CANDIDATE

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TEST

Quiz 5

Subject code	--
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Question	Status	Marks	Question type
1.1	Correct	1/1	Multiple Response
1.2	Correct	1/1	Multiple Choice
1.3	Correct	1/1	True / False
1.4	Correct	1/1	Text Entry
1.5	Correct	1/1	Multiple Response
1.6	Correct	1/1	Multiple Response
1.7	Correct	1/1	Multiple Response
2.1	Correct	1/1	True / False
2.2	Correct	1/1	Multiple Response
2.3	Correct	1/1	Multiple Choice

1.1 Suppose $f:S\to S$ and $g:S\to S$ are bijections.

Which of the following are true for all such functions?
Select all that apply

- [ ] $(f\circ g)^{-1}$ always exists and $(f\circ g)^{-1} = (f^{-1})\circ(g^{-1})$
- [x] $(f\circ g)^{-1}$ always exists and $(f\circ g)^{-1} = (g^{-1})\circ(f^{-1})$ ✔
- [ ] $(f;g)^{\leftarrow}$ always exists and $(f;g)^{\leftarrow} = (f^{\leftarrow});(g^{\leftarrow})$
- [x] $(f;g)^{\leftarrow}$ always exists and $(f;g)^{\leftarrow} = (g^{\leftarrow});(f^{\leftarrow})$ ✔
- [ ] None of the above

1.2 Let $A = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}$

Which of the following matrices is $AA - 2A^T$?

- $\begin{pmatrix} 7 & 2 \\ 8 & 3 \end{pmatrix}$
- $\begin{pmatrix} 7 & -4 \\ 14 & 3 \end{pmatrix}$ ✔
- $\begin{pmatrix} 3 & -1 \\ 8 & -1 \end{pmatrix}$
- $\begin{pmatrix} 3 & -7 \\ 14 & -1 \end{pmatrix}$
- None of the above

1.3 Suppose T(n) is defined recursively as:

- T(0) = 1
- T(n) = 3T(n/3) + O(n)

True or false: T(n) $\in$ O(n)

- False ✔
- True

1.4 Suppose f,g:{a,b}*→{a,b}* are defined recursively as follows:

- f(λ)=a
- g(λ)=b
- f(aw) = f(w)g(w)
- f(bw) = g(w)f(w)
- g(aw)=g(bw)=f(w)

What is f(bba)?

abaab ✔

1.5 Consider the following code snippet:

```
myFunc(n):
  if n==0:
     return 1
  else:
     return myFunc(n-1) + myFunc(n-1)
```

Which of the following hold with regard to the running time T(n) of this code
Select all that apply

- [ ] $T(n) \in O(\log n)$
- [x] $T(n) \in O(2^n)$ ✔
- [ ] None of these options
- [ ] $T(n) \in O(n^2)$
- [ ] $T(n) \in O(n)$

1.6 Consider the following code snippet:

```
myFunc(n):
  if n==0:
    return 1
  else:
    return 2*myFunc(n-1)
```

Which of the following hold with regard to the running time T(n) of this code

Select all that apply

- [x] $T(n) \in O(n)$
- [x] $T(n) \in O(2^n)$
- [x] $T(n) \in O(n^2)$
- [ ] None of these options
- [ ] $T(n) \in O(\log n)$

1.7 Recall the definition of a valid expression for the proof checker:

- $\emptyset$ and $\mathcal{U}$ are valid expressions
- Any single letter A-Z or a-z is a valid expression
- If E is a valid expression, then:
  - (E) is a valid expression
  - E^c is a valid expression
- If E_1 and E_2 are valid expressions, then:
  - $(E_1 \cap E_2)$ is a valid expression
  - $(E_1 \cup E_2)$ is a valid expression
  - $(E_1 \setminus E_2)$ is a valid expression
  - $(E_1 \oplus E_2)$ is a valid expression

Based on the above definition, which of the following are valid expressions?
Select all that apply:

- [ ] $((A \cup a) \cap (B \cup b) \cap (C \cup c))^c$
- [x] A^{ccc} ✔
- [x] $((A \cap B) \cup (C \cap (D)))$ ✔
- [x] $((A \cap B))$ ✔
- [ ] $((A1 \oplus A2)^c)^c$
- [x] $(((\mathcal{U} \setminus \emptyset) \setminus \mathcal{U}) \setminus \emptyset)$ ✔

2.1 Suppose T(n) is defined recursively as:

- $T(0) = 1$
- $T(n) = 3T(n-3) + O(n)$

True or false: $T(n) \in O(2^n)$

- ( ) False
- (•) True ✔

2.2 Consider the following code fragment that works on an array:

```
myFunc(A, lo, hi):
  if lo + 1 >= hi:
    return

  j = (lo + hi) / 2
  if A[lo] < A[hi]:
    myFunc(A, lo, j)
    myFunc(A, j, hi)
  else:
    myFunc(A, j, hi)
    myFunc(A, lo, j)

  for i ∈ [lo, hi):
    print(A[i])
```

In terms of n = hi - lo, which of the following hold for the running time T(n) of this code?

Select all that apply:

- [ ] $T(n) \in O(n)$
- [ ] $T(n) \in O(1)$
- [x] $T(n) \in O(2^n)$ ✓
- [x] $T(n) \in O(n^2)$ ✓
- [ ] $T(n) \in O(\log n)$
- [x] $T(n) \in O(n \log n)$ ✓

2.3 Consider a subset, EXP, of valid expressions (see Question 1.7), defined recursively as follows:

- $\varnothing$ and $\mathcal{U}$ are elements of EXP
- X,Y,Z $\in$ EXP
- If E $\in$ EXP, then:
  - (E) $\in$ EXP
  - $E^c \in$ EXP
- If $E_1, E_2 \in$ EXP, then:
  - $(E_1 \cap E_2) \in$ EXP
  - $(E_1 \cup E_2) \in$ EXP

Consider dual: EXP → EXP defined recursively as follows:

- dual($\varnothing$) = $\mathcal{U}$
- dual($\mathcal{U}$) = $\varnothing$
- dual(X) = X, dual(Y) = Y, dual(Z) = Z
- If E $\in$ EXP, then
  - dual((E)) = (dual(E))
  - dual(E^c) = dual(E) ^{c}
- If $E_1, E_2 \in$ EXP, then
  - dual($(E_1 \cap E_2)$) = (dual(E_1) $\cup$ dual(E_2))
  - dual($(E_1 \cup E_2)$) = (dual(E_1) $\cap$ dual(E_2))

What is dual($((X \cap Y^c) \cup \mathcal{U}$))?
Select one alternative:

- $((X^c \cap Y) \cup \varnothing)$
- $\varnothing$
- $((X \cup Y^c) \cap \varnothing)$
- $((X^c \cup Y) \cap \varnothing)$
- None of the above