CSC 535 – Probabilistic Graphical Models

Assignment Nine

Due: 11:59pm (*) Wednesday, Dec 05.

(*) There is grace until 8am the next morning, as the instructor will not grade assignment before then. However, once the instructor starts grading assignments, no more assignments will be accepted.

Weight is about 7 points

This assignment can be done in groups of 2-3. You should create one PDF for the entire group, but everyone should hand in a copy of the PDF to help me keep track. Make it clear on the first page of your PDF who your group is. Group reports are expected to be higher quality than individual ones. This assignment is only barely related to a previous assignment, so if you want to change groups, or create new ones, that is fine.

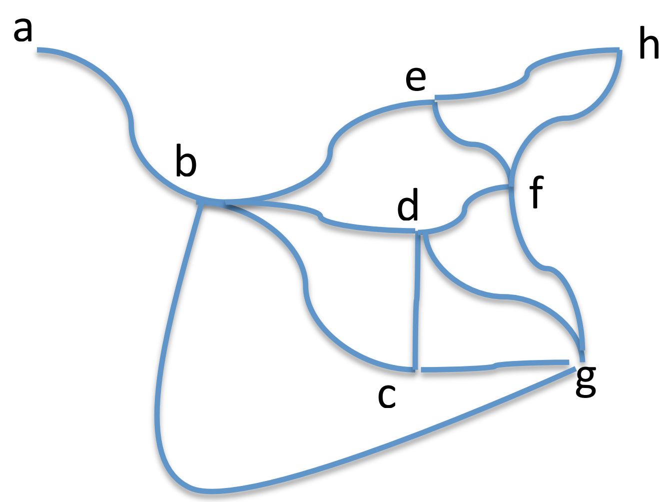
The purpose of this assignment is to learn about approximate inference for missing value problems by implementing the EM algorithm. There is one optional problem at the end.

Deliverables

Deliverables are specified below in more detail. For the high level perspective, you are to provide a program to output a few numbers and to create figures. You also need to create a PDF document that tells the story of the assignment including output, plots, and images that are displayed when the program runs. Even if the question does not explicitly remind you to put the resulting image into the PDF, if it is flagged with **($)**, you should do so. The instructor should not need to run the program to verify that you attempted the question. See

http://kobus.ca/teaching/grad-assignment-instructions.pdf

for more details about preparing write-ups. While it takes work, it is well worth getting better (and more efficient) at this. A substantive part of each assignment grade is reserved for exposition.



Hidden Markov Modeling

1)

a) Provide a state transition diagram for applying to college, and then being in the appropriate year by passing (or not) successive years, and then graduating. You should also model leaving college at any stage.

b) A certain (simplified) type of cell can be in two states: hibernation and growth. This may in response to environmental factors, but we have no way of observing the relevant ones, and so we will assume that it transitions from one state to the other due to unexplained stochastic processes within the cell. It is important the cell "knows" what state it is in because each one requires the coordination of many activities. In each state we observe a different distribution of expressed genes. We measure these expressions every hour, and the cell switches states about once a day (i.e., you can assume that 24 hours is the expected value). Create an HMM model for this data, explaining any assumptions you need to make.

2) Consider the trail network in the Figure. Suppose that a traveler who finds themselves at any junction, has equal probability of choosing any of the directions, including the one they just came from. However, they never stay at a junction (no resting!). Suppose that whenever the traveller passes through a junction, they telephone their significant other who writes down the time of the phone call. However, the traveller does not know which junction they are at. We can attempt to determine their route with the following information. At each junction there is a sensor at that responds with 85% probability if the traveller passes through. Further, at that time, all other sensors responds randomly to animals, leaves blowing in the wind, and so on, with probability 25%. The observations are thus the responses of all the sensors at the time of each phone call. Also suppose we know that the traveler started at node "a".

a) Write down the matrix of transition probabilities of going from each node to another.

b) Is our traveler's node sequence a Markov process? Explain.

c) Write down the **joint** probability for a particular sequence of observations **and** a particular path under this model in an algebraic form. It should be clear where any quantities needed come from. IE, if your model has a quantity T_{ij}, you could say something like "where T_{ij} is found in the matrix T specified earlier". However you do it, your formulation should be clear enough that someone could compute the right number without having to read this document, or take CSC 535.

d) Suppose the traveler took the route: R1=[a,b,c,d,b,c,d,b,e,f,h]. What is the probability of sensor response sequences of:

i) {a}, {b}, {c}, {d}, {b}, {c}, {d}, {b}, {e}, {f}, {h}

ii) {a,b}, {b,c}, {c,d}, {d,f}, {b,d}, {c,f}, {d,h}, {b,e}, {e,h}, {f,e}, {f,h}

iii) {}, {b}, {}, {f}, {}, {f}, {}, {h}, {}, {e}, {}

e) Consider a second route, R2=[a,b,d,f,d,f,h,e,h,e,f]. What is the ratio of the probability of R2, to that of R1, given sensor responses (i), (ii), (iii)?

Hint. Notice that in (d) we are considering the data given the route, but here we are looking at the reverse.

f) **OPTIONAL in F18**. Write a computer program to compute the most probable route given which sensors responded for a path through 11 nodes (ideally, N nodes). Run your program on the three sensor responses listed in problem (d). You are, of course, welcome to reuse any code you have written for this course so far, but you might find it easier to implement the more specific form of the inference covered in lecture 26.

3) Explain why (or why it is not possible) the following can be modeled as a standard, stationary, first order, HMM, assuming that we get noisy measurements about each described state. Also, for (a) and (b), if you argued that it can be done provide the transition diagram.

a) A baby MQL learning to walk. MQLs move according to a clock that ticks every second. When they are standing, both feet are magnetically bound to the surface. A step to the left involves releasing the magnetic force on the left foot (tick 1); moving the left foot outward (tick 2); activating the magnet (tick 3); releasing the magnet on the right foot (tick 4); drawing the legs together (tick 5); and activating the right foot magnet (tick 6). A step the right works analogously. If they are standing, at each clock tick they either continue to stand, or initiate a left step or a right step with some known probability distribution (the details do not matter). However, when they are babies, whenever they only have one magnet active, they are unstable, and with some probability they will try to regain their balance by moving the same arm as the unbound leg up on the next tick, followed by moving the arm down on the following tick. However, when their leg is unbound, and the corresponding arm is up, there is also some probability that they will fall over where they stay (until mom comes by, but you do not need to model that).

b) A simple metronome which malfunctions with age. When it is working, the state of the pendulum being on the left side is followed by the pendulum being on the right side. There is a small probability that the pendulum fails and stays in the same state for a transition. The probability that the pendulum fails increases asymptotically to unity (one) with increasing age of the device.

What to Hand In

Hand in the PDF file hw9.pdf with the story of your efforts into D2L. If you wrote code for this assignment, hand in a program hw9.<suffix> (e.g., hw9.m if you are working in Matlab). If you are working in groups, the code and the PDF can be the same for each person in the group, but each member should hand in a copy to help me keep track of things. PDFs from groups are expected to be of higher quality than individual ones.