

# Inflationary Redistribution vs. Trading Opportunities

Timothy Kam<sup>1</sup>, Junsang Lee<sup>2</sup>

<sup>1</sup>RSE, Australian National University

<sup>2</sup>Sungkyunkwan University

February 13, 2017  
*AETW2017, Auckland*

# Question

How big is the welfare cost and inequality effects of inflation?

... in a heterogenous agent monetarist model?

- ▶ A competitive (directed) search approach to incomplete markets—endogenous Baumol-Tobin model
- ▶ Role of market frictions and search externality
- ▶ Equilibrium trade-off in *intensive* vs. *extensive* margins w.r.t. inflation policy
- ▶ Consequence for average welfare and inequality of wealth (money holdings)
- ▶ A more flexible quantitative model with endogenous limited financial markets participation
  - ▶ can be taken to data

# What We Do

- ▶ A directed search model of money and *ex-post* heterogeneous agents:
  - ▶ Endogenous Baumol-Tobin liquidity channel
    - ▶ Menzio, Shi, Sun (MSS2013, JET): deterministic steady state;
    - ▶ no aggregate dynamics, characterize zero inflation monetary equilibrium
    - ▶ pointers to more work ... inflation, dynamics
- ▶ Characterize *stationary monetary equilibrium* (SME)
  - ▶ existence of SME with unique non-degenerate distribution of agents
- ▶ A computational geometry solution to nonconvexities arising from matching externalities
- ▶ Compute various non-stochastic steady state equilibrium regimes over different monetary policies

# Shoulders of Giants

# On the Shoulders of Giants

Baumol-Tobin liquidity channel of monetary transmission:

- ▶ Alvarez, Atkeson and Edmond (2009, QJE)
  - ▶ Money is introduced into GE via CIA constraint assumption; not a fundamental friction of markets
  - ▶ Exogenous measures of *active* households who can “go to the bank”
- ▶ c.f., MSS2013, when to top up liquidity is *endogenous* decision.

Alternative mechanisms:

- ▶ Rocheteau, Weill and Wong (2015, NBER):
  - ▶ Undirected search with random matching and bargaining.
  - ▶ Exogenous upper bound on labor supply—incomplete insurance in CM.
  - ▶ Money is non-neutral and generates redistribution (akin to our intensive margin).
  - ▶ c.f., no “extensive margin” trade-off.
- ▶ Head, Liu, Menzio, Wright (2012, JEEA):
  - ▶ Inflation affects price dispersion only in shifting support of price distribution
  - ▶ Money is neutral

Existence of SME with unique agent distribution:

- ▶ Kamihigashi-Stachurski (TE, 2014) sufficient (and necessary) conditions;
- ▶ ⇒ Hopenhayn-Prescott (1992, ECTA)

# Model: Big Picture

1. Ex-ante agents identical. Ex post, agents realize different wealth outcomes
2. Each period, each agent decides who to be:
  - ▶ An *anonymous buyer* who – search in **DM**: decentralised; competitive search; matching frictions, or,
  - ▶ a *worker/consumer* – **CM**: work; asset; neoclassical Walrasian market.
3. **Matching friction** in **DM**  $\Rightarrow$  *ex-post* agent **heterogeneity** of money holdings.
  - ▶ competitive firms create trading posts (some cost); post terms of trades
  - ▶ buyers direct their search decisions to these trading posts.
4. **Equilibrium spending cycle** — wealth accumulation at different rates (Baumol-Tobin)
  - ▶ monetary policy affect the real economy, through affecting agents' *uneven equilibrium liquidity demands*
5. **Costly participation** in the Walrasian **CM**  $\Rightarrow$  *ex-post* agent **heterogeneity** of money holdings.
6. Equilibrium trade-offs:
  - ▶ **Intensive margins**: quantities in both markets
  - ▶ **Extensive margins**: participation rates in **CM**; matching prob. **DM**

# Model

## Fiat money supply, measurement units

- ▶ Money supply growth:

$$\frac{M_{+1}}{M} = 1 + \tau$$

- ▶  $1/\omega$ : per-capita real money balance (in labor units)
  - ▶ If  $M$ : initial aggregate stock of money
  - ▶ Then:  $1/\omega = M \times 1/\omega M$  initial (per-capita) stock of money, *measured in units of labor*

# Model

## Agents and Goods in DM

- ▶ Finite *set of types* of individuals and firms/goods,  $I$
- ▶ Measure-one continuum of individuals of type  $i \in I$
- ▶ An individual consumes good  $i$  and produces good  $(i + 1) \bmod |I|$



# Model

## Firms, matching, submarket location in DM

- ▶ Firm chooses:
  - ▶ how many trading posts in submarket  $(x, q)(s)$  and labor hiring, given wage 1, as function of who visits post.
    - ▶ “who” indexed by current state of a buyer’s money balance (units of equiv. labor) and aggregate money supply growth rate:  $s := (m; \omega)$
  - ▶ each trading post costs  $k$  units of labor to create
  - ▶ zero expected profit

# Model

## Firms, matching, submarket location in DM

### ► Matching technology:

- Exists bijective relation between sellers' matching probability and buyers':  
 $s(s) = \mu[b(s)]$
- Trading post's matching probability (market tightness),  $\mu(b)$ :  $\mu' < 0$  and  $\mu'' < 0$

### ► Decision rule (equilibrium), at each fixed $s$ :

$$\mu(b)(x - q) - k \begin{cases} < 0 & \Rightarrow \text{not enter} \\ = 0 & \Rightarrow \text{indifferent on \#posts} \\ > 0 & \Rightarrow \text{not equilibrium} \end{cases}$$

### ► Implies quantity traded at a submarket with positive market tightness:

$$q(s) = Q[(x, b)(s)] \equiv x(s) - \frac{k}{\mu(b(s))}$$

# Model

## Ex-ante Individuals

- ▶ *ex-ante* decide to be buyer/worker, depending on current state of individual money balance (units of equiv. labor) and aggregate money supply growth rate:  $\mathbf{s} := (m; \omega)$
- ▶ *ex-post*, if *buyer*, then:
  - ▶ decides on *where to shop*  $\equiv$  *how much to pay and eat*  $(x, q)(\mathbf{s})$
- ▶ *ex-post*, if *worker*, then:
  - ▶ decides how much to work  $l^*(\mathbf{s})$

# Model

*Ex-post* CM worker/consumer

Worker named  $\mathbf{s} := (m; \omega)$ :

$$W(\mathbf{s}) = \max_{(C, l, y) \in \mathbb{R}_+^3} \left\{ U(C) - h(l) + \beta \bar{V}(\mathbf{s}_{+1}) : pC + y \leq m + l \right\},$$

- ▶ takes  $\omega$  as parametric
- ▶  $m_{+1} = \frac{\omega y + \tau}{\omega_{+1}(1 + \tau)}$

# Model

## Change of decision variables in DM (equilibrium)

- ▶ Trading post's matching probability a convex and decreasing function of buyer's matching probability:  $\mu(b)$
- ▶ Implies in any equilibrium, each active trading post trades in:

$$q(s) = Q[(x, b)(s)] \equiv x(s) - \frac{k}{\mu(b(s))}$$

# Model

## Ex-post DM buyer

Buyer named  $\mathbf{s} := (m; \omega)$ :

$$B(\mathbf{s}) = \max_{x \in [0, m], q \in \mathbb{R}_+} \left\{ \beta [1 - b(x, q)] \left[ \bar{V} \left( \frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right. \\ \left. + b(x, q) \left[ u(q) + \beta \bar{V} \left( \frac{\omega (m - x) + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right) \right] \right\}.$$

- If  $m \leq k$ , buyer can only afford to visit submarkets  $(\tilde{x}, \tilde{q})$  where  $\tilde{x} - \tilde{q} \leq m \leq k$ 
  - For *this* submarket, optimal to choose
$$b^*(\mathbf{s}) = x^*(\mathbf{s}) = 0 \Rightarrow B(\mathbf{s}) = \beta \bar{V} \left( \frac{\omega m + \tau}{\omega_{+1} (1 + \tau)}, \omega_{+1} \right)$$

# Model

Individuals: ex-ante individual

At start of date:

- ▶ Let  $\tilde{V}(s) = \max \left\{ B(s), W(m - \chi, \omega) \right\}$
- ▶ **Lemma:**  $\tilde{V}_m$  exists for all  $m > 0$
- ▶ But  $\tilde{V}$  not concave ...
  - ▶ Exists profitable solution where ex-ante agent can randomize between pure strategies  $\{buyer, seller\}$
  - ▶ Next we describe fair lotteries

# Model

Individuals: convexifying ex-ante preferences

Recall ex-ante value:

$$\bar{V}(\mathbf{s}) = \alpha W(\mathbf{s}) + (1 - \alpha) V(\mathbf{s}),$$

where  $V(\mathbf{s})$  is the value of playing a fair lottery  $(\pi_1, 1 - \pi_1)$  over the prizes  $\{z_1, z_2\}$ , i.e.,

$$V(\mathbf{s}) = \max_{\pi_1 \in [0,1], z_1, z_2} \left\{ \pi_1 \tilde{V}(z_1, \omega) + (1 - \pi_1) \tilde{V}(z_2, \omega) : \pi_1 z_1 + (1 - \pi_1) z_2 = m \right\};$$

and, given a lottery outcome  $z$ , the individual's value becomes

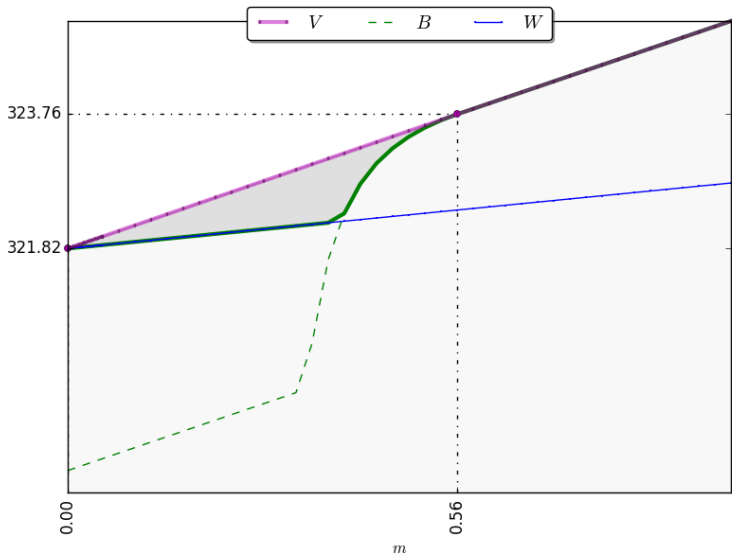
$$\tilde{V}(z, \omega) = \begin{cases} \max \{W(z - \chi, \omega), B(z, \omega)\}, & z - \chi \geq -y_{\max}(\omega; \tau) \\ B(z, \omega), & \text{otherwise} \end{cases},$$

where  $y_{\max}(\omega; \tau) := \min \{\bar{m} - \frac{\tau}{\omega}, \bar{m}\}$



# Model

Individuals: ex-ante individual



# Model

## Stationary Monetary Equilibrium

### Definition

A *stationary monetary equilibrium* (SME), given exogenous monetary policy  $\tau$ , is a

- ▶ list of value functions  $\mathbf{s} \mapsto (W, B, \bar{V})(\mathbf{s})$ , satisfying the Bellman functionals ...
- ▶ a list of policies  $\mathbf{s} \mapsto (l^*, y^*, b^*, x^*, q^*, z^*, \pi^*)(\mathbf{s})$  supporting the value functions;
- ▶ market tightness  $\mathbf{s} \mapsto \mu \circ b^*(\mathbf{s})$  given matching technology  $\mu$ : firms max. profits;
- ▶ an ergodic distribution of wealth  $G(\mathbf{s})$  s.t.

$$T(G)(E) = \int P(\mathbf{s}, \mathcal{E}) dG(\mathbf{s}) \quad \forall E \in \mathcal{E},$$

$\mathbf{s} \mapsto P(\mathbf{s}, \cdot)$  is a Markov kernel induced by  $(l^*, x^*, q^*, z^*, \pi^*)$  and  $\mu \circ b^*$  under  $\tau$ ; and,

- ▶ a wage rate function  $\mathbf{s} \mapsto \omega(\mathbf{s})$  satisfying labor market clearing

# SME: Existence w/ unique distro

### Theorem 1 (CM policies and value)

1.  $W(\cdot, \omega) \in \mathcal{V}[0, \bar{m}]$ , i.e., it is continuous, increasing and concave on  $[0, \bar{m}]$ . Moreover, it is linear on  $[0, \bar{m}]$ .
2. The partial derivative functions  $W_1(\cdot, \omega)$  and  $\bar{V}_1(\cdot, \omega_{+1})$  exist and satisfy the first-order condition

$$\frac{\beta}{1+\tau} \left( \frac{\omega}{\omega_{+1}} \right) \bar{V}_1 \left( \frac{\omega y^*(m, \omega) + \tau}{\omega_{+1} (1+\tau)}, \omega_{+1} \right) \begin{cases} \leq A, & y^*(m, \omega) \geq 0 \\ \geq A, & y^*(m, \omega) \leq y_{\max}(\omega; \tau) \end{cases},$$

and the envelop condition:

$$W_1(m, \omega) = A,$$

3. The stationary Markovian policy rules  $y^*(\cdot, \omega)$  and  $l^*(\cdot, \omega)$  are scalar-valued and continuous functions on  $[0, \bar{m}]$ .
  - ▶ The function  $y^*(\cdot, \omega)$ , is constant valued on  $[0, \bar{m}]$ .
  - ▶ The optimizer  $l^*(\cdot, \omega)$  is an affine and decreasing function on  $[0, \bar{m}]$ .
  - ▶ Moreover, for every  $(m, \omega)$ , the optimal choice  $l^*(m, \omega)$  is interior.

## Theorem 2 (DM policies and value)

1. For any  $\bar{V}(\cdot, \omega_{+1}) \in \mathcal{V}[0, \bar{m}]$ , the DM buyer's value function is increasing and continuous in  $m$ :  $B(\cdot; \omega) \in \mathcal{C}[0, \bar{m}]$ .
2. For any  $m \leq k$ , optimal decisions are  $b^*(m, \omega) = x^*(m, \omega) = q^*(m, \omega) = 0$ , and  $B(m, \omega) = \beta \bar{V}[\phi(m, \omega), \omega_{+1}]$ , where  $\phi(m, \omega) := (\omega m + \tau) / [\omega_{+1}(1 + \tau)]$ .
3. At any  $(m, \omega)$ , where  $m \in [k, \bar{m}]$  and  $b^*(m, \omega) > 0$ :
  - ▶  $(x^*, b^*, q^*)(m, \omega)$  are unique, continuous, and increasing in  $m$ .
  - ▶  $B_1(m, \omega)$  exists if and only if  $\bar{V}_1[\phi(m, \omega), \omega]$  exists.
  - ▶  $B(m, \omega)$  is strictly increasing in  $m$ .
  - ▶  $b^*$  and  $x^*$ , respectively, satisfy the FoCs

$$\begin{aligned} u \circ Q[x^*(m, \omega), b^*(m, \omega)] + b^*(m, \omega) (u \circ Q)_2[x^*(m, \omega), b^*(m, \omega)] \\ = \beta [\bar{V}(\phi(m, \omega), \omega_{+1}) - \bar{V}(\phi^*(m, \omega), \omega_{+1})], \end{aligned}$$

$$(u \circ Q)_1[x^*(m, \omega), b^*(m, \omega)] = \frac{\beta}{1 + \tau} \left( \frac{\omega}{\omega_{+1}} \right) \bar{V}_1[\phi^*(m, \omega), \omega_{+1}].$$

## Theorem 3 (Existence with unique distro)

There is a SME with a unique nondegenerate distribution  $G$ .

► **Lemmata:**

- **(L1):** Equilibrium policy functions are monotone in  $m$ , given  $\tau$  fixed, bounded on  $m$
- **(L2):**  $\varphi(\cdot|\tau_{-1})$  is an exogenous increasing Markov kernel, bounded in probability:  $\mathbb{E}|\tau| < +\infty$ 
  - (Meyn and Tweedie, Ch. 12)
- **(L1 and L2) gives (L3):** Therefore equilibrium Markov kernel  $P(s, \cdot)$  is increasing and bounded in probability
- **(L4):** Natural state space  $S := [0, \bar{m}(\tau)] \times [0, +\infty) \in (m; \tau)$  has least and greatest element
- **(L5):** For any  $(P, s)$ -Markov process,
  - there is a  $c \in S$  such that  $\Pr\{s_t \leq c\} > 0$ , and,
  - a  $d \in S$  such that  $\Pr\{s_t \geq d\} > 0$

► **Theorem:** Kamihigashi and Stachurski (TE, 2014, Theorem 3.1):

- (L3)-(L5) implies stability result.

# SME Behavior

Qualitative: Focus on DM

- ▶ Fixing money supply growth  $\tau$ :
  - ▶ Buyer with higher  $m$  spends more  $x^*(m; \omega)$  in each submarket to optimize on match probability
    - ▶ with positive probability every period until  $m = 0$ .
    - ▶ all subsequent submarket matches exhibit lower  $(x, q)$
- ▶ A higher  $\tau$  will speed up the buyer's spending in each spending round, and lowers worker's  $l^*(m; \omega)$



# SME: Computation

# Computation

## Parametrization

- ▶  $U(C) = \bar{U}_{CM} \frac{(C+0.001)^{1-\sigma_{CM}} - (0.001)^{1-\sigma_{CM}}}{1-\sigma_{CM}}$
- ▶  $u(q) = \frac{(q+0.001)^{1-\sigma_{DM}} - (0.001)^{1-\sigma_{DM}}}{1-\sigma_{DM}}$
- ▶  $\mu(b) = 1 - b.$

**Table:** Baseline parameterization

Parameter	Value	Unit	Description
$\beta$	0.99	-	Quarterly interest rate of $1/\beta - 1 \approx 1\%$
$\sigma_{CM}$	2	-	CM risk aversion
$\sigma_{DM}$	1.01	-	Normalized DM risk aversion
$A$	1	-	Scale on labor disutility function
$\bar{m}$	$0.95 \times U^{-1}(A)$	labor	See assumption (2.3)
$\chi$	$0.1 \times \bar{m} = 0.095$	labor	Fixed cost of CM participation
$\alpha$	0	-	Probability of costless CM entry
$k$	0.05	labor	Fixed cost of creating a trading post
$\bar{U}_{CM}$	0.002	-	CM preference scale parameter

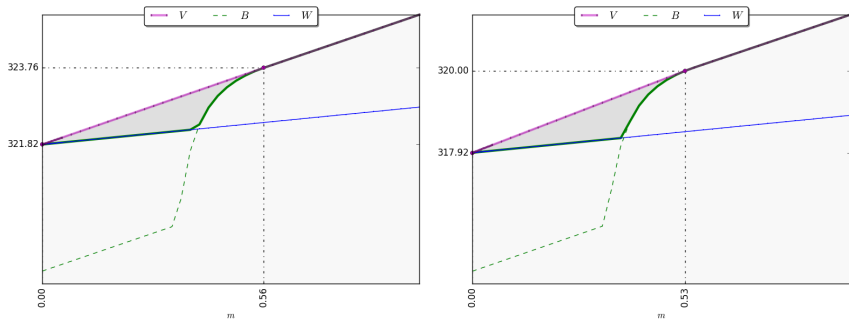
# Computation

- ▶ Program runs in Python 2.7/3.0
  - ▶ with optional OpenMPI parallelization capabilities
- ▶ Finding a steady state equilibrium:
  - ▶ ~90 seconds on workstation running on 18 parallel cores

# SME: Results

# Steady State

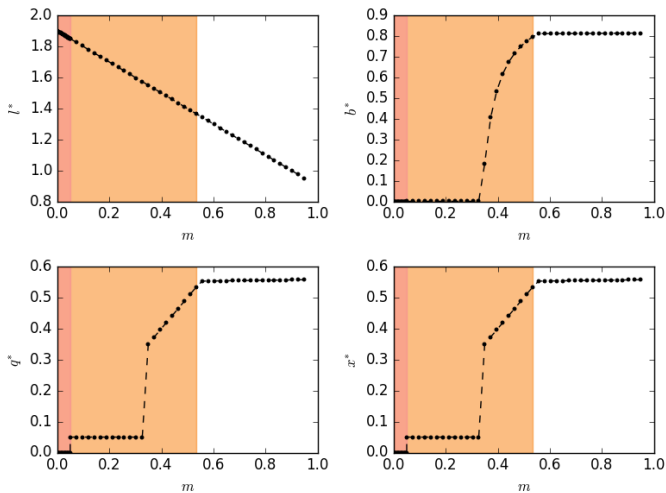
Value Functions: zero money growth



**Figure:** Value functions from two economies.

# Steady State

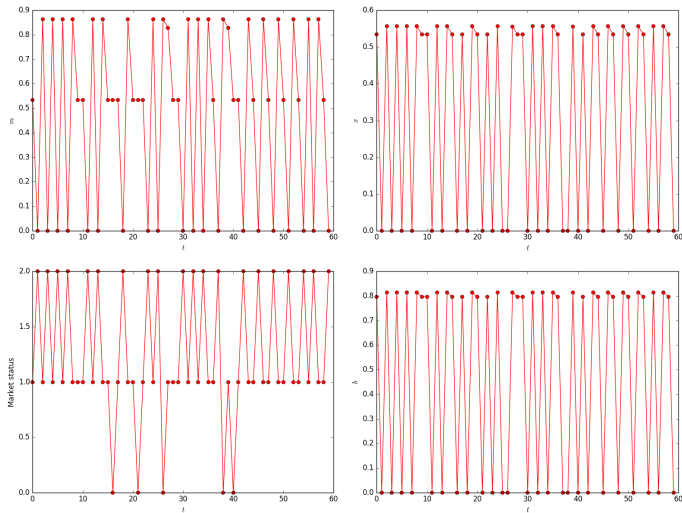
Markov Policy Functions: 10% money growth



**Figure:** Markov policy functions with  $\tau = 0.1$

# Steady State

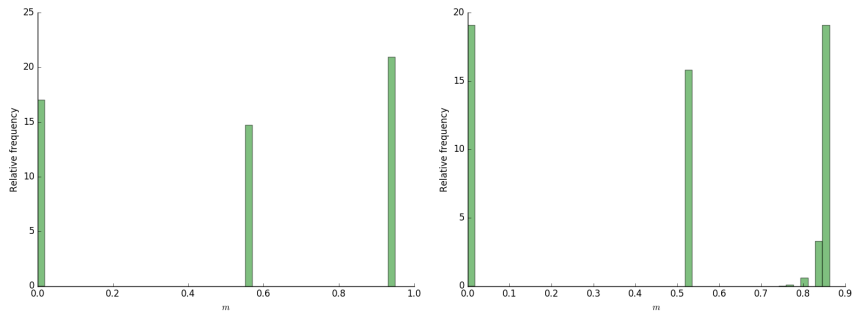
Agent sample path: 10% money growth



**Figure:** Agent sample path ( $\tau = 0.1$ )

# Steady State

## Money Distribution



**Figure:** Money distribution under different equilibrium inflation rates



# Comparative SME: Preliminary Results

# Effect on Inflation

## Comparative steady states: Equilibrium wage

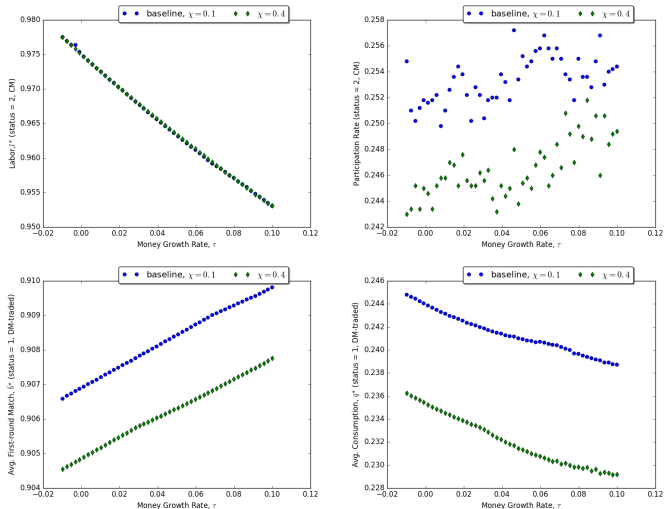


Figure: Comparative steady states — allocations

# Effect on Inflation

Comparative steady states:  $(\tau, \chi)$

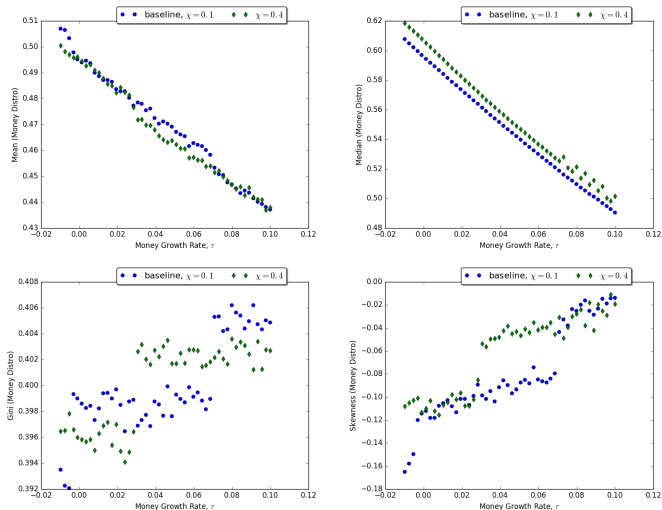
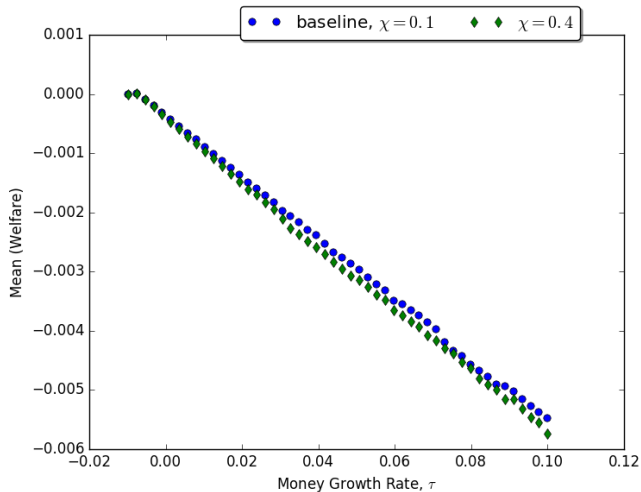


Figure: Comparative steady states — distribution

# Effect on Inflation

Comparative steady states: Utilitarian Welfare



**Figure: 2** — 2.4% consumption loss p.a. (0 — 10% inflation)

# Comparative Steady States

## Mechanism

- ▶ Buyer's submarket trade-off:  $Q_1(x, b) > 0$ ,  $Q_2(x, b) < 0$ ,  $Q(x, b)$  weakly concave,  $Q_{12}(x, b) = 0$ :
  - ▶ given  $b$ , the more a buyer willing to pay  $x$ , the more  $q$  consumable
  - ▶ given  $x$ , buyer who wants to match with higher probability  $b$ , must tolerate eating less  $q$
- ▶ Anticipated higher inflation, consider agents with  $m > k$ :
  - ▶ Higher inflation affects agents unequally since with risk aversion  $V$  inherits concavity
  - ▶ *Higher inflation* means *lower* (higher) *marginal valuation* of money to *high* (low)-balance agents
    - ▶ One hand (intensive margin): inflation, a redistribution of wealth from high-agents to low agents
    - ▶ Other hand (extensive margin): inflation raises the downside risk of agents not getting matched in DM; or to participate in CM—so more incentive to trade off  $q$  and  $l$  for higher DM-match and CM-participation rates.

# Comparative Steady States

## Mechanism

- ▶ So higher anticipated inflation means:
  - ▶ ex ante agents participate less in CM, but work more intensively in CM, spend more  $x$  in CM to maintain  $(C, q)$  (**intensive margins**) ...
  - ▶ **vs.** more likely to match  $b$  each round of trade with less  $q$ ; and participate in CM more, but work less intensively (opposite force: **extensive margins**)
- ▶ Extensive margins dominates intensive margins
  - ▶ Gini inequality rises with inflation rate (or  $\tau$ )
  - ▶ Relatively higher proportion of low balance agents dominate average welfare—thus average welfare falls with inflation

# Quantitative Next Steps

- ▶ Estimate model — discipline with real-world data:
    - ▶ not all trades use “money”—proportion of money exchanges small
    - ▶ some exchanges supported by credit and contracts
    - ▶ exogenous limited participation ( $\alpha$ ) model allows to do counterfactual studies on degree of market incompleteness
  - ▶ Re-consider welfare cost of inflation, by accounting for welfare along:
    - ▶ deterministic transition (with J. Lee)
    - ▶ stochastic transitions (with J. Lee, L. Wang, and A. Sun)
- answer may differ from comparative steady state analysis.