Our reference: FI6044

AUTHOR QUERY FORM

ELSEVIER	**Journal: FI** **Article Number: 6044**	**Please e-mail your responses and any corrections to:** **E-mail: correctionsaptara@elsevier.com**

Dear Author,

Please check your proof carefully and mark all corrections at the appropriate place in the proof (e.g., by using on-screen annotation in the PDF file) or compile them in a separate list. Note: if you opt to annotate the file with software other than Adobe Reader then please also highlight the appropriate place in the PDF file. To ensure fast publication of your paper please return your corrections within 48 hours.

Your article is registered as a regular item and is being processed for inclusion in a regular issue of the journal. If this is NOT correct and your article belongs to a Special Issue/Collection please contact p.huggins@elsevier.com immediately prior to returning your corrections.

For correction or revision of any artwork, please consult http://www.elsevier.com/artworkinstructions

Any queries or remarks that have arisen during the processing of your manuscript are listed below and highlighted by flags in the proof. Click on the '**Q**' link to go to the location in the proof.

Location in article	**Query / Remark: click on the Q link to go** **Please insert your reply or correction at the corresponding line in the proof**
Q1	AU: The author names have been tagged as given names and surnames (surnames are highlighted in teal color). Please confirm if they have been identified correctly. AQ:1: No Problem.
Q2	AU: Please confirm that the provided email "l_weizhen@163.com" is the correct address for official communication, else provide an alternate e-mail address to replace the existing one, because private e-mail addresses should not be used in articles as the address for communication. AQ:2: No Problem. If this email "l_weizhen@163.com" is not allowed, it can be revised to "weizhen.liu@hit.edu.cn".
Q3	AU: Please check funding information and confirm its correctness. AQ:3: No Problem.

Q4	AU: Refs. [24,33] were identical, the latter has been removed from the reference list and subsequent references have been renumbered. AQ:4: We agree to this revision.

Please check this box or indicate your approval if you have no corrections to make to the PDF file	

Thank you for your assistance.

Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Journal of the Franklin Institute xxx (xxxx) xxx
www.elsevier.com/locate/jfranklin

High-order command filtered adaptive backstepping control for second- and high-order fully actuated strict-feedback systems

Weizhen Liu[a,*], Guangren Duan[a,b], Mingzhe Hou[a]

[a] Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, China
[b] Center for Control Science and Technology, Southern University of Science and Technology, Shenzhen 518055, China

Received 9 October 2021; received in revised form 9 November 2022; accepted 12 January 2023
Available online xxx

Abstract

In this paper, a high-order command filtered adaptive backstepping (HOCFAB)-based approach is proposed in order to track a given reference signal for the second- and high-order strict-feedback systems (SFSs) with parametric uncertainties, where both their subsystems hold a common full-actuation structure, namely, high-order fully actuated (HOFA) SFSs. Unlike the prevailing traditional first-order state-space backstepping approach which suffers from the problem of "explosion of terms", the proposed HOCFAB approach circumvents the complexity arising owing to differentiating the virtual controllers repeatedly, and does not need to convert the high-order systems into first-order forms which is easier to carry out and demands fewer steps. Meanwhile, an error-compensating mechanism is constructed to reduce filtering errors. A critical analysis is theoretically proven which indicates that in both cases the entire system states are uniformly ultimately bounded under the proposed high-order controller, and the tracking error could be made arbitrarily small with predesigned parameters. Finally, the effectiveness of the proposed scheme is verified by a benchmark application in the robotic manipulator.

* Corresponding author.
E-mail addresses: l_weizhen@163.com (W. Liu), g.r.duan@hit.edu.cn (G. Duan), hithyt@hit.edu.cn (M. Hou).

https://doi.org/10.1016/j.jfranklin.2023.01.014

1. Introduction

Adaptive control is an old topic and flourish results have been obtained since the past several decades, e.g., model reference adaptive control [1], robust adaptive control [2], nonlinear adaptive control with low-triangular form ~~Krstic et al.~~ [3], multivariable adaptive control [4] and a special type of model-free adaptive control, namely, extremum seeking [5,6].

Even though the model of most physical systems by differential equations of second or higher order, those models were frequently turned into the first order and designed in the first-order state-space framework. Albeit the first-order state-space approach is more effective to obtain the solution of the state response and appropriate for observation (estimation), its contribution to controller design is limited [7]. Recently, the high-order system approach has been drawing considerable attention and a systematic high-order fully actuated (HOFA) system approach has been constructed to relieve the controller design [7–12]. By utilizing the fully actuated features, the HOFA system approach could directly deal with the analysis and control of high-order systems without converting the high-order systems into first-order ones. Fully actuated systems originate from the second-order mechanical system, and are regarded as a quite small amount of control systems by contrast with the under-actuated systems. Thus, fully actuated systems get little research attention until the concept of HOFA models is induced which can describe a large class of practical and theoretical systems. Moreover, the fully actuated feature can largely ease the controller design.

Unlike the feedback linearization method which requires precise models and often cancels some useful nonlinearities, the backstepping design approach is proposed which studies lower-triangular form nonlinear systems and utilizes recursive procedures to design virtual control law and feedback law [3,13]. Moreover, the backstepping design is flexible and avoids some useful nonlinearities cancelled when compared with the feedback linearization. Although backstepping has become mainstream of controller design methods for the lower-triangular systems, it possesses the explosion of complexity problem which is caused by the repeated differentiation of virtual controllers in each step design and inevitably leads to a complicated form with a heavy computation burden. It is worth noting that the complexity of the controller grows drastically if the backstepping steps are larger than three [14].

Later on, an effective approach named command filtered backstepping (CFB) was proposed by adopting command filters at each design step to compute the derivative of nonlinear functions and this technique avoids the phenomenon of explosion of complexity [14]. Also, this scheme was extended to command filtered adaptive backstepping (CFAB) where parameter uncertainties were considered [15]. The CFB has attracted tremendous attention and various accomplishments have been derived, such as SFSs with unmeasured states [16], finite-time CFB [17], CFB with unmodeled dynamics and external disturbances [18,19], CFB for nonlinear systems with time-varying parameters and disturbances [20], finite-time CFB for uncertain nonlinear systems with unknown actuator faults [21], CFB for a type of nonlinear MIMO systems with disturbances [22]. Moreover, fruitful results were obtained in the practical application by utilizing the CFB technique, such as in land vehicle control [23], vertical take-off and landing vehicles [24], multi-agent systems [25], control of islanded photovoltaic microgrid [26], control of hydraulic turbine regulating systems [27], nonlinear excitation control of diesel generator [28], control of AC/DC converter [29], control of dual-motor servo systems [30].

Despite the fact that many efforts have been made to study CFB, the majority of them confine the physical model to first-order state-space forms while ignoring the fully actuated properties of the original high-order systems. It means that the whole system is treated as a cascade of first-order subsystems. However, the dynamics of the system are usually governed by high-order differential equations in practical scenarios so the original description of the system dynamics is usually a cascade of high-order subsystems. As pointed out in Farrell et al. [14], calculation of the derivative of the virtual control signal can be quite complicated in applications when steps of backstepping are greater than three. Thus, reducing the steps and circumventing the explosion of complexity both play a significant role in practical applications. Compared with the existing first-order command filter backstepping results [14,15,31,32], the proposed high-order CFAB (HOCFAB) scheme needs to know the high-order derivative of the desired trajectory where the order of the derivative is twice the order of the first subsystem. This request can be explained that the system is expressed as a high-order form rather than the first-order state-space form, and the derivative of the desired trajectory should be known to achieve desired output trajectory tracking. As pointed out in Farrell et al. [14], the high-order derivative of the desired trajectory can be obtained by treating the trajectory as the input to a prefilter. Even though a recent study utilizes the CFB and the HOFA system approach to construct the controller in Liu et al. [12], this study and the recently published article belong to two completely different research areas since they consider different types of uncertainties, i.e., parameter uncertainties and nonlinear uncertainties. Moreover, they are totally different in the framework of stability analysis since this study introduces a high-order error-compensating mechanism.

This paper, by fusing the techniques of HOFA and CFAB, a direct design scheme is proposed to construct the controller along with introducing a high-order error-compensating mechanism, under which a new framework is derived. The contributions of the proposed HOCFAB scheme are outlined below:

- This result proposes a new perspective in designing the CFAB to deal with high-order SFSs under parameter uncertainties that do not need to convert the high-order systems into first-order forms.
- Compared with the recent studies for high-order robust command filtered backstepping in Liu et al. [12], this proposed HOCFAB scheme has a different structure in the organization and introduces high-order filters to generate certain compensating signals necessary to compute compensated tracking errors. While these introduced dynamics lead to a new framework for carrying out the stability analysis since a Lyapunov function consisting of all the dynamic states can not be chosen here.
- Similar to recent studies for high-order SFSs under bounded nonlinear uncertainties in Liu et al. [12] even though they consider different types of uncertainties, i.e., nonlinear uncertainties and parameter uncertainties, this proposed scheme can still achieve the properties that avoid complexity arising due to repeated differentiation and demand fewer backstepping steps.

The paper's rest organization is as follows. The forms of second- and high-order (mixed-order) SFSs are stated, and the necessary assumptions are given in Section 2. The design steps of the HOCFAB scheme are presented for the second- and high-order (mixed-order) SFSs in Sections 3 and 4, respectively, along with theoretical proof of uniformly ultimately bounded of all the system states. A benchmark application in robotic manipulators is provided in Section 5 to verify the validity of the proposed scheme. Finally, the conclusion is presented in Section 6.

2. Problem formulation

Consider a type of second-order SFSs with parametric uncertainties which are frequently used in Duan [8,10] is given by,

$$\begin{cases} \ddot{x}_1 = f_{10}\left(x_1^{(0\sim1)}\right) + f_1^{\mathrm{T}}\left(x_1^{(0\sim1)}\right)\theta_1 + g_1\left(x_1^{(0\sim1)}\right)x_2 \\ \ddot{x}_2 = f_{20}\left(x_{1\sim2}^{(0\sim1)}\right) + f_2^{\mathrm{T}}\left(x_{1\sim2}^{(0\sim1)}\right)\theta_2 + g_2\left(x_{1\sim2}^{(0\sim1)}\right)x_3 \\ \vdots \\ \ddot{x}_{n-1} = f_{(n-1)0}\left(x_{1\sim n-1}^{(0\sim1)}\right) + f_{n-1}^{\mathrm{T}}\left(x_{1\sim n-1}^{(0\sim1)}\right)\theta_{n-1} + g_{n-1}\left(x_{1\sim n-1}^{(0\sim1)}\right)x_n \\ \ddot{x}_n = f_{n0}\left(x_{1\sim n}^{(0\sim1)}\right) + f_n^{\mathrm{T}}\left(x_{1\sim n}^{(0\sim1)}\right)\theta_n + g_n\left(x_{1\sim n}^{(0\sim1)}\right)u, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R},\ i = 1, 2, \ldots, n$ are the state variables, $u \in \mathbb{R}$ is the control input, $f_{i0}\left(x_{1\sim i}^{(0\sim1)}\right)$ and $g_i\left(x_{1\sim i}^{(0\sim1)}\right) \in \mathbb{R},\ i = 1, 2, \ldots, n$ are two set of sufficiently smooth scalar functions, $\theta_i \in \mathbb{R}^{r_i}, i = 1, 2, \ldots, n$ are unknown constant vector parameters, and $f_i\left(x_{1\sim i}^{(0\sim1)}\right) \in \mathbb{R}^{r_i}, i = 1, 2, \ldots, n$ are a set of sufficiently smooth vector functions.

The proposed HOCFAB scheme aims to steer x_1 to track the given reference signal x_{1c} with the designing control signal u for the system Eq. (1) under certain initial condition. At the same time, all signals are uniformly ultimately bounded and the signal x_{1c} obeys the following assumption.

Assumption 1. The desired trajectory x_{1c} and its first to fourth derivative $\dot{x}_{1c}, \ddot{x}_{1c}, \dddot{x}_{1c}, x_{1d}^{(4)}$ are smooth, available and bounded.

A straightforward generalization of the above second-order SFSs Eq. (1) is the following high-order (mixed-order) SFSs with unknown parameters:

$$\begin{cases} x_1^{(m_1)} = f_{10}\left(x_1^{(0\sim m_1-1)}\right) + f_1^{\mathrm{T}}\left(x_1^{(0\sim m_1-1)}\right)\theta_1 + g_1\left(x_1^{(0\sim m_1-1)}\right)x_2 \\ x_2^{(m_2)} = f_{20}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right) + f_2^{\mathrm{T}}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right)\theta_2 + g_2\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right)x_3 \\ \vdots \\ x_{n-1}^{(m_{n-1})} = f_{(n-1)0}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n-1}\right) + f_{n-1}^{\mathrm{T}}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n-1}\right)\theta_{n-1} \\ \qquad + g_{n-1}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n-1}\right)x_n \\ x_n^{(m_n)} = f_{n0}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right) + f_n^{\mathrm{T}}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\theta_n + g_n\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)u, \end{cases} \tag{2}$$

where $m_i,\ i = 1, 2, \ldots, n$ are a set of positive integers. Throughout the paper, the arguments $g_i\left(x_{1\sim i}^{(0\sim1)}\right)$ in second-order SFSs or $g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)$ in high-order SFSs will be discarded, and expressed as g_i to ease the presentation.

The reference signal x_{1c} for the high-order SFSs Eq. (2) satisfies the following assumption.

Assumption 2. The desired trajectory x_{1c} and its first to $2m_1$th derivative $\dot{x}_{1c}, \ldots, x_{1c}^{(2m_1)}$ are smooth, available and bounded.

Remark 1. Compared with the traditional command filter backstepping, the proposed HOC-FAB scheme can largely reduce the unnecessary steps while the fourth order or ~~$(2m_i)$th~~ order derivative of the desired trajectory x_{1c} are needed for second-order SFSs or for high-order SFSs, respectively. Since state-space form command filter backstepping only requires knowledge of the intended trajectory x_{1c} and its first derivative, this assumption is a little more rigorous. However, when contrasted to the rewards of massively reducing the backstepping steps, those are rather mild request.

Assumption 3. Let Ω_d represent an open set that contains the origin, the initial condition $x(0)$, the trajectory x_{1c}. For the second-order SFSs Eq. (1) $f_{i0}^{(j)}\left(x_{1\sim i}^{(0\sim 1)}\right)$, $f_i^{(j)}\left(x_{1\sim i}^{(0\sim 1)}\right)$ and $g_i^{(j)}\left(x_{1\sim i}^{(0\sim 1)}\right)$ are bounded on $\bar{\Omega}_d$ for $j = 1, 2, \ldots, n-i$. Furthermore, $\left|g_i\left(x_{1\sim i}^{(0\sim 1)}\right)\right| \neq 0$ and $\left|g_i\left(x_{1\sim i}^{(0\sim 1)}\right)\right| \leq \rho, \forall x_i, \dot{x}_i \in \mathbb{R},\ i = 1, 2, \ldots, n$ for second-order SFSs or $\left|g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\right| \neq 0$ and $\left|g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\right| \leq \rho, \forall x_j^{(0\sim m_j-1)} \in \mathbb{R}^{m_j},\ i = 1, 2, \ldots, n$ for high-order SFSs where ρ is known positive constant.

Remark 2. The fully actuated strict-feedback systems only require that $\left|g_i\left(x_{1\sim i}^{(0\sim 1)}\right)\right| \neq 0$ or $\left|g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\right| \neq 0$. However, the condition $\left|g_i\left(x_{1\sim i}^{(0\sim 1)}\right)\right| \leq \rho$, $\left|g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\right| \leq \rho$ needs also to be satisfied since we aim to circumvent the "explosion of complexity" beyond only stabilization in classical backstepping. This assumption is universal in command filtered backstepping [14,15], and this is easy to verify that $g_i\left(x_{1\sim i}^{(0\sim 1)}\right)$ or $g_i\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)$ has a maximum ρ since it is continuous on the compact set $\bar{\Omega}_d$.

Certain symbols are provided to simplify the presentation,

$$x^{(0\sim n)} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(n)} \end{bmatrix}, x_{i\sim j}^{(0\sim n)} = \begin{bmatrix} x_i^{(0\sim n)} \\ x_{i+1}^{(0\sim n)} \\ \vdots \\ x_j^{(0\sim n)} \end{bmatrix}, j \geq i,$$

$$A^{0\sim n-1} = \begin{bmatrix} A_0 & A_1 & \ldots & A_{n-1} \end{bmatrix},$$

and

$$\Phi\left(A^{0\sim n-1}\right) = \begin{bmatrix} 0 & I & & \\ & & \ddots & \\ & & & I \\ -A_0 & -A_1 & \ldots & -A_{n-1} \end{bmatrix}.$$

Furthermore, $\lambda_{\max}(P_i)$ and $\lambda_{\min}(P_i)$ will be represented the largest and smallest eigenvalues of matrix P_i, respectively.

Lemma 1 [9]**.** *Let* $A \in \mathbb{R}^{r\times r}$ *and* $\Phi\left(A^{0\sim n-1}\right) \in \mathbb{R}^{nr\times nr}$ *satisfy*

$$\mathrm{Re}\,\lambda_i(\Phi) \leq -\frac{\mu}{2},\ \ i = 1, 2, \ldots, n, \tag{3}$$

where $\mu > 0$, then there exists a positive definite symmetric matrix $P(A^{0\sim n-1}) \in \mathbb{R}^{nr\times nr}$ satisfying

$$\Phi^{\mathrm{T}}(A^{0\sim n-1})P(A^{0\sim n-1}) + P(A^{0\sim n-1})\Phi(A^{0\sim n-1}) \le -\mu P(A^{0\sim n-1}). \tag{4}$$

3. HOCFAB for second-order SFSs

Different from the conventional approach which turns the system into a first-order SFSs form, a HOCFAB scheme for the second-order SFSs Eq. (1) is proposed. And this proposed scheme is simpler to implement and needs fewer steps.

Assume $A_i^{(0\sim 1)} \in \mathbb{R}^{1\times 2}, i = 1, 2, \dots, n$ are a set of matrices to make $\Phi(A_i^{0\sim 1}) \in \mathbb{R}^{2\times 2}, i = 1, 2, \dots, n$ stable and

$$P_i(A_i^{0\sim 1}) = \begin{bmatrix} P_{iF}(A_i^{0\sim 1}) & P_{iL}(A_i^{0\sim 1}) \end{bmatrix} \in \mathbb{R}^{2\times 2}. \tag{5}$$

Lemma 2 [16,24,33]**.** *Let $q(t)$ be an input signal to the command filter with*

$$\dot{z}_1 = z_2 \tag{6}$$

$$\dot{z}_2 = -2\zeta\omega z_2 - \omega^2(z_1 - q), \tag{7}$$

where $\omega \in \mathbb{R}_{>0}, \zeta \in (0, 1]$. If $q(t)$ satisfies $|q(t)| \le \beta, |\dot{q}(t)| \le \gamma$ and $|\ddot{q}(t)| \le \eta$ for all $t \ge 0$, where β, γ and η are positive constants and $z_1(0) = q(0), z_2(0) = 0$, then there exist $\sigma > 0$ such that $|z_1 - q| \le \sigma, |z_1|, |\dot{z}_1|, |\ddot{z}_1|$ and $|\dddot{z}_1|$ are bounded.

Remark 3. Even though this lemma obtains that the signals $|z_1 - q|, |z_1|, |\dot{z}_1|, |\ddot{z}_1|$ and $|\dddot{z}_1|$ are bounded by requiring bounded $|q(t)|, |\dot{q}(t)|$ and $|\ddot{q}(t)|$ without critical prove, the bounded of the input signal $|\alpha_i|, |\dot{\alpha}_i|$ and $|\ddot{\alpha}_i|$ used in command filter, which was defined later in Eqs. (9) and (10), will be obtained in Lemma 3 and this lemma is critically proved step by step.

3.1. Control law and parameter update law design

For the second-order SFSs Eq. (1), the tracking error of the command filtered backstepping is defined as

$$\tilde{x}_i = x_i - x_{ic}, \tag{8}$$

x_{1c} is the desired output trajectory, and $x_{ic}, i = 2, 3, \dots, n$ are derived by a range of command filters on the $(i-1)$th virtual control functions α_{i-1} which are defined later in Eq. (14)

$$\dot{z}_{i,1} = z_{i,2} \tag{9}$$

$$\dot{z}_{i,2} = -2\zeta\omega_i z_{i,2} - \omega_i^2(z_{i,1} - \alpha_{i-1}), \tag{10}$$

where $\omega_i \in \mathbb{R}_{>0}$ and $\zeta \in (0, 1]$ are frequency and damp ratio, respectively. Furthermore, the outputs of each command filer are defined as $x_{ic} = z_{i,1}, \dot{x}_{ic} = z_{i,2}$ and $\ddot{x}_{ic} = \dot{z}_{i,2} = -2\zeta\omega_i z_{i,2} - \omega_i^2(z_{i,1} - \alpha_{i-1})$.

The compensated tracking signals is a filtered version of $\left(x_{(i+1)c}-\alpha_i\right)$ which aims to reduce the effect of such unachieved portion $\left(x_{(i+1)c}-\alpha_i\right)$ with $i=1,2,\ldots,n-1$ and are defined as

$$\ddot{\xi}_i=-A_i^{0\sim1}\xi_i^{(0\sim1)}+g_i\xi_{i+1}+g_i\left(x_{(i+1)c}-\alpha_i\right), \tag{11}$$

$$\ddot{\xi}_n=-A_n^{0\sim1}\xi_n^{(0\sim1)}. \tag{12}$$

Moreover, define the compensated tracking error signals v_i as

$$v_i=\tilde{x}_i-\xi_i. \tag{13}$$

The virtual control function α_i of HOCFAB for system Eq. (1) is represented as below and for the i $(1\le i\le n-1)$ step,

$$\alpha_i=-\frac{1}{g_i}\left(A_i^{0\sim1}\tilde{x}_i^{(0\sim1)}+f_{i0}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)+f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\hat{\theta}_i-\ddot{x}_{ic}\right), \tag{14}$$

and the control law

$$u=-\frac{1}{g_n}\left(A_n^{0\sim1}\tilde{x}_n^{(0\sim1)}+f_{n0}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)+f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\hat{\theta}_n-\ddot{x}_{nc}\right). \tag{15}$$

The estimation of the unknown parameter θ_i is represented as $\hat{\theta}_i$, and $\hat{\theta}_i$ satisfies parameter update law below

$$\dot{\hat{\theta}}_i=r_if_i\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)P_{iL}^{\mathrm{T}}(A_i^{0\sim1})v_i^{(0\sim1)}+\pi_i\left(\theta_{ip}-\hat{\theta}_i\right), \tag{16}$$

where r_i, λ_i and π_i are predesigned constant values, and θ_{ip} is the predictive value of the unknown parameter θ_i which can make a warm start to the parameters with a possible prior knowledge. The estimation errors of the parameters are defined as

$$\tilde{\theta}_i=\theta_i-\hat{\theta}_i. \tag{17}$$

Lemma 3. *Given the Assumptions 1 and 3 , for the virtual control function α_i defined in Eq. (14) and the command filters defined in Eqs. (9) and (10), we have $|\alpha_i|$, $|\dot{\alpha}_i|$ and $|\ddot{\alpha}_i|$ are bounded.*

Proof. Refer to the Appendix for the critical proof. □

3.2. HOCFAB algorithm for second-order SFSs

The algorithm of the proposed HOCFAB is completed by the following n steps.

Step 1: Taking differentials of the Eq. (8) for $i=1$, and by utilizing the first equation in Eq. (1), it produces that

$$\ddot{\tilde{x}}_1=f_{10}\left(\tilde{x}_1^{(0\sim1)}\right)+f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim1)}\right)\theta_1+g_1(\tilde{x}_2+x_{2c})-\ddot{x}_{1c}. \tag{18}$$

In the condition of $i=1$, related parameter update law in Eq. (16) and virtual control law in Eq. (14) can be acquired,

$$\dot{\hat{\theta}}_1=r_1f_1\left(\tilde{x}_1^{(0\sim1)}\right)P_{1L}^{\mathrm{T}}(A_1^{0\sim1})v_1^{(0\sim1)}+\pi_1\left(\theta_{1p}-\hat{\theta}_1\right), \tag{19}$$

$$\alpha_1 = -\frac{1}{g_1}\left(A_1^{0\sim 1}\tilde{x}_1^{(0\sim 1)} + f_{10}\left(\tilde{x}_1^{(0\sim 1)}\right) + f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim 1)}\right)\hat{\theta}_1 - \ddot{x}_{1c}\right), \tag{20}$$

and the resulting closed-loop subsystem is obtained by substituting Eq. (20) into Eq. (18),

$$\ddot{\tilde{x}}_1 = -A_1^{0\sim 1}\tilde{x}_1^{(0\sim 1)} + g_1\tilde{x}_2 + g_1(x_{2c} - \alpha_1) + f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim 1)}\right)\tilde{\theta}_1. \tag{21}$$

Differentiating error signals v_1 and streamlining that outcome by putting Eq. (11) with $i = 1$ from Eq. (21), then the dynamics of the compensated tracking errors are derived as

$$\ddot{v}_1 = -A_1^{0\sim 1}v_1^{(0\sim 1)} + g_1 v_2 + f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim 1)}\right)\tilde{\theta}_1.$$

The above dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_1^{(0\sim 1)} = \Phi(A_1^{0\sim 1})v_1^{(0\sim 1)} + \begin{bmatrix} 0 \\ f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim 1)}\right)\tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ g_1 v_2 \end{bmatrix}. \tag{22}$$

Hence, define

$$V_1 = \frac{1}{2}\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})v_1^{(0\sim 1)} + \frac{1}{2r_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1.$$

According to the Lemma 1, Eq. (5) and the parameter update law Eq. (19), the time derivative of V_1 is given as

$$\begin{aligned}
\dot{V}_1 &= \frac{1}{2}\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}(A_1^{0\sim 1})P_1(A_1^{0\sim 1}) + P_1(A_1^{0\sim 1})\Phi(A_1^{0\sim 1})\right)v_1^{(0\sim 1)} \\
&\quad + \left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})\left(\begin{bmatrix} 0 \\ f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim 1)}\right)\tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ g_1 v_2 \end{bmatrix}\right) - \frac{1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\dot{\hat{\theta}}_1 \\
&\leq -\frac{\mu_1}{2}\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})v_1^{(0\sim 1)} + \left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_{1L}(A_1^{0\sim 1})g_1 v_2 - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right).
\end{aligned} \tag{23}$$

By utilizing Young's inequality, gives

$$\begin{aligned}
\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_{1L}(A_1^{0\sim 1})g_1 v_2 &= \left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})\begin{bmatrix} 0 \\ g_1 v_2 \end{bmatrix} \\
&\leq \left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})P_1^{\mathrm{T}}(A_1^{0\sim 1})v_1^{(0\sim 1)} + \frac{g_1^2}{4}v_2^2 \\
&\leq \lambda_{\max}(P_1)\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})v_1^{(0\sim 1)} \\
&\quad + \frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim 1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim 1})v_2^{(0\sim 1)}.
\end{aligned} \tag{24}$$

Substituting Eq. (24) into Eq. (23), we can obtain

$$\begin{aligned}
\dot{V}_1 &\leq -\frac{1}{2}(\mu_1 - 2\lambda_{\max}(P_1))\left(v_1^{(0\sim 1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim 1})v_1^{(0\sim 1)} \\
&\quad + \frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim 1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim 1})v_2^{(0\sim 1)} - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right).
\end{aligned}$$

To keep $\dot{V}_1$ with a uniform form with the following $\dot{V}_i$, we further add an extra positive part for $\dot{V}_1$ and rewrite it as

$$\dot{V}_1 \leq -\frac{1}{2}\left(\mu_1 - 2\lambda_{\max}(P_1) - \frac{g_0^2}{2\lambda_{\min}(P_1)}\right)\left(v_1^{(0\sim1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim1})v_1^{(0\sim1)} + \frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim1})v_2^{(0\sim1)} - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right), \tag{25}$$

where g_0 is a small positive constant value.

Step 2: Taking differentials of the Eq. (8) for $i = 2$, and by utilizing the second equation in Eq. (1), yields

$$\ddot{\tilde{x}}_2 = f_{20}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right) + f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\theta_2 + g_2(\tilde{x}_3 + x_{3c}) - \ddot{x}_{2c}. \tag{26}$$

In the condition of $i = 2$, related parameter update law in Eq. (16) and virtual control law in Eq. (14) can be derived,

$$\dot{\hat{\theta}}_2 = r_2 f_2\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)P_{2L}^{\mathrm{T}}(A_2^{0\sim1})v_2^{(0\sim1)} + \pi_2\left(\theta_{2p} - \hat{\theta}_2\right), \tag{27}$$

226

$$\alpha_2 = -\frac{1}{g_2}\left(A_2^{0\sim1}\tilde{x}_2^{(0\sim1)} + f_{20}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right) + f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\hat{\theta}_2 - \ddot{x}_{2c}\right), \tag{28}$$

and the related closed-loop subsystem is gained by substituting Eq. (28) into Eq. (26),

$$\ddot{\tilde{x}}_2 = -A_2^{0\sim1}\tilde{x}_2^{(0\sim1)} + g_2\tilde{x}_3 + g_2(x_{3c} - \alpha_2) + f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\tilde{\theta}_2. \tag{29}$$

Differentiating error signals v_2 and streamlining that outcome by putting Eq. (11) with $i = 2$ from Eq. (29), then the dynamics of the compensated tracking errors are derived as

$$\ddot{v}_2 = -A_2^{0\sim1}v_2^{(0\sim1)} + g_2 v_3 + f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\tilde{\theta}_2.$$

The above dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_2^{(0\sim1)} = \Phi(A_2^{0\sim1})v_2^{(0\sim1)} + \begin{bmatrix} 0 \\ f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\tilde{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 v_3 \end{bmatrix}. \tag{30}$$

Hence, define

$$V_2 = V_1 + \frac{1}{2}\left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim1})v_2^{(0\sim1)} + \frac{1}{2r_2}\tilde{\theta}_2^{\mathrm{T}}\tilde{\theta}_2,$$

According to the Lemma 1, Eq. (5) and the parameter update law Eq. (27), the time derivative of V_2 is given as

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \frac{1}{2}\left(v_2^{(0\sim1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}(A_2^{0\sim1})P_2(A_2^{0\sim1}) + P_2(A_2^{0\sim1})\Phi(A_2^{0\sim1})\right)v_2^{(0\sim1)} \\ &\quad + \left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim1})\left(\begin{bmatrix} 0 \\ f_2^{\mathrm{T}}\left(\tilde{x}_{1\sim2}^{(0\sim1)}\right)\tilde{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 v_3 \end{bmatrix}\right) - \frac{1}{r_2}\tilde{\theta}_2^{\mathrm{T}}\dot{\hat{\theta}}_2 \\ &\leq \dot{V}_1 - \frac{\mu_2}{2}\left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_2(A_2^{0\sim1})v_2^{(0\sim1)} + \left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_{2L}(A_2^{0\sim1})g_2 v_3 - \frac{\pi_2}{r_2}\tilde{\theta}_2^{\mathrm{T}}\left(\theta_{2p} - \hat{\theta}_2\right). \end{aligned} \tag{31}$$

Similar to Eq. (24), it follows that

$$\left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_{2L}\left(A_2^{0\sim1}\right) g_2 v_3 \leq \lambda_{\max}(P_2)\left(v_2^{(0\sim1)}\right)^{\mathrm{T}} P_2\left(A_2^{0\sim1}\right) v_2^{(0\sim1)} + \frac{g_2^2}{4\lambda_{\min}(P_3)}\left(v_3^{(0\sim1)}\right)^{\mathrm{T}} P_3\left(A_3^{0\sim1}\right) v_3^{(0\sim1)}. \tag{32}$$

Substituting Eq. (32) into Eq. (31), we can obtain

$$\dot{V}_2 \leq -\frac{1}{2}\sum_{j=1}^{2}\left(\mu_j - 2\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim1}\right) v_j^{(0\sim1)} - \sum_{j=1}^{2}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \hat{\theta}_j\right) + \frac{g_2^2}{4\lambda_{\min}(P_3)}\left(v_3^{(0\sim1)}\right)^{\mathrm{T}} P_3\left(A_3^{0\sim1}\right) v_3^{(0\sim1)}.$$

Step i: Using the ith equation in Eq. (1) and taking the differentials of the Eq. (8), it gives

$$\ddot{\tilde{x}}_i = f_{i0}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right) + f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\theta_i + g_i\left(\tilde{x}_{i+1} + x_{(i+1)c}\right) - \ddot{x}_{ic}. \tag{33}$$

In the condition of i, related parameter update law in Eq. (16) and virtual control law in Eq. (14) can be acquired,

$$\dot{\hat{\theta}}_i = r_i f_i\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right) P_{iL}^{\mathrm{T}}\left(A_i^{0\sim1}\right) v_i^{(0\sim1)} + \pi_i\left(\theta_{ip} - \hat{\theta}_i\right), \tag{34}$$

$$\alpha_i = -\frac{1}{g_i}\left(A_i^{0\sim1}\tilde{x}_i^{(0\sim1)} + f_{i0}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right) + f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\hat{\theta}_i - \ddot{x}_{ic}\right), \tag{35}$$

and the resulting closed-loop subsystem is established by putting Eq. (35) into Eq. (33),

$$\ddot{\tilde{x}}_i = -A_i^{0\sim1}\tilde{x}_i^{(0\sim1)} + g_i\tilde{x}_{i+1} + g_i\left(x_{(i+1)c} - \alpha_i\right) + f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\tilde{\theta}_i. \tag{36}$$

Differentiating error signals v_i and streamlining that outcome by putting Eq. (11) from Eq. (36), then the dynamics of the compensated tracking errors are derived as

$$\ddot{v}_i = -A_i^{0\sim1}v_i^{(0\sim1)} + g_i v_{i+1} + f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\tilde{\theta}_i.$$

The dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_i^{(0\sim1)} = \Phi\left(A_i^{0\sim1}\right) v_i^{(0\sim1)} + \begin{bmatrix} 0 \\ f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\tilde{\theta}_i \end{bmatrix} + \begin{bmatrix} 0 \\ g_i v_{i+1} \end{bmatrix}. \tag{37}$$

Hence, define

$$V_i = V_{i-1} + \frac{1}{2}\left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim1}\right) v_i^{(0\sim1)} + \frac{1}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i.$$

According to the Lemma 1, Eq. (5) and the parameter update law Eq. (34), the time derivative of V_i is given as

$$\dot{V}_i = \dot{V}_{i-1} + \frac{1}{2}\left(v_i^{(0\sim1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}\left(A_i^{0\sim1}\right) P_i\left(A_i^{0\sim1}\right) + P_i\left(A_i^{0\sim1}\right)\Phi\left(A_i^{0\sim1}\right)\right) v_i^{(0\sim1)} + \left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim1}\right)\left(\begin{bmatrix} 0 \\ f_i^{\mathrm{T}}\left(\tilde{x}_{1\sim i}^{(0\sim1)}\right)\tilde{\theta}_i \end{bmatrix} + \begin{bmatrix} 0 \\ g_i v_{i+1} \end{bmatrix}\right) - \frac{1}{r_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i$$

$$\leq \dot{V}_{i-1} - \frac{\mu_i}{2}\left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim1}\right)\left(v_i^{(0\sim1)}\right) + \left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim1}\right) g_i v_{i+1}$$
$$- \frac{\pi_i}{r_i}\tilde{\theta}_i^{\mathrm{T}}\left(\theta_{ip} - \hat{\theta}_i\right). \tag{38}$$

Similar to Eq. (24), it follows that

$$\left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim1}\right) g_i v_{i+1} \leq \lambda_{\max}(P_i)\left(v_i^{(0\sim1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim1}\right) v_i^{(0\sim1)}$$
$$+ \frac{g_i^2}{4\lambda_{\min}(P_{i+1})}\left(v_{i+1}^{(0\sim1)}\right)^{\mathrm{T}} P_{i+1}\left(A_{i+1}^{0\sim1}\right) v_{i+1}^{(0\sim1)}. \tag{39}$$

By Eq. (39), we have

$$\dot{V}_i \leq -\frac{1}{2}\sum_{j=1}^{i}\left(\mu_j - 2\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim1}\right) v_j^{(0\sim1)}$$
$$- \sum_{j=1}^{i}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \hat{\theta}_j\right) + \frac{g_i^2}{4\lambda_{\min}(P_{i+1})}\left(v_{i+1}^{(0\sim1)}\right)^{\mathrm{T}} P_{i+1}\left(A_{i+1}^{0\sim1}\right) v_{i+1}^{(0\sim1)}.$$

Step n: An exhaustive treatment of other steps is not given here for brevity and similar to the previous treatment, it follows that,

$$\ddot{\tilde{x}}_n = f_{n0}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right) + f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\theta_n + g_n u - \ddot{x}_{nc}. \tag{40}$$

Related parameter update law in Eq. (16) and control law in Eq. (15) could be obtained,

$$\dot{\hat{\theta}}_n = r_n f_n\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right) P_{nL}^{\mathrm{T}}\left(A_n^{0\sim1}\right) v_n^{(0\sim1)} + \pi_n\left(\theta_{np} - \hat{\theta}_n\right), \tag{41}$$

$$u = -\frac{1}{g_n}\left(A_n^{0\sim1}\tilde{x}_n^{(0\sim1)} + f_{n0}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right) + f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\hat{\theta}_n - \ddot{x}_{nc}\right), \tag{42}$$

and a closed-loop subsystem is established by putting Eq. (42) into Eq. (40),

$$\ddot{\tilde{x}}_n = -A_n^{0\sim1}\tilde{x}_n^{(0\sim1)} + f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\tilde{\theta}_n. \tag{43}$$

Differentiating error signals v_n and streamlining that outcome by putting Eq. (12) from Eq. (43), then the dynamics of the compensated tracking errors are derived as

$$\ddot{v}_n = -A_n^{0\sim1} v_n^{(0\sim1)} + f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\tilde{\theta}_n.$$

The dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_n^{(0\sim1)} = \Phi\left(A_n^{0\sim1}\right) v_n^{(0\sim1)} + \begin{bmatrix} 0 \\ f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim1)}\right)\tilde{\theta}_n \end{bmatrix}. \tag{44}$$

Hence, define

$$V_n = V_{n-1} + \frac{1}{2}\left(v_n^{(0\sim1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim1}\right) v_n^{(0\sim1)} + \frac{1}{2r_n}\tilde{\theta}_n^{\mathrm{T}}\tilde{\theta}_n.$$

According to the Lemma 1, Eq. (5) and the parameter update law Eq. (41), the time derivative of V_n is given as

$$
\begin{aligned}
\dot{V}_n &= \dot{V}_{n-1} + \frac{1}{2}\left(v_n^{(0\sim 1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}\left(A_n^{0\sim 1}\right)P_n\left(A_n^{0\sim 1}\right) + P_n\left(A_n^{0\sim 1}\right)\Phi\left(A_n^{0\sim 1}\right)\right)v_n^{(0\sim 1)} \\
&\quad + \left(v_n^{(0\sim 1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim 1}\right)\begin{bmatrix} 0 \\ f_n^{\mathrm{T}}\left(\tilde{x}_{1\sim n}^{(0\sim 1)}\right)\tilde{\theta}_n \end{bmatrix} - \frac{1}{r_n}\tilde{\theta}_n^{\mathrm{T}}\dot{\hat{\theta}}_n \\
&\le \dot{V}_{n-1} - \frac{\mu_n}{2}\left(v_n^{(0\sim 1)}\right)^{\mathrm{T}} P_n\left(A_n^{0\sim 1}\right)v_n^{(0\sim 1)} - \frac{\pi_n}{r_n}\tilde{\theta}_n^{\mathrm{T}}\left(\theta_{np} - \hat{\theta}_n\right).
\end{aligned}
\tag{45}
$$

In order to maintain a uniform form, we also added an additional positive portion for $\dot{V}_n$ and restated Eq. (45) as

$$
\begin{aligned}
\dot{V}_n \le & -\frac{1}{2}\sum_{j=1}^{n}\left(\mu_j - 2\lambda_{\max}\left(P_j\right) - \frac{g_{j-1}^2}{2\lambda_{\min}\left(P_j\right)}\right)\left(v_j^{(0\sim 1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim 1}\right)v_j^{(0\sim 1)} \\
& - \sum_{j=1}^{n}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \hat{\theta}_j\right).
\end{aligned}
\tag{46}
$$

According to Young's inequality, it follows that

$$
\begin{aligned}
-\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \hat{\theta}_j\right) &= -\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \theta_j + \theta_j - \hat{\theta}_j\right) \\
&= -\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp} - \theta_j\right) \\
&\le -\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \frac{\pi_j}{2r_j}\left(\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \left(\theta_{jp} - \theta_j\right)^{\mathrm{T}}\left(\theta_{jp} - \theta_j\right)\right) \\
&= -\frac{\pi_j}{2r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \frac{\pi_j}{2r_j}\left(\theta_{jp} - \theta_j\right)^{\mathrm{T}}\left(\theta_{jp} - \theta_j\right).
\end{aligned}
\tag{47}
$$

Substituting Eq. (47) into Eq. (46) gives

$$
\begin{aligned}
\dot{V}_n \le & -\frac{1}{2}\sum_{j=1}^{n}\left(\mu_j - 2\lambda_{\max}\left(P_j\right) - \frac{g_{j-1}^2}{2\lambda_{\min}\left(P_j\right)}\right)\left(v_j^{(0\sim 1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim 1}\right)v_j^{(0\sim 1)} \\
& - \sum_{j=1}^{n}\frac{\pi_j}{2r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \sum_{j=1}^{n}\frac{\pi_j}{2r_j}\left(\theta_{jp} - \theta_j\right)^{\mathrm{T}}\left(\theta_{jp} - \theta_j\right).
\end{aligned}
\tag{48}
$$

3.3. Stability analysis

From the above design process, the following result can be concluded on the stability of the resulted closed-loop system.

Lemma 4. *Let the input signal of the compensating signals satisfy* $\left|g_i\left(x_{(i+1)c} - \alpha_i\right)\right| \le \sigma_i\rho$ *with the help of Assumption 3, then the states are bounded by*

$$
\|\xi\| \le \sqrt{\frac{2b}{\lambda_{\min}(P_i)a}\left(1 - e^{-at}\right)}
\tag{49}
$$

where $a = \min\left\{\mu_1 - 4\lambda_{\max}(P_1) - \frac{g_0^2}{2\lambda_{\min}(P_1)}, \ldots, \mu_n - 4\lambda_{\max}(P_n) - \frac{g_{n-1}^2}{2\lambda_{\min}(P_n)}\right\}$, $b = \sum_{j=1}^{n-1} \frac{\rho^2\sigma_j^2}{4}$.

Proof. Refer to the Appendix for the critical proof. □

Theorem 1. *Suppose the second-order SFSs Eq. (1) under Assumptions 1 and 3 hold. Then, consider the proposed scheme constructed by adaptive control law Eq. (15) together with the virtual control function Eq. (14), the command filters Eqs. (9) and (10), the compensating signals Eqs. (11) and (12) and the parameter update laws Eq. (16). There exists suitable design parameters such that* $A_i^{0\sim 1}$, $i = 1, 2, \ldots, n$, $\omega_i, i = 2, 3, \ldots, n$, $\pi_i, i = 1, 2, \ldots, n$, $r_i, i = 1, 2, \ldots, n$ *and* $\mu_i, i = 1, 2, \ldots, n$, *then the following properties hold,*

1. *The signals* $v_i, \tilde{\theta}_i, \hat{\theta}_i, \xi_i, \tilde{x}_i$ x_{ic} *and* x_i *are bounded.*
2. *The tracking error could be made arbitrarily small with predesigned parameters.*

Proof. Based on the Eq. (48), it obtains

$$\dot{V}_n \le -\alpha V_n + \beta, \tag{50}$$

by letting

$$\alpha = \min\left\{\mu_1 - 2\lambda_{\max}(P_1) - \frac{g_0^2}{2\lambda_{\min}(P_1)}, \ldots, \mu_n - 2\lambda_{\max}(P_n) - \frac{g_{n-1}^2}{2\lambda_{\min}(P_n)}, \pi_1, \ldots, \pi_n\right\}, \tag{51}$$

$$\beta = \sum_{j=1}^{n} \frac{\pi_j}{2r_j}(\theta_{jp} - \theta_j)^{\mathrm{T}}(\theta_{jp} - \theta_j). \tag{52}$$

Furthermore, it clear to conclude the following result by comparison principle [34],

$$0 \le V_n(t) \le \frac{\beta}{\alpha} + \left[V_n(0) - \frac{\beta}{\alpha}\right]e^{-\alpha t}. \tag{53}$$

Thus, $v_i, \tilde{\theta}_i, i = 1, 2, \ldots, n$ are all uniformly ultimately bounded. Furthermore, $\hat{\theta}_i$ are bounded since $\hat{\theta}_i = \theta_i - \tilde{\theta}_i$ and θ_i are positive constants. □

Remark 4. It is a remarkable fact that the tracking error $\frac{\beta}{\alpha}$ can be predesigned as a quite small value since α can be designed as a quite large value from Eq. (51) and β can be designed as a quite small value from Eq. (52). By choosing the proper $A_i^{0\sim 1}$, the eigenvalues can be arbitrarily assigned, then μ_i can be chosen as a large value from Eq. (3). Even though a larger value of π_i may affect β, a much larger r_i can be freely chosen to eliminate this effect and make the value of β freely tuned. Thus, α and β can be freely chosen by properly tuning the predesigned parameters μ_i, π_i and r_i.

By utilizing the Lemma 4, it ensures the states ξ of Eq. (11) is bounded. Therefore, $\|\tilde{x}\|$ is bounded since

$$\tilde{x} = v + \xi,$$

and the bound is

$$\|\tilde{x}\| \le \sqrt{\frac{2b}{\lambda_{\min}(P_i)a}(1 - e^{-at})} + \|v\|.$$

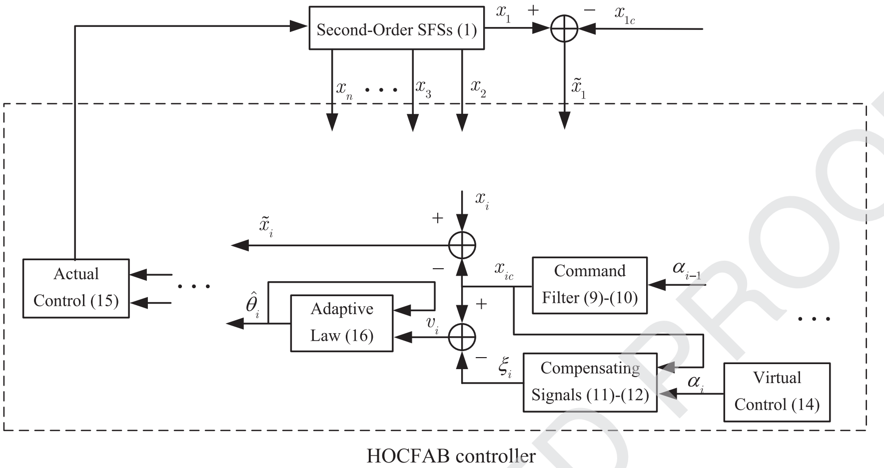

Fig. 1. Block diagram of the proposed HOCFAB scheme for second-order SFSs.

Furthermore, Lemma 3 ensures that $|\alpha_i|$, $|\dot{\alpha}_i|$ and $|\ddot{\alpha}_i|$ are bounded, then $|x_{ic}|$, $|\dot{x}_{ic}|$ and $|\ddot{x}_{ic}|$ are bounded by the Lemma 2. It also obtains that $|x|$ is bounded since $x_i = \tilde{x}_i + x_{ic}$. The tracking error

$$\lim_{t\to\infty} |\tilde{x}_1| \le \sqrt{\frac{2\beta}{\lambda_{\min}(P_i)\alpha}} + \sqrt{\frac{2b}{\lambda_{\min}(P_i)a}} \tag{54}$$

and α, β, a and b can be freely chosen by properly tuning the predesigned parameters μ_i, π_i and r_i to lead to an arbitrary small.

The control block of the proposed HOCFAB scheme for the second-order SFSs Eq. (1) is illustrated in Fig. 1.

Remark 5. This remark is addressed to show the relationship among the controller parameters $A_i^{0\sim1}$, command filter parameters ω_i, and adaptive parameters r_i, π_i on the system performance. It can be seen in Eq. (54) that large values of α, a and small values of β, b can result in a small tracking error as $t \to \infty$. Furthermore, large value of α in Eq. (51) and a in Eq. (105) are mainly determined by μ_i which are achieved by tuning a larger $A_i^{0\sim1}$. The small value of β in Eq. (52) is mainly determined by selecting smaller π_i and larger r_i, and the small value of b is mainly determined by σ_i which are achieved by tuning a larger ω_i. However, it is worth noting that very small π_i may result a small α if π_i are smaller than $\mu_i - 2\lambda_{\max}(P_i) - \frac{g_{i-1}^2}{2\lambda_{\min}(P_i)}$ according to the analysis in Eq. (51). Thus, we should select relatively big values of $A_i^{0\sim1}$, r_i and relatively small values of π_i in certain sense. As for how big the tuning parameters, an accurate answer can not be given since they depend on the feature of a system. It should also be noticed that too big values of $A_i^{0\sim1}$, ω_i and r_i may cause a large overshoot and increase the control cost, so these parameters should be selected according to practical demands.

Remark 6. In this study, we can not choose a Lyapunov function that contains all the systems states since the introduced dynamics of the compensating signals. Thus, inspired by the former studies in command filtered adaptive backstepping in Dong et al. [15], we first prove that the $v_i, \tilde{\theta}_i, i = 1, 2, \ldots, n$ are all uniformly ultimately bounded. Then, the compensated tracking signals ξ_i are proven to be bounded. Finally, the tracking errors are bounded since $\tilde{x} = v + \xi$. The study in Liu et al. [12] shows that a Lyapunov function consisting of all the dynamic states can be chosen to prove that all closed-loop signals are bounded since all the states are in the invariant set. Thus, the two studies utilized totally different perspectives to draw the stability of the system.

4. HOCFAB for high-order SFSs

Similar to the second-order SFSs, the scheme for the high-order SFSs Eq. (2) is discussed, and it is clear to show that the original m_i steps needed in traditional CFB are reduced to 1 step in each subsystem design.

Assume $A_i^{0\sim m_i-1} \in \mathbb{R}^{1\times m_i}, i = 1, 2, \ldots, n$ are a set of matrices to make $\Phi\left(A_i^{0\sim m_i-1}\right) \in \mathbb{R}^{m_i\times m_i}, i = 1, 2, \ldots, n$ stable and

$$P_i\left(A_i^{0\sim m_i-1}\right) = \left[P_{iF}\left(A_i^{0\sim m_i-1}\right) \quad \ldots \quad P_{iL}\left(A_i^{0\sim m_i-1}\right)\right], \tag{55}$$

where $P_i\left(A_i^{0\sim m_i-1}\right) \in \mathbb{R}^{m_i\times m_i}$, $P_{iF}\left(A_i^{0\sim m_i-1}\right)$ and $P_{iL}\left(A_i^{0\sim m_i-1}\right)$ are the first and last columns of $P_i\left(A_i^{0\sim m_i-1}\right)$.

4.1. Control law and parameter update law design

For the high-order SFSs Eq. (2), the tracking error of the command filtered backstepping is defined as

$$\tilde{x}_i = x_i - x_{ic}, \tag{56}$$

x_{1c} is the desired output trajectory, and $x_{ic}, i = 2, 3, \ldots, n$ is derived by a range of command filters on the $(i-1)$th virtual control functions α_{i-1} which are defined later in Eq. (62)

$$\dot{z}_{i,j} = z_{i,j+1}, \quad 1 \le j \le m_{i-1} \tag{57}$$

$$\dot{z}_{i,m_i} = -\omega_{i,m_i} z_{i,m_i} - \ldots - \omega_{i,2} z_{i,2} - \omega_{i,1}\left(z_{i,1} - \alpha_{i-1}\right), \tag{58}$$

where $\omega_{i,j} \in \mathbb{R}_{>0}$. Furthermore, the outputs of the each command filer are defined as $x_{ic}^{(j-1)} = z_{i,j}$, $1 \le j \le m_i$ and $x_{ic}^{(m_i)} = \dot{z}_{i,m_i}$.

Remark 7. Not only this high-order command filter can circumvent the "explosion of complexity" phenomenon, but also it has useful noise reduction properties as pointed out in Dong et al. [15].

The compensating signals aim to remove the effect of the known error between the signal $x_{(i+1)c}$ and the virtual control α_i with $i = 1, 2, \ldots, n-1$ and are defined as

$$\xi_i^{(m_i)} = -A_i^{0\sim m_i-1}\xi_i^{(0\sim m_i-1)} + g_i\xi_{i+1} + g_i\left(x_{(i+1)c} - \alpha_i\right), \tag{59}$$

$$\xi_n^{(m_n)} = -A_n^{0\sim m_n-1}\xi_n^{(0\sim m_n-1)}. \tag{60}$$

Moreover, define the compensated tracking error signals v_i as

$$v_i = \tilde{x}_i - \xi_i. \tag{61}$$

The virtual control function α_i of HOCFAB for system Eq. (2) is represented as below and for the $i(1 \le i \le n-1)$ step,

$$\alpha_i = -\frac{1}{g_i}\Big(A_i^{0\sim m_i-1}\tilde{x}_i^{(0\sim m_i-1)} + f_{i0}\Big(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\Big) + f_i^{\mathrm{T}}\Big(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\Big)\hat{\theta}_i - x_{ic}^{(m_i)}\Big), \tag{62}$$

and the control law

$$u = -\frac{1}{g_n}\Big(A_n^{0\sim m_n-1}\tilde{x}_n^{(0\sim m_n-1)} + f_{n0}\Big(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\Big) + f_n^{\mathrm{T}}\Big(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\Big)\hat{\theta}_n - x_{nc}^{(m_n)}\Big). \tag{63}$$

The estimation of the unknown parameter θ_i is expressed as $\hat{\theta}_i$, and $\hat{\theta}_i$ satisfies parameter update law below

$$\dot{\hat{\theta}}_i = r_i f_i\Big(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\Big)P_{iL}^{\mathrm{T}}\big(A_i^{0\sim m_i-1}\big)v_i^{(0\sim m_i-1)} + \pi_i\Big(\theta_{ip} - \hat{\theta}_i\Big), \tag{64}$$

where r_i, λ_i and π_i are ~~predesign~~ constant values, and θ_{ip} is the predictive value of the unknown parameter θ_i. The estimation errors of the parameters are defined as

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i. \tag{65}$$

4.2. HOCFAB algorithm for high-order SFSs

Similar to the HOCFAB algorithm for second-order SFSs, the algorithm for high-order SFSs is completed by the following n steps.

Step 1: Taking differentials of the Eq. (56) for $i = 1$, and by utilizing the first equation in Eq. (2), yields

$$\tilde{x}_1^{(m_1)} = f_{10}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big) + f_1^{\mathrm{T}}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)\theta_1 + g_1(\tilde{x}_2 + x_{2c}) - x_{1c}^{(m_1)}. \tag{66}$$

Related parameter update law in Eq. (64) and virtual control law in Eq. (62) is obtained as $i = 1$,

$$\dot{\hat{\theta}}_1 = r_1 f_1\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)P_{1L}^{\mathrm{T}}\big(A_1^{0\sim m_1-1}\big)v_1^{(0\sim m_1-1)} + \pi_1\Big(\theta_{1p} - \hat{\theta}_1\Big), \tag{67}$$

$$\alpha_1 = -\frac{1}{g_1}\Big(A_1^{0\sim m_1-1}\tilde{x}_1^{(0\sim m_1-1)} + f_{10}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big) + f_1^{\mathrm{T}}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)\hat{\theta}_1 - x_{1c}^{(m_1)}\Big), \tag{68}$$

and the resulting closed-loop subsystem is obtained by substituting Eq. (68) into Eq. (66),

$$\tilde{x}_1^{(m_1)} = -A_1^{0\sim m_1-1}\tilde{x}_1^{(0\sim m_1-1)} + g_1\tilde{x}_2 + g_1(x_{2c} - \alpha_1) + f_1^{\mathrm{T}}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)\tilde{\theta}_1. \tag{69}$$

Differentiating error signals v_1 and streamlining that outcome by putting Eq. (59) with $i = 1$ from Eq. (69), then the dynamics of the compensated tracking errors are derived as

$$v_1^{(m_1)} = -A_1^{0\sim m_1-1}v_1^{(0\sim m_1-1)} + g_1 v_2 + f_1^{\mathrm{T}}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)\tilde{\theta}_1.$$

The dynamics of the compensated tracking errors is expressed as

$$\dot{v}_1^{(0\sim m_1-1)} = \Phi\big(A_1^{0\sim m_1-1}\big)v_1^{(0\sim m_1-1)} + \begin{bmatrix} 0_{(m_1-1)\times 1} \\ f_1^{\mathrm{T}}\Big(\tilde{x}_1^{(0\sim m_1-1)}\Big)\tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0_{(m_1-1)\times 1} \\ g_1 v_2 \end{bmatrix}. \tag{70}$$

Hence, define

$$V_1 = \frac{1}{2}\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)v_1^{(0\sim m_1-1)} + \frac{1}{2r_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1.$$

According to the Lemma 1, Eq. (55) and the parameter update law Eq. (67), the time derivative of V_1 is given as

$$\begin{aligned}\dot{V}_1 = &\frac{1}{2}\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}\left(A_1^{0\sim m_1-1}\right)P_1\left(A_1^{0\sim m_1-1}\right) + P_1\left(A_1^{0\sim m_1-1}\right)\Phi\left(A_1^{0\sim m_1-1}\right)\right)v_1^{(0\sim m_1-1)}\\ &+\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)\left(\begin{bmatrix}0_{(m_1-1)\times 1}\\ f_1^{\mathrm{T}}\left(\tilde{x}_1^{(0\sim m_1-1)}\right)\tilde{\theta}_1\end{bmatrix} + \begin{bmatrix}0_{(m_1-1)\times 1}\\ g_1 v_2\end{bmatrix}\right) - \frac{1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\dot{\hat{\theta}}_1\\ &\le -\frac{\mu_1}{2}\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)v_1^{(0\sim m_1-1)} + \left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_{1L}\left(A_1^{0\sim m_1-1}\right)g_1 v_2\\ &\quad - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right).\end{aligned} \tag{71}$$

By utilizing Young's inequality, gives

$$\begin{aligned}\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_{1L}\left(A_1^{0\sim m_1-1}\right)g_1 v_2 \le\ &\lambda_{\max}(P_1)\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)v_1^{(0\sim m_1-1)}\\ &+ \frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}} P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)}.\end{aligned} \tag{72}$$

Substituting Eqs. (72) into Eq. (71), we can obtain

$$\begin{aligned}\dot{V}_1 \le &-\frac{1}{2}(\mu_1 - 2\lambda_{\max}(P_1))\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)v_1^{(0\sim m_1-1)}\\ &+\frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}} P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)} - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right).\end{aligned}$$

An extra positive part for $\dot{V}_1$ is added to make $\dot{V}_1$ organized as a uniform form with the following $\dot{V}_i$,

$$\begin{aligned}\dot{V}_1 \le &-\frac{1}{2}\left(\mu_1 - 2\lambda_{\max}(P_1) - \frac{g_0^2}{2\lambda_{\min}(P_1)}\right)\left(v_1^{(0\sim m_1-1)}\right)^{\mathrm{T}} P_1\left(A_1^{0\sim m_1-1}\right)v_1^{(0\sim m_1-1)}\\ &+\frac{g_1^2}{4\lambda_{\min}(P_2)}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}} P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)} - \frac{\pi_1}{r_1}\tilde{\theta}_1^{\mathrm{T}}\left(\theta_{1p} - \hat{\theta}_1\right),\end{aligned} \tag{73}$$

where g_0 is a small positive constant value.

Step 2: Taking differentials of the Eq. (56) with $i = 2$, and by using the second equation in Eq. (2), gives

$$\tilde{x}_2^{(m_2)} = f_{20}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right) + f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\theta_2 + g_2(\tilde{x}_3 + x_{3c}) - x_{2c}^{(m_2)}. \tag{74}$$

Related parameter update law in Eq. (64) and virtual control law in Eq. (62) is derived for $i = 2$,

$$\dot{\hat{\theta}}_2 = r_2 f_2\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)P_{2L}^{\mathrm{T}}\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)} + \pi_2\left(\theta_{2p} - \hat{\theta}_2\right), \tag{75}$$

$$\alpha_2 = -\frac{1}{g_2}\left(A_2^{0\sim m_2-1}\tilde{x}_2^{(0\sim m_2-1)} + f_{20}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\right.$$

$$+f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\hat{\theta}_2-x_{2c}^{(m_2)}\Big), \tag{76}$$

and the closed-loop subsystem is established by putting Eq. (76) into Eq. (74),

$$\tilde{x}_2^{(m_2)}=-A_2^{0\sim m_2-1}\tilde{x}_2^{(0\sim m_2-1)}+g_2\tilde{x}_3+g_2(x_{3c}-\alpha_2)+f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\tilde{\theta}_2. \tag{77}$$

Differentiating error signals v_2 and streamlining that outcome by putting Eq. (59) with $i=2$ from Eq. (77), then the dynamics of the compensated tracking errors are derived as

$$v_2^{(m_2)}=-A_2^{0\sim m_2-1}v_2^{(0\sim m_2-1)}+g_2v_3+f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\tilde{\theta}_2.$$

The dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_2^{(0\sim m_2-1)}=\Phi\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)}+\begin{bmatrix}0_{(m_2-1)\times 1}\\ f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\tilde{\theta}_2\end{bmatrix}+\begin{bmatrix}0_{(m_2-1)\times 1}\\ g_2v_3\end{bmatrix}. \tag{78}$$

Define

$$V_2=V_1+\frac{1}{2}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)}+\frac{1}{2r_2}\tilde{\theta}_2^{\mathrm{T}}\tilde{\theta}_2.$$

According to the Lemma 1, Eq. (55) and the parameter update law Eq. (75), the time derivative of V_2 is given as

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+\frac{1}{2}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}\left(A_2^{0\sim m_2-1}\right)P_2\left(A_2^{0\sim m_2-1}\right)+P_2\left(A_2^{0\sim m_2-1}\right)\Phi\left(A_2^{0\sim m_2-1}\right)\right)v_2^{(0\sim m_2-1)}\\&\quad+\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_2\left(A_2^{0\sim m_2-1}\right)\left(\begin{bmatrix}0\\ f_2^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim 2}\right)\tilde{\theta}_2\end{bmatrix}+\begin{bmatrix}0_{(m_2-1)\times 1}\\ g_2v_3\end{bmatrix}\right)-\frac{1}{r_2}\tilde{\theta}_2^{\mathrm{T}}\dot{\hat{\theta}}_2\\&\le\dot{V}_1-\frac{\mu_2}{2}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)}+\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_{2L}\left(A_2^{0\sim m_2-1}\right)g_2v_3\\&\quad-\frac{\pi_2}{r_2}\tilde{\theta}_2^{\mathrm{T}}\left(\theta_{2p}-\hat{\theta}_2\right).\end{aligned} \tag{79}$$

Similar to Eq. (72), it follows that

$$\begin{aligned}\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_{2L}\left(A_2^{0\sim m_2-1}\right)g_2v_3&\le\lambda_{\max}(P_2)\left(v_2^{(0\sim m_2-1)}\right)^{\mathrm{T}}P_2\left(A_2^{0\sim m_2-1}\right)v_2^{(0\sim m_2-1)}\\&\quad+\frac{g_2^2}{4\lambda_{\min}(P_3)}\left(v_3^{(0\sim m_3-1)}\right)^{\mathrm{T}}P_3\left(A_3^{0\sim m_3-1}\right)v_3^{(0\sim m_3-1)}.\end{aligned} \tag{80}$$

Putting Eq. (80) into Eq. (79), we derive

$$\begin{aligned}\dot{V}_2&\le-\frac{1}{2}\sum_{j=1}^{2}\left(\mu_j-2\lambda_{\max}(P_j)-\frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim m_j-1)}\right)^{\mathrm{T}}P_j\left(A_j^{0\sim m_j-1}\right)v_j^{(0\sim m_j-1)}\\&\quad-\sum_{j=1}^{2}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp}-\hat{\theta}_j\right)+\frac{g_2^2}{4\lambda_{\min}(P_3)}\left(v_3^{(0\sim m_3-1)}\right)^{\mathrm{T}}P_3\left(A_3^{0\sim m_3-1}\right)v_3^{(0\sim m_3-1)}.\end{aligned}$$

Step i: Using the *i*th equation in Eq. (2) and taking the differentials of the Eq. (56), it gives

$$\tilde{x}_i^{(m_i)}=f_{i0}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)+f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\theta_i+g_i\left(\tilde{x}_{i+1}+x_{(i+1)c}\right)-x_{ic}^{(m_i)}. \tag{81}$$

Related parameter update law in Eq. (64) and virtual control law in Eq. (62) is given as, 392

$$\dot{\hat{\theta}}_i = r_i f_i\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)P_{iL}^{\mathrm{T}}\left(A_i^{0\sim m_i-1}\right)v_i^{(0\sim m_i-1)} + \pi_i\left(\theta_{ip} - \hat{\theta}_i\right), \tag{82}$$

393

$$\alpha_i = -\frac{1}{g_i}\left(A_i^{0\sim m_i-1}\tilde{x}_i^{(0\sim m_i-1)} + f_{i0}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\right. \\ \left. + f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\hat{\theta}_i - x_{ic}^{(m_i)}\right), \tag{83}$$

and the related closed-loop subsystem is gained by substituting Eq. (83) into Eq. (81), 394

$$\tilde{x}_i^{(m_i)} = -A_i^{0\sim m_i-1}\tilde{x}_i^{(0\sim m_i-1)} + g_i\tilde{x}_{i+1} + g_i\left(x_{(i+1)c} - \alpha_i\right) + f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\tilde{\theta}_i. \tag{84}$$

Differentiating error signals v_i and streamlining that outcome by putting Eq. (59) from 395
Eq. (84), then the dynamics of the compensated tracking errors are derived as 396

$$v_i^{(m_i)} = -A_i^{0\sim m_i-1}v_i^{(0\sim m_i-1)} + g_i v_{i+1} + f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\tilde{\theta}_i.$$

The dynamics of the compensated tracking errors can be rewritten as 397

$$\dot{v}_i^{(0\sim m_i-1)} = \Phi\left(A_i^{0\sim m_i-1}\right)v_i^{(0\sim m_i-1)} + \begin{bmatrix} 0_{(m_i-1)\times 1} \\ f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\tilde{\theta}_i \end{bmatrix} + \begin{bmatrix} 0_{(m_i-1)\times 1} \\ g_i v_{i+1} \end{bmatrix}. \tag{85}$$

Then, define 398

$$V_i = V_{i-1} + \frac{1}{2}\left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim m_i-1}\right)v_i^{(0\sim m_i-1)} + \frac{1}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i.$$

According to the Lemma 1, Eq. (55) and the parameter update law Eq. (82), the time derivative 399
of V_i is given as 400

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + \frac{1}{2}\left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}\left(A_i^{0\sim m_i-1}\right)P_i\left(A_i^{0\sim m_i-1}\right) + P_i\left(A_i^{0\sim m_i-1}\right)\Phi\left(A_i^{0\sim m_i-1}\right)\right)v_i^{(0\sim m_i-1)} \\ &\quad + \left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim m_i-1}\right)\left(\begin{bmatrix} 0_{(m_i-1)\times 1} \\ f_i^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim i}\right)\tilde{\theta}_i \end{bmatrix} + \begin{bmatrix} 0_{(m_i-1)\times 1} \\ g_i v_{i+1} \end{bmatrix}\right) - \frac{1}{r_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i \\ &\le \dot{V}_{i-1} - \frac{\mu_i}{2}\left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim m_i-1}\right)\left(v_i^{(0\sim m_i-1)}\right) + \left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim m_i-1}\right)g_i v_{i+1} \\ &\quad - \frac{\pi_i}{r_i}\tilde{\theta}_i^{\mathrm{T}}\left(\theta_{ip} - \hat{\theta}_i\right). \end{aligned} \tag{86}$$

Similar to Eq. (72), it follows that 401

$$\begin{aligned} \left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_{iL}\left(A_i^{0\sim m_i-1}\right)g_i v_{i+1} &\le \lambda_{\max}(P_i)\left(v_i^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_i\left(A_i^{0\sim m_i-1}\right)v_i^{(0\sim m_i-1)} \\ &\quad + \frac{g_i^2}{4\lambda_{\min}(P_{i+1})}\left(v_{i+1}^{(0\sim m_{i+1}-1)}\right)^{\mathrm{T}} P_{i+1}\left(A_{i+1}^{0\sim m_{i+1}-1}\right)v_{i+1}^{(0\sim m_{i+1}-1)}. \end{aligned} \tag{87}$$

By Eq. (87), we have 402

$$\dot{V}_i \le -\frac{1}{2}\sum_{j=1}^{i}\left(\mu_j - 2\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim m_i-1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim m_j-1}\right)v_j^{(0\sim m_i-1)}$$

$$-\sum_{j=1}^{i}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp}-\hat{\theta}_j\right)+\frac{g_i^2}{4\lambda_{\min}(P_{i+1})}\left(v_{i+1}^{(0\sim m_{i+1}-1)}\right)^{\mathrm{T}}P_{i+1}\left(A_{i+1}^{0\sim m_{i+1}-1}\right)v_{i+1}^{(0\sim m_{i+1}-1)}. \tag{88}$$

Step n: An exhaustive treatment of other steps is not given here for brevity and similar to the previous treatment, it follows that,

$$\tilde{x}_n^{(m_n)}=f_{n0}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)+f_n^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\theta_n+g_n u-x_{nc}^{(m_n)}. \tag{89}$$

Related parameter update law in Eq. (64) and control law in Eq. (63) are given as,

$$\dot{\hat{\theta}}_n=r_n f_n\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)P_{nL}^{\mathrm{T}}(A_n^{0\sim m_n-1})v_n^{(0\sim m_n-1)}+\pi_n\left(\theta_{np}-\hat{\theta}_n\right), \tag{90}$$

$$\begin{aligned}u=&-\frac{1}{g_n}\left(A_n^{0\sim m_n-1}\tilde{x}_n^{(0\sim m_n-1)}+f_{n0}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\right.\\&\left.+f_n^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\hat{\theta}_n-x_{nc}^{(m_n)}\right),\end{aligned} \tag{91}$$

and a closed-loop subsystem is established by putting Eq. (91) into Eq. (89),

$$\tilde{x}_n^{(m_n)}=-A_n^{0\sim m_n-1}\tilde{x}_n^{(0\sim m_n-1)}+f_n^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\tilde{\theta}_n, \tag{92}$$

Differentiating error signals v_n and streamlining that outcome by putting Eq. (60) from Eq. (92), then the dynamics of the compensated tracking errors are derived as

$$v_n^{(m_n)}=-A_n^{0\sim m_n-1}v_n^{(0\sim m_n-1)}+f_n^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\tilde{\theta}_n.$$

The dynamics of the compensated tracking errors can be rewritten as

$$\dot{v}_n^{(0\sim m_n-1)}=\Phi(A_n^{0\sim m_n-1})v_n^{(0\sim m_n-1)}+\begin{bmatrix}0_{(m_n-1)\times 1}\\ f_n^{\mathrm{T}}\left(\tilde{x}_j^{(0\sim m_j-1)}\big|_{j=1\sim n}\right)\tilde{\theta}_n\end{bmatrix}. \tag{93}$$

Hence, define

$$V_n=V_{n-1}+\frac{1}{2}(v_n^{(0\sim m_n-1)})^{\mathrm{T}}P_n(A_n^{0\sim m_n-1})v_n^{(0\sim m_n-1)}+\frac{1}{2r_n}\tilde{\theta}_n^{\mathrm{T}}\tilde{\theta}_n.$$

According to the Lemma 1, Eq. (55) and the parameter update law Eq. (90), the time derivative of V_n is given as

$$\dot{V}_n=\dot{V}_{n-1}-\frac{\mu_n}{2}(v_n^{(0\sim m_n-1)})^{\mathrm{T}}P_n(A_n^{0\sim m_n-1})v_n^{(0\sim m_n-1)}+\frac{\pi_n}{r_n}\tilde{\theta}_n^{\mathrm{T}}\hat{\theta}_n. \tag{94}$$

In order to maintain a uniform form, we also added an additional positive portion for $\dot{V}_n$ and restated Eq. (94) as

$$\begin{aligned}\dot{V}_n\le&-\frac{1}{2}\sum_{j=1}^{n}\left(\mu_j-2\lambda_{\max}(P_j)-\frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim m_j-1)}\right)^{\mathrm{T}}P_j\left(A_j^{0\sim m_j-1}\right)v_j^{(0\sim m_j-1)}\\&-\sum_{j=1}^{n}\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp}-\hat{\theta}_j\right).\end{aligned} \tag{95}$$

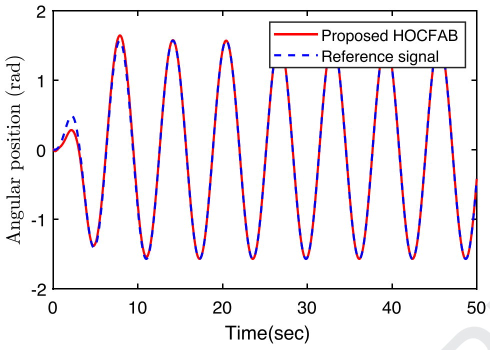

Fig. 2. Tracking performance of the proposed HOCFAB.

According to Young's inequality, it follows that

$$-\frac{\pi_j}{r_j}\tilde{\theta}_j^{\mathrm{T}}\left(\theta_{jp}-\hat{\theta}_j\right) \leq -\frac{\pi_j}{2r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \frac{\pi_j}{2r_j}\left(\theta_{jp}-\theta_j\right)^{\mathrm{T}}\left(\theta_{jp}-\theta_j\right). \tag{96}$$

Substituting Eq. (96) into Eq. (95) gives

$$\begin{aligned}\dot{V}_n \leq & -\frac{1}{2}\sum_{j=1}^{n}\left(\mu_j - 2\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(v_j^{(0\sim m_j-1)}\right)^{\mathrm{T}} P_j\left(A_j^{0\sim m_j-1}\right) v_j^{(0\sim m_j-1)} \\ & -\sum_{j=1}^{n}\frac{\pi_j}{2r_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j + \sum_{j=1}^{n}\frac{\pi_j}{2r_j}\left(\theta_{jp}-\theta_j\right)^{\mathrm{T}}\left(\theta_{jp}-\theta_j\right).\end{aligned} \tag{97}$$

4.3. Stability analysis

For the sake of simplicity, an exhaustive proof of stability is not stated, while identical procedures may be found for the case of second-order SFSs in Section 3.3.

Theorem 2. *Suppose the high-order SFSs Eq. (2) under Assumptions 2 and 3 hold. Then, consider the proposed scheme constructed by adaptive control law Eq. (63) together with the virtual control function Eq. (62), the command filters Eqs. (57) and (58), and the parameter update laws (64). There exist suitable design parameters such that* $A_i^{0\sim m_i-1}$, $i = 1, 2, \ldots, n$, $\omega_{i,j}$, $j = 1, 2, \ldots, m_i$, $i = 2, 3, \ldots, n$, π_i, $i = 1, 2, \ldots, n$, r_i, $i = 1, 2, \ldots, n$ *and* μ_i, $i = 1, 2, \ldots, n$, *then the tracking error could be made arbitrarily small with predesigned parameters and the proposed HOCFAB scheme ensures that all closed-loop signals are bounded.*

5. An example in robotic manipulator

Consider the following description of the trajectory tracking control of a single link robotic manipulator driven by a direct current motor. The dynamics of this system is a popular benchmark example and can be described as follows [35,36],

$$\begin{cases} M\ddot{q} + B\dot{q} + N\sin(q) = I \\ L\dot{I} + RI + K_B\dot{q} = V, \end{cases} \tag{98}$$

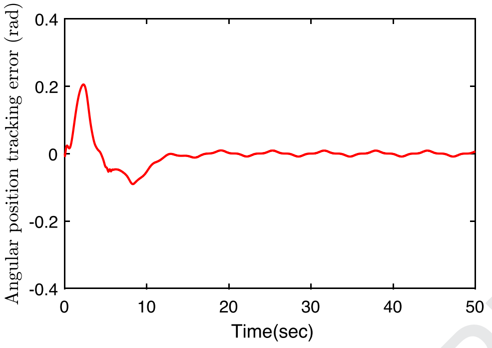

Fig. 3. Tracking error between the proposed HOCFAB and the reference signal.

where q is the angular position of the link, I is the motor armature current, V is the input control voltage, L is the armature inductance, R is the armature resistance, K_B is the back-emf coefficient, and M, B and N are constants given by

$$
\begin{aligned}
M &= \left(J + mL_0^2/3 + M_0L_0^2 + 2M_0R_0^2/5\right)/K_\tau,\\
B &= B_0/K_\tau,\\
N &= (mL_0g/2 + M_0L_0g)/K_\tau,
\end{aligned}
$$

in which J is the rotor inertia, m is the link mass, L_0 is the link length, M_0 is the load mass, R_0 is the radius of the load, B_0 is the coefficient of viscous friction at the joint, g is the gravitational acceleration, and K_τ is the coefficient which characterizes the electromechanical conversion of armature current to torque. $J = 1.625 \times 10^{-3}$ kg m^2, $m = 0.506$ kg, $M_0 = 0.434$ kg, $L_0 = 0.305$ m, $R_0 = 0.023$ m, $B_0 = 16.25 \times 10^{-3}$ N m s/rad, $L = 25 \times 10^{-3}$H, $R = 5\Omega$, $K_\tau = K_B = 0.9$ N m/A and $g = 9.8$ m/s^2 are chosen here for simulation. The proposed HOCFAB scheme may handle the system directly, in contrast to the conventional method of transforming the system Eq. (98) into first-order state-space form. The system parameters B, N, K_B and R are assumed to be unknown in controller design, therefore Eq. (98) is rebuilt as Eq. (2), with $x_1 = q$, $x_2 = I$ and $u = V$ as the new system states.

$$
\begin{cases}
x_1^{(m_1)} = f_{11}\left(x_1^{(0\sim m_1-1)}\right)\theta_{11} + f_{12}\left(x_1^{(0\sim m_1-1)}\right)\theta_{12} + g_1\left(x_1^{(0\sim m_1-1)}\right)x_2\\
x_2^{(m_2)} = f_{21}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right)\theta_{21} + f_{22}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right)\theta_{22} + g_2\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right)u,
\end{cases}
\tag{99}
$$

where $m_1 = 2$, $m_2 = 1$, $f_{11}\left(x_1^{(0\sim1)}\right) = -\dot{x}_1$, $f_{12}\left(x_1^{(0\sim1)}\right) = -\sin x_1$, $f_{21}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right) = -\dot{x}_1$, $f_{22}\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right) = -x_2$, $g_1\left(x_1^{(0\sim1)}\right) = \frac{1}{M}$, $g_2\left(x_j^{(0\sim m_j-1)}\big|_{j=1\sim2}\right) = \frac{1}{L}$, $\theta_{11} = \frac{B}{M}$, $\theta_{12} = \frac{N}{M}$, $\theta_{21} = \frac{K_B}{L}$ and $\theta_{22} = \frac{R}{L}$. The control objective is to design the input control voltage such that the link angular position q can track its desired signal $x_{1c} = \frac{\pi}{2}\left(1 - e^{-0.1t^2}\right)\sin(t)$. The simulation is initialized with $x_1(0) = \pi/6$, $\dot{x}_1(0) = x_2(0) = 0$, $\xi_1(0) = \dot{\xi}_1(0) = \xi_2(0) = 0$, $x_{2d}(0) = 10$, $\hat{\theta}_{11}(0) = 0.2$, $\hat{\theta}_{12}(0) = 30$, $\hat{\theta}_{21}(0) = 30$, $\hat{\theta}_{22}(0) = 100$, $\theta_{1p,1} = 0.2$, $\theta_{1p,2} = 35$, $\theta_{2p,1} = 35$ and $\theta_{2p,2} = 200$, and the parameters are set to be $\pi_1 = 0.1$, $\pi_2 = 0.2$, $\omega_2 = 50$, $P_1 = \begin{bmatrix} 273558/5 & 31118/91 \\ 31118/91 & 26091/58 \end{bmatrix}$, $A_1^{0\sim1} = \begin{bmatrix} 120 & 22 \end{bmatrix}$, $r_1 = r_2 = 1$, $P_2 = 1$ and $A_2 = 20$. Con-

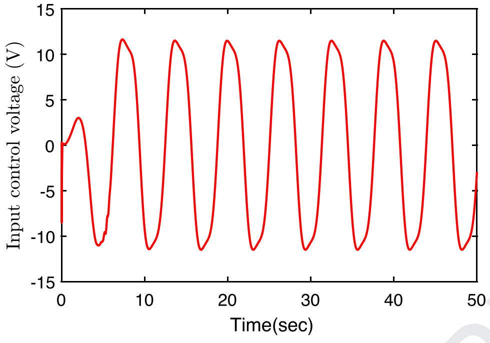

Fig. 4. Input control voltage of the proposed HOCFAB.

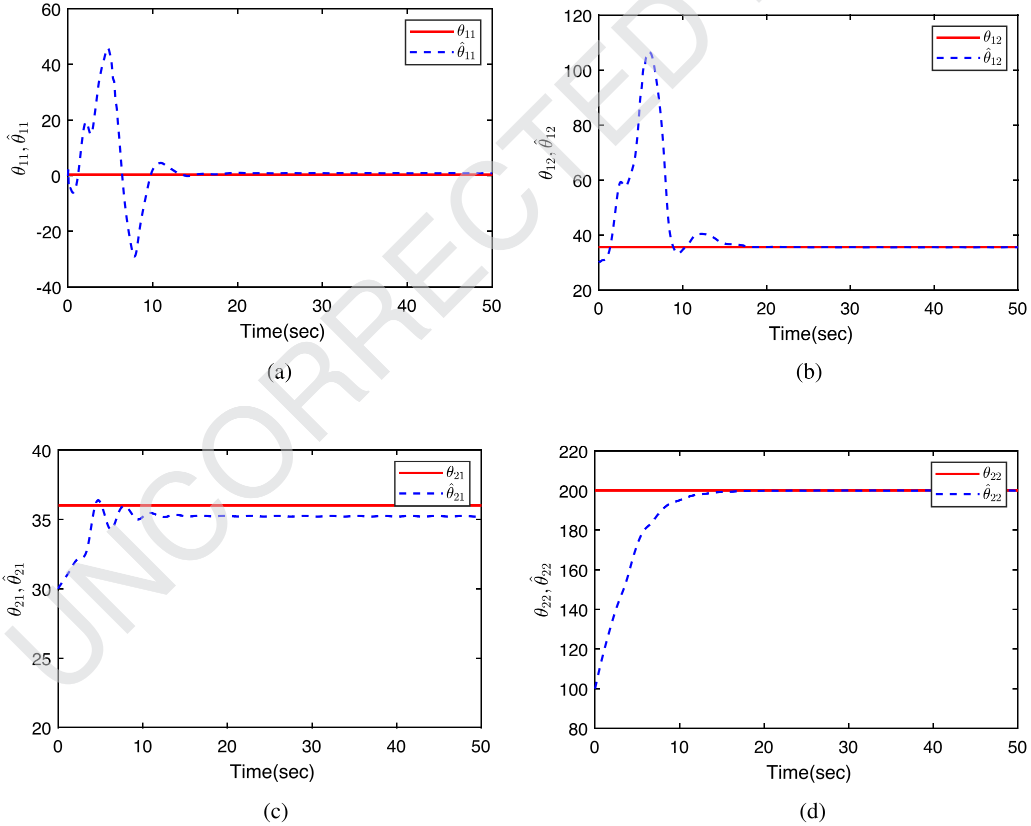

Fig. 5. Estimation performance of the unknown system parameters with the proposed adaptive law.

trol performances by the proposed HOCFAB are shown in Figs. 2–5. It can be observed that only two steps are needed with the proposed HOCFAB scheme, and the trajectory tracking is achieved under parameter uncertainties.

6. Conclusion

This paper has proposed a HOCFAB-based scheme to track a given reference signal for the second- and high-order SFSs with parametric uncertainties, where both the two HOFA systems share an identical full-actuation structure. Not only the proposed HOCFAB scheme can circumvent the complexity and demands fewer steps, but an error-compensating mechanism is also constructed to reduce filtering errors. A critical analysis is theoretically proven which shows that in both cases the entire system states are uniformly ultimately bounded under the proposed high-order controller, and the tracking error could be made arbitrarily small with predesigned parameters. To demonstrate the usefulness of the proposed HOCFAB scheme, a benchmark application in robotic manipulators is provided where this direct design scheme only needs two steps and without the explosion of terms. Further studies will focus on how to extend the proposed scheme to systems with unknown nonlinearities and unknown control directions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Weizhen Liu: Conceptualization, Investigation, Methodology, Validation, Writing – original draft. **Guangren Duan:** Supervision, Funding acquisition, Investigation, Methodology, Writing – review & editing. **Mingzhe Hou:** Methodology, Writing – review & editing.

Acknowledgments

This work was supported by the Science Center Program of the National Natural Science Foundation of China under grant no. 62188101, Major Program of National Natural Science Foundation of China under Grants nos. 61690210, 61690212, National Natural Science Foundation of China under Grant no. 62073096, Self-Planned Task (No. SKLRS201716A) of State Key Laboratory of Robotics and System (HIT), and the Program of Heilongjiang Touyan Team.

Appendix A

We now prove Lemma 3.

Proof. Inspired by the proof of Lemma 2 in Dong et al. [15], this proof is completed by an induction way.

Step 1: Firstly, there exists positive constants $\beta_{c1}, \gamma_{c1}, \eta_{c1}$ and φ_{c1} satisfying $|x_{1c}| \leq \beta_{c1}, |\dot{x}_{1c}| \leq \gamma_{c1}$ $|\ddot{x}_{1c}| \leq \eta_{c1}$ and $|\dddot{x}_{1c}| \leq \varphi_{c1}$ by Assumption 2. It is worth noting that the Lemma 1 in Dong et al. [15] that if the Assumption 3 and related conditions are satisfied, then $x(t)$ is bounded over any time interval. Based on the definition of α_1 in Eq. (14), there exist positive constants $\beta_{\alpha 1}, \gamma_{\alpha 1}$ and $\eta_{\alpha 1}$ satisfying $|\alpha_1| \leq \beta_{\alpha 1}, |\dot{\alpha}_1| \leq \gamma_{\alpha 1}$ and $|\ddot{\alpha}_1| \leq \eta_{\alpha 1}$.

Step 2: By putting the bounded $|\dot{\alpha}_1|$ and $|\ddot{\alpha}_1|$ into Eqs. (9) and (10) with $i = 2$ and utilizing the Lemma 2, there exist positive constants $\beta_{c2}, \gamma_{c2}, \eta_{c2}$ and φ_{c2} satisfying $|x_{2c}| \leq \beta_{c2}, |\dot{x}_{2c}| \leq \gamma_{c2}, |\ddot{x}_{2c}| \leq \eta_{c2}$ and $|\dddot{x}_{2c}| \leq \varphi_{c2}$. Similar with step 1, there exist positive constants $\beta_{\alpha 2}, \gamma_{\alpha 2}$ and $\eta_{\alpha 2}$ satisfying $|\alpha_2| \leq \beta_{\alpha 2}, |\dot{\alpha}_2| \leq \gamma_{\alpha 2}$ and $|\ddot{\alpha}_2| \leq \eta_{\alpha 2}$.

Step i: Then, with the same induction, the ~~bounded~~ of the $|\alpha_i|$, $|\dot{\alpha}_i|$ and $|\ddot{\alpha}_i|$ are obtained. We have simplified certain simple yet arduous deductions in order to simplify the presentation.

□

We now prove Lemma 4.

Proof. Consider the following Lyapunov function as

$$V_\xi = \frac{1}{2}\sum_{j=1}^{n}\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)}$$

then the derivative of V_ξ along Eq. (11) is

$$\begin{aligned}
\dot{V}_\xi &= \frac{1}{2}\sum_{j=1}^{n}\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}}\left(\Phi^{\mathrm{T}}(A_j^{0\sim1})P_j(A_j^{0\sim1}) + P_j(A_j^{0\sim1})\Phi(A_j^{0\sim1})\right)\left(\xi_j^{(0\sim1)}\right) \\
&\quad + \sum_{j=1}^{n-1}\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\begin{bmatrix} 0 \\ g_j\xi_{j+1}^{(0\sim1)} + g_j(x_{(j+1)c} - \alpha_j) \end{bmatrix} \\
&\leq -\frac{1}{2}\sum_{j=1}^{n}\mu_j\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} \\
&\quad + \sum_{j=1}^{n-1}\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\begin{bmatrix} 0 \\ g_j\xi_{j+1}^{(0\sim1)} + g_j(x_{(j+1)c} - \alpha_j) \end{bmatrix}.
\end{aligned} \tag{100}$$

By utilizing Young's inequality, we have

$$\begin{aligned}
\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\begin{bmatrix} 0 \\ g_j\xi_{j+1}^{(0\sim1)} \end{bmatrix} &\leq \lambda_{\max}(P_j)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} + \frac{g_j^2}{4}\left(\xi_{j+1}^{(0\sim1)}\right)^{\mathrm{T}}\xi_{j+1}^{(0\sim1)} \\
&\leq \lambda_{\max}(P_j)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} \\
&\quad + \frac{g_j^2}{4\lambda_{\min}(P_{j+1})}\left(\xi_{j+1}^{(0\sim1)}\right)^{\mathrm{T}} P_{j+1}(A_j^{0\sim1})\xi_{j+1}^{(0\sim1)},
\end{aligned} \tag{101}$$

and

$$\begin{aligned}
&\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\begin{bmatrix} 0 \\ g_j(x_{(j+1)c} - \alpha_j) \end{bmatrix} \\
&\leq \lambda_{\max}(P_j)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} + \frac{g_j^2}{4}(x_{(j+1)c} - \alpha_j)^2 \\
&\leq \lambda_{\max}(P_j)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} + \frac{\rho^2\sigma_j^2}{4}.
\end{aligned} \tag{102}$$

Substituting Eqs. (101) and (102) into Eq. (100), we have

$$\dot{V}_\xi \le -\frac{1}{2}(\mu_1 - 4\lambda_{\max}(P_1))\left(\xi_1^{(0\sim1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim1})\xi_1^{(0\sim1)} - \frac{1}{2}\sum_{j=2}^{n}\left(\mu_j - 4\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} + \sum_{j=1}^{n-1}\frac{\rho^2\sigma_j^2}{4}.$$

In order to maintain a uniform form, we also added an additional positive portion $\frac{g_0^2}{4\lambda_{\min}(P_1)}\left(\xi_1^{(0\sim1)}\right)^{\mathrm{T}} P_1(A_1^{0\sim1})\xi_1^{(0\sim1)}$ for $\dot{V}_\xi$ and rewritten the above equation as

$$\dot{V}_\xi \le -\frac{1}{2}\sum_{j=1}^{n}\left(\mu_j - 4\lambda_{\max}(P_j) - \frac{g_{j-1}^2}{2\lambda_{\min}(P_j)}\right)\left(\xi_j^{(0\sim1)}\right)^{\mathrm{T}} P_j(A_j^{0\sim1})\xi_j^{(0\sim1)} + \sum_{j=1}^{n-1}\frac{\rho^2\sigma_j^2}{4}, \tag{103}$$

where g_0 is a small positive constant.

Based on the Eq. (103), it obtains

$$\dot{V}_\xi \le -aV + b, \tag{104}$$

by letting

$$a = \min\left\{\mu_1 - 4\lambda_{\max}(P_1) - \frac{g_0^2}{2\lambda_{\min}(P_1)}, \ldots, \mu_n - 4\lambda_{\max}(P_n) - \frac{g_{n-1}^2}{2\lambda_{\min}(P_n)}\right\}, \tag{105}$$

$$b = \sum_{j=1}^{n-1}\frac{\rho^2\sigma_j^2}{4}. \tag{106}$$

Thus, by setting $V_\xi(0) = 0$, it obtains that

$$V_\xi(t) \le \frac{b}{a}(1 - e^{-at}),$$

and it can be shown that $\|\xi\|^2 \le \frac{2b}{\lambda_{\min}(P_i)a}(1 - e^{-at})$. $\square$

References

[1] G. Tao, Adaptive Control Design and Analysis, John Wiley & Sons, 2003.
[2] P.A. Ioannou, J. Sun, Robust Adaptive Control, Courier Corporation, 2012.
[3] M. Krstic, P.V. Kokotovic, I. Kanellakopoulos, Nonlinear and Adaptive Control Design, John Wiley & Sons, Inc., 1995.
[4] G. Tao, Multivariable adaptive control: a survey, Automatica 50 (11) (2014) 2737–2764.
[5] M. Krstić, H.H. Wang, Stability of extremum seeking feedback for general nonlinear dynamic systems, Automatica 36 (4) (2000) 595–601.
[6] W. Liu, X. Huo, K. Ma, S. Chen, Performance improvement in fast extremum seeking with adaptive phase compensator and high-gain optimizer, IEEE Trans. Syst., Man, Cybern. 52 (11) (2022) 6774–6788.
[7] G.R. Duan, High-order fully actuated system approaches: part I. Models and basic procedure, Int. J. Syst. Sci. 52 (2) (2021a) 422–435.
[8] G.R. Duan, High-order fully actuated system approaches: part II. Generalized strict-feedback systems, Int. J. Syst. Sci. 52 (3) (2021b) 437–454.
[9] G.R. Duan, High-order fully actuated system approaches: part III. Robust control and high-order backstepping, Int. J. Syst. Sci. 52 (5) (2021c) 952–971.
[10] G.R. Duan, High-order fully actuated system approaches: part IV. Adaptive control and high-order backstepping, Int. J. Syst. Sci. 52 (5) (2021d) 972–989.

[11] G.R. Duan, High-order fully actuated system approaches: part V. Robust adaptive control, Int. J. Syst. Sci. 52 (10) (2021e) 2129–2143.
[12] W. Liu, G.-R. Duan, M. Hou, High-order robust command filtered backstepping design for strict-feedback systems: a high-order fully-actuated system approach, Int. J. Robust Nonlinear Control 32 (18) (2022) 10251–10270.
[13] I. Kanellakopoulos, P.V. Kokotovic, A.S. Morse, Systematic design of adaptive controllers for feedback linearizable systems, IEEE Trans. Autom. Control 36 (11) (1991) 1241–1253.
[14] J.A. Farrell, M. Polycarpou, M. Sharma, W. Dong, Command filtered backstepping, IEEE Trans. Autom. Control 54 (6) (2009) 1391–1395.
[15] W. Dong, J.A. Farrell, M.M. Polycarpou, V. Djapic, M. Sharma, Command filtered adaptive backstepping, IEEE Trans. Control Syst. Technol. 20 (3) (2011) 566–580.
[16] J. Yu, P. Shi, W. Dong, H. Yu, Observer and command-filter-based adaptive fuzzy output feedback control of uncertain nonlinear systems, IEEE Trans. Ind. Electron. 62 (9) (2015) 5962–5970.
[17] J. Yu, P. Shi, L. Zhao, Finite-time command filtered backstepping control for a class of nonlinear systems, Automatica 92 (2018) 173–180.
[18] X. Zheng, X. Yang, Command filter and universal approximator based backstepping control design for strict-feedback nonlinear systems with uncertainty, IEEE Trans. Autom. Control 65 (3) (2019) 1310–1317.
[19] G. Zhu, J. Du, Y. Kao, Command filtered robust adaptive NN control for a class of uncertain strict-feedback nonlinear systems under input saturation, J. Frankl. Inst. 355 (15) (2018) 7548–7569.
[20] Y.X. Li, Command filter adaptive asymptotic tracking of uncertain nonlinear systems with time-varying parameters and disturbances, IEEE Trans. Autom. Control 67 (6) (2022) 2973–2980.
[21] Y.X. Li, Finite time command filtered adaptive fault tolerant control for a class of uncertain nonlinear systems, Automatica 106 (2019) 117–123.
[22] R. Rashad, A. Aboudonia, A. El-Badawy, A novel disturbance observer-based backstepping controller with command filtered compensation for a MIMO system, J. Frankl. Inst. 353 (16) (2016) 4039–4061.
[23] V. Djapic, J. Farrell, W. Dong, Land vehicle control using a command filtered backstepping approach, in: 2008 American Control Conference, 2008, pp. 2461–2466.
Q4 [24] J. Hu, H. Zhang, Immersion and invariance based command-filtered adaptive backstepping control of VTOL vehicles, Automatica 49 (7) (2013) 2160–2167.
[25] Q. Shen, P. Shi, Distributed command filtered backstepping consensus tracking control of nonlinear multiple-agent systems in strict-feedback form, Automatica 53 (2015) 120–124.
[26] D. Xu, Y. Dai, C. Yang, X. Yan, Adaptive fuzzy sliding mode command-filtered backstepping control for islanded PV microgrid with energy storage system, J. Frankl. Inst. 356 (4) (2019) 1880–1898.
[27] A. Kanchanaharuthai, E. Mujjalinvimut, Fixed-time command-filtered backstepping control design for hydraulic turbine regulating systems, Renew. Energy 184 (2022) 1091–1103.
[28] R. Patel, F. Hafiz, A. Swain, A. Ukil, Nonlinear excitation control of diesel generator: a command filter backstepping approach, IEEE Trans. Ind. Inf. 17 (7) (2020) 4809–4817.
[29] C. Fu, C. Zhang, G. Zhang, J. Song, C. Zhang, B. Duan, Disturbance observer-based finite-time control for three-phase AC–DC converter, IEEE Trans. Ind. Electron. 69 (6) (2021) 5637–5647.
[30] B. Wang, M. Iwasaki, J. Yu, Finite-time command filtered backstepping control for dual-motor servo systems with LuGre friction, IEEE Trans. Ind. Inform. (Early access) (2022), doi:10.1109/TII.2022.3182341.
[31] J. Yu, P. Shi, X. Chen, G. Cui, Finite-time command filtered adaptive control for nonlinear systems via immersion and invariance, Sci. China-Inf. Sci. 64 (9) (2021) 1–14.
[32] G. Cui, J. Yu, Q.G. Wang, Finite-time adaptive fuzzy control for MIMO nonlinear systems with input saturation via improved command-filtered backstepping, IEEE Trans. Syst., Man, Cybern. 52 (2) (2022) 980–989.
[33] J. Yu, L. Zhao, H. Yu, C. Lin, Barrier Lyapunov functions-based command filtered output feedback control for full-state constrained nonlinear systems, Automatica 105 (2019) 71–79.
[34] H.K. Khalil, Nonlinear Systems, third ed., Prentice Hall, New Jewsey, 2002.
[35] Z. Zhang, G.-R. Duan, M. Hou, An improved adaptive dynamic surface control approach for uncertain nonlinear systems, Int. J. Adapt. Control Signal Process. 32 (5) (2018) 713–728.
[36] Y. Pan, H. Yu, Dynamic surface control via singular perturbation analysis, Automatica 57 (2015) 29–33.