Inflationary Redistribution vs. Trading Opportunities

Costs of Inflation and Financial Innovation

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Paper: github.com/phantomachine/csim

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Motivation

Yellen (2016) in *Macroeconomic Research After the Crisis*:

> *[O]nce heterogeneity is taken into account, other important channels emerge. For example, spending by many households ... quite sensitive to changes in ... income [... and etc. (sic) ...] that in turn affects their access to credit ...*

> *[S]tudying monetary models with heterogeneous agents **more closely** could help us shed new light on these aspects of the monetary transmission mechanism.*

What We Do

- An integrated New Monetarist Heterogenous Agent Framework:
  - in the spirit of Lagos and Wright (2005, JPE) but with costly choice of Complete Markets participation
  - with tractable heterogeneity from Menzio, Shi and Sun (2014, JET)
  - provable existence of unique non-degenerate agent distro
  - lower computational complexity, c.f., random matching models
- An application: Welfare cost and inequality consequences
  - of ***inflation***?
  - (Corrollary) and of "***financial innovation***"?

Why Cost of Inflation (again)?

Some literature

Taxonomy. Welfare Costs/benefits of inflation

1. Intertemporal distortionary tax, distorting consumption (all models): *raises cost*
2. Precautionary liquidity management activities: *raises cost*
3. Redistributes money wealth from “rich” to “poor”: *lowers cost*

Models:

- İmhoroğlu-Prescott (1991); Kaplan *et al* (2017):
  - #3 reduced heterogeneity, small inflation cost (unless other *ad-hoc/parametric frictions*)
- Lagos-Wright (2005) (quasilinearity + bargaining):
  - kill #3 (RA model) and accentuate #2, large inflation cost
- Molico (2006) (relaxed LW quasilinearity):
  - brings back #3, reduced heterogeneity, smaller inflation cost
- Molico-Chiu (2010) (Costly CM participation): accentuate #2
  - but #3 still strong, reduced heterogeneity, smaller inflation cost

Key Takeaways

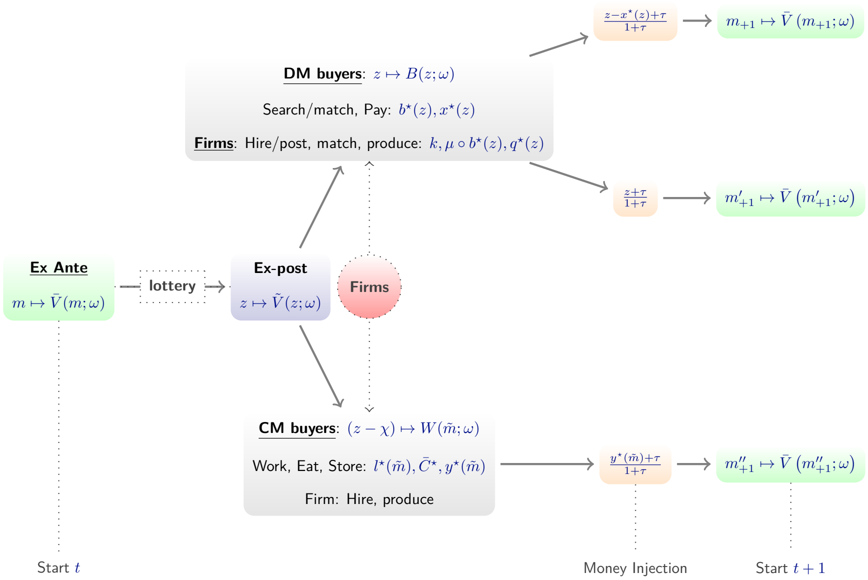
1. **Exists** *a stationary monetary equilibrium*. Equilibrium features *unique* non-degenerate distribution of agents

2. **Economic insight**:
   - **What we already know:** *Redistributive inflation*
     - Rich to poor, but reduces average welfare
     - Mechanism: ***intensive margin*** (all HA models)
   - **What's new:** Search externality affects *trading probabilities*
     - Sufficiently high inflation: ***extensive margins*** (DM **trading opportunities***; CM participation) dominate redistribution effect
     - and result in possible reduction of average welfare
     - a *rise in inequality* of real liquid asset holdings (*contra.* Molico and Chiu, 2010)
   - **"*Financial innovation*"**: raise average welfare, but increase liquid wealth inequality

3. `Python` **Library**: *Open-source*, exploits computational geometry for convexification
   - fast solution: highly parallelized and JIT-able (for structural estimation)
   - more efficient algorithm (with provable convergence properties), c.f., heuristics in Krusell-Smith methods in standard and random-matching HA models

An Heterogeneous Agent New Monetarist Model

Model



Firms in DM | Money supply | In prose

Model

Limiting cases

- Agents must go to CM after DM (no ex-ante participation choice) and $\chi = 0$

  $\Rightarrow$ a version of **Lagos-Wright** (JPE2005) with competitive search

- CM Participation cost $\chi = 0$, no CM utility of consumption, and strictly convex disutility of labor

  $\Rightarrow$ **Menzio-Shi-Sun** (JET2013)

Giants

Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ, is a

- list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- a list of policies $\mathbf{s} \mapsto (l^\star, y^\star, b^\star, x^\star, q^\star, z^\star, \pi^\star)(\mathbf{s})$ supporting the value functions;
- market tightness $\mathbf{s} \mapsto \mu \circ b^\star(\mathbf{s})$ given matching technology μ: firms max. profits;
- an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$G(E) = \int P(\mathbf{s}, E) \mathrm{d}G(\mathbf{s}) \qquad \forall E \in \mathcal{E},$$

  $\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^\star, x^\star, q^\star, z^\star, \pi^\star)$ and $\mu \circ b^\star$under τ; and,
- a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G.

SME: Computation

Incomplete Information Estimation

Method of Simulated Moments

- CM: $U\left(C\right)=\bar{U}_{CM}\frac{C^{1-\sigma_{CM}}-(0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$ and $-A\cdot l$
- DM: $u\left(q\right)=\frac{(q+0.001)^{1-\sigma_{DM}}-(0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- DM matching technology: $\mu(b)=1-b$.

Table: Benchmark estimates

Parameter	Value	Empirical Targets	Description (MSM*)
β	0.99	-	$1/\beta-1\approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized
A	1	-	Normalized
$\bar{m}$	$0.95\times U^{-1}(A)$	-	Sufficient condition (2.3)
χ	$0.132\times\bar{m}=0.125$	Aux reg. (π,M/PY)	Fixed cost, CM participation *
k	0.05103	Aux reg. (π,M/PY)	Fixed cost, trading post *
$\bar{U}_{CM}$	0.002710	Mean Hours ($\frac{1}{3}$)	CM preference scale *

Incomplete Information Estimation

Method of Simulated Moments: Result

Table: Implied Target Estimators

	Auxiliary Stats	Model Stats*	Description (MSM*)
(χ, k)	0.60	0.70	Mean M/PY
	-0.31	-0.21	Aux reg (π,M/PY) slope at mean
$\bar{U}_{CM}$	0.33	0.34	Mean Hours ($\frac{1}{3}$)

Comparative SME: Costs of Inflation

Comparative steady states

Intensive vs Extensive Margins

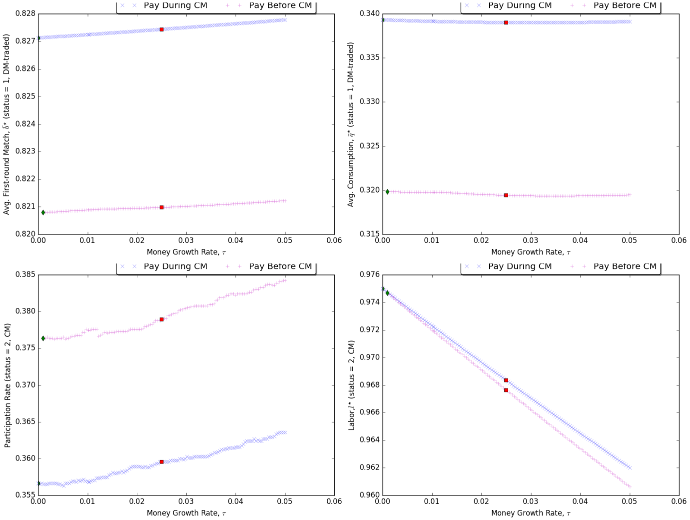


Figure: Extensive margins (left) vs Intensive margins (right)

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$

- With anticipated higher inflation and V concave, *higher* (lower)-balance agents have *lower* (higher) *marginal valuation* of liquid asset
- One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
  - standard insight in HA-CIA (İmrohoroğlu-Prescott, 1991) or HANK (Kaplan-Moll-Violante, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
  - *feature of competitive search*

General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ($\chi > 0$):

- more incentive to trade off q and l for higher DM-match (**DM extensive**) *and CM-participation rates* (**CM extensive**)

Comparative Steady States

Economic Mechanism - numerical result

- So higher anticipated inflation means:
  - ex ante agents participate less in CM, but work more intensively in CM, spend more x in DM to maintain (C, q) (**intensive margins**) ...
  - **vs.** more likely to match b each round of trade with less q; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- With sufficiently high inflation, extensive margins dominates intensive margins
  - Gini inequality may rise with inflation rate (or τ), if paying χ during CM (baseline economy)
  - Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Effect on Distribution

Comparative steady states

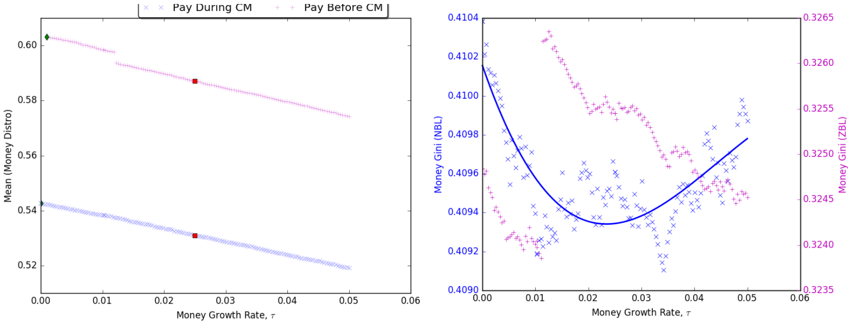


Figure: Distributional consequences

Effect on Welfare

Comparative steady states: Utilitarian Welfare

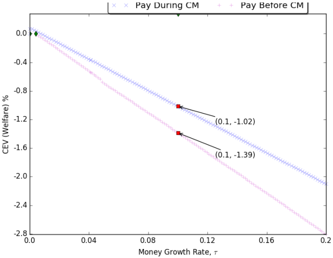


Table: Size matters

Economy	Welfare Cost*	Remarks
CSIM (This Model)	-0.28	$\chi \searrow 0$
	1.02	$\chi = 0.125$
	1.39	$\chi = 0.125$, ZBL
İmrohoroğlu-Prescott (1991, JMCB)	0.90	Bewley-CIA-HA
Chiu-Molico (2010, JME)	0.41	RM-HA
Lagos-Wright (2005, JPE)	1.32	RM-TIOLI-RA
Lucas (2000, ECTA)	0.87	CIA-RA

* *Note*: % CEV, $0\% - 10\%$ inflation p.a.

Comparative SME: "Financial Innovation"

Comparative Steady States

Financial Innovation

Interpretation 1: *more access to CM borrowing*

- From Pay χ *Before CM* ($\Longrightarrow$ ZBL) to *During CM* ($\Longrightarrow$ NBL)

Table: FI #1

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) **ZBL**	0.33	0.22
(After) **NBL**	0.41	0.20

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*

2. **`CSIM Python` Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium

3. With high enough inflation, *extensive margins* (DM **trading opportunities***; CM participation) can dominate redistribution effect; and result in
   - a reduction of average welfare
   - a rise in inequality of money balances (if some can "borrow" to overcome CM participation cost)

4. *"Financial innovation"* raises average welfare, but raises liquid wealth inequality

Back Pocket

On the Shoulders of Giants

Go Back Literature

Baumol-Tobin liquidity channel of monetary transmission:

- Alvarez, Atkeson and Edmond (2009, QJE)
- Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - Exogenous measures of *active* households who can "go to the bank"
- c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- Rocheteau, Weill and Wong (2015, NBER):
  - Undirected search: random matching and bargaining.
  - Exogenous upper bound on labor supply—incomplete insurance in CM.
  - Money is non-neutral and generates redistribution (akin to our intensive margin).
  - c.f., no "extensive margin" trade-off.
- Head, Liu, Menzio, Wright (2012, JEEA):
  - Inflation affects price dispersion only in shifting support of price distribution
  - Money is neutral

Existence of SME with unique agent distribution:

- Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- $\Rightarrow$ Hopenhayn-Prescott (1992, ECTA)

Big Picture

1. Each period, agent market participation:
   - *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
   - *worker/consumer* in **CM:** work; precautionary asset (money); neoclassical Walrasian market
2. **Matching friction** in **DM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings
   - competitive firms create trading posts (some cost); post terms of trades
   - buyers direct their search decisions to these trading posts
   - Equilibrium spending cycle
     - liquid wealth accumulation at different rates (Baumol-Tobin)
     - monetary policy has real effects via *uneven equilibrium liquidity demands*
3. **Costly participation** in the Walrasian **CM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
4. Equilibrium trade-offs:
   - **Intensive margins**: quantities in both markets
   - **Extensive margins**: participation rates in **CM**; matching prob. **DM**

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Model

Fiat money supply, measurement units

- Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- Labor is the numeraire good.
  - Denote ωM as the nominal wage rate.
  - A dollar's worth of money is equivalent to $1/\omega M$.
- $1/\omega$: per-capita real money balance (in labor units)
  - If M: initial aggregate stock of money
  - Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*
- Nominal wage growth rate γ:

$$\gamma(\tau) \equiv \omega_+ M_+ / \omega M$$

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Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m, \omega)$:

$$W(\mathbf{s}) = \max_{(C,l,y)\in\mathbb{R}^3_+} \left\{ U(C) - A \cdot l + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- takes ω as given
- $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1+\tau)}$

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Model

***Ex-post* DM buyer**

Buyer named $\mathbf{s} := (m, \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0,m], q \in \mathbb{R}_+} \left\{ \beta \left[1 - b(x,q)\right] \left[\bar{V}\left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right.$$
$$\left. + \, b(x,q) \left[u(q) + \beta \bar{V}\left(\frac{\omega(m-x)+\tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets $(\tilde{x}, \tilde{q})$ where $\tilde{x} - \tilde{q} \leq m \leq k$
  - For *this* submarket, optimal to choose $b^{\star}(\mathbf{s}) = x^{\star}(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V}\left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right)$

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Model

Ex-ante market participation decisions

Ex-ante value of fair lottery $(\pi_1, 1-\pi_1)$ over prizes $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1-\pi_1)\tilde{V}(z_2, \omega) : \right.$$
$$\left. \pi_1 z_1 + (1-\pi_1) z_2 = m \right\},$$

where *ex-post* value is

$$\tilde{V}(z, \omega) = \begin{cases} \max\{W(z-\chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega; \tau) := \begin{cases} \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} & \text{if pay } \chi \text{ in CM} \\ A \in \left[0, \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\}\right) & \text{if pay } \chi \text{ before entering CM} \end{cases}$$

Model

Firms, matching, submarket location in DM

Matching technology:

- Sellers and buyers: $s(\mathbf{s}) = \mu\left[b(\mathbf{s})\right]$, given buyer type: $\mathbf{s} := (m;\omega)$
  - $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

Expected profit-max. decision rule:

- Create trading posts $(x,q)(\mathbf{s})$, and labor hiring from competitive CM market:
- $$\mu\left[b(\mathbf{s})\right]\left[x(\mathbf{s}) - q(\mathbf{s})\right] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

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SME properties

Bounded labor implies NBL

- Sufficient condition (primitives):

$$\exists \bar{m} \in \left(0, U_1^{-1}(A)\right) \iff A < U_1(\bar{m}) < U_1(0)$$

- Implies **bounded** and **interior** *optimal labor choice*:

$$l^{\star}(m, \omega) = \bar{y}^{\star}(\omega) - m + \bar{C}^{\star} \in (0, l_{\mathsf{max}}(\omega, \tau))$$

where

$$\begin{aligned} l_{\mathsf{max}}(\omega, \tau) := y_{\mathsf{max}}\ (\omega; \tau) - 0 + \bar{C}^{\star} \\ = \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} + U^{-1}(A) < 2U^{-1}(A) \end{aligned}$$

- Equivalent interpretation: $y_{\mathsf{max}}\ (\omega; \tau)$ is a **N**atural **B**orrowing **L**imit.

Go Back

SME properties

Theorem 1 (CM policies and value)

1. $W(\cdot,\omega) \in \mathcal{V}[0,\bar{m}]$, i.e., it is continuous, increasing and concave on $[0,\bar{m}]$. Moreover, it is linear on $[0,\bar{m}]$.
2. The partial derivative functions $W_1(\cdot,\omega)$ and $\bar{V}_1(\cdot,\omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left(\frac{\omega y^\star(m,\omega)+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)\begin{cases}\leq A, & y^\star(m,\omega)\geq 0\\ \geq A, & y^\star(m,\omega)\leq y_{\max}(\omega;\tau)\end{cases},$$

   and the envelop condition:

$$W_1(m,\omega)=A,$$

3. The stationary Markovian policy rules $y^\star(\cdot,\omega)$ and $l^\star(\cdot,\omega)$ are scalar-valued and continuous functions on $[0,\bar{m}]$.
   - The function $y^\star(\cdot,\omega)$, is constant valued on $[0,\bar{m}]$.
   - The optimizer $l^\star(\cdot,\omega)$ is an affine and decreasing function on $[0,\bar{m}]$.
   - Moreover, for every (m,ω), the optimal choice $l^\star(m,\omega)$ is interior.

SME properties

Theorem 2 (DM policies and value)

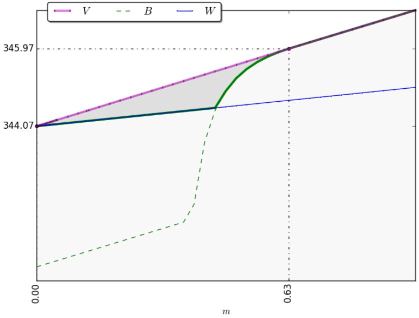
1. Fo any $\bar{V}(\cdot,\omega_{+1}) \in \mathcal{V}[0,\bar{m}]$, the DM buyer's value function is increasing and continuous in m: $B(\cdot;\omega) \in \mathcal{C}[0,\bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^\star(m,\omega) = x^\star(m,\omega) = q^\star(m,\omega) = 0$, and $B(m,\omega) = \beta\bar{V}[\phi(m,\omega),\omega_{+1}]$, where $\phi(m,\omega) := (\omega m + \tau)/[\omega_{+1}(1+\tau)]$.
3. At any (m,ω), where $m \in [k,\bar{m}]$ and $b^\star(m,\omega) > 0$:
   - $(x^\star, b^\star, q^\star)(m,\omega)$ are unique, continuous, and increasing in m.
   - $B_1(m,\omega)$ exists if and only if $\bar{V}_1[\phi(m,\omega),\omega]$ exists.
   - $B(m,\omega)$ is strictly increasing in m.
   - $b^\star$and $x^\star$, respectively, satisfy the FoCs

$$
\begin{aligned}
u \circ Q\left[x^\star(m,\omega), b^\star(m,\omega)\right] + b^\star(m,\omega)\left(u \circ Q\right)_2\left[x^\star(m,\omega), b^\star(m,\omega)\right] \\
= \beta\left[\bar{V}\left(\phi\left(m,\omega\right),\omega_{+1}\right) - \bar{V}\left(\phi^\star\left(m,\omega\right),\omega_{+1}\right)\right],
\end{aligned}
$$

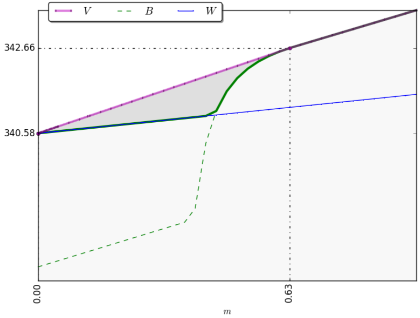
$$
\left(u \circ Q\right)_1\left[x^\star(m,\omega), b^\star(m,\omega)\right] = \frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left[\phi^\star(m,\omega),\omega_{+1}\right].
$$

SME

Value Functions (Theorems 1 and 2)



(a) $\tau = 0$ (0% inflation)



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(b) $\tau = 0.1$ (10% inflation)

Figure: Value functions from two economies.

Steady State

Markov Policy Functions: Theorems 1 and 2

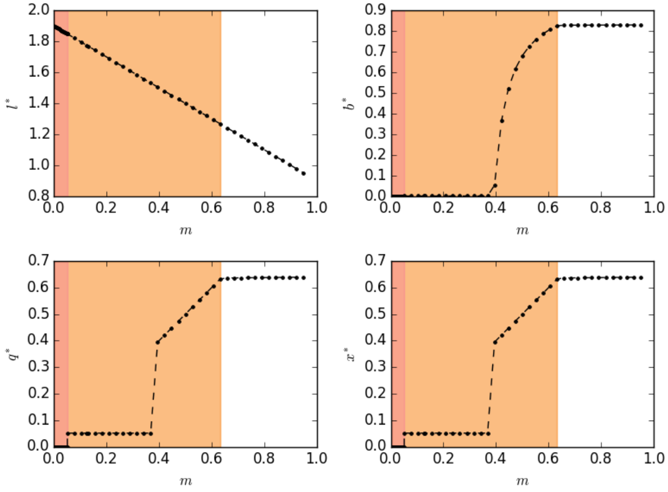
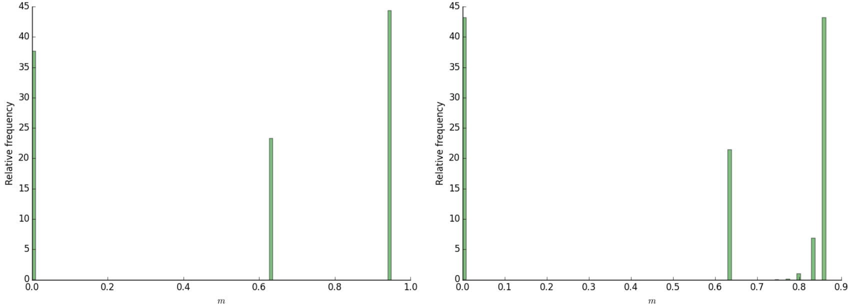


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Money Distribution



(a) $\tau = 0$ (0% inflation) (b) $\tau = 0.1$ (10% inflation)

Figure: Money distribution under different equilibrium inflation rates

Steady State

Agent sample path: 10% money growth

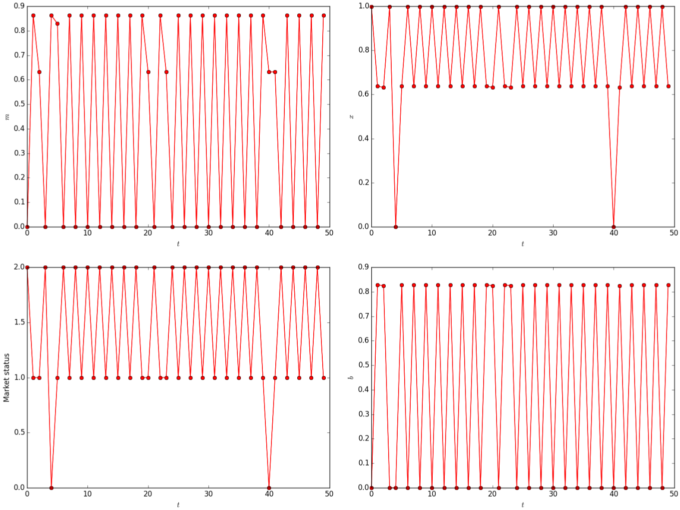


Figure: Agent sample path ($\tau = 0.1$)