}]$.
 - ▶ Moreover, for every (m, ω) , the optimal choice $l^*(m, \omega)$ is interior.

Theorem 2 (DM policies and value)

1. For any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m : $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^*(m, \omega) = x^*(m, \omega) = q^*(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω) , where $m \in [k, \bar{m}]$ and $b^*(m, \omega) > 0$:
 - ▶ $(x^*, b^*, q^*)(m, \omega)$ are unique, continuous, and increasing in m .
 - ▶ $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
 - ▶ $B(m, \omega)$ is strictly increasing in m .
 - ▶ b^* and x^* , respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q[x^*(m, \omega), b^*(m, \omega)] + b^*(m, \omega) (u \circ Q)_2[x^*(m, \omega), b^*(m, \omega)] \\ = \beta [\bar{V}(\phi(m, \omega), \omega_{+1}) - \bar{V}(\phi^*(m, \omega), \omega_{+1})], \end{aligned}$$

$$(u \circ Q)_1[x^*(m, \omega), b^*(m, \omega)] = \frac{\beta}{1 + \tau} \left(\frac{\omega}{\omega_{+1}} \right) \bar{V}_1[\phi^*(m, \omega), \omega_{+1}].$$

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G .

► **Lemmata:**

- **(L1):** Equilibrium policy functions are monotone in m , given τ fixed, bounded on m
- **(L2):** $\varphi(\cdot|\tau_{-1})$ is an exogenous increasing Markov kernel, bounded in probability: $\mathbb{E}|\tau| < +\infty$
 - (Meyn and Tweedie, Ch. 12)
- **(L1 and L2) gives (L3):** Therefore equilibrium Markov kernel $P(s, \cdot)$ is increasing and bounded in probability
- **(L4):** Natural state space $S := [0, \bar{m}(\tau)] \times [0, +\infty) \in (m; \tau)$ has least and greatest element
- **(L5):** For any (P, s) -Markov process,
 - there is a $c \in S$ such that $\Pr\{s_t \leq c\} > 0$, and,
 - a $d \in S$ such that $\Pr\{s_t \geq d\} > 0$

► **Theorem:** Kamihigashi and Stachurski (TE, 2014, Theorem 3.1):

- (L3)-(L5) implies stability result.

SME Behavior

Qualitative: Focus on DM

- ▶ Fixing money supply growth τ :
 - ▶ Buyer with higher m spends more $x^*(m; \omega)$ in each submarket to optimize on match probability
 - ▶ with positive probability every period until $m = 0$.
 - ▶ all subsequent submarket matches exhibit lower (x, q)
- ▶ A higher τ will speed up the buyer's spending in each spending round, and lowers worker's $l^*(m; \omega)$

SME: Computation

Computation

Parametrization

- ▶ $U(C) = \bar{U}_{CM} \frac{(C+0.001)^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$
- ▶ $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- ▶ $\mu(b) = 1 - b.$

Table: Baseline parameterization

Parameter	Value	Unit	Description
β	0.99	-	Quarterly interest rate of $1/\beta - 1 \approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized DM risk aversion
A	1	-	Scale on labor disutility function
\bar{m}	$0.95 \times U^{-1}(A)$	labor	See assumption (2.3)
χ	$0.1 \times \bar{m} = 0.095$	labor	Fixed cost of CM participation
α	0	-	Probability of costless CM entry
k	0.05	labor	Fixed cost of creating a trading post
\bar{U}_{CM}	0.002	-	CM preference scale parameter

Computation

- ▶ Program runs in Python 2.7/3.0
 - ▶ with optional OpenMPI parallelization capabilities
- ▶ Finding a steady state equilibrium:
 - ▶ ~90 seconds on workstation running on 18 parallel cores

SME: Results

Steady State

Value Functions: zero money growth

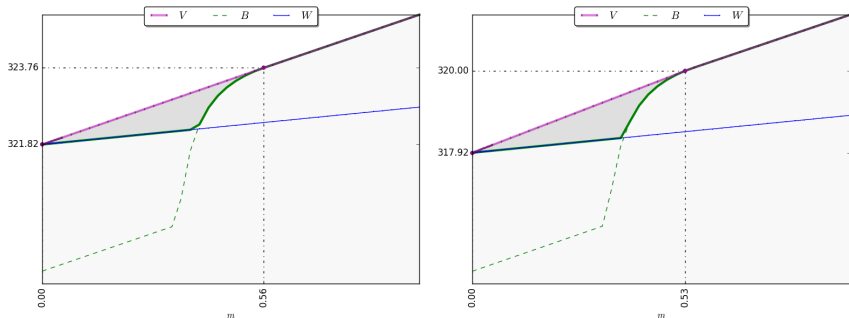


Figure: Value functions from two economies.

Steady State

Markov Policy Functions: 10% money growth

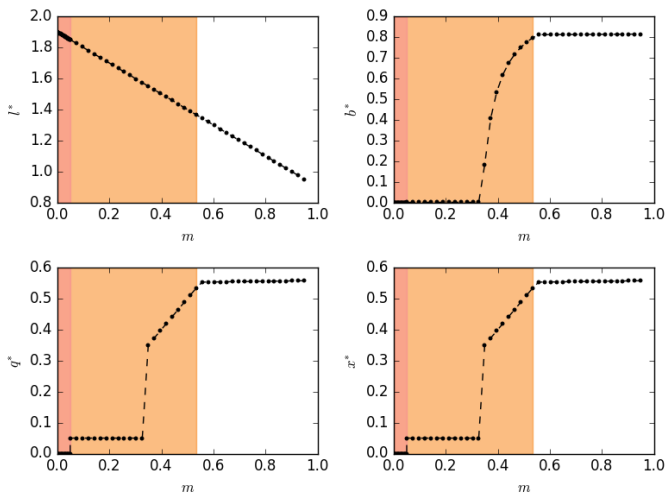


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Agent sample path: 10% money growth

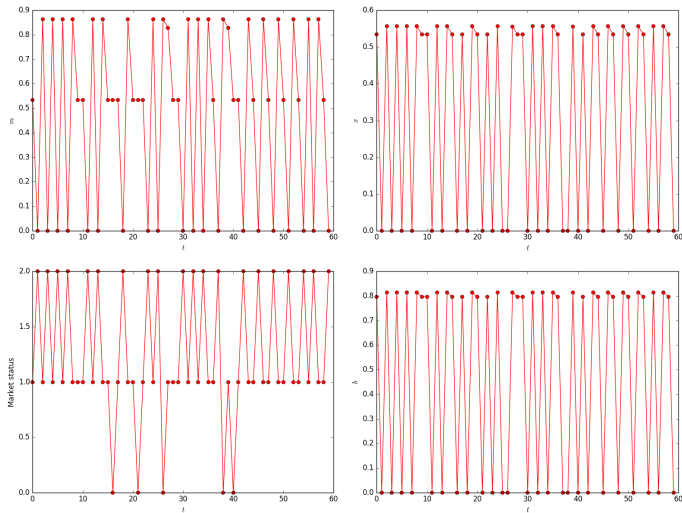


Figure: Agent sample path ($\tau = 0.1$)

Steady State

Money Distribution

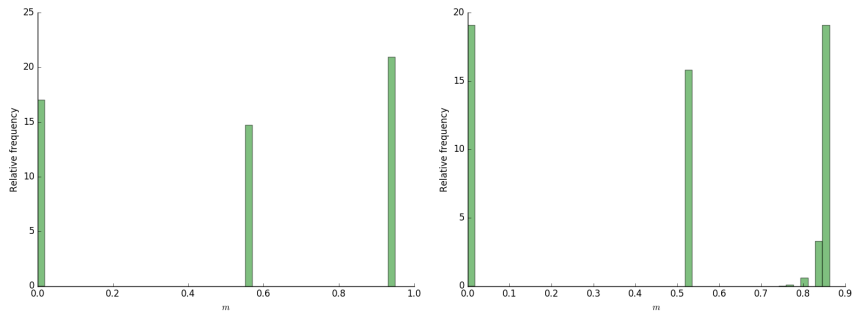


Figure: Money distribution under different equilibrium inflation rates

Comparative SME: Preliminary Results

Effect on Inflation

Comparative steady states: Equilibrium wage

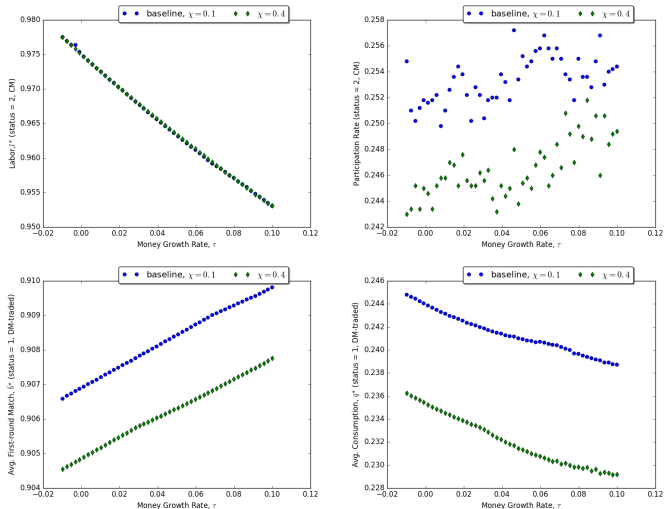


Figure: Comparative steady states — allocations

Effect on Inflation

Comparative steady states: (τ, χ)

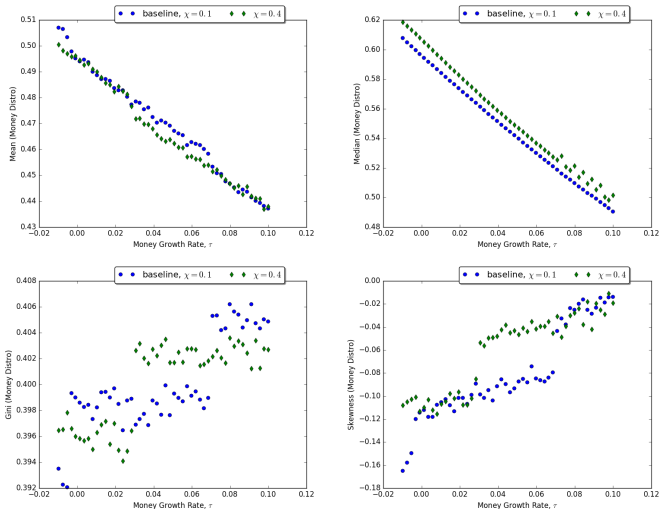


Figure: Comparative steady states — distribution

Effect on Inflation

Comparative steady states: Utilitarian Welfare

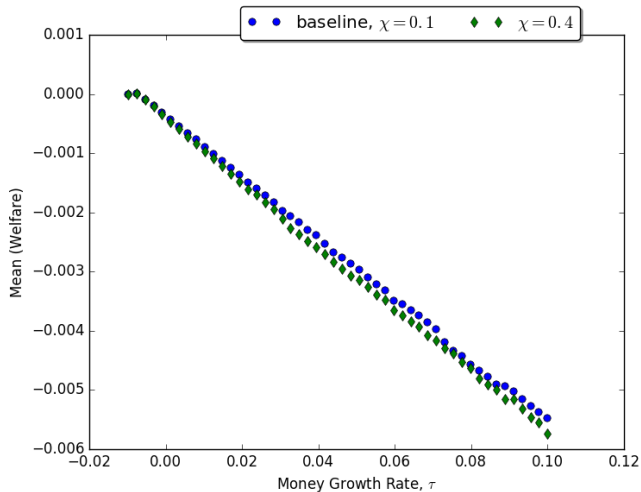


Figure: 2 — 2.4% consumption loss p.a. (0 — 10% inflation)

Comparative Steady States

Mechanism

- ▶ Buyer's submarket trade-off: $Q_1(x, b) > 0$, $Q_2(x, b) < 0$, $Q(x, b)$ weakly concave, $Q_{12}(x, b) = 0$:
 - ▶ given b , the more a buyer willing to pay x , the more q consumable
 - ▶ given x , buyer who wants to match with higher probability b , must tolerate eating less q
- ▶ Anticipated higher inflation, consider agents with $m > k$:
 - ▶ Higher inflation affects agents unequally since with risk aversion V inherits concavity
 - ▶ *Higher inflation* means *lower* (higher) *marginal valuation* of money to *high* (low)-balance agents
 - ▶ One hand (intensive margin): inflation, a redistribution of wealth from high-agents to low agents
 - ▶ Other hand (extensive margin): inflation raises the downside risk of agents not getting matched in DM; or to participate in CM—so more incentive to trade off q and l for higher DM-match and CM-participation rates.

Comparative Steady States

Mechanism

- ▶ So higher anticipated inflation means:
 - ▶ ex ante agents participate less in CM, but work more intensively in CM, spend more x in CM to maintain (C, q) (**intensive margins**) ...
 - ▶ **vs.** more likely to match b each round of trade with less q ; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- ▶ Extensive margins dominates intensive margins
 - ▶ Gini inequality rises with inflation rate (or τ)
 - ▶ Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Quantitative Next Steps

- ▶ Estimate model — discipline with real-world data:
 - ▶ not all trades use “money”—proportion of money exchanges small
 - ▶ some exchanges supported by credit and contracts
 - ▶ exogenous limited participation (α) model allows to do counterfactual studies on degree of market incompleteness
 - ▶ Re-consider welfare cost of inflation, by accounting for welfare along:
 - ▶ deterministic transition (with J. Lee)
 - ▶ stochastic transitions (with J. Lee, L. Wang, and A. Sun)
- answer may differ from comparative steady state analysis.