Inflationary Redistribution vs. Trading Opportunities

with Competitive Search Incomplete Markets

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Question

Welfare cost and inequality effects of inflation?

... in a heterogenous agent monetarist model?

- Endogenous Baumol-Tobin + Segmented Asset Markets model
- Role of market participation frictions and search externality
  - Equilibrium trade-off in *intensive* vs. *extensive* margins w.r.t. inflation policy

What We Do

- A competitive search model of money with *ex-post* heterogeneous agents:
  - Endogenous Baumol-Tobin liquidity channel (Menzio, Shi, Sun JET2013)
    - augmented with limited centralized (asset) market participation
    - We study inflationary equilibria to quantify welfare and inequality
- Characterize *stationary monetary equilibrium* (SME)
  - existence of SME with unique non-degenerate distribution of agents
- A computational geometry solution to nonconvexities arising from matching externalities
- Compute steady state equilibrium as function of money supply growth

Model: Big Picture

1. Ex-ante agents identical. Ex post, agents realize different wealth outcomes
2. Each period, each agent decides which market to enter:
   - A *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
   - a *worker/consumer* in **CM:** work; precautionary asset (money); neoclassical Walrasian market.
3. **Matching friction** in **DM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
   - competitive firms create trading posts (some cost); post terms of trades
   - buyers direct their search decisions to these trading posts.
4. **Equilibrium spending cycle** — wealth accumulation at different rates (Baumol-Tobin)
   - monetary policy affect the real economy, through affecting agents' *uneven equilibrium liquidity demands*
5. **Costly participation** in the Walrasian **CM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
6. Equilibrium trade-offs:
   - **Intensive margins**: quantities in both markets
   - **Extensive margins**: participation rates in **CM**; matching prob. **DM**

CSIM

Competitive **S**earch **I**ncomplete **M**arkets

a Heterogeneous Agent New Monetarist model

Model: Big Picture

Intersecting special model classes

- Agents must go to CM after DM (no ex-ante participation choice) and $\chi = 0$

  $\Rightarrow$ **Lagos-Wright** (JPE2005) with competitive search, or **Rocheteau-Wright** (ECTA2008)

- CM Participation cost $\chi \nearrow \infty$ and strictly convex disutility of labor

  $\Rightarrow$ a version of **Menzio-Shi-Sun** (JET2013)

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*
2. `CSIM Python` **Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium
3. With inflation, *extensive margins* (DM **trading opportunities***; CM participation) tend to dominate redistribution effect; and result in
   - a reduction of average welfare
   - a rise in inequality of money balances (if some can "borrow" to overcome CM participation cost)

Model

Fiat money supply, measurement units

- Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- Labor is the numeraire good.
  - Denote ωM as the nominal wage rate.
  - A dollar's worth of money is equivalent to $1/\omega M$.
- $1/\omega$: per-capita real money balance (in labor units)
  - If M: initial aggregate stock of money
  - Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*
- Nominal wage growth rate γ:

$$\gamma(\tau) \equiv \omega_+ M_+ / \omega M$$

Model

Agents and Firms in DM

Agents (when in DM):

- Finite *set of types* of individuals and firms/goods, I
- Measure-one continuum of individuals of type $i \in I$
- An individual consumes good i and produces good $(i+1)$ mod-$|I|$

Firm (in DM) chooses:

- how many trading posts in submarket $(x, q)(\mathbf{s})$, and labor hiring
- $\mathbf{s} := (m; \omega)$
  - each trading post costs k units of labor to create
  - zero expected profit

Model

Firms, matching, submarket location in DM

Matching technology:

- Sellers' matching probability and buyers': $s(\mathbf{s}) = \mu\left[b(\mathbf{s})\right]$
  - $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

Expected profit-max. decision rule:

$$\mu\left[b(\mathbf{s})\right]\left[x(\mathbf{s}) - q(\mathbf{s})\right] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

- Implies quantity traded at a submarket with positive market tightness:

$$q(\mathbf{s}) = Q\left[(x, b)(\mathbf{s})\right] \equiv x(\mathbf{s}) - \frac{k}{\mu\left[b(\mathbf{s})\right]}$$

Model

Ex-ante Individuals

Consider *ex-post* agent named $\mathbf{s} := (m; \omega)$

- If *DM buyer*, action is
  - *where to shop* $\equiv$ *how much to pay* and *eat* $(x, q)(\mathbf{s})$
- If *CM consumer/worker/saver*,
  - how much to work $l(\mathbf{s})$ and accumulate money balance $y(\mathbf{s})$ (precautionary)

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m; \omega)$:

$$W(\mathbf{s}) = \max_{(C,l,y) \in \mathbb{R}^3_+} \left\{ U(C) - A \cdot l + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- takes ω as given
- $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1+\tau)}$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m; \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0,m], q \in \mathbb{R}_+} \left\{ \beta \left[1 - b\left(x, q\right)\right] \left[\bar{V} \left(\frac{\omega m + \tau}{\omega_{+1}\left(1+\tau\right)}, \omega_{+1} \right) \right] + b\left(x, q\right) \left[u(q) + \beta \bar{V} \left(\frac{\omega\left(m - x\right) + \tau}{\omega_{+1}\left(1+\tau\right)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets $(\tilde{x}, \tilde{q})$ where $\tilde{x} - \tilde{q} \leq m \leq k$
  - For *this* submarket, optimal to choose $b^{\star}(\mathbf{s}) = x^{\star}(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right)$

Model

Ex-ante market participation decisions

Ex-ante value of playing a fair lottery $(\pi_1, 1-\pi_1)$ over the prizes $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \Big\{ \pi_1 \tilde{V}(z_1, \omega) + (1-\pi_1)\tilde{V}(z_2, \omega) : \\ \pi_1 z_1 + (1-\pi_1) z_2 = m \Big\},$$

where *ex-post* value is non-concave in lottery realization z:

$$\tilde{V}(z, \omega) = \begin{cases} \max\{W(z-\chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega; \tau) := \begin{cases} \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} & \text{if NBL} \\ A \in \left[0, \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\}\right) & \text{if ad-hoc BL} \end{cases}$$

Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ, is a

- list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- a list of policies $\mathbf{s} \mapsto (l^\star, y^\star, b^\star, x^\star, q^\star, z^\star, \pi^\star)(\mathbf{s})$ supporting the value functions;
- market tightness $\mathbf{s} \mapsto \mu \circ b^\star(\mathbf{s})$ given matching technology μ: firms max. profits;
- an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$T(G)(E) = \int P(\mathbf{s}, \mathcal{E}) \mathrm{d}G(\mathbf{s}) \qquad \forall E \in \mathcal{E},$$

  $\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^\star, x^\star, q^\star, z^\star, \pi^\star)$ and $\mu \circ b^\star$under τ; and,
- a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME: Existence w/ unique distro

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G.

SME: Computation

Computation

Parametrization

- $U(C) = \bar{U}_{CM} \frac{(C+0.001)^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$
- $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- $\mu(b) = 1 - b$.

Table: Baseline parameterization

Parameter	Value	Unit	Description
β	0.99	-	Quarterly interest rate of $1/\beta - 1 \approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized DM risk aversion
A	1	-	Scale on labor disutility function
$\bar{m}$	$0.95 \times U^{-1}(A)$	labor	See assumption (2.3)
χ	$0.1 \times \bar{m} = 0.095$	labor	Fixed cost of CM participation
α	0	-	Probability of costless CM entry
k	0.05	labor	Fixed cost of creating a trading post
$\bar{U}_{CM}$	0.002	-	CM preference scale parameter

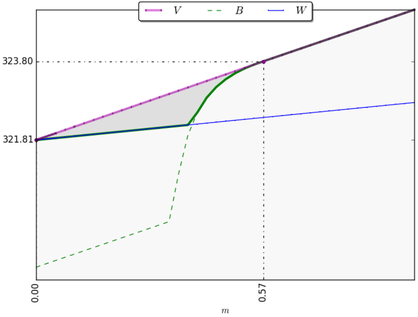
Computation

- Program runs in `Python` 2.7/3.6
  - with optional `OpenMPI` parallelization capabilities via `mpi4py` library
- Finding a steady state equilibrium:
  - $\sim$90 seconds on workstation running on 18 parallel cores

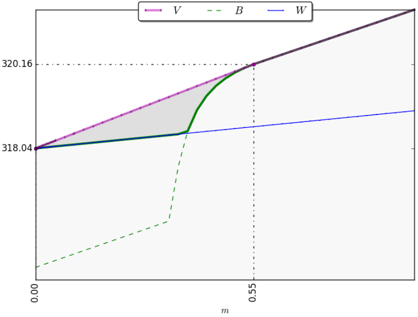
SME: Results

Steady State

Value Functions: zero money growth



(a) $\tau = 0$ (0% inflation)



(b) $\tau = 0.1$ (10% inflation)

Figure: Value functions from two economies.

Steady State

Markov Policy Functions: 10% money growth

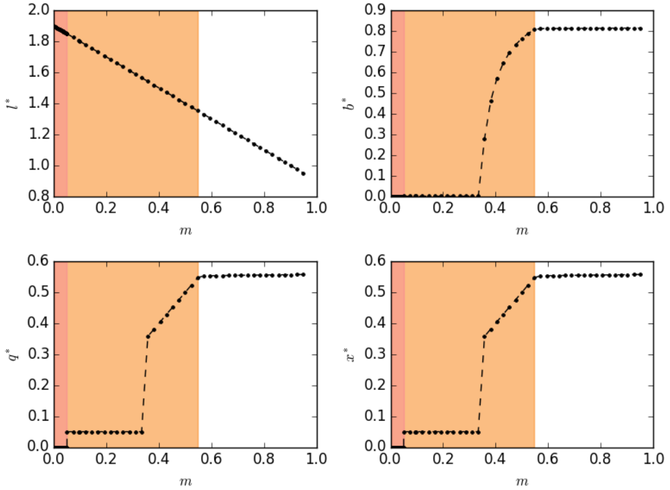
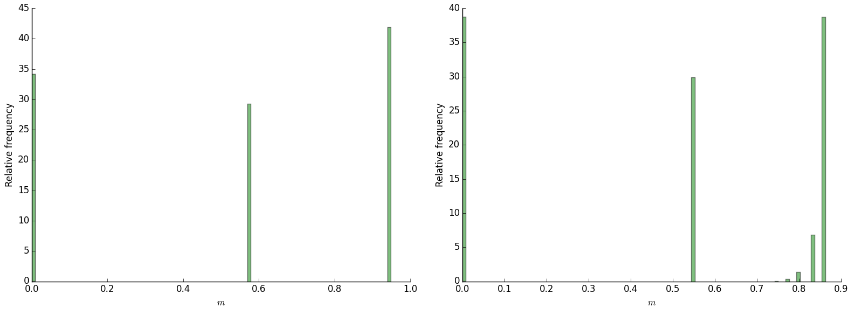


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Money Distribution



(a) $\tau = 0$ (0% inflation) (b) $\tau = 0.1$ (10% inflation)

Figure: Money distribution under different equilibrium inflation rates

Comparative SME: Results

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$

- Near-zero limited **CM** asset market participation
- **DM** poses "virtually only" trading friction ...
- **DM Search externality:** intensive-extensive trade-off
  - given b, if buyer pays more x, then more q consumable
  - given x, buyer wanting greater trading opportunity b, eats less q

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$ (cont'd)

- With anticipated higher inflation and V concave, *lower* (higher) *marginal valuation* of money to *high* (low)-balance agents
- One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
  - standard insight in HA-CIA (Imhoroglu-Prescott, 1991) or HANK (Kaplan-Violante-Moll, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
  - *feature of competitive search*

General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ($\chi > 0$):

- more incentive to trade off q and l for higher DM-match (**DM extensive**) *and CM-participation rates* (**CM extensive**)

Comparative Steady States

Economic Mechanism - numerical result

- So higher anticipated inflation means:
  - ex ante agents participate less in CM, but work more intensively in CM, spend more x in DM to maintain (C, q) (**intensive margins**) ...
  - **vs.** more likely to match b each round of trade with less q; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- Extensive margins dominates intensive margins
  - Gini inequality rises with inflation rate (or τ), if *natural borrowing limit* for paying χ
  - Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Effect on DM

Comparative steady states

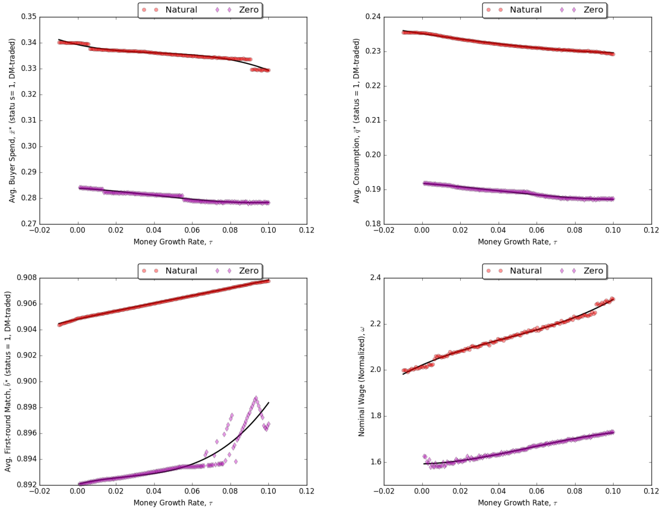


Figure: Comparative steady states — DM allocations

Effect on Distribution

Comparative steady states

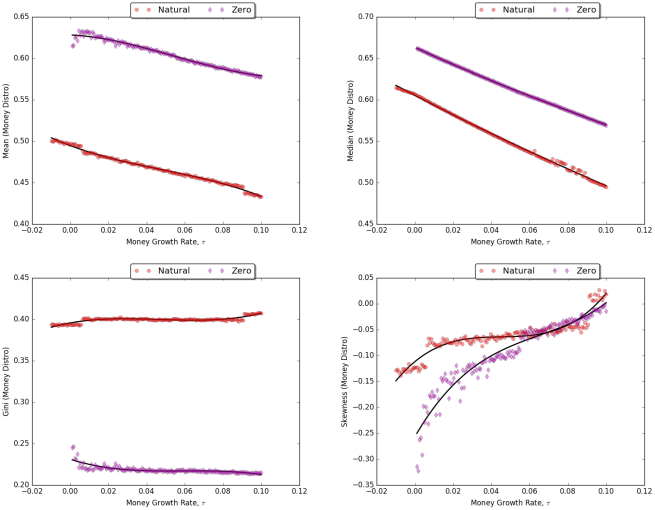


Figure: Comparative steady states — distribution

Effect on Welfare

Comparative steady states: Utilitarian Welfare

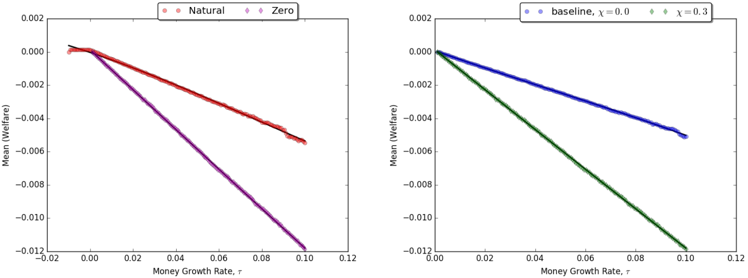


Figure: 2 — 2.4% consumption loss p.a. (0 — 10% inflation)

Key Takeaways

```
from csim import basemod as model
```

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2. `CSIM Python` **Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium

3. With inflation, *extensive margins* (DM **trading opportunities***; CM participation) tend to dominate redistribution effect; and result in
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   - a rise in inequality of money balances (if some can "borrow" to overcome CM participation cost)

Back Pocket

On the Shoulders of Giants

Baumol-Tobin liquidity channel of monetary transmission:

- Alvarez, Atkeson and Edmond (2009, QJE)
  - Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - Exogenous measures of *active* households who can "go to the bank"
- c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- Rocheteau, Weill and Wong (2015, NBER):
  - Undirected search with random matching and bargaining.
  - Exogenous upper bound on labor supply—incomplete insurance in CM.
  - Money is non-neutral and generates redistribution (akin to our intensive margin).
  - c.f., no "extensive margin" trade-off.
- Head, Liu, Menzio, Wright (2012, JEEA):
  - Inflation affects price dispersion only in shifting support of price distribution
  - Money is neutral

Existence of SME with unique agent distribution:

- Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- $\Rightarrow$ Hopenhayn-Prescott (1992, ECTA)

SME properties

Theorem 1 (CM policies and value)

1. $W(\cdot,\omega) \in \mathcal{V}[0,\bar{m}]$, i.e., it is continuous, increasing and concave on $[0,\bar{m}]$. Moreover, it is linear on $[0,\bar{m}]$.
2. The partial derivative functions $W_1(\cdot,\omega)$ and $\bar{V}_1(\cdot,\omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left(\frac{\omega y^\star(m,\omega)+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)\begin{cases}\leq A, & y^\star(m,\omega)\geq 0\\ \geq A, & y^\star(m,\omega)\leq y_{\max}(\omega;\tau)\end{cases},$$

   and the envelop condition:

$$W_1(m,\omega)=A,$$

3. The stationary Markovian policy rules $y^\star(\cdot,\omega)$ and $l^\star(\cdot,\omega)$ are scalar-valued and continuous functions on $[0,\bar{m}]$.
   - The function $y^\star(\cdot,\omega)$, is constant valued on $[0,\bar{m}]$.
   - The optimizer $l^\star(\cdot,\omega)$ is an affine and decreasing function on $[0,\bar{m}]$.
   - Moreover, for every (m,ω), the optimal choice $l^\star(m,\omega)$ is interior.

SME properties

Theorem 2 (DM policies and value)

1. For any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m: $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^\star(m, \omega) = x^\star(m, \omega) = q^\star(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω), where $m \in [k, \bar{m}]$ and $b^\star(m, \omega) > 0$:
    - $(x^\star, b^\star, q^\star)(m, \omega)$ are unique, continuous, and increasing in m.
    - $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
    - $B(m, \omega)$ is strictly increasing in m.
    - $b^\star$and $x^\star$, respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q[x^\star(m,\omega), b^\star(m,\omega)] + b^\star(m,\omega)\,(u \circ Q)_2[x^\star(m,\omega), b^\star(m,\omega)] \\ = \beta\left[\bar{V}(\phi(m,\omega), \omega_{+1}) - \bar{V}(\phi^\star(m,\omega), \omega_{+1})\right], \end{aligned}$$

$$(u \circ Q)_1[x^\star(m,\omega), b^\star(m,\omega)] = \frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1[\phi^\star(m,\omega), \omega_{+1}].$$

Steady State

Agent sample path: 10% money growth

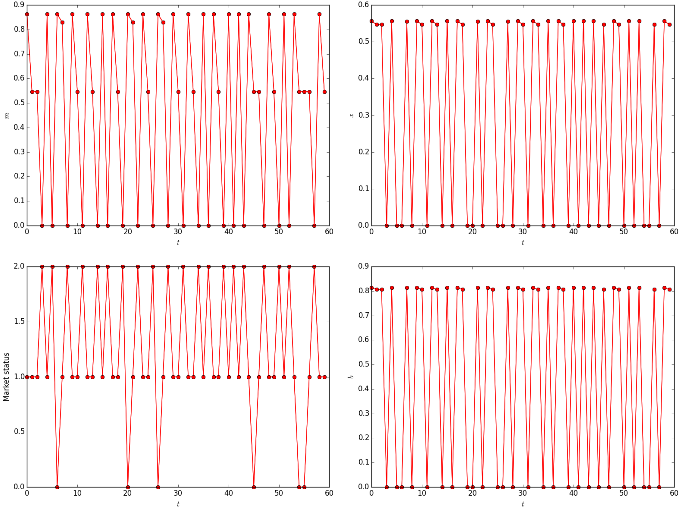

Figure: Agent sample path ($\tau = 0.1$)