

Inflationary Redistribution vs. Trading Opportunities

Costs of Inflation and Financial Innovation

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Paper: github.com/phantomachine/csim

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Motivation

Yellen (2016) in *Macroeconomic Research After the Crisis*:

[O]nce heterogeneity is taken into account, other important channels emerge. For example, spending by many households ... quite sensitive to changes in ... income [... and etc. (sic) ...] that in turn affects their access to credit ...

*[S]tudying monetary models with heterogeneous agents **more closely** could help us shed new light on these aspects of the monetary transmission mechanism.*

What We Do

- ▶ An integrated New Monetarist Heterogenous Agent Framework:
 - ▶ in the spirit of Lagos and Wright (2005, JPE) but with costly choice of Complete Markets participation
 - ▶ with tractable heterogeneity from Menzio, Shi and Sun (2014, JET)
 - ▶ provable existence of unique non-degenerate agent distro
 - ▶ lower computational complexity, c.f., random matching models
- ▶ An application: Welfare cost and inequality consequences
 - ▶ of *inflation*?
 - ▶ (Corollary) and of “*financial innovation*”?

Why Cost of Inflation (again)?

Some literature

Taxonomy. Welfare Costs/benefits of inflation

1. Intertemporal distortionary tax, distorting consumption (all models): *raises cost*
2. Precautionary liquidity management activities: *raises cost*
3. Redistributes money wealth from “rich” to “poor”: *lowers cost*

Models:

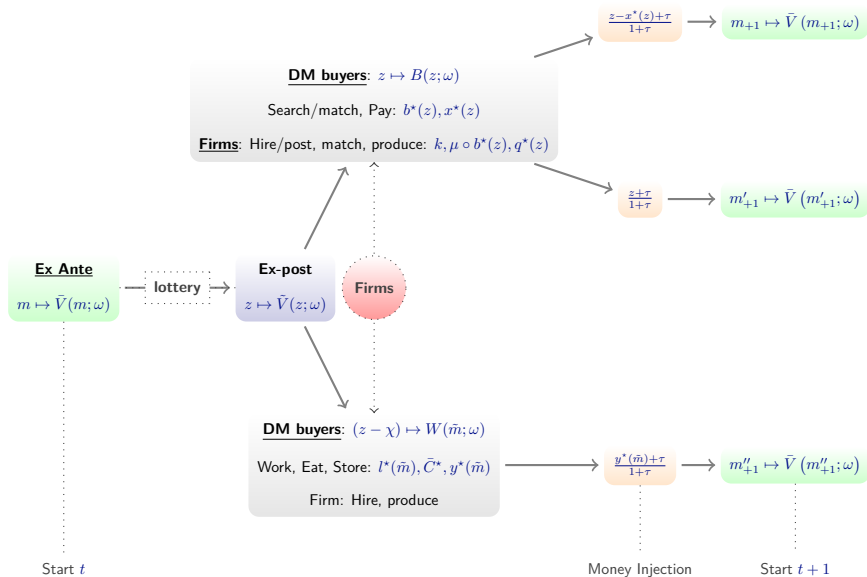
- ▶ İmhoroglu-Prescott (1991); Kaplan *et al* (2017):
 - ▶ #3 reduced heterogeneity, small inflation cost (unless other *ad-hoc/parametric frictions*)
- ▶ Lagos-Wright (2005) (quasilinearity + bargaining):
 - ▶ kill #3 (RA model) and accentuate #2, large inflation cost
- ▶ Molico (2006) (relaxed LW quasilinearity):
 - ▶ brings back #3, reduced heterogeneity, smaller inflation cost
- ▶ Molico-Chiu (2010) (Costly CM participation): accentuate #2
 - ▶ but #3 still strong, reduced heterogeneity, smaller inflation cost

Key Takeaways

1. **Exists** a *stationary monetary equilibrium*. Equilibrium features *unique* non-degenerate distribution of agents
2. **Economic insight:**
 - ▶ **What we already know:** *Redistributive inflation*
 - ▶ Rich to poor, but reduces average welfare
 - ▶ Mechanism: *intensive margin* (all HA models)
 - ▶ **What's new:** Search externality affects *trading probabilities*
 - ▶ Sufficiently high inflation: *extensive margins* (DM **trading opportunities***; CM participation) dominate redistribution effect
 - ▶ and result in possible reduction of average welfare
 - ▶ a *rise in inequality* of real liquid asset holdings (*contra*. Molico and Chiu, 2010)
 - ▶ **“Financial innovation”**: raise average welfare, but increase liquid wealth inequality
3. **Python Library:** *Open-source*, exploits computational geometry for convexification
 - ▶ fast solution: highly parallelized and JIT-able (for structural estimation)
 - ▶ more efficient algorithm (with provable convergence properties), c.f., heuristics in Krusell-Smith methods in standard and random-matching HA models

An Heterogeneous Agent New Monetarist Model

Model



Firms in DM

Money supply

In prose

Model

Limiting cases

- ▶ Agents must go to CM after DM (no ex-ante participation choice) and $\chi = 0$

⇒ a version of **Lagos-Wright** (JPE2005) with competitive search

- ▶ CM Participation cost $\chi = 0$, no CM utility of consumption, and strictly convex disutility of labor

⇒ **Menzio-Shi-Sun** (JET2013)

Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ , is a

- ▶ list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- ▶ a list of policies $\mathbf{s} \mapsto (l^*, y^*, b^*, x^*, q^*, z^*, \pi^*)(\mathbf{s})$ supporting the value functions;
- ▶ market tightness $\mathbf{s} \mapsto \mu \circ b^*(\mathbf{s})$ given matching technology μ : firms max. profits;
- ▶ an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$G(E) = \int P(\mathbf{s}, E) dG(\mathbf{s}) \quad \forall E \in \mathcal{E},$$

$\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^*, x^*, q^*, z^*, \pi^*)$ and $\mu \circ b^*$ under τ ; and,

- ▶ a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G .

SME: Computation

Incomplete Information Estimation

Method of Simulated Moments

- ▶ CM: $U(C) = \bar{U}_{CM} \frac{C^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$ and $-A \cdot l$
- ▶ DM: $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- ▶ DM matching technology: $\mu(b) = 1 - b$.

Table: Benchmark estimates

Parameter	Value	Empirical Targets	Description (MSM*)
β	0.99	-	$1/\beta - 1 \approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized
A	1	-	Normalized
\bar{m}	$0.95 \times U^{-1}(A)$	-	Sufficient condition (2.3)
χ	$0.132 \times \bar{m} = 0.125$	Aux reg. $(\pi, M/PY)$	Fixed cost, CM participation *
k	0.05103	Aux reg. $(\pi, M/PY)$	Fixed cost, trading post *
\bar{U}_{CM}	0.002710	Mean Hours $(\frac{1}{3})$	CM preference scale *

Incomplete Information Estimation

Method of Simulated Moments: Result

Table: Implied Target Estimators

	Auxiliary Stats	Model Stats*	Description (MSM*)
(χ, k)	0.60	0.70	Mean M/PY
	-0.31	-0.21	Aux reg $(\pi, M/PY)$ slope at mean
\bar{U}_{CM}	0.33	0.34	Mean Hours ($\frac{1}{3}$)

Comparative SME: Costs of Inflation

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$

- ▶ With anticipated higher inflation and V concave, *higher* (lower)-balance agents have *lower* (higher) *marginal valuation* of liquid asset
- ▶ One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
 - ▶ standard insight in HA-CIA (İmrohoroglu-Prescott, 1991) or HANK (Kaplan-Moll-Violante, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- ▶ Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
 - ▶ *feature of competitive search*

General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ($\chi > 0$):

- ▶ more incentive to trade off q and l for higher DM-match (**DM extensive**) and CM-participation rates (**CM extensive**)

Comparative Steady States

Economic Mechanism - numerical result

- ▶ So higher anticipated inflation means:
 - ▶ ex ante agents participate less in CM, but work more intensively in CM, spend more x in DM to maintain (C, q) (**intensive margins**) ...
 - ▶ **vs.** more likely to match b each round of trade with less q ; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- ▶ With sufficiently high inflation, extensive margins dominates intensive margins
 - ▶ Gini inequality may rise with inflation rate (or τ), if paying χ during CM (baseline economy)
 - ▶ Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Effect on Distribution

Comparative steady states

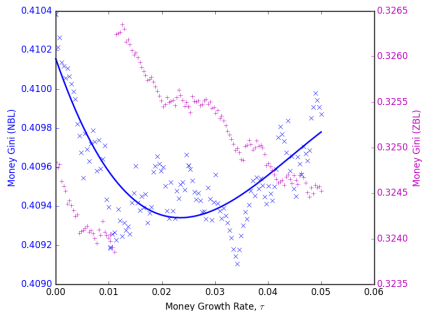
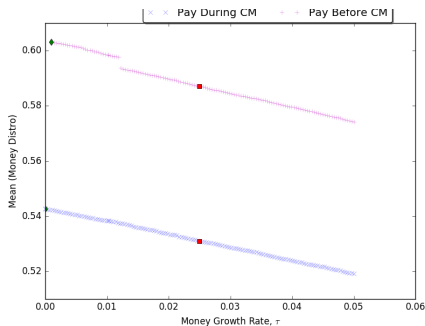


Figure: Distributional consequences

Effect on Welfare

Comparative steady states: Utilitarian Welfare

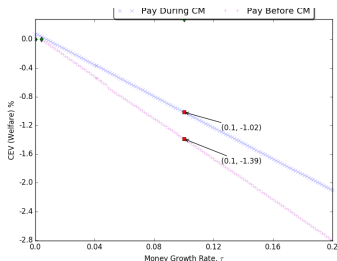


Table: Size matters

Economy	Welfare Cost*	Remarks
CSIM (This Model)	-0.28	$\chi = 0.001$
	1.02	$\chi = 0.125$
	1.39	$\chi = 0.125$, ZBL
İmrohoroglu-Prescott (1991, JMCB)	0.90	Bewley-CIA-HA
Chiu-Molico (2010, JME)	0.41	RM-HA
Lagos-Wright (2005, JPE)	1.32	RM-TIOLI-RA
Lucas (2000, ECTA)	0.87	CIA-RA

* Note: % CEV, 0% – 10% inflation p.a.

Comparative SME: “Financial Innovation”

Comparative Steady States

Financial Innovation

Interpretation 1: *more access to CM borrowing*

- From Pay χ *Before CM* (\Rightarrow ZBL) to *During CM* (\Rightarrow NBL)

Table: FI #1

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) ZBL	0.33	0.22
(After) NBL	0.41	0.20

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*
2. **CSIM Python Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium
3. With high enough inflation, *extensive margins* (DM **trading opportunities***; CM participation) can dominate redistribution effect; and result in
 - ▶ a reduction of average welfare
 - ▶ a rise in inequality of money balances (if some can “borrow” to overcome CM participation cost)
4. “*Financial innovation*” raises average welfare, but raises liquid wealth inequality

Back Pocket

On the Shoulders of Giants

[Go Back](#)

[Literature](#)

Baumol-Tobin liquidity channel of monetary transmission:

- ▶ Alvarez, Atkeson and Edmond (2009, QJE)
- ▶ Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
 - ▶ Exogenous measures of *active* households who can “go to the bank”
- ▶ c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- ▶ Rocheteau, Weill and Wong (2015, NBER):
 - ▶ Undirected search: random matching and bargaining.
 - ▶ Exogenous upper bound on labor supply—incomplete insurance in CM.
 - ▶ Money is non-neutral and generates redistribution (akin to our intensive margin).
 - ▶ c.f., no “extensive margin” trade-off.
- ▶ Head, Liu, Menzio, Wright (2012, JEEA):
 - ▶ Inflation affects price dispersion only in shifting support of price distribution
 - ▶ Money is neutral

Existence of SME with unique agent distribution:

- ▶ Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- ▶ \Rightarrow Hopenhayn-Prescott (1992, ECTA)

Big Picture

1. Each period, agent market participation:
 - ▶ *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
 - ▶ *worker/consumer* in **CM**: work; precautionary asset (money); neoclassical Walrasian market
2. **Matching friction** in **DM** \Rightarrow *ex-post* agent **heterogeneity** of money holdings
 - ▶ competitive firms create trading posts (some cost); post terms of trades
 - ▶ buyers direct their search decisions to these trading posts
 - ▶ Equilibrium spending cycle
 - ▶ liquid wealth accumulation at different rates (Baumol-Tobin)
 - ▶ monetary policy has real effects via *uneven equilibrium liquidity demands*
3. **Costly participation** in the Walrasian **CM** \Rightarrow *ex-post* agent **heterogeneity** of money holdings.
4. Equilibrium trade-offs:
 - ▶ **Intensive margins**: quantities in both markets
 - ▶ **Extensive margins**: participation rates in **CM**; matching prob. **DM**

Model

Fiat money supply, measurement units

- ▶ Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- ▶ Labor is the numeraire good.

- ▶ Denote ωM as the nominal wage rate.
 - ▶ A dollar's worth of money is equivalent to $1/\omega M$.

- ▶ $1/\omega$: per-capita real money balance (in labor units)

- ▶ If M : initial aggregate stock of money
 - ▶ Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*

- ▶ Nominal wage growth rate γ :

$$\gamma(\tau) \equiv \omega_+ M_+ / \omega M$$

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m, \omega)$:

$$W(\mathbf{s}) = \max_{(C, l, y) \in \mathbb{R}_+^3} \left\{ U(C) - A \cdot l + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- ▶ takes ω as given
- ▶ $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1 + \tau)}$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m, \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0, m], q \in \mathbb{R}_+} \left\{ \beta [1 - b(x, q)] \left[\bar{V} \left(\frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right. \\ \left. + b(x, q) \left[u(q) + \beta \bar{V} \left(\frac{\omega (m - x) + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets (\tilde{x}, \tilde{q}) where $\tilde{x} - \tilde{q} \leq m \leq k$
 - For *this* submarket, optimal to choose
$$b^*(\mathbf{s}) = x^*(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left(\frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right)$$

Model

Ex-ante market participation decisions

Ex-ante value of fair lottery $(\pi_1, 1 - \pi_1)$ over prizes $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1 - \pi_1) \tilde{V}(z_2, \omega) : \right. \\ \left. \pi_1 z_1 + (1 - \pi_1) z_2 = m \right\},$$

where *ex-post* value is

$$\tilde{V}(z, \omega) = \begin{cases} \max \{W(z - \chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega; \tau) := \begin{cases} \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\} & \text{if pay } \chi \text{ in CM} \\ A \in [0, \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\}) & \text{if pay } \chi \text{ before entering CM} \end{cases}$$

Model

Firms, matching, submarket location in DM

Matching technology:

- ▶ Sellers and buyers: $s(\mathbf{s}) = \mu [b(\mathbf{s})]$, given buyer type: $\mathbf{s} := (m; \omega)$
 - ▶ $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

Expected profit-max. decision rule:

- ▶ Create trading posts $(x, q)(\mathbf{s})$, and labor hiring from competitive CM market:
- ▶

$$\mu [b(\mathbf{s})] [x(\mathbf{s}) - q(\mathbf{s})] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

SME properties

Bounded labor implies NBL

- Sufficient condition (primitives):

$$\exists \bar{m} \in (0, U_1^{-1}(A)) \iff A < U_1(\bar{m}) < U_1(0)$$

- Implies **bounded** and **interior optimal labor choice**:

$$l^*(m, \omega) = \bar{y}^*(\omega) - m + \bar{C}^* \in (0, l_{\max}(\omega, \tau))$$

where

$$\begin{aligned} l_{\max}(\omega, \tau) &:= y_{\max}(\omega; \tau) - 0 + \bar{C}^* \\ &= \min \left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\} + U^{-1}(A) < 2U^{-1}(A) \end{aligned}$$

- Equivalent interpretation: $y_{\max}(\omega; \tau)$ is a **Natural Borrowing Limit**.

SME properties

Theorem 1 (CM policies and value)

1. $W(\cdot, \omega) \in \mathcal{V}[0, \bar{m}]$, i.e., it is continuous, increasing and concave on $[0, \bar{m}]$. Moreover, it is linear on $[0, \bar{m}]$.
2. The partial derivative functions $W_1(\cdot, \omega)$ and $\bar{V}_1(\cdot, \omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau} \left(\frac{\omega}{\omega_{+1}} \right) \bar{V}_1 \left(\frac{\omega y^*(m, \omega) + \tau}{\omega_{+1} (1+\tau)}, \omega_{+1} \right) \begin{cases} \leq A, & y^*(m, \omega) \geq 0 \\ \geq A, & y^*(m, \omega) \leq y_{\max}(\omega; \tau) \end{cases},$$

and the envelop condition:

$$W_1(m, \omega) = A,$$

3. The stationary Markovian policy rules $y^*(\cdot, \omega)$ and $l^*(\cdot, \omega)$ are scalar-valued and continuous functions on $[0, \bar{m}]$.
 - ▶ The function $y^*(\cdot, \omega)$, is constant valued on $[0, \bar{m}]$.
 - ▶ The optimizer $l^*(\cdot, \omega)$ is an affine and decreasing function on $[0, \bar{m}]$.
 - ▶ Moreover, for every (m, ω) , the optimal choice $l^*(m, \omega)$ is interior.

SME properties

Theorem 2 (DM policies and value)

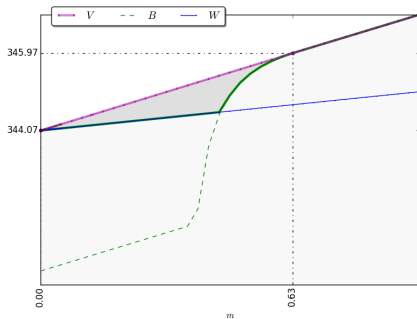
1. For any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m : $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^*(m, \omega) = x^*(m, \omega) = q^*(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω) , where $m \in [k, \bar{m}]$ and $b^*(m, \omega) > 0$:
 - ▶ $(x^*, b^*, q^*)(m, \omega)$ are unique, continuous, and increasing in m .
 - ▶ $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
 - ▶ $B(m, \omega)$ is strictly increasing in m .
 - ▶ b^* and x^* , respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q[x^*(m, \omega), b^*(m, \omega)] + b^*(m, \omega) (u \circ Q)_2[x^*(m, \omega), b^*(m, \omega)] \\ = \beta [\bar{V}(\phi(m, \omega), \omega_{+1}) - \bar{V}(\phi^*(m, \omega), \omega_{+1})], \end{aligned}$$

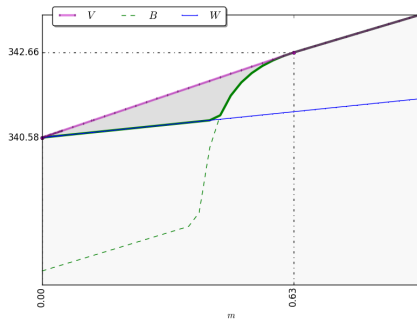
$$(u \circ Q)_1[x^*(m, \omega), b^*(m, \omega)] = \frac{\beta}{1 + \tau} \left(\frac{\omega}{\omega_{+1}} \right) \bar{V}_1[\phi^*(m, \omega), \omega_{+1}].$$

SME

Value Functions (Theorems 1 and 2)



(a) $\tau = 0$ (0% inflation)



(b) $\tau = 0.1$ (10% inflation)

Figure: Value functions from two economies.

Steady State

Markov Policy Functions: Theorems 1 and 2

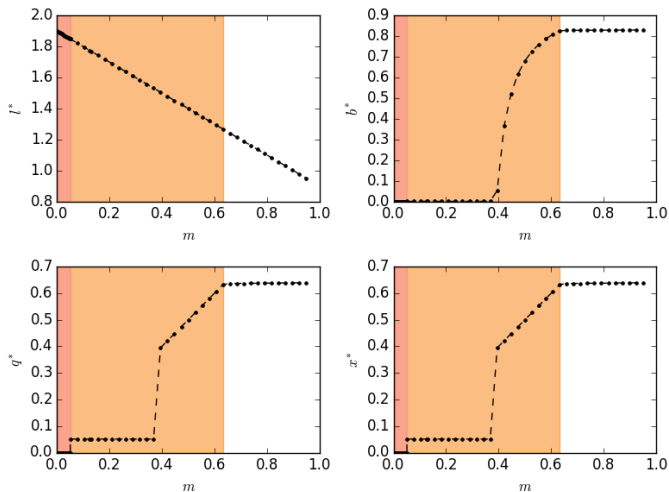
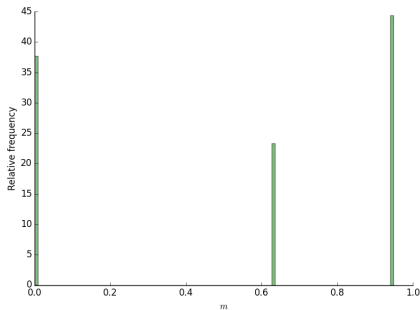


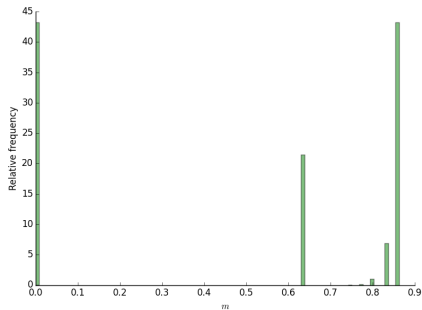
Figure: Markov policy functions with $\tau = 0.1$

Steady State

Money Distribution



(a) $\tau = 0$ (0% inflation)



(b) $\tau = 0.1$ (10% inflation)

Figure: Money distribution under different equilibrium inflation rates

Steady State

Agent sample path: 10% money growth

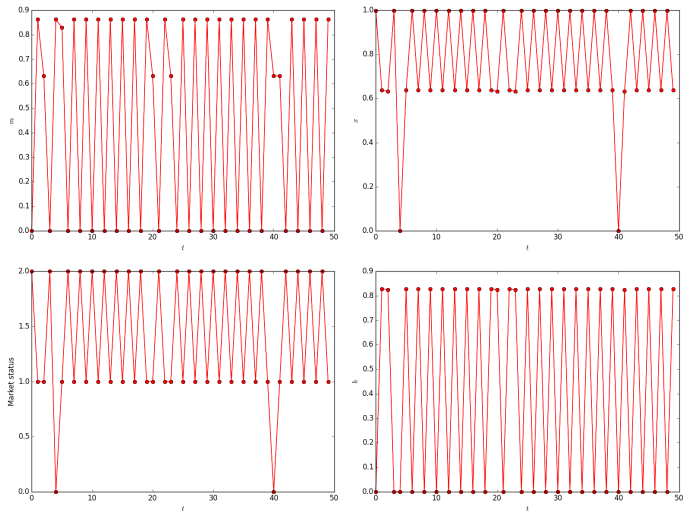


Figure: Agent sample path ($\tau = 0.1$)

Comparative steady states

Intensive vs Extensive Margins

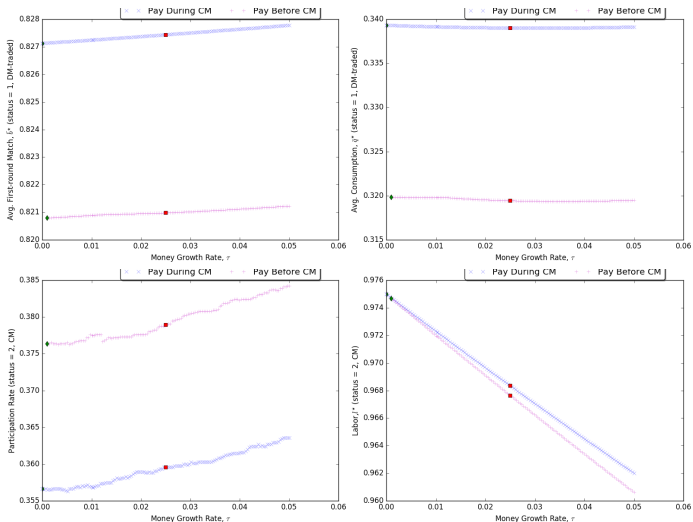


Figure: Extensive margins (left) vs Intensive margins (right)