Fairness in Participatory Budgeting via Equality of Resources

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University of Amsterdam

AAMAS 2023

1. Introduction

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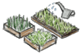

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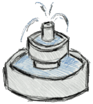

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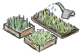

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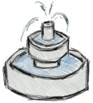

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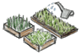
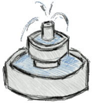
7000$ 					
	1000$	2000$	2500$	2500$	5000$
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Satisfaction-Based Fairness for Participatory Budgeting

Fairness is about distributing some *measure* fairly among the agents.

↳ What is a good measure in the case of participatory budgeting?

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✓ The satisfaction of an agent is obvious

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We aim at *equity of resources* among the agents.

2. The Share

Definition

The share of an agent:
the ressources spent on
that specific agent

$$share(\pi, A_i) = \sum_{p \in \pi \cap A_i} \frac{c(p)}{|\{A' \in \boldsymbol{A} \mid p \in A'\}|}$$

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The budget allocation

The agent's ballot

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Cost of the project

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The budget allocation

The agent's ballot

Number of voters
approving of p

An Example

$= 10$	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	
	✓		✓			
	✓			✓		

$= 10$	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			
	✓			✓		

An Example

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	✓			✓		4

3. Providing Fair Share

The Perfect Situation

Every agent is provided their *fair share*, *i.e.*:

$$share(\pi, A_i) \geq \min\left\{share(A_i, i), \frac{b}{n}\right\}$$

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	✓	✓		✓	✓	
	✓		✓			
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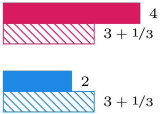

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A First Problem

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; $3 + 1/3$
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	✓	✓		✓	✓	7; $3 + 1/3$
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	✓			✓		0; $3 + 1/3$

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	7; $3 + 1/3$
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	✓			✓		2; $3 + 1/3$

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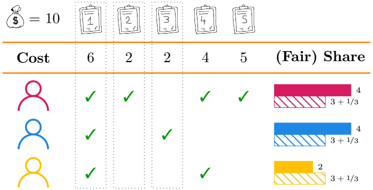

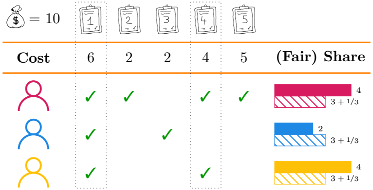

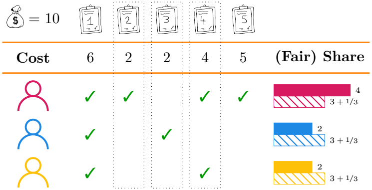

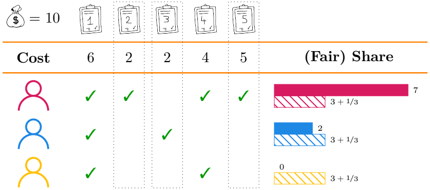

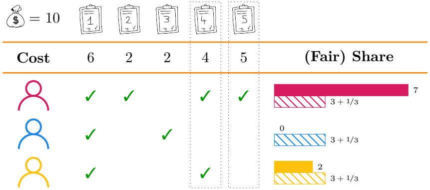

It is not possible to always provide fair share to everyone (and hard to know if we can).

4. Experimental Analysis of the Share

Experimental Analysis

Instances: 353 instances from Pabulib with up to 65 projects.

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Measure of Interest:

The average normalised L_1 distance to fair share: $$1 - \frac{1}{n} \sum_{i \in \mathcal{N}} \frac{|share(\pi, i) - fairshare(i)|}{fairshare_i}$$

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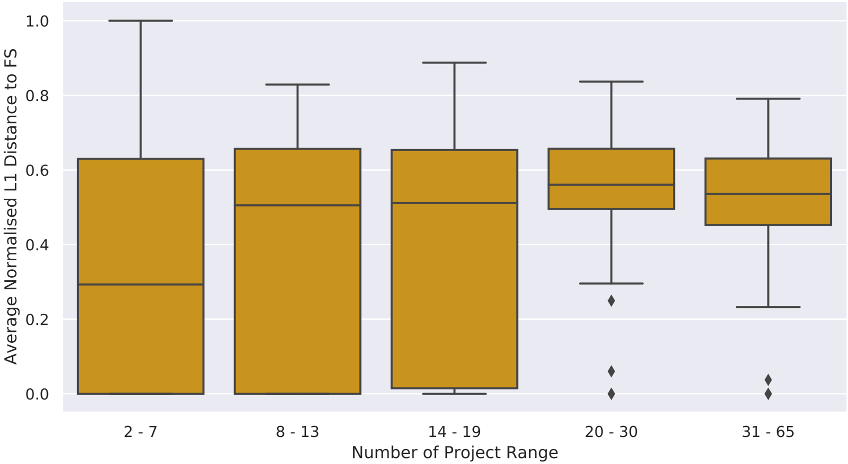
Fair share can be provided in only one instance out of the 353 considered (with 3 projects and 198 voters).

Optimal L_1 Distance to Fair Share



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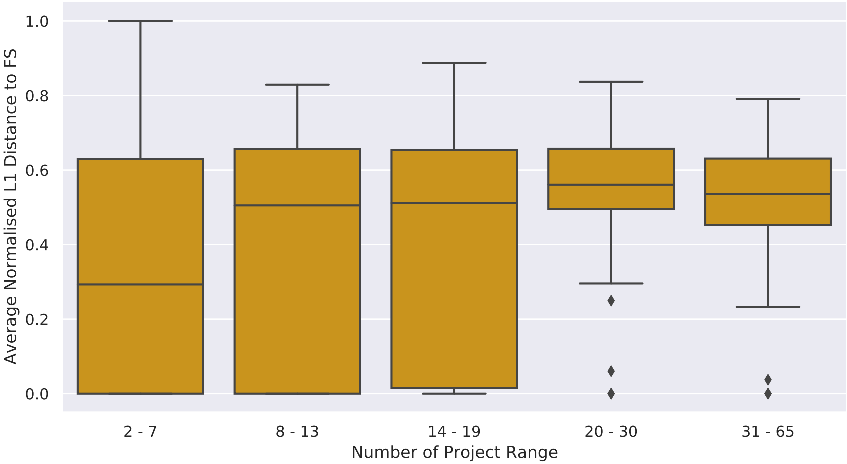
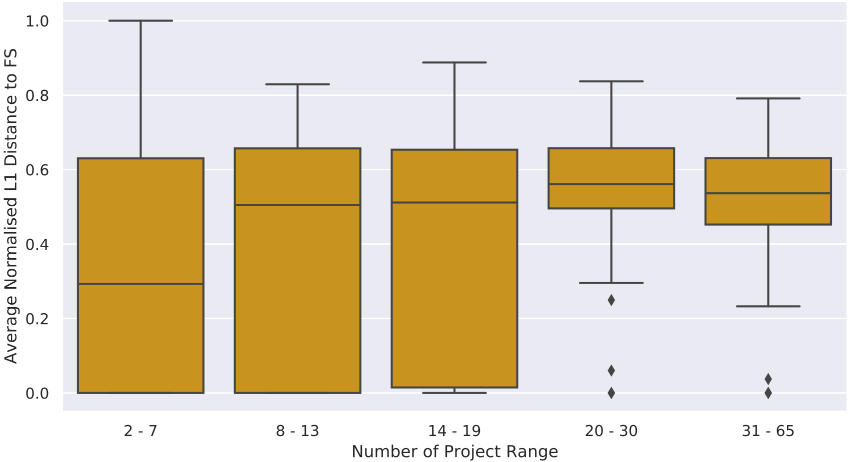


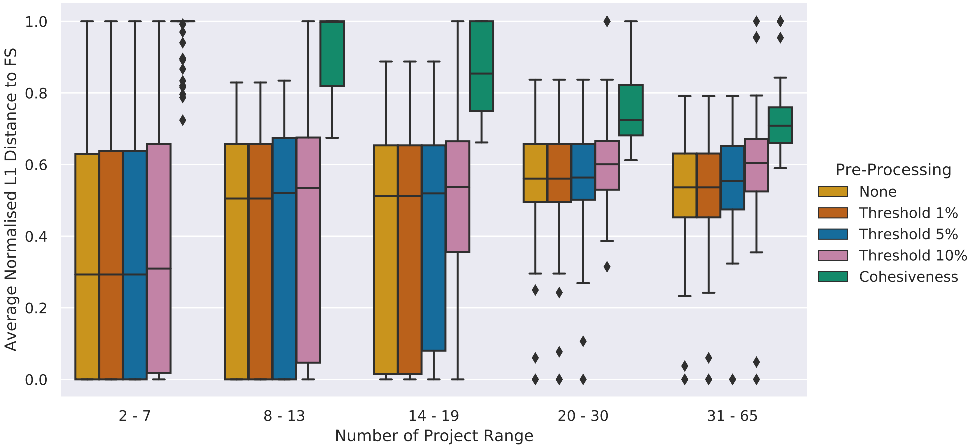
↳ We are far from achieving fair share.

Optimal L_1 Distance to Fair Share



- We are far from achieving fair share.
- It gets easier as the number of projects increase.

Optimal L_1 Distance to Fair Share – Preprocessing



↳ Fair Share is hard to satisfy, *structurally* hard.

5. Approximate Fair Share

Two Relaxations — Fair Share up to One Project

Every agent is provided their *fair share up to one project*, *i.e.*, for each agent there exists a project $p \in \mathcal{P}$ such that:

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↳ This is however still unsatisfiable (and again, hard to check whether it can be satisfied)...

Two Relaxations — Local Fair Share

A budget allocation π provides *local fair share* if there is no project $p \in \mathcal{P} \setminus \pi$ such that for every agent i approving of p we have:

$$\mathit{share}(\pi \cup \{p\}, A_i) < \mathit{fairshare}_i$$

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↪ An explanation? If such a p exists, all supporters of p receive less than their fair share and:

- Either p can be selected without exceeding the budget limit; let's select it then!
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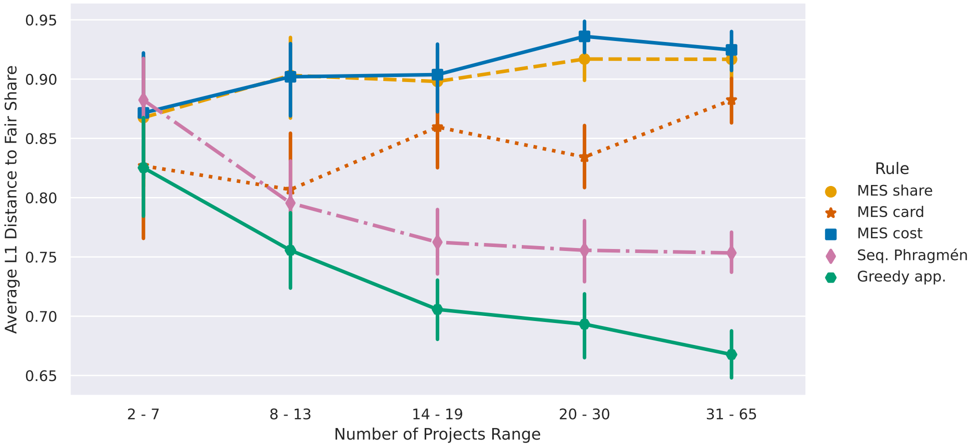
↳ But how does MES performs in terms of fair share?

6. Achieved Fair Share by Common Rules

Distance to Fair Share



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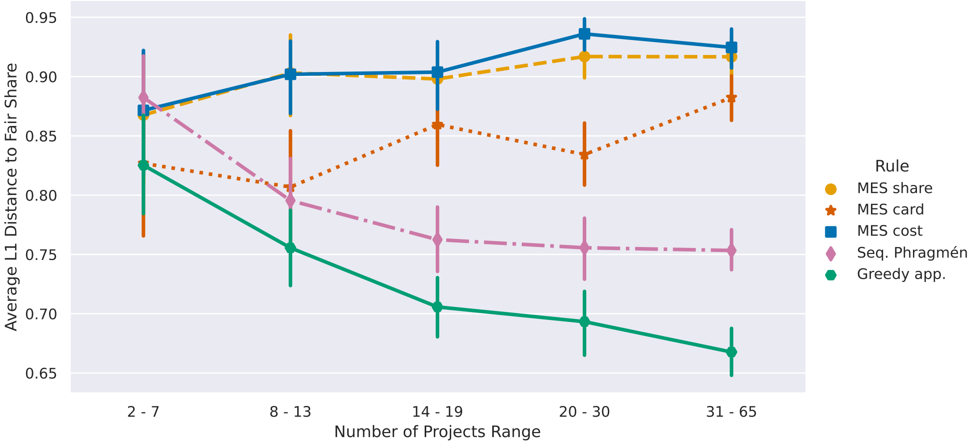

↳ MES rules approach fair share nicely, and MES_{cost} is particularly attractive.

7. Conclusion

We have:

- Argued for fairness in terms of equity of resources;
- Presented the share, one way to do it;
- Discussed fair share in theoretical and experimental terms.

I want:

- Non-sequential rules satisfying strong requirements (when they exist);
- Rules providing satisfaction-based and effort-based fairness together.

THANKS!