Practice No. 1

Good electric conductor must have a high concentration of charge carriers capable to move easily throughout the entire volume of medium.

γ - electrical conductivity ; basic parameter characterizing material's ability to conduct electric current

$$\boxed{\gamma = q n \mu}$$

q - electrical charge

n - volume concentration of carriers

μ - carrier mobility

E - intensity of electric field

$$\boxed{\mu = \frac{v_d}{E}}$$

γ - electrical conductivity is defined as the conductance of material with length l and cross-section area S

$$① \quad \boxed{\gamma = \frac{1}{R} \cdot \frac{l}{S}} \quad \left[\frac{S}{m}\right]$$

ρ - electrical resistivity

$$② \quad \boxed{\rho = \frac{1}{\gamma}}$$

① ②

$$\left\{ \rho = \frac{1}{\gamma} = R \cdot \frac{S}{l} \right\}$$

The highest value of conductivity is characterized by metals and metal alloys. The best conductors:

- Ag ; silver
- Cu ; copper
- Au ; gold
- Al ; aluminum

Metals and metal alloys ~~are~~ have good electrical and thermal ~~associated~~ conductivity. This is a results ~~of~~ from special interatomic bonds ensured by the great number of free valence electrons. These bonds are called metallic bonds.

Concentraction n of the conductivity electrons is very large and doesn't depend on temperature and other external conditions.

Positive ions neutralize the negative charge of Fermi's quantum gas. This is equivalent to the assumption that positive charges are distributed over the entire volume of metal.

The interaction between electrons is ignored!

Due to high concentraction, electron gas in metals is subjected to Fermi-Dirac dist. and Pauli excl. princ. i.e. electrons cannot occupy the same energy state.

At $T = 0K$ electrons occupy the states with the lowest energies.

The maximum energy of molecules at $T = 0K$ is called Fermi E_F energy.

Electric field with intensity $\vec{E}$ causes an exertion of the force to the electron. Force tends to shift electrons towards a higher potential. $\vec{F} = e\vec{E}$

If this would be the only force working on electrons we would see accelerated movement of electrons.

In fact, we observe a DC current in fixed electric field, this is a result of collisions of electrons with phonons and imperfections of

As a resoult of these collisions, arises a state of equilibrium which is characterized by a fixed drift velocity $\vec{v}_d$. The factor determining a certain drift speed $\vec{v}_d$ is the influence of ions despite the accelerating effect of $\vec{E}$.

As the temperature rises, the amplitude of thermal vibrations increases and deviations become stronger.

Defects in the crystal structure causes local changes in potential. Electrons encountering those changes are being scattered which reduces their average speed. This mean that electrons are subject to constant acceleration between scatterings which are decreasing their speed.

τ - average time between collisions.

$$\boxed{\gamma = \frac{e^2}{m} n \tau}$$

Only small amounts of electrons with energies close to Fermi energy E_F are involved in the conduction of electrical current.

Knowing the time between colisions (τ) and speed of electrons (v_F) we can calculate the mean free path l (średnia droga pomiędzy kolizjami)

$$\boxed{l = \tau \cdot v_F}$$

Ideal crystal lattice, in which the ions are immovable and which doesn't contain defects, there is no electrical resistance. This is due to the wave nature of electron and the fact that its wavelength corresponding to E_F is much greater than the inter-atomic distances, so the electron doesn't interact with ions and doesn't lose its energy.

The scattering of electron energy in metals occurs as a result of 2 phenomena:

- electron - phonon colision (thermal oscillations of the network)
- collision of the electron with crystal lattice defect

The resistances resulting from independent scattering processes are additive - Mathiessen's rule

$$\rho = \rho_1 + \rho_2 + \dots + \rho_n$$

Total resistivity of metals is therefore equal to:

$$\rho(T) = \rho_r + \rho_f(T)$$

ρ_r - residual resistivity; independent of T, resulting of scatterings on defects of crystal lattice

ρ_f - phonon resistivity; dependent of T, result of electron - phonon colisions

Influence of T on electrican conductivity of metals

from privious formulas: $\gamma = \frac{e^2}{m} n \tau$ and $l = \tau v_F$
1 2

and the relation $\gamma = \frac{1}{\rho}$

$$\gamma = \frac{1}{\rho} = \frac{e^2 n}{m} \cdot \frac{l}{v_F}$$ (1) (2)

Temperature affects the electrical conductivity of metals (γ) only by changing the average free path of electrons (l)

The concentraction of electrons (n) and the velocity of Fermi (v_F) are practically independent of T

$$\rho_f \sim T^5 \text{ - at low } T$$

In most cases the scattering of electrons on phonons in low temerature range doesn't play a significant role.

The scattering processes are caused not only by interactions with phonons but also:

- local changes in lattice constant parameter caused by atoms of different sizes etc.
- local differences in charge, caused by atoms of diff. valence in relation to valence of matrix atom

The concentraction of defects this type is independent of T so the electrical resistivity (ρ) caused by scattering on the impurities or dopants is also independent of T

Electrial resistance (ρ) in low temp. doesn't depend on T because it is effect of scattering on impurities or dopants. And the scattering on phonons at low T doesn't make a significant contribution to ρ due to low amplitude of vibrations. As a T rises, the additive contribution of phonon scattering increases, causing $\rho \sim T^5$ at low T and $\rho \sim T$ at higher T

Practice No 1M

The measuring range - largest value that a multimeter can show.

The measurement error value is sum of 2 components: one dependent on the ~~val~~ measured value and the other on the selected measuring range.

Absolute error: $\Delta = |\text{actual value} - \text{indicated value}|$

Relative error: quotient of the absolute error by the real value expresed as %

$$\delta = \frac{\Delta}{\text{atual value}} \cdot 100\%$$

Resolution - the smallest value that can be indicated in a given measuring range

The number of digits - number of significant numbers displayed on the display
eg. $3\frac{1}{2}$ means that max displace value is 1999
for $4\frac{1}{2}$ it is 19999

The meter accuracy class - The largest permissible relative measurement error

$$\Delta_{max} = \pm (class/100) \cdot range$$

Limiting error of measurement with a digital multimeter

The accurancy of the measurement with a digital multimeter can be expressed as: (a% of display + b% of range)

The measurement error: $\Delta_{gr} = \frac{a \cdot indication + b \cdot range}{100}$

LSB - last significant bit, resolution

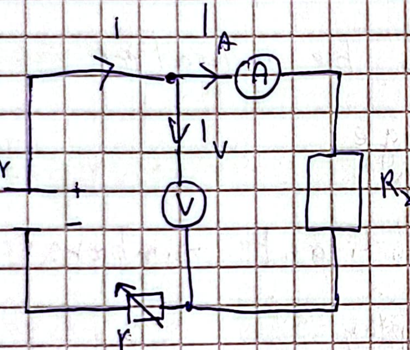
Measurement methods

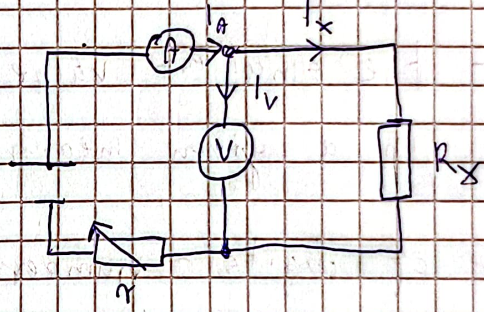
Direct measurement – the value of the measured value is read directly from the instrument

Indirect measurement – the values of measured quantities read from instrument are associated with the unknown parameter with the functional dependence from which it can be calculated

For less accurate measurements, the resistance of meters is not taken into account. e.g instead of the exact value of R_x, the approximate value R_x' is calculated

while $R_x > R_{xgr}$





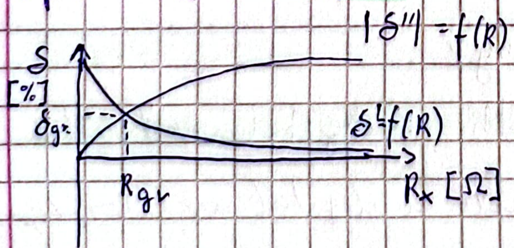
while $R_x < R_{xgr}$

- large resistances compared to resistance of ammeter
- relative error $\delta_p' > 0$ $(R_x' > R_x)$

$\delta_p' \downarrow \sim R_x \uparrow \wedge R_A \downarrow$

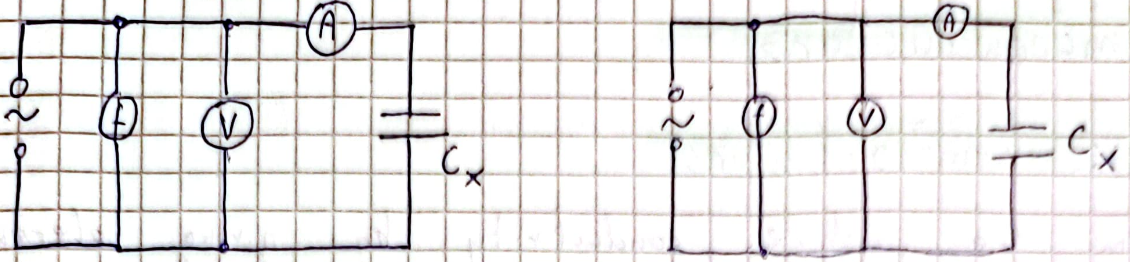
- small resistance in relation to resistance of voltmeter
- relative error $\delta_p'' < 0$ $(R_x'' < R_x)$

$|\delta_p''| \downarrow \sim R_x \downarrow \wedge R_V \uparrow$



The intersection point of the curves determines the so-called limiting resistance (R_{xgr}) and limiting error (δ_{gr})

R_{xgr} is the value of measured resistance at which the error from omitting the resistance of meters is the same, regardless of the connection $\delta_p' = |\delta_p''|$



The system diagram for measuring the capacitance by technical method

- at the set current value
- at the set voltage value

- for low f of C_x with capacities below several dozen μF and acustic f below several μF

The effective value and average value of the measured signal are the values characterizing the waveforms varying in time.

The effective value ~~is~~ of AC is equal to the value of DC that, over time equal to one period of AC, produces the same thermal effect as the given AC signal

The average value of AC ~~is~~ waveform equals that value of DC for which, over a time equal to one period of AC, the same electric charge flows that flowed at the same time during AC flow.

$$U_{sk} > U_{sr}$$

The shape factor is the ratio of the effective value of the signal to its average value $k = \frac{U_{sk}}{U_{sr}}$

The crest factor is the ratio of the peak signal to its effective value $s = \frac{U_m}{U_{sk}}$

Semiconductors

Intrinsic semiconductors

In them we achive conductivity by giving electrons energy eg. by heat. By giving them energy electrons jump from valence band to conduction band.

~~Ex~~ Extrinsic (doped) semiconductors

We can achive conducting electricity in semiconductors also by adding doping (impurities) to them.

N-type - This is type to which we add impurities those atoms containing one more valence electron, thanks to that all interatomic bonds are saturated and we are left with extra electron that can move easily. Electrons are charge carriers

P-type - Here we add impurity ~~the~~ which atoms have one less valence electron, that causes that not all bonds are saturated and there become holes. Current is conducted here because electrons can jump to the holes leaving holes behind them. We say that holes are ~~*~~ carriers here.

Computer measuring system - set of measuring apparatus connected with computer equipment, equipped with specialized software for processing measuring information for the purpuse of obtaining, presenting, archiving, transmitting and possible further processing.

- **diagnostic systems**: identyfing the reason for discrepancy
- **control systems**: checking whether value is within accepted range

The way of connecting elements of system determines a way of information exchange between its elements. **3 main configurations**:

- **star**:
  - elements conected directly to central unit and all inf. exchange take place through it.
- **common bus**:
  - elements conected to central unit via single interface
  - units identified by addressing
- **loop**:
  - inf. sent by central unit goes to all units and after checking for addres incompatibility is forwarded further.
  - identified by addressing
  - lowest speed
  - requires the least data lines

Structure of measuring system

central unit - main role: coordinate the work of system

sensor - convert non-electrical quantities into measurable electrical quantities

data acquisition module - most important of elements; collect and convert analog signals to digital form

data processing module - implements Digital Signal Processing process

user communication module - allows observation of results and parameterization of process

Measuring interfaces - conect instruments to central unit

serial transmission - sending data bit by bit, asynchronously or in clock time synch.

parallel transmission - sending data in form of a string of words, asynchronously or in rhythm of sync signals

RS-232C - serial transmission, to connect 2 end devices DTE we need make it via modems DCE

Advantages
- simplicity
- low cost of hardware implementation

disadvantages
- low speed of data transfer

types of transmition between DTE terminals:
- simplex, in one-way
- half duplex, two-way non-simultaneous
- duplex, two-way simultaneous

due to method of synchronization of transmited bits:
- asynchronus using signs - most commonly used
- synchronous

DB-9

To connect 2 DTE devices without DCE modems we need null modem cable

logic 1 +3V - +15V

logic 0 -15V - -3V

USB - universal serial bus - most popular interface for connecting peripherals with computer

(1) Its versatility and flexibility elables us to connect wide range of peripherals to computer ~~USB interface can automaticly~~

(2) Automatic recognition by host (computer) of connected device is special feature of USB interface. When conecting device equipped with USB interface it sends unique string that uniquely defines the type of it and basics on this computer select parameters for cooperation with the device.

(3) Network architecture of USB - connected devices has a star topology. Maximal number of devices in network can consists of up to 127 devices

(4) Only one device can act as bus controller, so it is not possible to connect 2 computers via USB.

(5) USB vesion 1.x and 2.0 has four cabels: power supply +5V, mass and 2 data lines (D-, D+

(6) To convert binary data into electrical signal ~~symbol~~ The NRZI method is used where logical 1 is represented by no change in signal level and logical 0 by a change of signal level

(7) Due to much higher data transfer rate USB 3.X uses additional 2 pairs of signal lines Conectors for USB3.X standard must have 9 pins and remain compatible with conectors for earlier standard

IEEE-488

Typical system connected via IEEE-488 interface has bus configuration.
Each device connected to the bus must have a unique 5-bit address from 0 to 30, which can be set by user.
Up to 15 devices (14 + controller) can be connected
length of cables cannot exceed 20 meters, max 2 meters between 2 devices
This transmition is parallel, asynchronous with confirmation of receipt (handshake). Handshake mode allows to adjust the transmission speed to slowest recipient.

standard
SCPI (IEEE-488.2)

unify syntax and form of command and format of data sent between measuring devices and computer.
It derives directly from work on standardization of IEEE-488 interface but it is not limited to specific physical interface. It is used as language for communication through other interfaces.
Commands are sent as text strings composed of characters in ASCII code.

No. 2

Dielectrics are solids, liquids and volatles, which in there structure don't contain free charges

There isn't electron conduction in dielectric only ionic

Ideal dielectric have no free electric charges.

non-polar particles ; centre of „gravity" of „+" charges is in centre of „-" charges

polar particles ; centre of „gravity" of + doesn't concide in centre of -

electric dipole ; opposite charges q and -q separated by distance l

electric polarization

- electric field causes separation of + and - centers of ~~each~~ inducing dipole moment (elastic polarization)
- electric field orientates dipoles towards the field (relaxing polarization)

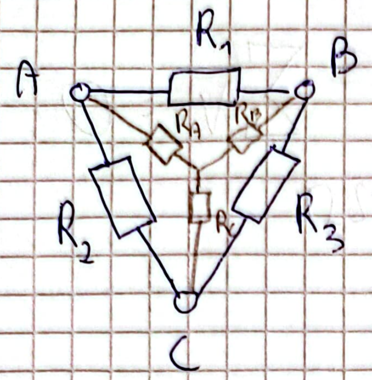
relative permittivity $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Value of permeability depends on external factors such as humidity and temperature. In alternating field depends also on frequency of changes of field

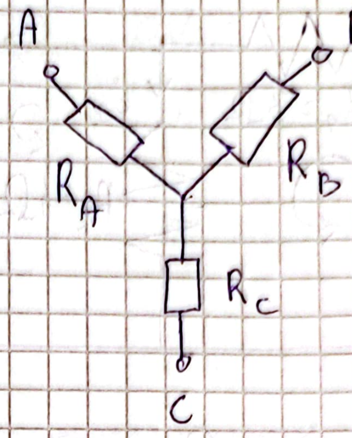
In real crystallines dielectrics, the source of free charge carriers are point defects and ions from impurities

Under influence of thermal stimulation, some ions can pass to interstitial space. This process creates a pair of almost free carriers: an ion and empty space. This pair of carriers is called Frankel's heat defect

$\Delta \Rightarrow Y$ EFE



$\Rightarrow$

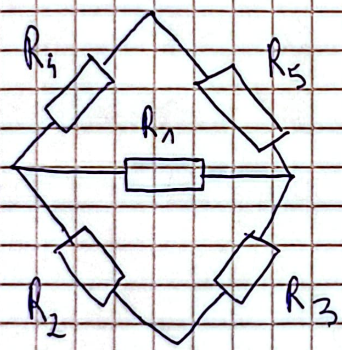


$$R_A = \frac{R_1 \, R_2}{R_1 + R_2 + R_3}$$

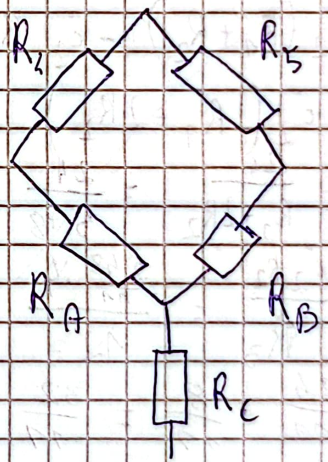
$$R_B = \frac{R_1 \, R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

ex.



$\Rightarrow$

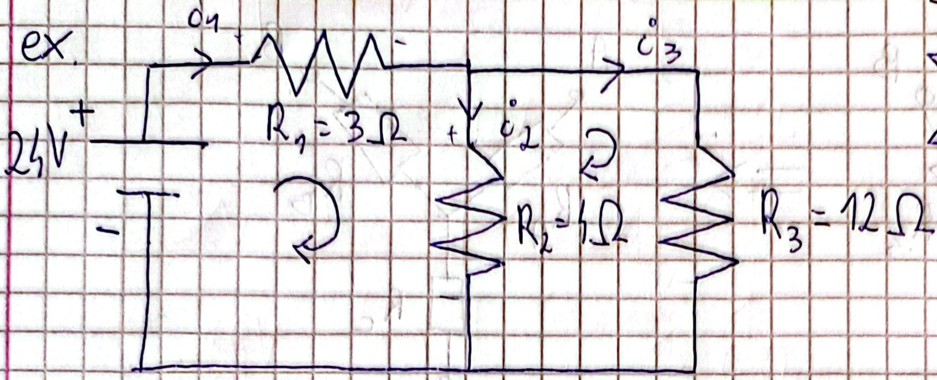


KVL & KCL

$\sum i_{in} = \sum i_{out}$

$\sum V = 0$

ex.



1) $i_1 - i_2 - i_3 = 0 \Rightarrow i_1 = i_2 + i_3$

2) $24V - i_1 R_1 - i_2 R_2 = 0 \Rightarrow$

3) $i_2 R_2 - i_3 R_3 = 0$

2) $24 - i_2 R_1 - i_3 R_1 - i_2 R_2 = 0 \Rightarrow 24 - i_2 (R_1 + R_2) - i_3 R_1 =$

$i_2 R_2 - i_3 R_3 = 0$

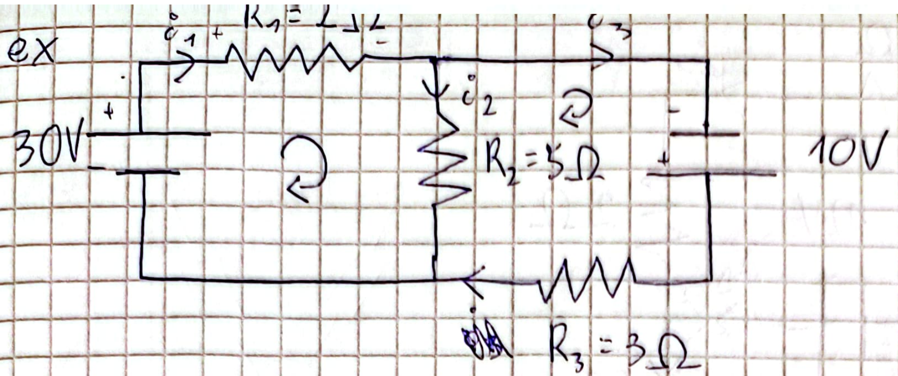
$24 - i_2 (R_1 + R_2) - i_2 R_2 = 0$

$24 - i_2 (R_1 + 2R_2) = 0$

$i_2 = \frac{24}{R_1 + 2R_2} = \frac{24}{3+8} = \frac{24}{11}$ A

$i_3 = \frac{i_2 R_2}{R_3} = \frac{\frac{24}{11} \cdot 4}{12} = \frac{8}{11}$ A

$i_1 = \frac{24}{11} + \frac{8}{11} = \frac{32}{11}$ A ex



$$\begin{cases} i_1 - i_2 - i_3 = 0 \\ 30 - i_1 R_1 - i_2 R_2 = 0 \\ 10 - i_3 R_3 + i_2 R_2 = 0 \end{cases}$$

$$\begin{cases} i_3 = i_1 - i_2 \\ 30 - 2i_1 - 5i_2 = 0 \\ 10 - 3(i_1 - i_2) + 5i_2 = 0 \end{cases}$$

$$\begin{cases} 30 - 2i_1 - 5i_2 = 0 & / \cdot 3 \\ 10 - 3i_1 + 8i_2 = 0 & / \cdot -2 \end{cases}$$

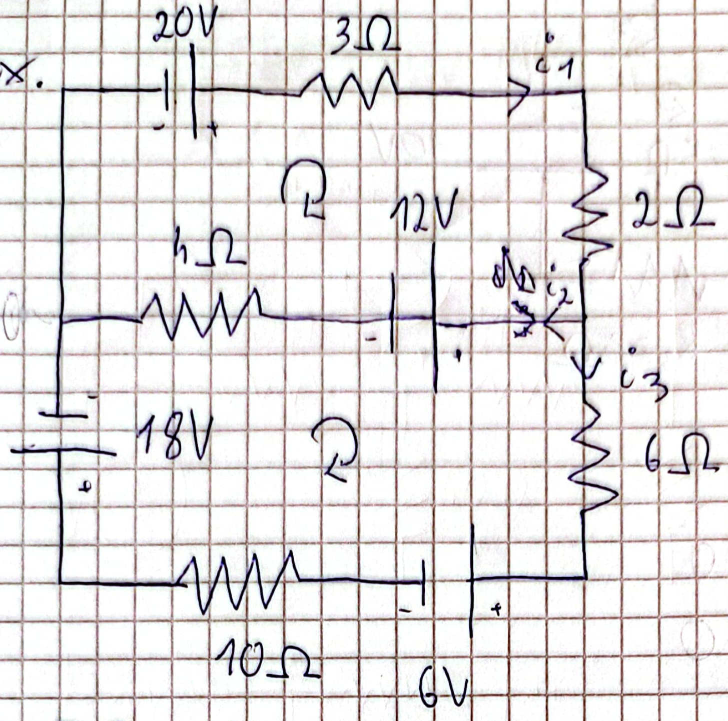
$$+\begin{cases} 90 - 6i_1 - 15i_2 = 0 \\ -20 + 6i_1 - 16i_2 = 0 \end{cases}$$

$$70 - 31i_2 = 0$$

$$\begin{cases} i_2 = \frac{70}{31} A \\ i_1 = \frac{30 - 5 \cdot i_2}{2} = \frac{30 - 5\frac{70}{31}}{2} \\ i_3 = i_1 - i_2 \end{cases}$$

physical medium

ex.



$$\begin{cases} i_1 \mp i_2 - i_3 = 0 \\ \cancel{20 - 3i_1 - 2i_1 - 12 + 4i_3 = 0} \\ \cancel{12 - 6i_3 - 6 + 10i_3 - 18 - 4i_3} \\ 20 - 3i_1 - 2i_1 - 12 - 4i_2 = 0 \\ 12 - 6i_3 - 6 - 10i_3 - 18 + 4i_2 = 0 \end{cases}$$

$$\begin{cases} i_1 \mp i_2 = i_3 \\ 8 - 5i_1 - 4i_2 = 0 \\ -12 - 16i_3 + 4i_2 = 0 \end{cases}$$

$$\begin{cases} 8 - 5i_1 - 4i_2 \qquad \cancel{/\cdot(-3)} \\ -12 - 16i_1 + \cancel{4}20i_2 = 0 \end{cases}$$

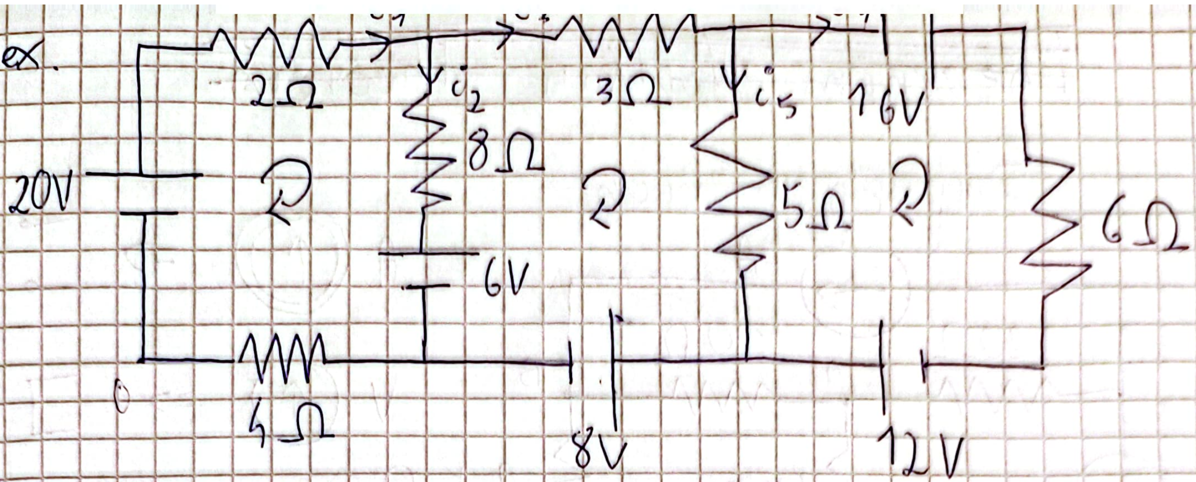
$$\cancel{\begin{cases} -24 + 15i_1 + 12i_2 = 0 \\ -12 - 16i_1 - 12i_2 = 0 \end{cases}}$$

$\cancel{-36 - i_1 = 0}$

$\cancel{i_1 = -36\,A}$

$\cancel{i_2 = 8 + 5 \cdot 36\ A}$

ex.



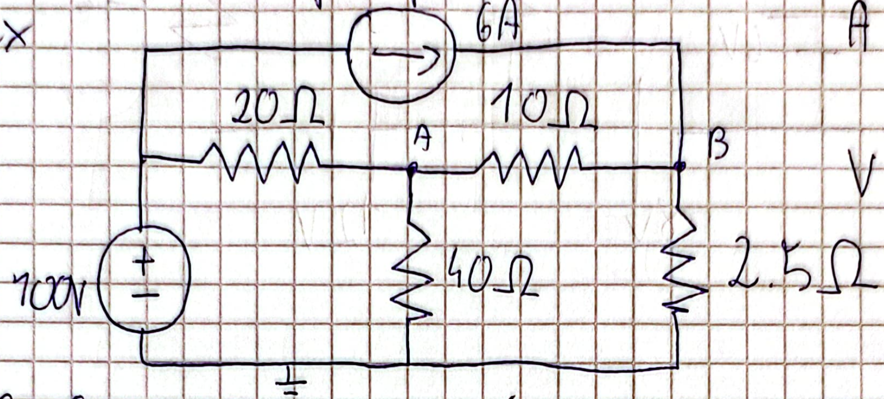
$$\begin{cases} i_1 - i_3 - i_2 = 0 \\ i_3 - i_4 - i_5 = 0 \\ 20 - 2i_1 - 8i_2 - 6 \cancel{+4(i_2 + i_3 + i)} - 4i_1 = 0 \\ 6 + 8i_2 - 3i_3 - 5i_3 - 8 = 0 \\ 16 - 6i_4 + 12 + 5i_5 = 0 \end{cases}$$

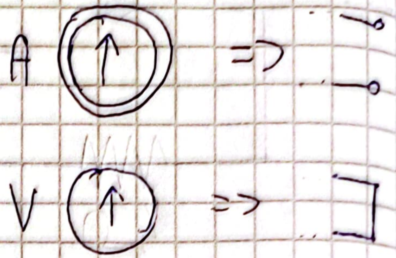
$$\begin{cases} i_1 - i_3 - i_2 = 0 \\ i_3 - i_4 - i_5 = 0 \\ 14 - 6i_1 - 8i_2 = 0 \\ -2 + 8i_2 - 8i_3 = 0 \\ 28 - 6i_4 + 5i_5 = 0 \end{cases}$$

~~Thevenim Northon Theorem~~

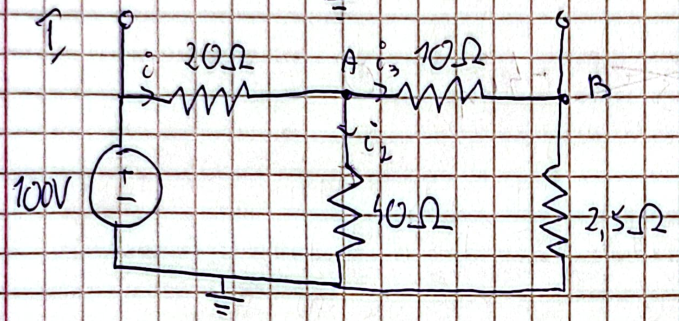
Superposition

ex





I,



$R_2 = \frac{12,5 \cdot 40}{52,5} + 20 = 29,52\,\Omega$

$i = \frac{100}{29,52} = 3,39\,A$

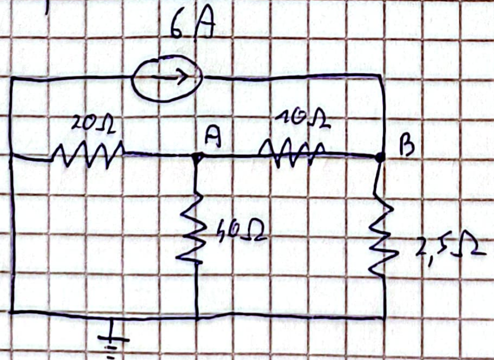
$V_A = 100 - 3,39 \cdot 20 = 32,25\,V$

$i_2 = \frac{32,25}{\cancel{3,39}\,40} = 0,8\,A$

$V_B = 32,25 - 33,9 = -1,68\,V$

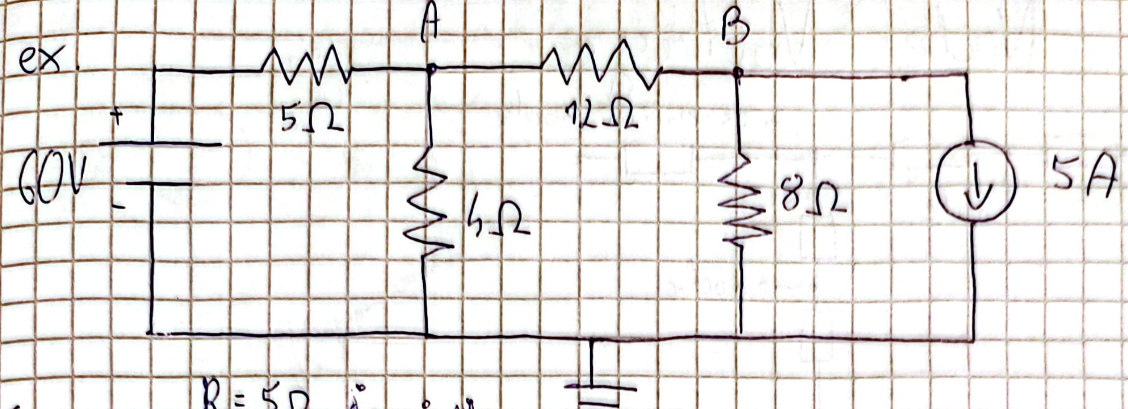
$i_3 = 2,59\,A$

II ~~V~~

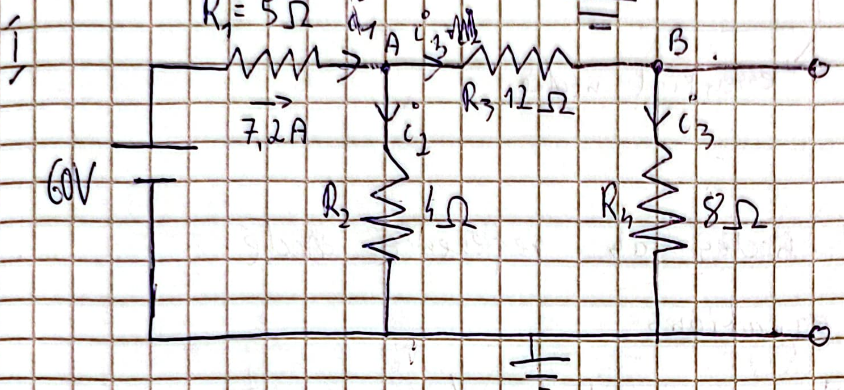


R

ex.



I



$R_{34} = 20\,\Omega$

$R_{234} = \frac{4 \cdot 20}{24} = \frac{10}{3}\,\Omega$

$R_z = \frac{10}{3} + 5 = 8\frac{1}{3}\,\Omega$

$i = \frac{V}{R} = \frac{60}{8\frac{1}{3}} = 7{,}2\,A$

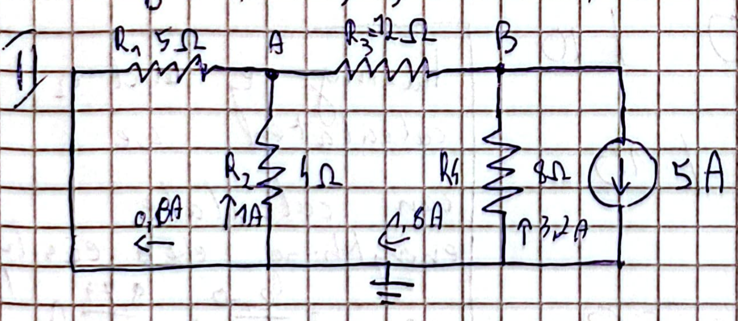
$V_A = 60 - 5 \cdot 7{,}2 = 24\,V$

$i_2 = \frac{V_A}{R_2} = \frac{24}{4} = 6\,A$

$i_3 = i_1 - i_2 = 1{,}2\,A$

$V_B = V_A - i_3 R_3 = 24 - 1{,}2 \cdot 12 = 9{,}6\,V$

II



$R_{12} = \cancel{9\,\Omega}\ \frac{20}{9}\,\Omega$

$R_{123} = \cancel{\frac{9 \cdot 12}{21}} = \cancel{\frac{108}{21}} = \cancel{\frac{36}{7}\,\Omega}\ 14\frac{2}{9}\,\Omega$

$R_z = \frac{14{,}2 \cdot 8}{22{,}2}\,\Omega = 5{,}1\,\Omega$

$V_B = -3{,}2 \cdot 8 = -25{,}6\,V$

$V_A = -1 \cdot 4 = -4\,V$

$V_A = 24 - 4 = 20\,V$

$V_B = 9{,}6\,V - 25{,}6\,V = -16\,V$

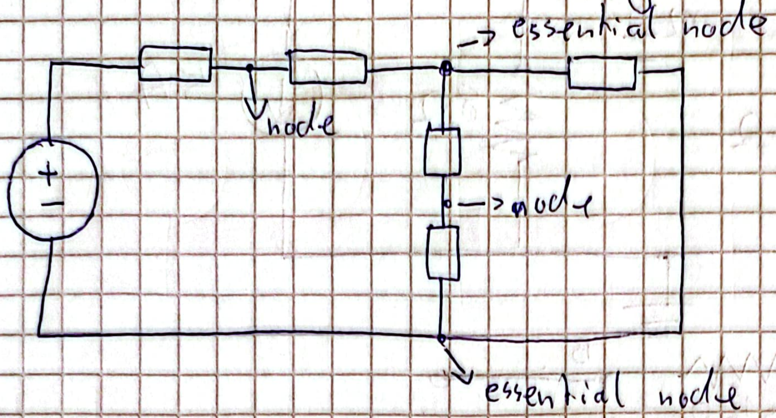
$i_1 = 7{,}2 + 0{,}8 = 8\,A$

$i_2 = 6 - 1 = 5\,A$

$i_3 = \ldots 12 - 3{,}2 = -2\,A \quad 8 - 5 = 3\,A$

$i_4 = 2\,A$

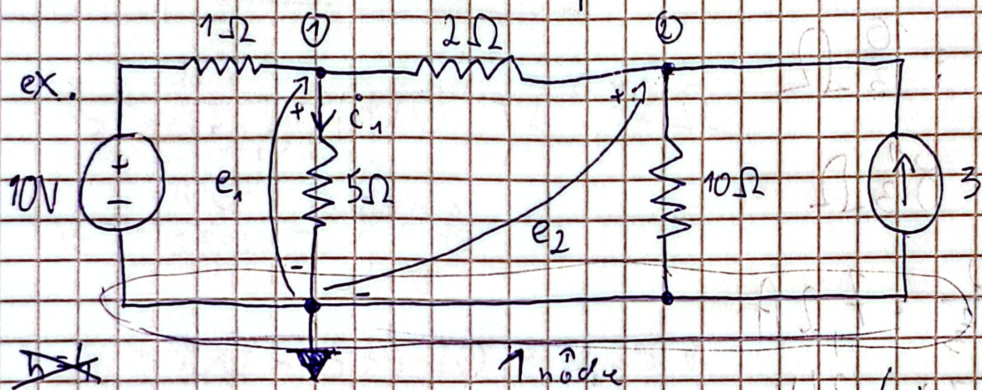
Node Voltage



1. Find essential nodes
2. Define one of those nodes as reference node
3. Write node voltage equations

~~WA~~ To complifly describe n essential node connection we need n − 1 equations

ex.



~~n=4~~

$n = 3 \Rightarrow 2$ equations

$= \frac{V}{R}$

①

$$\left\{ i = \frac{e_1 - 10}{R=1} = e_1 - 10 \right.$$

$$\begin{cases} ① \ e_1 - 10 + \frac{e_1 - 0}{5} + \frac{e_1 - e_2}{2} = 0 & / \cdot 10 \\ ② \ \frac{e_2 - e_1}{2} + \frac{e_2}{10} + (-3) = 0 & / \cdot 10 \end{cases}$$

$$\begin{cases} 10e_1 - 100 + 2e_1 + 5e_1 - 5e_2 = 0 \\ 5e_2 - 5e_1 + e_2 - 30 = 0 \end{cases}$$

$$\begin{cases} 17e_1 - 5e_2 - 100 = 0 \\ -5e_1 + 6e_2 - 30 = 0 \end{cases}$$

$$\begin{cases} e_1 = 9{,}74 \text{ V} \\ e_2 = 13{,}12 \text{ V} \end{cases}$$

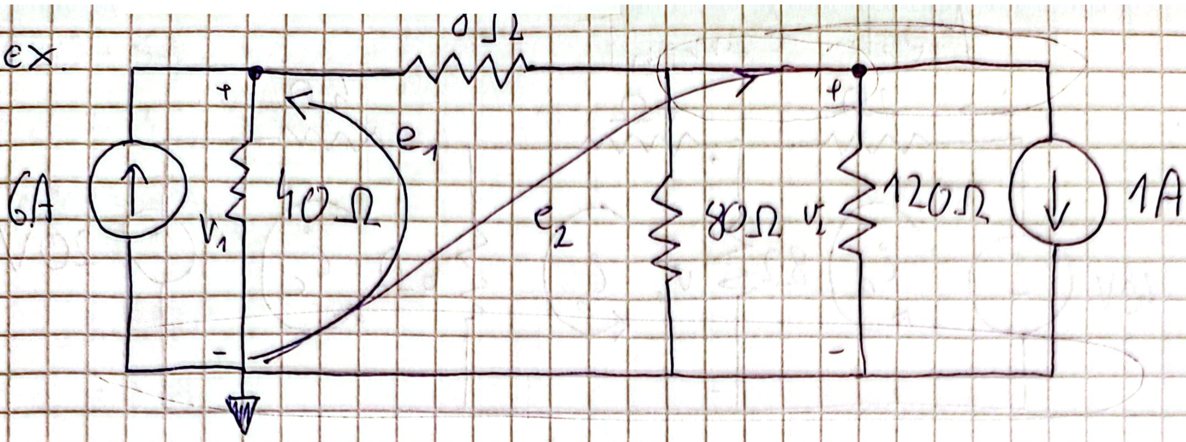
I always pretend that currents are going out of a node

np. i_1, i_2, i_3

Having e_1 and e_2 calculated we can calculate everything else easily

np. $i_1 = \frac{e_1 - 0}{R_{5\Omega}} = \frac{9{,}74}{5}$ A

ex.



$n = 3$

$$\begin{cases} \dfrac{e_1 - e_2}{8} + \dfrac{e_1}{40} - 6 = 0 & / \cdot 40 \\ 1 + \dfrac{e_2}{120} + \dfrac{e_2}{80} + \dfrac{e_2 - e_1}{8} = 0 & / \cdot 120 \end{cases}$$

$$\begin{cases} 5e_1 - 5e_2 + e_1 - 120 = 0 \\ 120 + e_2 + \frac{3}{2} e_2 + 15e_2 - 15e_1 = 0 \end{cases}$$

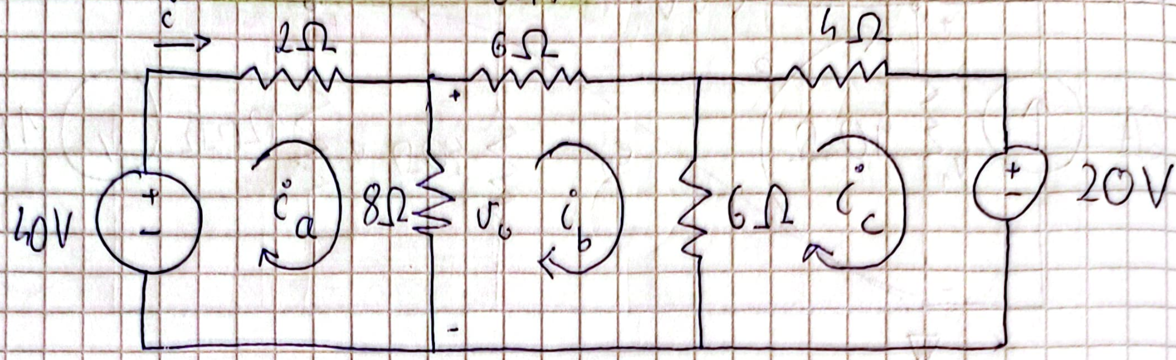
$$\begin{cases} 6e_1 - 5e_2 = 120 \\ 17{,}5e_2 - 15e_1 = -120 \end{cases}$$

$e_1 = 120$ V

$e_2 = 96$ V

Mesh Current

ex.



Find i and v_0 ...

Mesh A:

$$40 - 2i_a - 8(i_a - i_b) = 0$$

Mesh B:

$$-6i_b - 6(i_B - i_c) - 8\cancel{\Omega}(i_b - i_a) = 0$$

Mesh C:

$$+20 + 6(i_c - i_b) + 4i_c = 0$$

A: $40 - 2i_a - 8i_a + 8i_b = 0$

$40 = 10i_a - 8i_b \quad /:2$

$\underline{20 = 5i_a - 4i_b}$

B: $-6i_B - 6i_b + 6i_c - 8i_b + 8i_a = 0$

$8i_a + 6i_c - 20i_B = 0 \quad /:2$

$\underline{4i_a + 3i_c - 10i_b = 0}$

C: $+20 + 6i_c - 6i_b + 4i_c = 0$

$10i_c - 6i_b = -20 \quad /:2$

$\underline{5i_c - 3i_b = -10}$

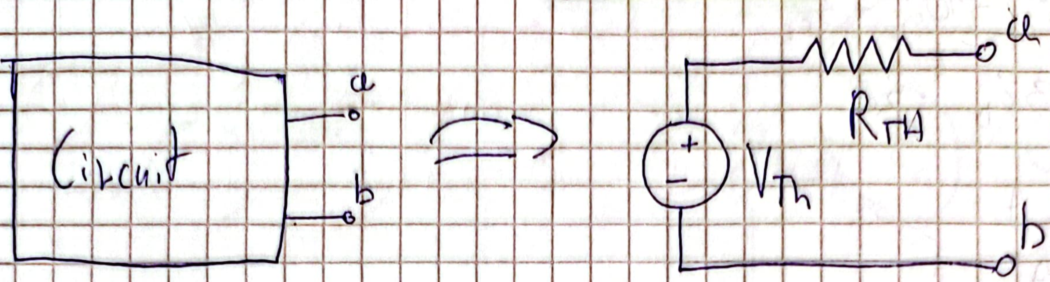
$$\begin{cases} 20 = 5i_a - 4i_b \\ 0 = 4i_a - 10i_b + 3i_c \\ -10 = -3i_b + 5i_c \end{cases}$$

$$\begin{cases} i_a = 5{,}6\ A \\ i_b = 2\ A \\ i_c = -0{,}8\ A \end{cases}$$

$i = 5{,}6\ A$

$v_o = (i_a - i_b)\,8 = 3{,}6 \cdot 8 = 28{,}8\ V$ //

Thevenim & Norton



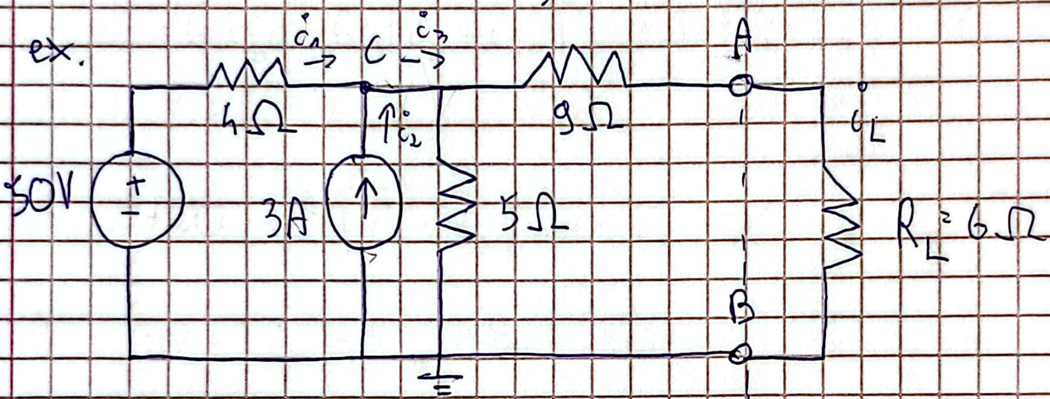
Thevenin Equivalent:

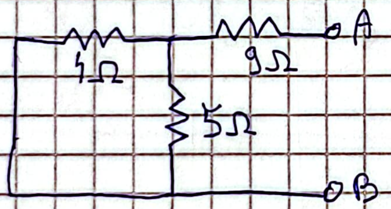
V_{TH}: Find open circuit voltage bet a -> b.

R_{TH}: Find the short circuit current bet a -> b.

Then $R_{TH} = \frac{V_{TH}}{i_{sc}}$

ex.





$$R_? = \frac{4 \cdot 5}{4+5} + 9 = \frac{20}{9} + 9 = 11\frac{2}{9}\,\Omega$$

$V_C = V_A = V_{Th}$

$i_1 + i_2 - i_3 = 0$

$$\begin{cases} i_1 = i_3 - 3 \\ i_1 = \frac{50 - V_c}{4} \\ i_3 = \frac{V_c}{5} \end{cases}$$

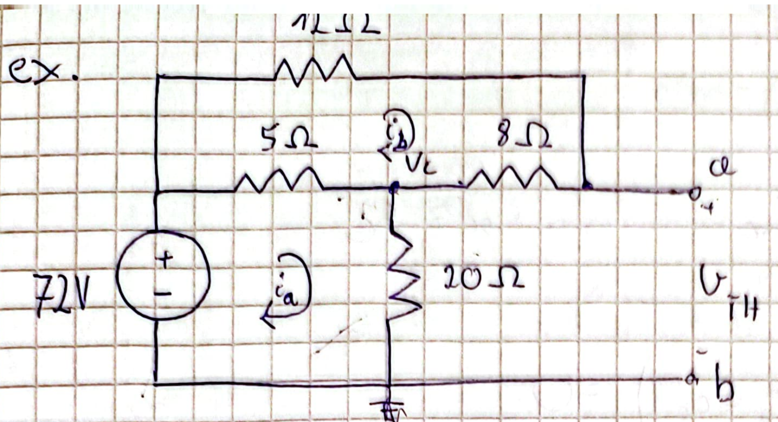
$\frac{50 - V_c}{4} = \frac{V_c}{5} - 3 \quad / \cdot 20$

$250 - 5V_c = 4V_c - 60$

$9V_c = 310$

$V_{Th} = V_c = \frac{310}{9}\,V$

ex.



~~$72 - 5 i_a$~~

$$\begin{cases} 72 - 5(i_a - i_b) - 20 i_a = 0 \\ -5(i_b - i_a) - 12 i_b - 8 i_b = 0 \end{cases}$$

$$\begin{cases} 72 - 25 i_a + 5 i_b = 0 \\ -25 i_b + 5 i_a = 0 \quad /:5 \end{cases}$$

$$+\begin{cases} 5 i_b - 25 i_a = -72 \\ -5 i_b + i_a = 0 \end{cases}$$

$-24 i_a = -72$

$i_a = \frac{72}{24} A = 3 A$

$i_b = \frac{1}{5} \cdot \frac{72}{24} = \frac{3}{5} A$

$72 - V_c = \left(3 - \frac{3}{5}\right) \cdot 5$

$V_c = 72 - 5 \cdot 2{,}4 = 60 V$

$V_c - V_a = 0{,}6 \cdot 8$

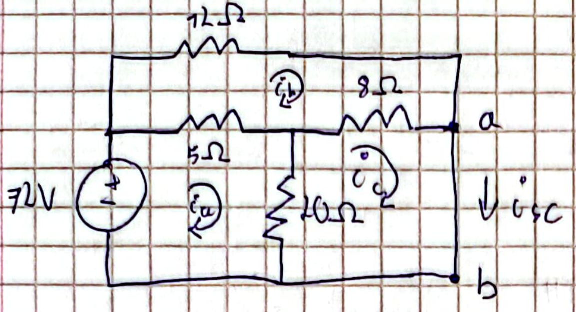
$V_a = 60 - 0{,}6 \cdot 8 = 55{,}2 V$

$V_{Th} = 55{,}2 V$

$V_{Th} = V_8 + V_{20}$

$V_{Th} = 0{,}6 \cdot 8 + 3 \cdot 20$

$V_{Th} = 64{,}8 V$

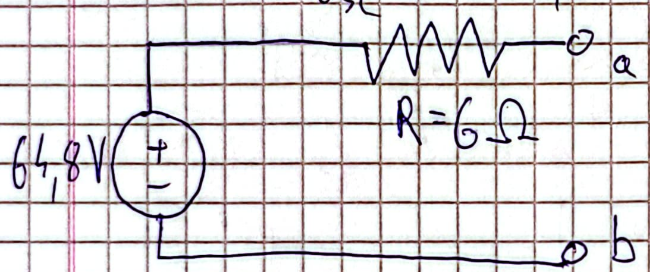


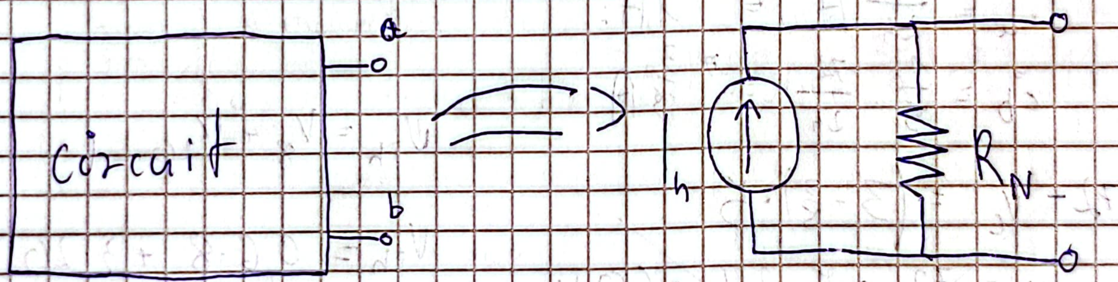
$i_{sc} = i_c$

$$\begin{cases} 72 - 5(i_a - i_b) - 20(i_a - i_c) = 0 \\ -5(i_b - i_a) - 12\, i_b - 8(i_b - i_c) = 0 \\ -20(i_c - i_a) - 8(i_c - i_b) = 0 \end{cases}$$

$i_{sc} = 10{,}8\,A$

$R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{64{,}8}{10{,}8} = 6\,\Omega$



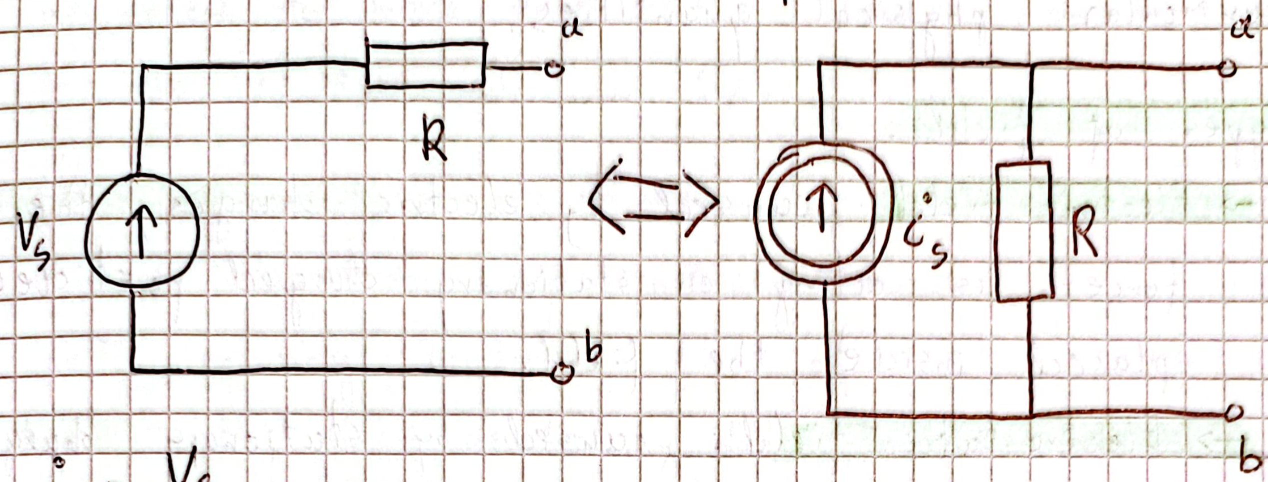


Norton equivalent:

$R_N = R_{TH}$

$I_N = \frac{V_{TH}}{R_{TH}}$

Source Transformations



$$i_s = \frac{V_s}{R}$$

Definitions

- **Structure of Atom:**
  - smallest particle of an element
  - nucleus consists of protons (+) and uncharged particles called neutrons
  - electrons (-) orbit nucleus (e = p => atom is neutral)
  - electron with the highest energy are on valence shell and are called valence electrons
  - if valence electron acquires enough external energy it leaves the atom and is called free electron; process - ionization
  - categories of materials (conductivity) depends on number of valence electrons.
- **Electric charge:** Feature of elementary particle which causes that the particles are subjects to electromagnetic operations.
- **The law of conservation of charge:** electric charge can neither be created or destroyed. In a closed system, the amount of charge remains the same.
- **Types of materials:**
  - insulators; glass, rubber, plastic, air - many valence electrons
  - conductors; copper, aluminium, gold, iron - low number of valence
  - semiconductors; silicon, germanium - moderate number of valence

- **Field** → state of the space defined by particular physical quantities.

- **Types of fields**:
  - → **Electric field**: caused by electric charges, the force is acting on stationary charged particles placed inside the field.
  - → **Electrostatic field**: caused by stationary charges
  - → **Magnetic field**: caused by moving electric charges, force is acting on moving charged particles inside field
  - *→ **magnetostatic field**: caused by DC current or lodestone

- **Maxwell equations** → $\vec{E}$ - electric field vector $[\frac{V}{m}]$, $\vec{B}$ - magnetic field vector $[T]$, ρ - charge density $[\frac{C}{m^3}]$, ε_0
  - → **Gauss's Law**: The flux of vector D crossing through any closed surface is equal to algebraic sum of all the charges enclosed by the surface.
  - → **Faraday's induction law**: A voltage along the closed curve c is equal to electromotive force induced by time changes of magnetic flux passing through the surface bounded by this curve.

- **Current** → An orderly movement of particles carrying electric charge
  → Scalar quantity beeing the ratio of elementary charge dq that passes through the plane cross-section surface in time dt to dt

  $$i(t) = \frac{dq}{dt}$$

  $$1A = \frac{1C}{1s}$$

  - flow of current is traditionally represented as a flow of positive charges

· **Voltage** → the voltage across an element is work requiered to move unit positive charge from the (-) terminal to the (+) terminal $1V = \frac{1J}{1C}$

◦ **Electric circuit** → interconection of electrical elements linked together by conductive wires in a closed path ensuring continous flow of electric current.

◦ **Node** → the place denoted by dot where elements endings are connected

◦ **Branch** → the part of circuit between nodes

◦ **Path** → sequence of branches which connect a sequence of nodes which are all distinct from one another

◦ **Loop** → path with common beginning node and end node

◦ **Graph** → graphical circuit mapping created by replacing all elements between nodes by segments

◦ **Oriented Graph** → graph containing additional information about reference directions of currents and voltages

◦ **Electric device** → the physical object that produces or uses electricity ex. capacitor, transistor, motor, alternator ...

◦ **KVL** → for any electric circuit, for any closed node sequence and any time, the algebraic sum of all node-to-node voltages around the chosen node is equal to 0

◦ **KCL** → for any electric circuit, any of its nodes, at any time, the algebraic sum of all the branch currents meeting at the node is 0.

- **Unique Kirchhoff's equations** → For any circuit having n nodes and b branches we formulate:
  - **$n-1$** independent KCL eq.
  - **$b-n+1$** independent KVL eq.
  - total number of unique linearly independent Kirchhoff's eq.

    $n-1+b-n+1 = \mathbf{b}$

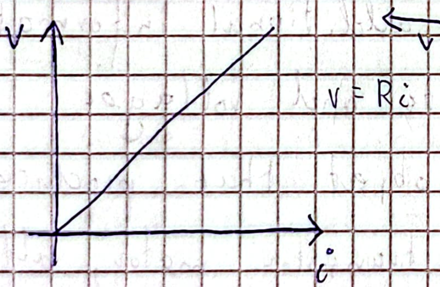
- **Model of circuit element** → description of those properties of a device we suppose are important for designer, constructor and user. Frequently model consists of an equation relating to the voltage and current

~~Oper~~

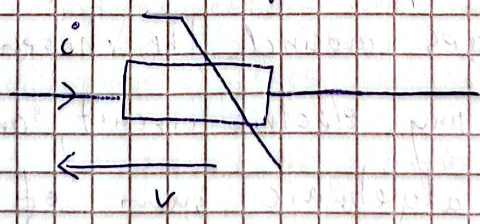
- **Resistance** → physical property of an element or device that impedes the flow of current.

- **Linear Resistor**

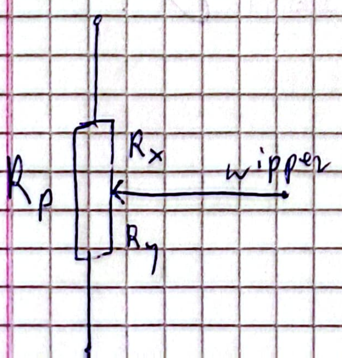
$v = Ri$



- **Nonlinear Resistor** → any resistor whose v-i characteristic is not a straight line through the origin is classif as nonlinear



- **Potentiometr**



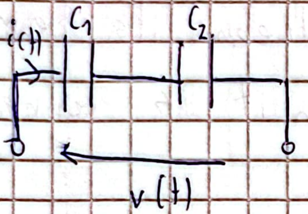
- 3 terminal resistor
- terminal 3 (wipper) can be shifted along the resistor, dividing R_p into R_x and R_y

- **Capacitor** → System of 2 metal electrodes (plates) isolated with dielectic
  → a two-terminal element which stores an electic charge

$C = \frac{q}{U}$

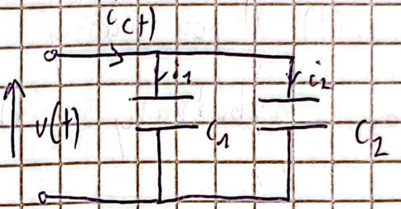
continuity property → voltage across any linear capacitor is a continuous function of time, it means that this voltage cannot jump instantaneously from one value to another

capacitors connected in series



$$\boxed{\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C}}$$

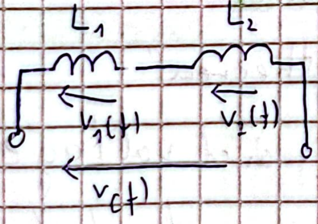
capacitors connected parallel



$$\boxed{C = C_1 + C_2}$$

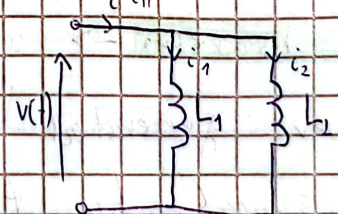
- **Inductor** → electrical component that stores energy in a magnetic field when current flows throught it

Series connection of Indutors



$$\boxed{L = L_1 + L_2}$$

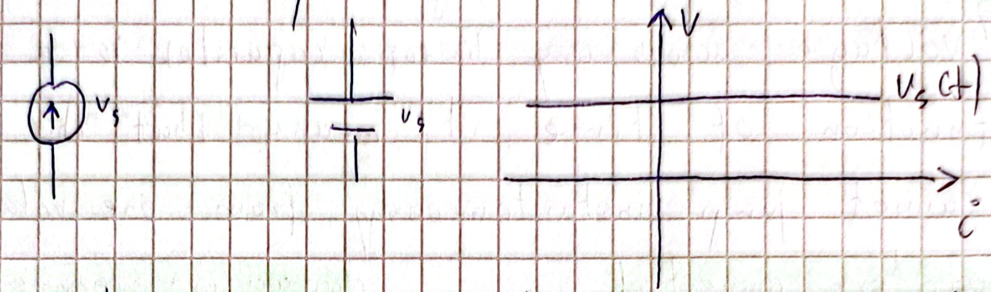
Parallel connection of inductors



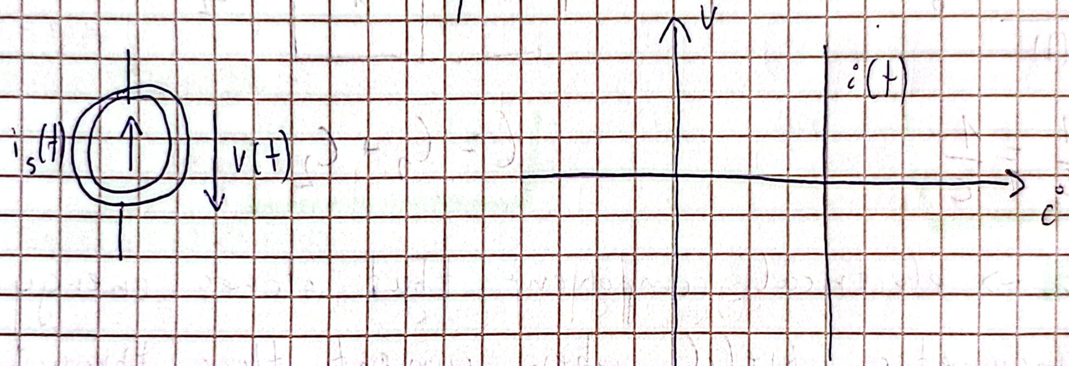
$$\boxed{\frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{L}}$$

- **Steady state** – when math description is constant in time.

- Ideal voltage source → maintains a prescribed voltage between its terminals:
  - for any current flowing through the source
  - in an arbitrary circuit to which it is connected

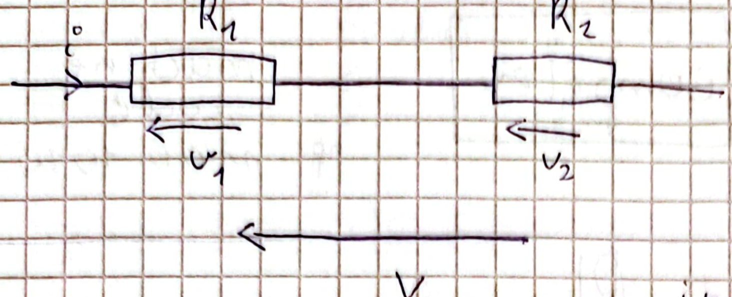


- Ideal current source → maintains a prescribed current $i_s(t)$ 1) for any voltage $v(t)$ between its terminals 2) in an arbitrary circuit to which it is connected



- Circuits P and Q are called equivalent if their mathematical description is the same
- Tellegen's theorem → aka „superposition theorem" total power generated by all current and voltage sources is equal to the total power absorbed by all the resistive loads. It is valid only for linear networks.
- Simple linear resistive circuits → circuits consisting of resistors and sources

→ Voltage divider



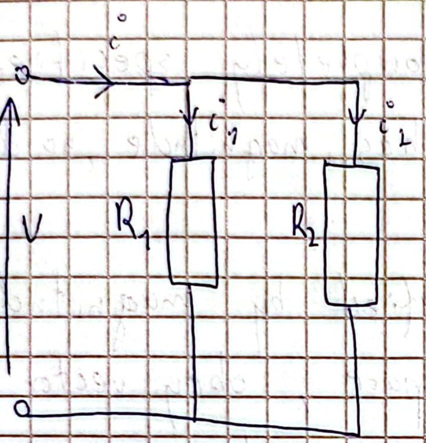
$v_1 = R_1 i$

$v_2 = R_2 i$

$\Rightarrow \boxed{\frac{v_1}{v_2} = \frac{R_1}{R_2}}$

Voltage v is divided in proportion to R_1 and R_2

→ Current divider



$i_1 = \frac{V}{R_1}$

$i_2 = \frac{V}{R_2}$

$\Rightarrow \boxed{\frac{i_1}{i_2} = \frac{R_2}{R_1}}$

Complex number basic

Cartesian form : $A = a + jb$

$a \to \mathrm{Re}(A)$, $b \to \mathrm{Im}(A)$

Polar form $A = |A| e^{j\alpha}$

$\boxed{|A| = \sqrt{a^2 + b^2}}$ $\boxed{\alpha = \arctan \frac{b}{a}}$

Trigonometric form $A = |A| (\cos\alpha + j \sin\alpha)$

Euler's formula $\boxed{e^{j\alpha} = \cos\alpha + j \sin\alpha}$