Inflationary Redistribution vs. Trading Opportunities

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Paper: github.com/phantomachine/csim

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Question

How big is the welfare cost and inequality effects of inflation?

... in a heterogenous agent monetarist model?

- A competitive (directed) search approach to incomplete markets—endogenous Baumol-Tobin model
- Role of market frictions and search externality
- Equilibrium trade-off in *intensive* vs. *extensive* margins w.r.t. inflation policy
- Consequence for average welfare and inequality of wealth (money holdings)
- A more flexible quantitative model with endogenous limited financial markets participation
  - can be taken to data

What We Do

- A directed search model of money and *ex-post* heterogeneous agents:
  - Endogenous Baumol-Tobin liquidity channel
    - Menzio, Shi, Sun (MSS2013, JET): deterministic steady state;
    - no aggregate dynamics, charaterize zero inflation monetary equilibrium
    - pointers to more work ... inflation, dynamics
- Characterize *stationary monetary equilibrium* (SME)
  - existence of SME with unique non-degenerate distribution of agents
- A computational geometry solution to nonconvexities arising from matching externalities
- Compute various non-stochastic steady state equilibrium regimes over different monetary policies

Shoulders of Giants

On the Shoulders of Giants

Baumol-Tobin liquidity channel of monetary transmission:

- Alvarez, Atkeson and Edmond (2009, QJE)
  - Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - Exogenous measures of *active* households who can "go to the bank"
- c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- Rocheteau, Weill and Wong (2015, NBER):
  - Undirected search with random matching and bargaining.
  - Exogenous upper bound on labor supply—incomplete insurance in CM.
  - Money is non-neutral and generates redistribution (akin to our intensive margin).
  - c.f., no "extensive margin" trade-off.
- Head, Liu, Menzio, Wright (2012, JEEA):
  - Inflation affects price dispersion only in shifting support of price distribution
  - Money is neutral

Existence of SME with unique agent distribution:

- Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- $\Rightarrow$ Hopenhayn-Prescott (1992, ECTA)

Model: Big Picture

1. Ex-ante agents identical. Ex post, agents realize different wealth outcomes
2. Each period, each agent decides who to be:
   - An *anonymous buyer* who – search in **DM**: decentralised; competitive search; matching frictions, or,
   - a *worker/consumer* – **CM:** work; asset; neoclassical Walrasian market.
3. **Matching friction** in **DM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
   - competitive firms create trading posts (some cost); post terms of trades
   - buyers direct their search decisions to these trading posts.
4. **Equilibrium spending cycle** — wealth accumulation at different rates (Baumol-Tobin)
   - monetary policy affect the real economy, through affecting agents' *uneven equilibrium liquidity demands*
5. **Costly participation** in the Walrasian **CM** $\Rightarrow$ *ex-post* agent **heterogeneity** of money holdings.
6. Equilibrium trade-offs:
   - **Intensive margins**: quantities in both markets
   - **Extensive margins**: participation rates in **CM**; matching prob. **DM**

Model

Fiat money supply, measurement units

- Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- $1/\omega$: per-capita real money balance (in labor units)
  - If M: initial aggregate stock of money
  - Then: $1/\omega = M \times 1/\omega M$ initial (per-capita) stock of money, *measured in units of labor*

Model

Agents and Goods in DM

- Finite *set of types* of individuals and firms/goods, I
- Measure-one continuum of individuals of type $i \in I$
- An individual consumes good i and produces good $(i+1)$ mod-$|I|$

Model

Firms, matching, submarket location in DM

- Firm chooses:
  - how many trading posts in submarket $(x, q)(\mathbf{s})$ and labor hiring, given wage 1, as function of who visits post.
    - "who" indexed by current state of a buyer's money balance (units of equiv. labor) and aggregate money supply growth rate: $\mathbf{s} := (m; \omega)$
  - each trading post costs k units of labor to create
  - zero expected profit

Model

Firms, matching, submarket location in DM

- **Matching technology:**
  - Exists bijective relation between sellers' matching probability and buyers': $s(\mathbf{s}) = \mu\left[b(\mathbf{s})\right]$
  - Trading post's matching probability (market tightness), $\mu(b)$: $\mu' < 0$ and $\mu'' < 0$
- Decision rule (equilibrium), at each fixed $\mathbf{s}$:

$$\mu(b)(x-q)-k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

- Implies quantity traded at a submarket with positive market tightness:

$$q(\mathbf{s}) = Q\left[(x,b)(\mathbf{s})\right] \equiv x(\mathbf{s}) - \frac{k}{\mu(b(\mathbf{s}))}$$

Model

Ex-ante Individuals

- *ex-ante* decide to be buyer/worker, depending on current state of individual money balance (units of equiv. labor) and aggregate money supply growth rate: $\mathbf{s} := (m; \omega)$
- *ex-post*, if *buyer*, then:
  - decides on *where to shop* $\equiv$ *how much to pay* and *eat* $(x, q)(\mathbf{s})$
- *ex-post*, if *worker*, then:
  - decides how much to work $l^{\star}(\mathbf{s})$

Model

Ex-post CM worker/consumer

Worker named $\mathbf{s} := (m; \omega)$:

$$W(\mathbf{s}) = \max_{(C,l,y)\in\mathbb{R}^3_+} \left\{ U(C) - h(l) + \beta \bar{V}\left(\mathbf{s}_{+1}\right) : pC + y \leq m + l \right\},$$

- takes ω as parametric
- $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1+\tau)}$

Model

Change of decision variables in DM (equilibrium)

- Trading post's matching probability a convex and decreasing function of buyer's matching probability: $\mu(b)$
- Implies in any equilibrium, each active trading post trades in:

$$q(\mathbf{s}) = Q\left[(x,b)(\mathbf{s})\right] \equiv x(\mathbf{s}) - \frac{k}{\mu(b(\mathbf{s}))}$$

Model

Ex-post DM buyer

Buyer named $\mathbf{s} := (m; \omega)$:

$$B(\mathbf{s}) = \max_{x \in [0,m], q \in \mathbb{R}_+} \left\{ \beta \left[1 - b(x,q) \right] \left[\bar{V} \left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right.$$
$$\left. + \, b(x,q) \left[u(q) + \beta \bar{V} \left(\frac{\omega (m - x) + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right) \right] \right\}.$$

- If $m \leq k$, buyer can only afford to visit submarkets $(\tilde{x}, \tilde{q})$ where $\tilde{x} - \tilde{q} \leq m \leq k$
  - For *this* submarket, optimal to choose $b^{\star}(\mathbf{s}) = x^{\star}(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left(\frac{\omega m + \tau}{\omega_{+1}(1+\tau)}, \omega_{+1} \right)$

Model

Individuals: ex-ante individual

At start of date:

- Let $\tilde{V}(\mathbf{s}) = \max\Big\{B(\mathbf{s}), W(m-\chi, \omega)\Big\}$
- **Lemma**: $\tilde{V}_m$ exists for all $m > 0$
- But $\tilde{V}$ not concave ...
  - Exists profitable solution where ex-ante agent can randomize between pure strategies $\{buyer, seller\}$
  - Next we describe fair lotteries

Model

Individuals: convexifying ex-ante preferences

Recall ex-ante value:

$$\bar{V}(\mathbf{s}) = \alpha W(\mathbf{s}) + (1-\alpha) V(\mathbf{s}),$$

where $V(\mathbf{s})$ is the value of playing a fair lottery $(\pi_1, 1-\pi_1)$ over the prizes $\{z_1, z_2\}$, i.e.,

$$V(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1-\pi_1) \tilde{V}(z_2, \omega) : \pi_1 z_1 + (1-\pi_1) z_2 = m \right\};$$

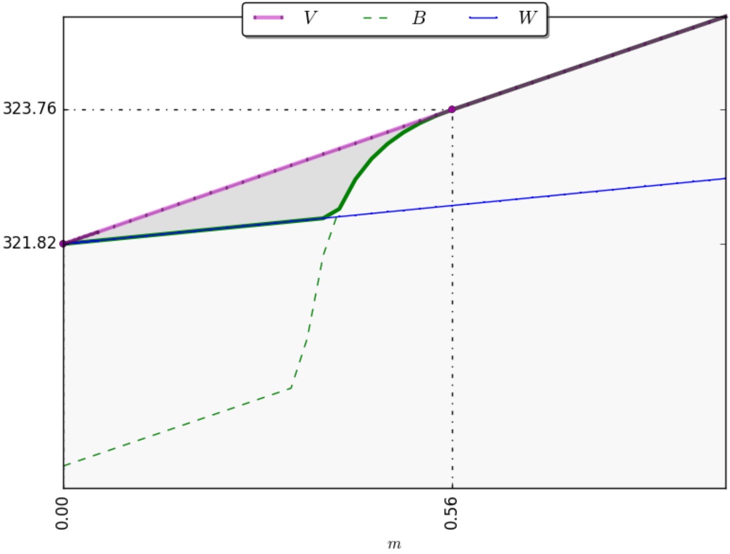
and, given a lottery outcome z, the individual's value becomes

$$\tilde{V}(z, \omega) = \begin{cases} \max\{W(z-\chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

where $y_{\max}(\omega; \tau) := \min\left\{\bar{m} - \frac{\tau}{\omega}, \bar{m}\right\}$

Model

Individuals: ex-ante individual



Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy τ, is a

- list of value functions $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$, satisfying the Bellman functionals ...
- a list of policies $\mathbf{s} \mapsto (l^\star, y^\star, b^\star, x^\star, q^\star, z^\star, \pi^\star)(\mathbf{s})$ supporting the value functions;
- market tightness $\mathbf{s} \mapsto \mu \circ b^\star(\mathbf{s})$ given matching technology μ: firms max. profits;
- an ergodic distribution of wealth $G(\mathbf{s})$ s.t.

$$T(G)(E) = \int P(\mathbf{s}, \mathcal{E})\mathrm{d}G(\mathbf{s}) \qquad \forall E \in \mathcal{E},$$

  $\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$ is a Markov kernel induced by $(l^\star, x^\star, q^\star, z^\star, \pi^\star)$ and $\mu \circ b^\star$under τ; and,
- a wage rate function $\mathbf{s} \mapsto \omega(\mathbf{s})$ satisfying labor market clearing

SME: Existence w/ unique distro

SME

Theorem 1 (CM policies and value)

1. $W(\cdot,\omega) \in \mathcal{V}[0,\bar{m}]$, i.e., it is continuous, increasing and concave on $[0,\bar{m}]$. Moreover, it is linear on $[0,\bar{m}]$.
2. The partial derivative functions $W_1(\cdot,\omega)$ and $\bar{V}_1(\cdot,\omega_{+1})$ exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1\left(\frac{\omega y^\star(m,\omega)+\tau}{\omega_{+1}(1+\tau)},\omega_{+1}\right)\begin{cases}\leq A, & y^\star(m,\omega)\geq 0\\ \geq A, & y^\star(m,\omega)\leq y_{\max}(\omega;\tau)\end{cases},$$

   and the envelop condition:

$$W_1(m,\omega)=A,$$

3. The stationary Markovian policy rules $y^\star(\cdot,\omega)$ and $l^\star(\cdot,\omega)$ are scalar-valued and continuous functions on $[0,\bar{m}]$.
   - The function $y^\star(\cdot,\omega)$, is constant valued on $[0,\bar{m}]$.
   - The optimizer $l^\star(\cdot,\omega)$ is an affine and decreasing function on $[0,\bar{m}]$.
   - Moreover, for every (m,ω), the optimal choice $l^\star(m,\omega)$ is interior.

SME

Theorem 2 (DM policies and value)

1. For any $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$, the DM buyer's value function is increasing and continuous in m: $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$.
2. For any $m \leq k$, optimal decisions are $b^{\star}(m, \omega) = x^{\star}(m, \omega) = q^{\star}(m, \omega) = 0$, and $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$, where $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$.
3. At any (m, ω), where $m \in [k, \bar{m}]$ and $b^{\star}(m, \omega) > 0$:
   - $(x^{\star}, b^{\star}, q^{\star})(m, \omega)$ are unique, continuous, and increasing in m.
   - $B_1(m, \omega)$ exists if and only if $\bar{V}_1[\phi(m, \omega), \omega]$ exists.
   - $B(m, \omega)$ is strictly increasing in m.
   - $b^{\star}$and $x^{\star}$, respectively, satisfy the FoCs

$$
\begin{aligned}
u \circ Q[x^{\star}(m, \omega), b^{\star}(m, \omega)] + b^{\star}(m, \omega)\,(u \circ Q)_2[x^{\star}(m, \omega), b^{\star}(m, \omega)] \\
= \beta\left[\bar{V}(\phi(m, \omega), \omega_{+1}) - \bar{V}(\phi^{\star}(m, \omega), \omega_{+1})\right],
\end{aligned}
$$

$$
(u \circ Q)_1[x^{\star}(m, \omega), b^{\star}(m, \omega)] = \frac{\beta}{1 + \tau}\left(\frac{\omega}{\omega_{+1}}\right)\bar{V}_1[\phi^{\star}(m, \omega), \omega_{+1}].
$$

SME

Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution G.

SME

- **Lemmata**:
  - **(L1):** Equilibrium policy functions are monotone in m, given τ fixed, bounded on m
  - **(L2):** $\varphi(\cdot|\tau_{-1})$ is an exogenous increasing Markov kernel, bounded in probability: $\mathbb{E}|(\tau)| < +\infty$
    - (Meyn and Tweedie, Ch. 12)
  - **(L1 and L2) gives (L3):** Therefore equilibrium Markov kernel $P(\mathbf{s}, \cdot)$ is increasing and bounded in probability
  - **(L4):** Natural state space $S := [0, \bar{m}(\tau)] \times [0, +\infty) \in (m; \tau)$ has least and greatest element
  - **(L5):** For any $(P, \mathbf{s})$-Markov process,
    - there is a $c \in S$ such that $\Pr\{\mathbf{s}_t \leq c\} > 0$, and,
    - a $d \in S$ such that $\Pr\{\mathbf{s}_t \geq d\} > 0$
- **Theorem:** Kamihigashi and Stachurski (TE, 2014, Theorem 3.1):
  - (L3)-(L5) implies stability result.

SME Behavior

Qualitative: Focus on DM

- Fixing money supply growth τ:
  - Buyer with higher m spends more $x^{\star}(m;\omega)$ in each submarket to optimize on match probability
    - with positive probability every period until $m = 0$.
    - all subsequent submarket matches exhibit lower (x, q)
- A higher τ will speed up the buyer's spending in each spending round, and lowers worker's $l^{\star}(m;\omega)$

SME: Computation

Computation

Parametrization

- $U(C) = \bar{U}_{CM} \frac{(C+0.001)^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$
- $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- $\mu(b) = 1 - b.$

Table: Baseline parameterization

Parameter	Value	Unit	Description
β	0.99	-	Quarterly interest rate of $1/\beta - 1 \approx 1\%$
σ_{CM}	2	-	CM risk aversion
σ_{DM}	1.01	-	Normalized DM risk aversion
A	1	-	Scale on labor disutility function
$\bar{m}$	$0.95 \times U^{-1}(A)$	labor	See assumption (2.3)
χ	$0.1 \times \bar{m} = 0.095$	labor	Fixed cost of CM participation
α	0	-	Probability of costless CM entry
k	0.05	labor	Fixed cost of creating a trading post
$\bar{U}_{CM}$	0.002	-	CM preference scale parameter

Computation

- Program runs in Python 2.7/3.0
  - with optional OpenMPI parallelization capabilities
- Finding a steady state equilibrium:
  - $\sim$90 seconds on workstation running on 18 parallel cores

SME: Results

Steady State

Value Functions: zero money growth

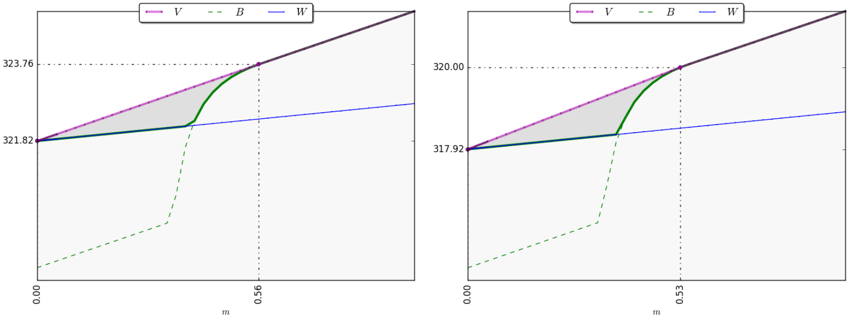


Figure: Value functions from two economies.

Steady State

Markov Policy Functions: 10% money growth

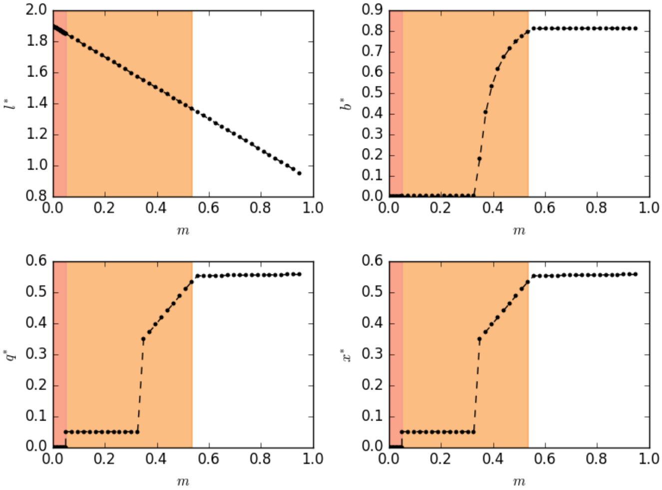


Figure: Markov policy functions with $\tau = 0.1$

Steady State

Agent sample path: 10% money growth

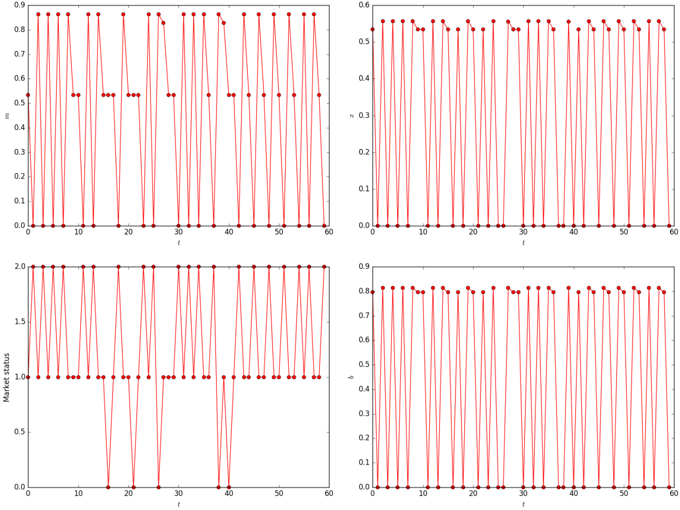


Figure: Agent sample path ($\tau = 0.1$)

Steady State

Money Distribution

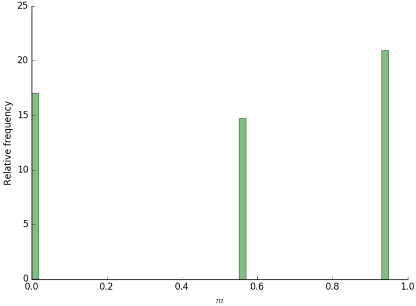


Figure: Money distribution under different equilibrium inflation rates

Comparative SME: Preliminary Results

Effect on Inflation

Comparative steady states: Equilibrium wage

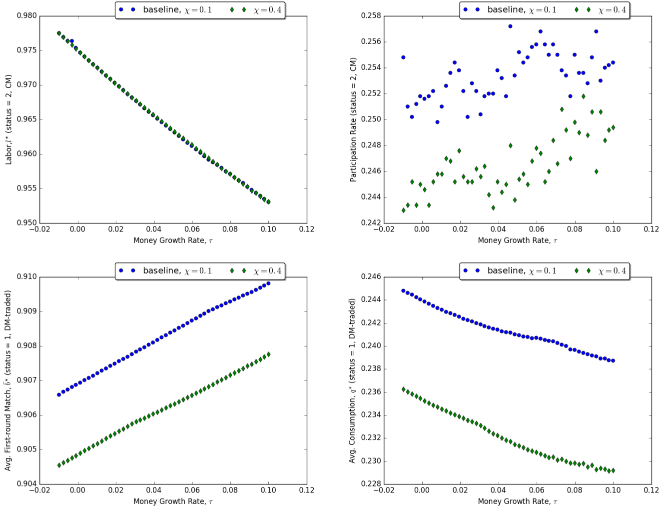


Figure: Comparative steady states — allocations

Effect on Inflation

Comparative steady states: (τ, χ)

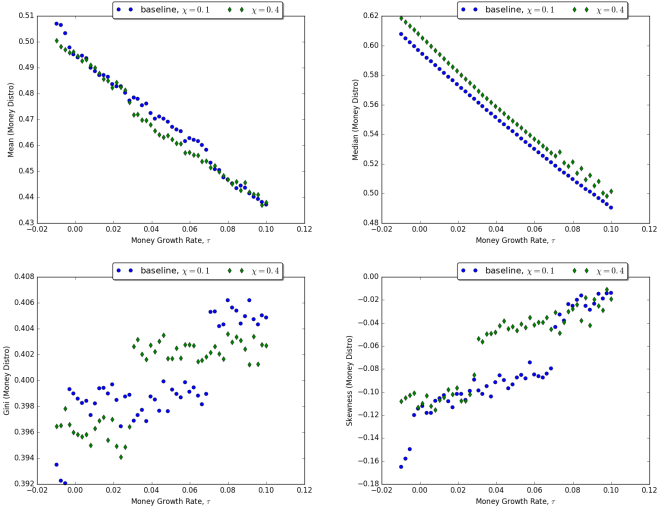


Figure: Comparative steady states — distribution

Effect on Inflation

Comparative steady states: Utilitarian Welfare

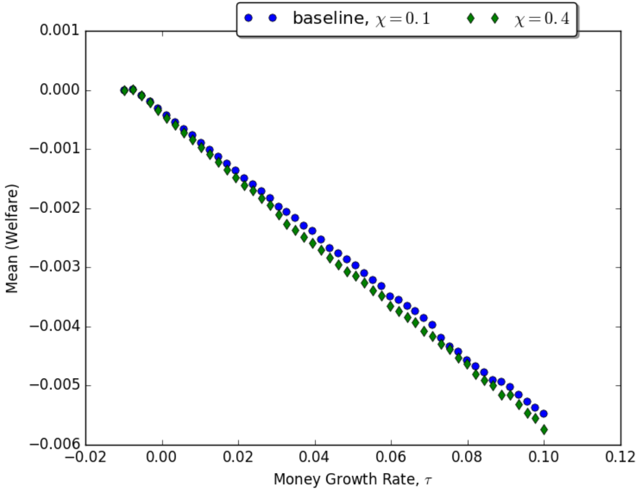


Figure: 2 — 2.4% consumption loss p.a. (0 — 10% inflation)

Comparative Steady States

Mechanism

- Buyer's submarket trade-off: $Q_1(x,b) > 0$, $Q_2(x,b) < 0$, $Q(x,b)$ weakly concave, $Q_{12}(x,b) = 0$:
  - given b, the more a buyer willing to pay x, the more q consumable
  - given x, buyer who wants to match with higher probability b, must tolerate eating less q
- Anticipated higher inflation, consider agents with $m > k$:
  - Higher inflation affects agents unequally since with risk aversion V inherits concavity
  - *Higher inflation* means *lower* (higher) *marginal valuation* of money to *high* (low)-balance agents
    - One hand (intensive margin): inflation, a redistribution of wealth from high-agents to low agents
    - Other hand (extensive margin): inflation raises the downside risk of agents not getting matched in DM; or to participate in CM—so more incentive to trade off q and l for higher DM-match and CM-participation rates.

Comparative Steady States

Mechanism

- So higher anticipated inflation means:
  - ex ante agents participate less in CM, but work more intensively in CM, spend more x in CM to maintain (C, q) (**intensive margins**) ...
  - **vs.** more likely to match b each round of trade with less q; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- Extensive margins dominates intensive margins
  - Gini inequality rises with inflation rate (or τ)
  - Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

Quantitative Next Steps

- Estimate model — discipline with real-world data:
  - not all trades use "money"—proportion of money exchanges small
  - some exchanges supported by credit and contracts
  - exogenous limited participation (α) model allows to do counterfactual studies on degree of market incompleteness
- Re-consider welfare cost of inflation, by accounting for welfare along:
  - deterministic transition (with J. Lee)
  - stochastic transitions (with J. Lee, L. Wang, and A. Sun)

  answer may differ from comparative steady state analysis.