

解决99%二叉树问题的算法—— 分治法 Divide & Conquer

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分治法 Divide & Conquer

将大规模问题拆分为若干个小规模的**同类型问题**去处理的算法 分治法和二分法(Binary Search)有什么区别?



分治法 Divide & Conquer

二分法会每次丢弃掉一半分治法分完以后两边都要处理



什么样的数据结构适合分治法?

数组:一个大数组可以拆分为若干个不相交的子数组 归并排序,快速排序,都是基于数组的分治法

二叉树: 整棵树的左子树和右子树都是二叉树

二叉树的大部分题都可以使用分治法解决



独孤九剑——破枪式

碰到二叉树的问题,就想想整棵树在该问题上的结果和左右儿子在该问题上的结果之间的联系是什么



二叉树的高度是多少?

A: O(n)

B: O(logn)

C: O(h)



二叉树的高度是多少?

最坏 O(n) 最好 O(logn)

一般用 O(h) 来表示更合适

二叉树考点剖析



考察形态:二叉树上求值,求路径

代表例题: http://www.lintcode.com/problem/subtree-with-maximum-average/

考点本质: 深度优先搜索(Depth First Search)

考察形态:二叉树结构变化

代表例题: http://www.lintcode.com/problem/invert-binary-tree/

考点本质:深度优先搜索(Depth First Search)

考察形态:二叉查找树(Binary Search Tree)

代表例题: http://www.lintcode.com/problem/validate-binary-search-tree/

考点本质: 深度优先搜索(Depth First Search)

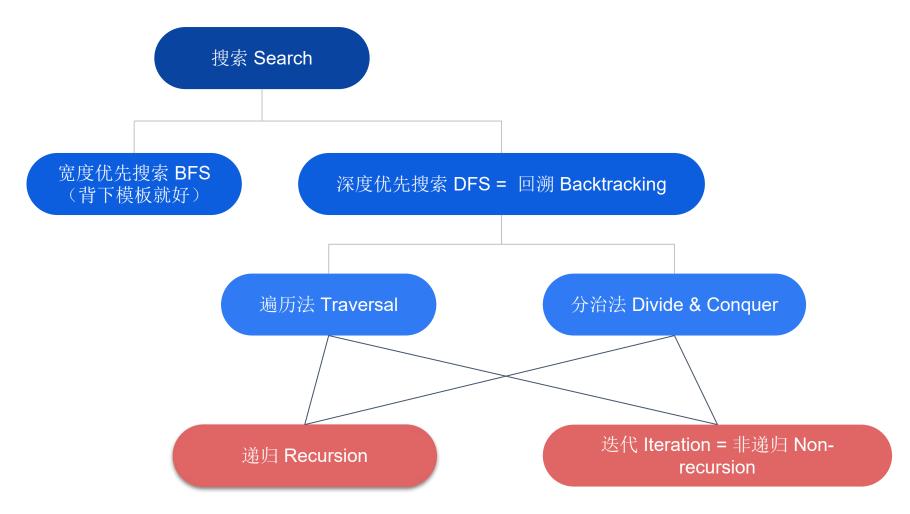


Tree-based Depth First Search

不管二叉树的题型如何变化 考点都是基于树的深度优先搜索

一张图搞明白: 递归, DFS, 回溯, 遍历, 分治, 迭代





将递归和非递归理解为算法的一种实现方式而不是算法



第一类考察形态

二叉树上求值,求路径 Maximum / Minimum / Average / Sum / Paths



Minimum Subtree

http://www.lintcode.com/problem/minimum-subtree/http://www.jiuzhang.com/solutions/minimum-subtree/求和最小的子树



一棵二叉树有多少棵子树

A: O(n)

B: O(2ⁿ)

C: O(n^2)

D: O(n) - 叶子节点个数

代码-使用了全局变量的分治法



```
public class Solution {
    private int minSum;
    private TreeNode minRoot;
    public TreeNode findSubtree(TreeNode root) {
        minSum = Integer.MAX VALUE;
       minRoot = null;
       getSum(root);
        return minRoot;
    private int getSum(TreeNode root) {
        if (root == null) {
            return 0;
        int sum = getSum(root.left) + getSum(root.right) + root.val;
        if (sum < minSum) {</pre>
            minSum = sum;
            minRoot = root;
        return sum;
```

```
class Solution:
   def findSubtree(self, root):
       self.minimum_weight = float('inf')
       self.minimum subtree root = None
       self.getTreeSum(root)
       return self.minimum_subtree_root
   # 得到 root 为根的二叉树的所有节点之和
   # 顺便打个擂台求出 minimum subtree
   def getTreeSum(self, root):
       if root is None:
           return 0
       left_weight = self.getTreeSum(root.left)
       right_weight = self.getTreeSum(root.right)
       root weight = left weight + right weight + root.val
       if root weight < self.minimum weight:</pre>
           self.minimum weight = root weight
           self.minimum_subtree_root = root
       return root_weight
```



全局变量的坏处

函数不"纯粹",容易出 Bug 不利于多线程化,对共享变量加锁带来效率下降



```
class Solution:
   def findSubtree(self, root):
       minimum, subtree, sum of root = self.helper(root)
        return subtree
   def helper(self, root):
       if root is None:
            return sys.maxsize, None, 0
        left_minimum, left_subtree, left_sum = self.helper(root.left)
        right minimum, right subtree, right sum = self.helper(root.right)
        sum_of_root = left_sum + right_sum + root.val
        if left_minimum == min(left_minimum, right_minimum, sum_of_root):
           return left minimum, left subtree, sum of root
        if right_minimum == min(left_minimum, right_minimum, sum_of_root):
            return right_minimum, right_subtree, sum_of_root
        return sum_of_root, root, sum_of_root
```

代码 - Java 纯分治的方法



```
public ResultType helper(TreeNode node) {
   if (node == null) {
                                                          public TreeNode findSubtree(TreeNode root) {
       return new ResultType(null, Integer.MAX VALUE, 0);
                                                                ResultType result = helper(root);
                                                                return result.minSubtree;
   ResultType leftResult = helper(node.left);
   ResultType rightResult = helper(node.right);
   ResultType result = new ResultType(
       node.
       leftResult.sum + rightResult.sum + node.val,
       leftResult.sum + rightResult.sum + node.val
                                                          class ResultType {
                                                              public TreeNode minSubtree;
   if (leftResult.minSum <= result.minSum) {</pre>
                                                              public int sum, minSum;
       result.minSum = leftResult.minSum;
                                                              public ResultType(TreeNode minSubtree, int minSum, int sum) {
       result.minSubtree = leftResult.minSubtree;
                                                                  this.minSubtree = minSubtree;
                                                                  this.minSum = minSum;
                                                                  this.sum = sum;
   if (rightResult.minSum <= result.minSum) {</pre>
       result.minSum = rightResult.minSum;
       result.minSubtree = rightResult.minSubtree;
   return result;
```



Lowest Common Ancestor

http://www.lintcode.com/problem/lowest-common-ancestor/

http://www.jiuzhang.com/solutions/lowest-common-ancestor/

with parent pointer vs no parent pointer

follow up: LCA II & III



问法1: 如果有父指针

http://www.lintcode.com/problem/lowest-common-ancestor-ii/

http://www.jiuzhang.com/solutions/lowest-common-ancestor-ii/

使用 HashSet 记录从 A 到根的所有点

访问从 B 到根的所有点,第一个出现在 HashSet 中的就是



```
public class Solution {
   public ParentTreeNode lowestCommonAncestorII(ParentTreeNode root,
                                        ParentTreeNode A,
                                        ParentTreeNode B) {
      Set<ParentTreeNode> parentSet = new HashSet<>();
                                                 def lowestCommonAncestorII(self, root, A, B):
      // 把A的祖先节点都加入到哈希表中
                                                      parentSet = set()
      ParentTreeNode curr = A;
      while (curr != null) {
                                                      # 把A的祖先节点都加入到哈希表中
          parentSet.add(curr);
                                                      curr = A
         curr = curr.parent;
                                                     while (curr is not None):
      // 遍历B的祖先节点, 第一个在哈希表中出现的即为答案
                                                          parentSet.add(curr)
      curr = B;
                                                          curr = curr.parent
      while (curr != null) {
         if (parentSet.contains(curr)) {
                                                      # 遍历B的祖先节点,第一个在哈希表中出现的即为答案
             return curr;
                                                      curr = B
                                                     while (curr is not None):
         curr = curr.parent;
                                                          if (curr in parentSet):
      return null;
                                                               return curr
                                                          curr = curr.parent
                                                      return None
```



问法2: 两个节点都在树里

http://www.lintcode.com/problem/lowest-common-ancestor/

http://www.jiuzhang.com/solutions/lowest-common-ancestor/

给你 root, A, B 三个点的信息

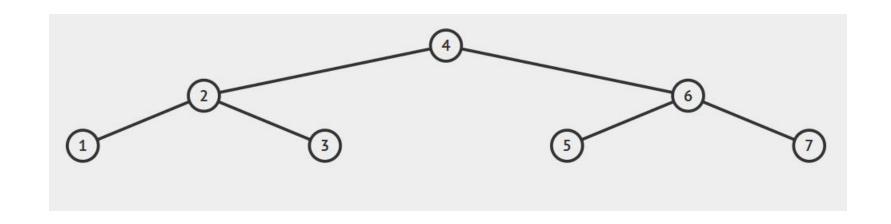
A和B保证都在 root 的下面



方法一: 直接遍历的方法

找出两个节点遍历之间深度最小的节点





					遍	历师	序						
node	4	2	1	2	3	2	4	6	5	6	7	6	4
depth	1	2	3	2	3	2	1	2	3	2	3	2	1



方法2: 分治法

定义返回值:

A,B 都存在 -> return LCA(A,B)

只有A --> return A

只有B --> return B

A,B 都不存在 --> return null

代码



```
public TreeNode lowestCommonAncestor(TreeNode root, TreeNode A, TreeNode B) {
   if (root == null){
       return null;
   if (root == A | root == B){
       return root;
   TreeNode left = lowestCommonAncestor(root.left, A, B);
   TreeNode right = lowestCommonAncestor(root.right, A, B);
   if (left != null && right != null){
                                                            lowestCommonAncestor(self, root, A, B):
       return root;
                                                            if root is None:
                                                                return None
   if (left != null){
                                                            if root == A or root == B:
       return left;
                                                                return root
                                                            left = self.lowestCommonAncestor(root.left, A, B)
   if (right != null){
       return right;
                                                            right = self.lowestCommonAncestor(root.right, A, B)
                                                            if left and right:
   return null;
                                                                return root
                                                            if left:
                                                                return left
                                                            if right:
                                                                return right
                                                            return None
```



问法3: 两个节点不一定都在树里

http://www.lintcode.com/problem/lowest-common-ancestor-iii/

http://www.jiuzhang.com/solutions/lowest-common-ancestor-iii/

root, p, q

但是不保证 root 里一定有 p 和 q



```
public TreeNode lowestCommonAncestor3(TreeNode root, public ResultType helper(TreeNode root, TreeNode A, TreeNode B) {
                                                         if (root == null)
                                      TreeNode A.
                                                             return new ResultType(false, false, null);
                                      TreeNode B) {
    ResultType rt = helper(root, A, B);
                                                         ResultType left rt = helper(root.left, A, B);
   if (rt.a_exist && rt.b_exist)
                                                         ResultType right rt = helper(root.right, A, B);
        return rt.node;
   else
                                                         boolean a exist = left rt.a exist | right rt.a exist | root == A;
       return null;
                                                         boolean b_exist = left_rt.b_exist || right rt.b exist || root == B;
                                                         if (root == A | root == B)
                                                             return new ResultType(a exist, b exist, root);
class ResultType {
                                                         if (left rt.node != null && right rt.node != null)
    public boolean a exist, b exist;
                                                             return new ResultType(a exist, b exist, root);
    public TreeNode node;
                                                         if (left rt.node != null)
    ResultType(boolean a, boolean b, TreeNode n) {
                                                             return new ResultType(a exist, b exist, left rt.node);
        a exist = a;
                                                         if (right rt.node != null)
       b exist = b;
                                                             return new ResultType(a_exist, b_exist, right_rt.node);
       node = n;
                                                         return new ResultType(a exist, b exist, null);
```



```
import copy
class Solution:

def lowestCommonAncestor3(self, root, A, B):
    a, b, lca = self.helper(root, A, B)
    if a and b:
        return lca
    else:
        return None
```

```
helper(self, root, A, B):
if root is None:
    return False, False, None
left a, left b, left node = self.helper(root.left, A, B)
right_a, right_b, right_node = self.helper(root.right, A, B)
a = left_a or right_a or root == A
b = left b or right b or root == B
if root == A or root == B:
    return a, b, root
if left_node is not None and right_node is not None:
    return a, b, root
if left node is not None:
    return a, b, left node
if right_node is not None:
    return a, b, right node
return a, b, None
```



TARE A BREAK



第二类考察形态

二叉树结构变化



Flatten Binary Tree to Linked List

http://www.lintcode.com/problem/flatten-binary-tree-to-linked-list/

http://www.jiuzhang.com/solutions/flatten-binary-tree-to-linked-list/

将二叉树拆成链表

进行前序遍历,将上一个节点的右指针指向当前节点



```
public class Solution {
   TreeNode prevNode = null;
    public void flatten(TreeNode root) {
        if (root == null) {
            return;
        if (prevNode != null) {
            prevNode.left = null;
            prevNode.right = root;
        prevNode = root;
        TreeNode right = root.right;
        flatten(root.left);
        flatten(right);
```

```
class Solution:
    prev node = None
   def flatten(self, root):
       if root is None:
            return
       if self.prev node is not None:
            self.prev node.left = None
            self.prev node.right = root
       self.prev node = root
       right = root.right
       self.flatten(root.left)
       self.flatten(right)
```



尽可能避免使用全局变量

容易写出 BUG 可以把需要修改的变量作为参数传入到函数里 或者是放在 return value 里

代码



```
public class Solution {
   public void flatten(TreeNode root) {
       flatten and return last node(root);
   private TreeNode flatten and return last node(TreeNode root) {
        if (root == null) {
            return null;
       TreeNode left last = flatten and return last node(root.left);
       TreeNode right last = flatten and return last node(root.right);
        // connect
       if (left last != null) {
            left last.right = root.right;
            root.right = root.left;
            root.left = null;
       if (right last != null) {
            return right last;
        if (left last != null) {
            return left last;
        if (root != null) {
            return root;
       return null;
```

```
lass Solution:
   def flatten(self, root):
       self.flatten and return last node(root)
   def flatten and return last node(self, root):
       if root is None:
           return None
       left last = self.flatten and return last node(root.left)
       right last = self.flatten and return last node(root.right)
       # connect
       if left last is not None:
           left last.right = root.right
           root.right = root.left
           root.left = None
       return right last or left last or root
```



第三类考察形态

二叉搜索树 Binary Search Tree

BST 基本性质



- 从定义出发:
 - 左子树都比根节点小
 - 右子树都不小于根节点
- 从效果出发:
 - 中序遍历 in-order traversal 是"不下降"序列
 - 如图,中序遍历为12345



- 性质:
 - 如果一棵二叉树的中序遍历不是"不下降"序列,则一定不是BST
 - 如果一棵二叉树的中序遍历是不下降,也未必是BST
 - 比如下面这棵树就不是 BST, 但是它的中序遍历是不下降序列。
 - 1
 - / \
 - 1 1



BST的高度是多少

A: O(n)

B: O(logn)

C: O(h)



BST的高度是多少

同样是最坏 O(n) 最好 O(logn) 用 O(h) 表示更合适

只有 Balanced Binary Tree (平衡二叉树) 才是 O(logn)



红黑树 Red-black Tree

红黑树是一种 Balanced BST C++/Java中有一个数据结构的实现用的是 Balanced BST 你知道是什么吗?



Java: TreeMap / TreeSet

C++: map / set

Python: 不好意思我没有

有一定要用 TreeMap 来做的面试题么? ——没有

关于红黑树, 你需要掌握的是



- 知道他是一个 Balanced BST
- 知道他能干嘛
 - O(LogN) 的时间内实现增删查改
 - O(LogN) 的时间内实现找最大找最小
 - O(LogN) 的时间内实现找比某个数小的最大值(upperBound)和比某个数大的最小值(lowerBound)
- 知道他的工程应用价值
 - Java 1.8 中的 HashMap 的实现里同时用到了 TreeMap 和 LinkedList

就面试而言, 你不需要掌握他的代码实现, 因为写出来200+行, 没人能在面试的时候写完



Kth Smallest Element in BST

https://www.lintcode.com/problem/kth-smallest-element-in-a-bst/https://www.jiuzhang.com/solution/kth-smallest-element-in-a-bst/时间复杂度如何分析?



```
public class Solution {
    public int kthSmallest(TreeNode root, int k) {
        Stack<TreeNode> stack = new Stack<>();
        while (root != null) {
            stack.push(root);
            root = root.left;
        for (int i = 0; i < k - 1; i++) {
            TreeNode node = stack.peek();
            if (node.right == null) {
                node = stack.pop();
                while (!stack.isEmpty() && stack.peek().right == node) {
                    node = stack.pop();
             else {
                node = node.right;
                while (node != null) {
                    stack.push(node);
                    node = node.left;
        return stack.peek().val;
```

```
class Solution:
   def kthSmallest(self, root, k):
       stack = []
       while root != None:
           stack.append(root)
           root = root.left
       for i in range(k - 1):
           node = stack[-1]
           if node.right == None:
               node = stack.pop(-1)
               while len(stack) != 0 and stack[-1].right == node:
                   node = stack.pop(-1)
           else:
               node = node.right
               while node != None:
                   stack.append(node)
                   node = node.left
       return stack[-1].val
```



时间复杂度分析

O(k + h)

当 k 是 1 的时候 ==> O(h)

当 k 是 n 的时候 ==> O(n)

k和h两者取大值



Follow up: 二叉树经常被修改

如何优化 kthSmallest 这个操作?

优化方法



在 TreeNode 中增加一个 counter, 代表整个树的节点个数

也可以用一个 HashMap<TreeNode, Integer> 来存储某个节点为代表的子树的节点个数

在增删查改的过程中记录不断更新受影响节点的 counter

在 kthSmallest 的实现中用类似 Quick Select 的算法去找到 kth smallest element

时间复杂度为 O(h), h 为树的高度。

Strong Hire: 能够答出 Follow Up 的算法,并写出kthSmallest核心代码(不需要写增删查改,45分钟写不完的), bug free or minor bug,不需要提示

Hire / Weak Hire:能够答出 Follow up 的算法,大致写出 kthSmallest 核心代码,存在一定bug,或者需要提示

No Hire: 答不出 follow up

Strong No: 连第一问的 Inorder traversal 都不会写



Closest Binary Search Tree Value

https://www.lintcode.com/problem/closest-binary-search-tree-value/

http://www.jiuzhang.com/solution/closest-binary-search-tree-value/

如果使用中序遍历,时间复杂度是多少?

如果使用 lowerBound / upperBound 的方法,时间复杂度是多少?



Follow up: 寻找 k 个最接近的值

https://www.lintcode.com/problem/closest-binary-search-tree-value-ii/

https://www.jiuzhang.com/solution/closest-binary-search-tree-value-ii/

如果是用中序遍历得到从小到大的所有值,接下来的问题相当于之前学过的哪个题?

有没有更快的办法?



方法1暴力做法

先用 inorder traversal 求出中序遍历 找到第一个 >= target 的位置 index 从 index-1 和 index 出发,设置两根指针一左一右,获得最近的 k 个整数



```
public List<Integer> closestKValues(TreeNode root, double target, int k) {
   List<Integer> values = new ArrayList<>();
                                                                  private void traverse(TreeNode root, List<Integer> values) {
   traverse(root, values);
                                                                      if (root == null) {
   int i = 0, n = values.size();
                                                                           return;
   for (; i < n; i++) {
       if (values.get(i) >= target) {
           break;
                                                                      traverse(root.left, values);
                                                                      values.add(root.val);
                                                                      traverse(root.right, values);
   if (i >= n) {
       return values.subList(n - k, n);
   int left = i - 1, right = i;
   List<Integer> result = new ArrayList<>();
   for (i = 0; i < k; i++) {
       if (left >= 0 && (right >= n || target - values.get(left) < values.get(right) -
           target)) {
           result.add(values.get(left));
           left--;
       } else {
           result.add(values.get(right));
           right++;
   return result;
```

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```
closestKValues(self, root, target, k):
if root is None or k == 0:
    return []
nums = self.get inorder(root)
left = self.find_lower_index(nums, target)
right = left + 1
results = []
for _ in range(k):
    if self.is_left_closer(nums, left, right, target):
        results.append(nums[left])
       left -= 1
   else:
        results.append(nums[right])
       right += 1
return results
```

```
get_inorder(self, root):
dummy = TreeNode(0)
dummy.right = root
stack = [dummy]
inorder = []
while stack:
   node = stack.pop()
   if node.right:
        node = node.right
        while node:
            stack.append(node)
            node = node.left
   if stack:
        inorder.append(stack[-1].val)
return inorder
```



```
find_lower_index(self, nums, target):
   start, end = 0, len(nums) - 1
   while start + 1 < end:
       mid = (start + end) // 2
       if nums[mid] < target:</pre>
            start = mid
       else:
            end = mid
   if nums[end] < target:</pre>
       return end
   if nums[start] < target:</pre>
       return start
   return -1
def is_left_closer(self, nums, left, right, target):
   if left < 0:
       return False
   if right >= len(nums):
       return True
   return target - nums[left] < nums[right] - target</pre>
```



方法2 使用两个 Iterator

一个 iterator move forward 另一个iterator move backward 每次 i++ 的时候根据 stack,挪动到 next node 每次i--的时候根据 stack,挪动到 prev node



```
public List<Integer> closestKValues(TreeNode root, double target, int k) {
   List<Integer> values = new ArrayList<>();
                                                                        private Stack<TreeNode> getStack(TreeNode root, double target) {
                                                                            Stack<TreeNode> stack = new Stack<>();
   if (k == 0 | root == null) {
       return values:
                                                                            while (root != null) {
                                                                                stack.push(root);
   Stack<TreeNode> lowerStack = getStack(root, target);
                                                                                if (target < root.val) {</pre>
   Stack<TreeNode> upperStack = new Stack<>();
                                                                                    root = root.left;
   upperStack.addAll(lowerStack);
                                                                                } else {
   if (target < lowerStack.peek().val) {</pre>
                                                                                    root = root.right;
       moveLower(lowerStack);
   } else {
       moveUpper(upperStack);
                                                                            return stack:
   for (int i = 0; i < k; i++) {
        if (lowerStack.isEmpty() ||
               !upperStack.isEmpty() && target - lowerStack.peek().val > upperStack
                   .peek().val - target) {
           values.add(upperStack.peek().val);
           moveUpper(upperStack);
        } else {
           values.add(lowerStack.peek().val);
           moveLower(lowerStack);
   return values;
```

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```
public void moveUpper(Stack<TreeNode> stack) {
    TreeNode node = stack.peek();
    if (node.right == null) {
        node = stack.pop();
       while (!stack.isEmpty() && stack.peek().right == node) {
            node = stack.pop();
        return;
                                              public void moveLower(Stack<TreeNode> stack) {
                                                  TreeNode node = stack.peek();
                                                  if (node.left == null) {
   node = node.right;
                                                      node = stack.pop();
   while (node != null) {
                                                      while (!stack.isEmpty() && stack.peek().left == node) {
        stack.push(node);
                                                          node = stack.pop();
        node = node.left;
                                                      return;
                                                  node = node.left;
                                                  while (node != null) {
                                                      stack.push(node);
                                                      node = node.right;
```

代码



```
closestKValues(self, root, target, k):
if root is None or k == 0:
   return []
lower_stack = self.get_stack(root, target)
upper_stack = list(lower_stack)
if lower_stack[-1].val < target:</pre>
    self.move upper(upper stack)
else:
    self.move lower(lower stack)
result = []
for i in range(k):
    if self.is_lower_closer(lower_stack, upper_stack, target):
        result.append(lower_stack[-1].val)
        self.move lower(lower stack)
    else:
        result.append(upper stack[-1].val)
        self.move upper(upper stack)
return result
get_stack(self, root, target):
stack = []
while root:
    stack.append(root)
    if target < root.val:</pre>
        root = root.left
    else:
        root = root.right
return stack
```

```
move_upper(self, stack):
if stack[-1].right:
    node = stack[-1].right
    while node:
        stack.append(node)
        node = node.left
else:
    node = stack.pop()
    while stack and stack[-1].right == node:
        node = stack.pop()
move lower(self, stack):
if stack[-1].left:
    node = stack[-1].left
    while node:
        stack.append(node)
        node = node.right
else:
    node = stack.pop()
    while stack and stack[-1].left == node:
        node = stack.pop()
is lower closer(self, lower stack, upper stack, target):
if not lower stack:
    return False
if not upper stack:
    return True
return target - lower stack[-1].val < upper stack[-1].val - target
```

Closest Binary Search Tree Value 评分标准



Strong Hire

找1个点和找k个点都答出来,且找 k 个点的能用 O(k + logn) 的时间复杂度

Hire

找1个点和找k个点都答出来,且找 k 个点的能用 O(klogn) 的时间复杂度完成,少 bug,无需提示

Weak Hire:

找1个点和找k个点都答出来,且找 k 个点的能分别用 O(klogn) 和 O(n) 的时间复杂度完成,bug 多,需要提示

No Hire

答出1个点,答不出 k 个点非O(n)的算法

Strong No Hire

啥都答不出来

Related Questions



- Search Range in Binary Search Tree
- http://www.lintcode.com/problem/search-range-in-binary-search-tree/
- Insert Node in a Binary Search Tree
- http://www.lintcode.com/problem/insert-node-in-a-binary-search-tree/
- Remove Node in a Binary Search Tree
- http://www.lintcode.com/problem/remove-node-in-binary-search-tree/
- http://www.mathcs.emory.edu/~cheung/Courses/171/Syllabus/9-BinTree/BST-delete.html

在第7周的互动课中继续学习如下二叉树的内容



- 第 34 章 后序遍历非递归与 Morris 算法
 - 不作为必须要掌握的知识点,但是学了可以提高 Coding 能力
- 第 35 章 二叉查找树的增删查改
 - 必须掌握增查改,删除操作不作要求