CANDIDATE

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TEST

Quiz 6

Subject code	--
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Question	Status	Marks	Question type
1.1	Correct	1/1	Multiple Response
1.2	Correct	1/1	True / False
1.3	Correct	1/1	Multiple Response
1.4	Correct	1/1	Multiple Response
1.5	Correct	1/1	Multiple Response
2.1	Correct	1/1	True / False
2.2	Correct	1/1	True / False
2.3	Correct	1/1	Multiple Response
2.4	Correct	1/1	Multiple Response
2.5	Correct	1/1	Multiple Response

1.1 Suppose you are trying to prove that the square of every natural number has remainder 0 or 1 on division by 4. If P(n) is the proposition:

$$P(n): \qquad n^2 =_{(4)} 0 \quad \textbf{or} \quad n^2 =_{(4)} 1$$

which of the following would be sufficient to prove the result by induction?

Select all that apply:

- [ ] $P(0)$; and for all $k \in \mathbb{N}$ $P(k^2) \Rightarrow P((k+1)^2)$
- [x] $P(0)$; $P(1)$; and for all $k \in \mathbb{N}$ $P(k) \Rightarrow P(k+2)$ ✔
- [ ] None of the other options
- [ ] $P(0)$; $P(1)$; and for all $k \in \mathbb{N}$ $P(k) \Rightarrow P(k+4)$
- [ ] $P(0)$; for all $k \in \mathbb{N}$ $P(k) \Rightarrow P(k+2)$; and for all $k \in \mathbb{N}$ $P(k+1) \Rightarrow P(k+3)$
- [ ] $P(0)$; and for all $k \in \mathbb{N}$ $P(k^2) \Rightarrow P(k^2+1)$

1.2 Suppose $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined recursively as follows for all $m,n \in \mathbb{N}$:

- $f(0,m) = m$
- $f(n+1,m) = 1 + f(n,m)$

True or false:

$f(m,n) = f(n,m)$ for all $m,n \in \mathbb{N}$

- (•) True ✔
- ( ) False

1.3 Let C(n) be the maximum number of elementary operations required to compute A+B where A and B are n×n matrices.

What can be said about the asymptotic behaviour of C(n)?

Select all that apply

- [ ] $C(n) \in O(\log n)$
- [x] $C(n) \in O(n^3)$
- [ ] None of these options
- [ ] $C(n) \in O(1)$
- [ ] $C(n) \in O(n)$
- [x] $C(n) \in O(n^2)$

1.4 Consider the following code snippet:

```
myFunc(n):
  i = 1
  while i < n:
    j = n
    while j > 0:
      print('*')
      j = j/3
    i = i+2
```

Which of the following hold with regard to the running time T(n) of this code (assume / is integer division)

Select all that apply

- [x] $T(n) \in O(n^2)$ ✔
- [x] $T(n) \in O(n \log n)$ ✔
- [ ] $T(n) \in O(\log n)$
- [ ] None of these options
- [ ] $T(n) \in O(n)$

1.5 Consider the following code snippet:

```
myFunc(n):
    if n==0:
        return 1
    i = 0
    x = 0
    while i < n:
        x = x + myFunc(0)
        i = i+1
    return x
```

Which of the following hold with regard to the running time T(n) of this code

Select all that apply

- [x] $T(n) \in O(2^n)$
- [x] $T(n) \in O(n \log n)$
- [x] $T(n) \in O(n)$
- [ ] None of these options
- [x] $T(n) \in O(n^2)$

2.1 Let EXP be as defined in Quiz 6, Question M3. That is:

- $\emptyset$ and $\mathcal{U}$ are elements of EXP
- X,Y,Z $\in$ EXP
- If E $\in$ EXP, then:
  - (E) $\in$ EXP
  - E^c $\in$ EXP
- If E_1, E_2 $\in$ EXP, then:
  - $(E_1 \cap E_2)$ $\in$ EXP
  - $(E_1 \cup E_2)$ $\in$ EXP

Suppose we can show that the following holds for a proposition P:

- P($\emptyset$) holds
- P($\mathcal{U}$) holds
- P(X), P(Y), and P(Z) hold
- If P(E) holds then P((E)) holds
- If P(E) holds then P(E^c) holds
- If P(E_1) **and** P(E_2) hold, then P($(E_1 \cap E_2)$) holds
- If P(E_1) **or** P(E_2) hold, then P($(E_1 \cup E_2)$) holds

True or false: P(E) holds for all E $\in$ EXP

Select one alternative:

- False
- True

2.2 Suppose f:$\mathbb{N}\times\mathbb{N}\rightarrow\mathbb{N}$ is defined recursively as follows for all m,n $\in$ $\mathbb{N}$:

- f(0,m) = 0
- f(n+1,m) = m + f(n,m)

True or false:
f(m,n) = f(n,m) for all m,n $\in$ $\mathbb{N}$

- False
- True

2.3 Which of the following are true for all functions $f:\mathbb{N}\rightarrow\mathbb{N}$ with $f(n)\in\Theta(n)$?

Select all that apply

- [ ] $f\circ f \in \Omega(n^2)$
- [x] $f\circ f \in O(n^2)$
- [x] $f\circ f \in \Omega(n)$
- [x] $f\circ f \in O(n)$

2.4 Consider the following code fragment that works on a (non-empty) array:

```
myFunc(A):
  if length(A) <= 5:
    return A[0]

  i = 0
  while i < length(A):
    B = A[i, i+5)  # Take the i - (i+4)th elements of A
    C[i/5] = myFunc(B)
    i = i+5

  return myFunc(C)
```

If n = length(A), which of the following hold for T(n), the running time of the above code?

Select all that apply

- [ ] $T(n) \in O(\log n)$
- [ ] $T(n) \in O(1)$
- [ ] None of these options
- [x] $T(n) \in O(n^2)$
- [x] $T(n) \in O(n \log n)$
- [x] $T(n) \in O(n)$

2.5 Consider the following code snippet:

```
myFunc(n):
    if n==0:
        return 1
    else:
        return myFunc(n-1) + myFunc2(n-1)

myFunc2(n):
    if n==0:
        return 0
    else:
        return myFunc(n-1)
```

Which of the following hold with regard to the running time T(n) of this code
Select all that apply

- [ ] $T(n) \in O(n)$
- [ ] $T(n) \in O(n^2)$
- [x] $T(n) \in O(2^n)$
- [ ] None of these options
- [ ] $T(n) \in O(\log n)$