Inflationary Redistribution vs. Trading Opportunities

Costs of Inflation and Financial Innovation

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Paper: github.com/phantomachine/csim

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Motivation

Yellen (2016) in *Macroeconomic Research After the Crisis*:

> *[O]nce heterogeneity is taken into account, other important channels emerge. For example, spending by many households ... quite sensitive to changes in ... income [... and etc. (sic) ...] that in turn affects their access to credit ...*

> *[S]tudying monetary models with heterogeneous agents* ***more closely*** *could help us shed new light on these aspects of the monetary transmission mechanism.*

Cost of Inflation

Some literature

Taxonomy. Welfare Costs/benefits of inflation

1. Intertemporal distortionary tax, distorting consumption (all models): *raises cost*
2. Precautionary liquidity management activities: *raises cost*
3. Redistributes money wealth from "rich" to "poor": *lowers cost*

Models:

- Lagos-Wright (2005) (quasilinearity + bargaining):
  - kill #3 (RA model) and accentuate #2, large inflation cost
- Molico (2006) (relaxed LW quasilinearity):
  - brings back #3, reduced heterogeneity, smaller inflation cost
- Molico-Chiu (2010) (Costly CM participation): accentuate #2
  - but #3 still strong, reduced heterogeneity, smaller inflation cost

Question

What happens to welfare cost

- of ***inflation***?
- (Corrollary) and of "***financial innovation***"?

when agents

1. do not choose to always participate in complete-insurance markets (CM), and,
2. face *new trade-off*: extensive (trading probability) vs. intensive (terms of trade) in frictional trades (DM)?

Key Takeaways

1. **Exists** *stationary monetary equilibrium* with *unique* non-degenerate distribution of agents

2. `CSIM Python` **Library**: *Open-source*, exploits computational geometry for convexification, fast solution: highly parallelized and JIT-able (for structural estimation)

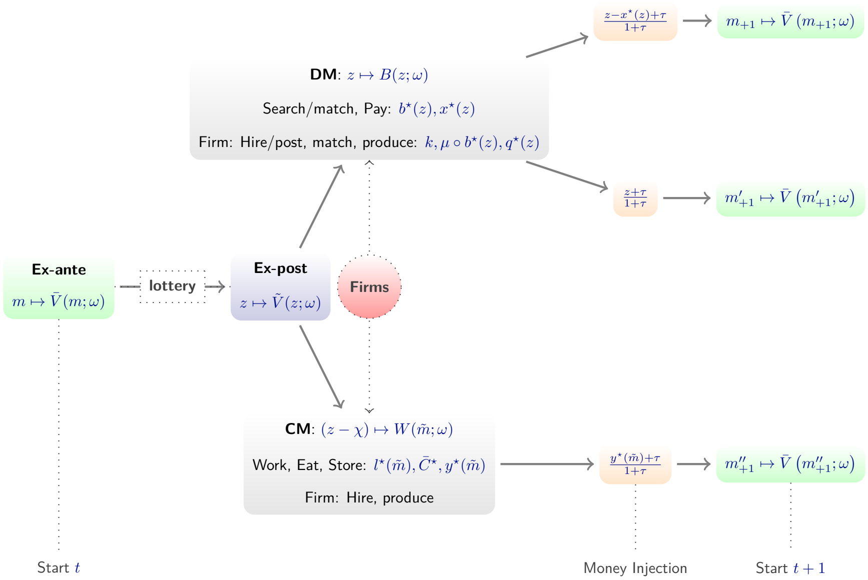
3. **Economic insight**:
   - **What we already know:** *Redistributive inflation*
     - Rich to poor, but reduces average welfare.
     - Mechanism: ***intensive margin*** (all HA models):
     - CIA and HANK (Imhoroglu and Prescott, 1992; Kaplan *et al*, 2017)
     - Random-matching (Molico, 2006; Molico and Chiu, 2010)
   - **What's new:** Search externality affects *trading probabilities*
     - Sufficiently high inflation: ***extensive margins*** (DM **trading opportunities***; CM participation) dominate redistribution effect
     - and result in possible reduction of average welfare
     - a *rise in inequality* of real liquid asset holdings (*contra.* Molico and Chiu, 2010)
   - "***Financial innovation***": raise average welfare, but increase liquid wealth inequality

CSIM

Competitive **S**earch **I**ncomplete **M**arkets

a Heterogeneous Agent New Monetarist model

Model



Firms in DM Money supply In prose

Model

- Agents must go to CM after DM (no ex-ante participation choice) and $\chi = 0$

  $\Rightarrow$ **Lagos-Wright** (JPE2005) with competitive search, or **Rocheteau-Wright** (ECTA2008)

- CM Participation cost $\chi = 0$, no CM utility of consumption, and strictly convex disutility of labor

  $\Rightarrow$ **Menzio-Shi-Sun** (JET2013)

Giants

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m, \omega)$:

$$W(\mathbf{s}) = \max_{(C,l,y)\in\mathbb{R}^3_+} \left\{ U(C) - A \cdot l + \beta \bar{V}\left(\mathbf{s}_{+1}\right) : pC + y \leq m + l \right\},$$

- takes ω as given
- $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1+\tau)}$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m, \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0,m], q \in \mathbb{R}_+} \left\{ \beta \left[1 - b(x,q)\right] \left[\bar{V}\left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right.$$
$$\left. + \, b(x,q) \left[u(q) + \beta \bar{V}\left(\frac{\omega(m-x) + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets $(\tilde{x}, \tilde{q})$ where $\tilde{x} - \tilde{q} \leq m \leq k$
  - For *this* submarket, optimal to choose $b^{\star}(\mathbf{s}) = x^{\star}(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V}\left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right)$

Model

Ex-ante market participation decisions

Ex-ante value of fair lottery $(\pi_1, 1-\pi_1)$ over prizes $\{z_1, z_2\}$

$$\bar{V}(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1-\pi_1) \tilde{V}(z_2, \omega) : \right.$$

$$\left. \pi_1 z_1 + (1-\pi_1) z_2 = m \right\},$$

where *ex-post* value is

$$\tilde{V}(z,\omega) = \begin{cases} \max\left\{ W(z-\chi,\omega), B(z,\omega) \right\}, & z-\chi \geq -y_{\max}(\omega;\tau) \\ B(z,\omega), & \text{otherwise} \end{cases},$$

and,

$$y_{\max}(\omega;\tau) := \begin{cases} \min\left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\} & \text{if pay } \chi \text{ in CM} \\ A \in \left[0, \min\left\{ \bar{m} - \frac{\tau}{\omega}, \bar{m} \right\}\right) & \text{if pay } \chi \text{ before entering CM} \end{cases}$$

Model

Stationary Monetary Equilibrium

Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ, is a

- list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- a list of policies $\mathbf{s} \mapsto (l^\star, y^\star, b^\star, x^\star, q^\star, z^\star, \pi^\star)(\mathbf{s})$ supporting the value functions;
- market tightness $\mathbf{s} \mapsto \mu \circ b^\star(\mathbf{s})$ given matching technology μ: firms max. profits;
- an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

  $$G(E) = \int P(\mathbf{s}, E)\mathrm{d}G(\mathbf{s}) \qquad \forall E \in \mathcal{E},$$

  $\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^\star, x^\star, q^\star, z^\star, \pi^\star)$ and $\mu \circ b^\star$under τ; and,
- a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G.

SME: Computation

Computation

- Program runs in Python 2.7/3.6
  - with optional OpenMPI parallelization capabilities
- Finding a steady state equilibrium
  - $\sim$90 seconds on workstation running on 18 parallel cores

Incomplete Information Estimation

Method of Simulated Moments

- CM: $U(C) = \bar{U}_{CM} \frac{C^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$ and $-A \cdot l$
- DM: $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- DM matching technology: $\mu(b) = 1 - b$.

Table: Benchmark estimates

Parameter	Value	Empirical Targets	Description (MSM*)
β	0.99	-	$1/\beta - 1 \approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized
A	1	-	Normalized
$\bar{m}$	$0.95 \times U^{-1}(A)$	-	Sufficient condition (2.3)
χ	$0.132 \times \bar{m} = 0.125$	Aux reg. (π,M/PY)	Fixed cost, CM participation *
k	0.05103	Aux reg. (π,M/PY)	Fixed cost, trading post *
$\bar{U}_{CM}$	0.002710	Mean Hours ($\frac{1}{3}$)	CM preference scale *

Incomplete Information Estimation

Method of Simulated Moments: Result

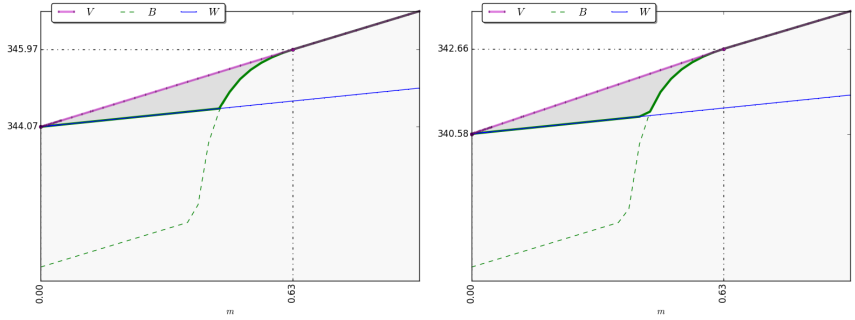
Table: Implied Target Estimators

	Auxiliary Stats	**Model Stats***	**Description (MSM*)**
(χ, k)	0.60	0.70	Mean M/PY
	-0.31	-0.21	Aux reg (π,M/PY) slope at mean
$\bar{U}_{CM}$	0.33	0.34	Mean Hours ($\frac{1}{3}$)

SME: Results

SME

Value Functions (Theorems 1 and 2)



(a) $\tau = 0$ (0% inflation)   (b) $\tau = 0.1$ (10% inflation)

Figure: Value functions from two economies.

Steady State

Markov Policy Functions: Theorems 1 and 2

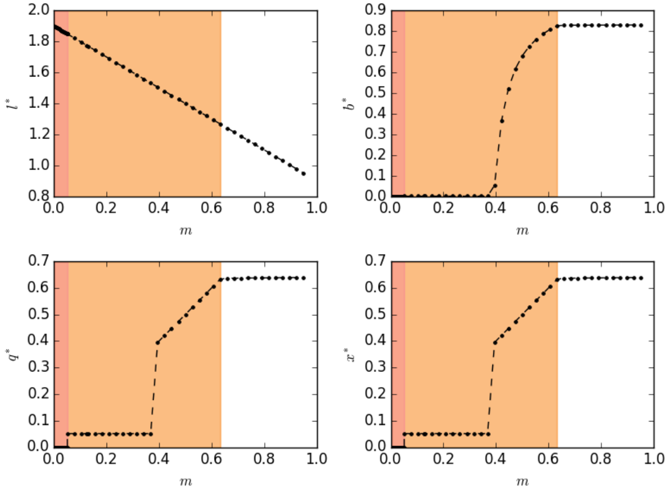
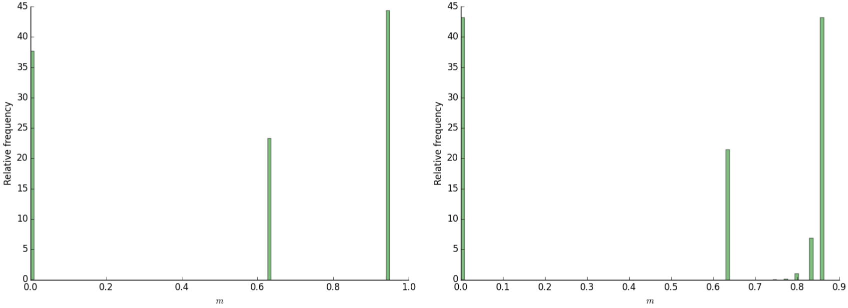


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Money Distribution



(a) $\tau = 0$ (0% inflation) (b) $\tau = 0.1$ (10% inflation)

Figure: Money distribution under different equilibrium inflation rates

Comparative SME: Costs of Inflation

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$

- Near-zero limited **CM** asset market participation
- **DM** poses "virtually only" trading friction ...
- **DM Search externality:** intensive-extensive trade-off
  - given b, if buyer pays more x, then more q consumable
  - given x, buyer wanting greater trading opportunity b, eats less q

Comparative Steady States

Economic Mechanism (a.k.a intuition)

Limiting case, $\chi \searrow 0$ (cont'd)

- With anticipated higher inflation and V concave, *higher* (lower)-balance agents have *lower* (higher) *marginal valuation* of liquid asset
- One hand (**DM or CM intensive** margin): inflation, a redistribution of wealth from high-agents to low agents
  - standard insight in HA-CIA (İmrohoroğlu-Prescott, 1991) or HANK (Kaplan-Moll-Violante, 2016), and in HA-random-matching (Molico, 2006; Chiu-Molico 2014).
- Other hand (**DM extensive** margin): inflation raises the downside risk of agents not getting matched in DM
  - *feature of competitive search*

General case, $\chi > 0$

Now, combine with non-zero **CM participation cost** ($\chi > 0$):

- more incentive to trade off q and l for higher DM-match (**DM extensive**) *and CM-participation rates* (**CM extensive**)

Comparative Steady States

Economic Mechanism - numerical result

- So higher anticipated inflation means:
  - ex ante agents participate less in CM, but work more intensively in CM, spend more x in DM to maintain (C, q) (**intensive margins**) ...
  - **vs.** more likely to match b each round of trade with less q; and participate in CM more, but work less intensively (opposite force: **extensive margin**s)
- With sufficiently high inflation, extensive margins dominates intensive margins
  - Gini inequality may rise with inflation rate (or τ), if paying χ during CM (baseline economy)
  - Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Comparative steady states

Intensive vs Extensive Margins

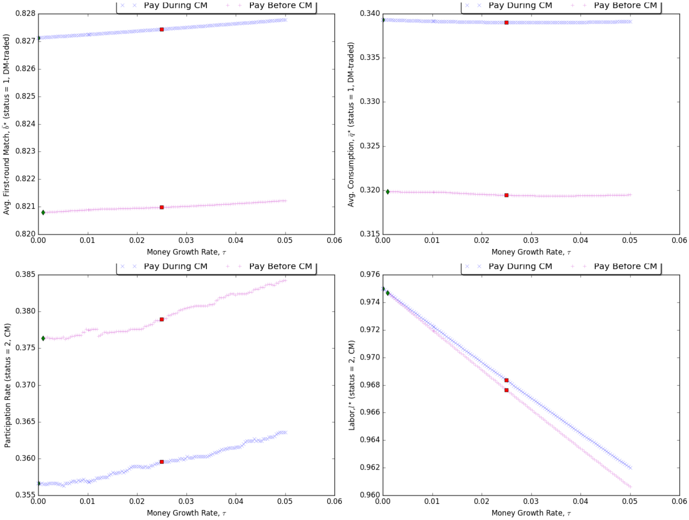


Figure: Extensive margins (left) vs Intensive margins (right)

Effect on Distribution

Comparative steady states

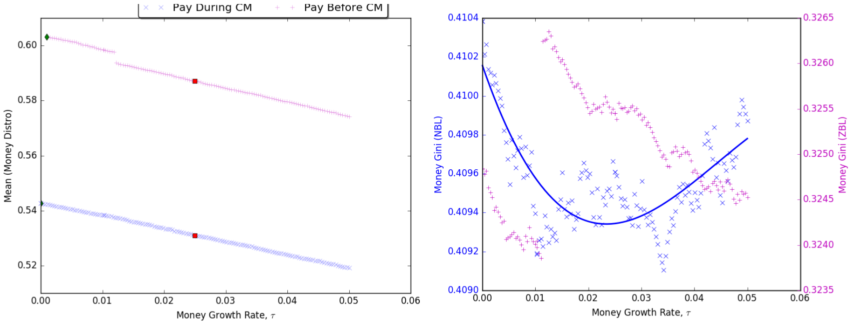


Figure: Distributional consequences

Effect on Welfare

Comparative steady states: Utilitarian Welfare

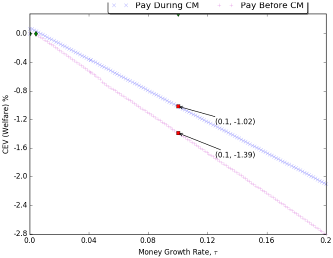


Table: Size matters

Economy	Welfare Cost*	Remarks
CSIM (This Model)	-0.28	$\chi = 0.001$
	1.02	$\chi = 0.125$
	1.39	$\chi = 0.125$, ZBL
İmrohoroğlu-Prescott (1991, JMCB)	0.90	Bewley-CIA-HA
Chiu-Molico (2010, JME)	0.41	RM-HA
Lagos-Wright (2005, JPE)	1.32	RM-TIOLI-RA
Lucas (2000, ECTA)	0.87	CIA-RA

* *Note*: % CEV, $0\% - 10\%$ inflation p.a.

Comparative SME: “Financial Innovation”

Comparative Steady States

Financial Innovation

Interpretation 1: *more access to CM borrowing*

- From Pay χ *Before CM* ($\Longrightarrow$ ZBL) to *During CM* ($\Longrightarrow$ NBL)

Table: FI #1

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) **ZBL**	0.33	0.22
(After) **NBL**	0.41	0.20

Interpretation 2: *lower fixed cost of CM participation*

- From $\chi > 0$ to $\chi = 0$, fix: Pay χ before CM ($\Longrightarrow$ ZBL)

Table: FI #2

Economy	Gini (Liquid Assets)	Gini (Consumption)
(Before) **ZBL**	0.33	0.22
(After) **ZBL** but $\chi = 0$	0.41	0.22

Key Takeaways

1. **Existence** of *stationary monetary equilibrium* with *unique non-degenerate distribution*

2. `CSIM` `Python` **Library**: An *open-source computational-geometry method* to quickly solve for a monetary equilibrium

3. With high enough inflation, *extensive margins* (DM **trading opportunities***; CM participation) can dominate redistribution effect; and result in
   - a reduction of average welfare
   - a rise in inequality of money balances (if some can "borrow" to overcome CM participation cost)

4. *"Financial innovation"* raises average welfare, but raises liquid wealth inequality

Quantitative Next Steps

- Not all trades use "money"—proportion of money exchanges small
  - some exchanges supported by credit and contracts
  - claims on capital may further exacerbate inequality effects
- Re-consider welfare cost of inflation, by accounting for welfare along:
  - deterministic transition (with J. Lee, L. Wang and A. Sun)
  - stochastic transitions (business cycles) (with J. Lee, L. Wang, and A. Sun)

  Qantitative answer may differ from comparative steady state analysis.

Back Pocket

On the Shoulders of Giants

Go Back Literature

Baumol-Tobin liquidity channel of monetary transmission:

- Alvarez, Atkeson and Edmond (2009, QJE)
- Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - Exogenous measures of *active* households who can "go to the bank"
- c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- Rocheteau, Weill and Wong (2015, NBER):
  - Undirected search: random matching and bargaining.
  - Exogenous upper bound on labor supply—incomplete insurance in CM.
  - Money is non-neutral and generates redistribution (akin to our intensive margin).
  - c.f., no "extensive margin" trade-off.
- Head, Liu, Menzio, Wright (2012, JEEA):
  - Inflation affects price dispersion only in shifting support of price distribution
  - Money is neutral

Existence of SME with unique agent distribution:

- Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- $\Rightarrow$ Hopenhayn-Prescott (1992, ECTA)

Big Picture

1. Each period, agent market participation:
    - *buyer* — search in **DM**: decentralised (anonymity); competitive search; matching frictions, or,
    - *worker/consumer* in **CM:** work; precautionary asset (money); neoclassical Walrasian market
2. **Matching friction** in **DM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings
    - competitive firms create trading posts (some cost); post terms of trades
    - buyers direct their search decisions to these trading posts
    - Equilibrium spending cycle
        - liquid wealth accumulation at different rates (Baumol-Tobin)
        - monetary policy has real effects via *uneven equilibrium liquidity demands*
3. **Costly participation** in the Walrasian **CM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
4. Equilibrium trade-offs:
    - **Intensive margins**: quantities in both markets
    - **Extensive margins**: participation rates in **CM**; matching prob. **DM**

Go Back

Model

Fiat money supply, measurement units

- Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- Labor is the numeraire good.
  - Denote ωM as the nominal wage rate.
  - A dollar's worth of money is equivalent to $1/\omega M$.
- $1/\omega$: per-capita real money balance (in labor units)
  - If M: initial aggregate stock of money
  - Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*
- Nominal wage growth rate γ:

$$\gamma(\tau) \equiv \omega_+ M_+ / \omega M$$

Go Back

Model

Firms, matching, submarket location in DM

Matching technology:

- Sellers and buyers: $s(\mathbf{s}) = \mu\left[b(\mathbf{s})\right]$, given buyer type: $\mathbf{s} := (m; \omega)$
  - $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$

Expected profit-max. decision rule:

- Create trading posts $(x, q)(\mathbf{s})$, and labor hiring from competitive CM market:
- $$\mu\left[b(\mathbf{s})\right]\left[x(\mathbf{s}) - q(\mathbf{s})\right] - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

Go Back

SME properties

Bounded labor implies NBL

- Sufficient condition (primitives):

$$\exists \bar{m} \in \left(0, U_1^{-1}(A)\right) \iff A < U_1(\bar{m}) < U_1(0)$$

- Implies **bounded** and **interior** *optimal labor choice*:

$$l^\star(m, \omega) = \bar{y}^\star(\omega) - m + \bar{C}^\star \in (0, l_{\max}(\omega, \tau))$$

where

$$\begin{aligned} l_{\max}(\omega, \tau) &:= y_{\max}\ (\omega; \tau) - 0 + \bar{C}^\star \\ &= \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\} + U^{-1}(A) < 2U^{-1}(A) \end{aligned}$$

- Equivalent interpretation: $y_{\max}\ (\omega; \tau)$ is a **N**atural **B**orrowing **L**imit.

Go Back

SME properties

Theorem 1 (CM policies and value)

1. $W(\cdot,\omega) \in \mathcal{V}[0,\bar{m}]$, i.e., it is continuous, increasing and concave on $[0,\bar{m}]$. Moreover, it is linear on $[0,\bar{m}]$.
2. The partial derivative functions $W_1(\cdot,\omega)$ and $\bar{V}_1(\cdot,\omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left(\frac{\omega y^{\star}(m,\omega)+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)\begin{cases}\leq A, & y^{\star}(m,\omega)\geq 0\\ \geq A, & y^{\star}(m,\omega)\leq y_{\max}(\omega;\tau)\end{cases},$$

   and the envelop condition:

$$W_1(m,\omega) = A,$$

3. The stationary Markovian policy rules $y^{\star}(\cdot,\omega)$ and $l^{\star}(\cdot,\omega)$ are scalar-valued and continuous functions on $[0,\bar{m}]$.
   - The function $y^{\star}(\cdot,\omega)$, is constant valued on $[0,\bar{m}]$.
   - The optimizer $l^{\star}(\cdot,\omega)$ is an affine and decreasing function on $[0,\bar{m}]$.
   - Moreover, for every (m,ω), the optimal choice $l^{\star}(m,\omega)$ is interior.

SME properties

Theorem 2 (DM policies and value)

1. Fo any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m: $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^\star(m, \omega) = x^\star(m, \omega) = q^\star(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω), where $m \in [k, \bar{m}]$ and $b^\star(m, \omega) > 0$:
   - $(x^\star, b^\star, q^\star)(m, \omega)$ are unique, continuous, and increasing in m.
   - $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
   - $B(m, \omega)$ is strictly increasing in m.
   - $b^\star$and $x^\star$, respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q\left[x^\star(m,\omega), b^\star(m,\omega)\right] + b^\star(m,\omega)\left(u \circ Q\right)_2\left[x^\star(m,\omega), b^\star(m,\omega)\right] \\ = \beta\left[\bar{V}\left(\phi\left(m,\omega\right), \omega_{+1}\right) - \bar{V}\left(\phi^\star\left(m,\omega\right), \omega_{+1}\right)\right], \end{aligned}$$

$$(u \circ Q)_1\left[x^\star(m,\omega), b^\star(m,\omega)\right] = \frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left[\phi^\star(m,\omega), \omega_{+1}\right].$$

Steady State

Agent sample path: 10% money growth

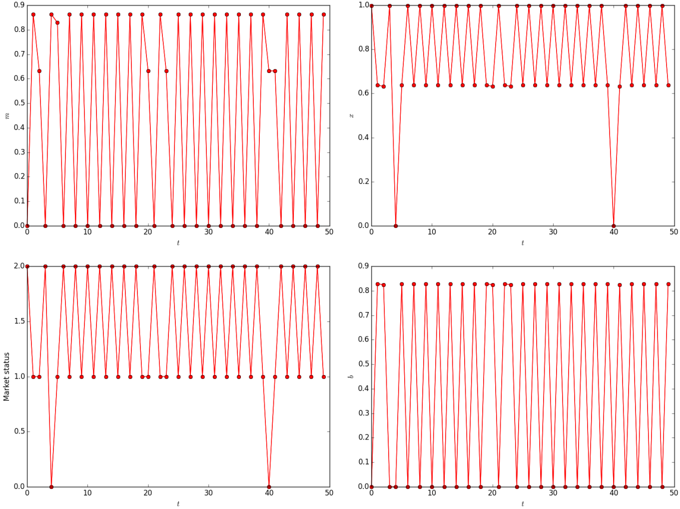


Figure: Agent sample path ($\tau = 0.1$)