H 6.1

a) $$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}$$

$$\mathcal{L}^{Lept}_{Yukawa} = -(Y_e)_{ij} \ \bar{L}_i \Phi \ R_j \ -(Y_e)_{ij} \ \bar{R}_j \Phi \ L_i$$

$$= -(Y_e)_{ij} \ (\bar{\nu}_L \ \ \bar{e}_L)_i \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} (e_R)_j - \frac{(Y_e)_{ij}}{\sqrt{2}} (\bar{e}_R)_j \ (0 \quad v+h(x)) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i$$

$$= -\frac{1}{\sqrt{2}} (Y_e)_{ij} \ (\bar{e}_L)_i (v+h(x)) (e_R)_j - \frac{1}{\sqrt{2}} (Y_e)_{ij} (\bar{e}_R)_j \ (v+h(x)) (e_L)_i$$

b) $$\mathcal{L} \supset -\frac{v}{\sqrt{2}} (Y_e)_{ij} \ (\bar{e}_L)_i (e_R)_j$$

$$\Rightarrow (m_l)_{ij} = \frac{(Y_l)_{ij}}{\sqrt{2}} v$$

$$Y_l = U_e Y_e^{diag} K_l^\dagger$$

$$\rightarrow \quad (m_e)_{ij} = (U_e Y_l^{dig} K_e^\dagger)_{ij} \frac{v}{\sqrt{2}}$$

$$\Rightarrow \mathcal{L} \supset -\frac{v}{\sqrt{2}} \ \underbrace{\bar{e}_L \ U_e}_{\substack{= e_L^\dagger \gamma_0 U_e \\ = (U_e^\dagger \gamma_0 e_L)^\dagger \\ = \hat{\bar{e}}_L}} Y_l^{diag} \underbrace{K_e^\dagger \ e_R}_{= \hat{e}_R}$$

|| not in same space ?

$$= -\frac{v}{\sqrt{2}} \ \hat{\bar{e}}_L \ \ Y_l^{diag} \ \ e_R$$

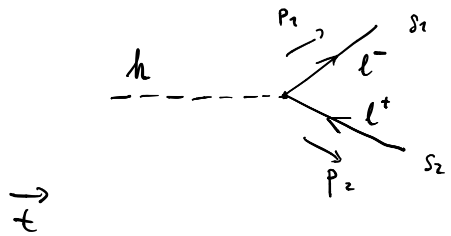
$$\Rightarrow \quad m_e = \frac{v}{\sqrt{2}} Y_l^{dig} = diag(m_e, m_\mu, m_\tau)$$

c) $$\mathcal{L}^{lept}_{Yukawa} = - \hat{\bar{e}}_L \ m_l (1+\tfrac{h}{v}) \hat{e}_R \ - \hat{\bar{e}}_R \ m_e (1+\tfrac{h}{v}) \ \hat{e}_L$$

$$= - \ (1+\tfrac{h}{v}) \begin{pmatrix} \hat{\bar{e}}_L \\ \hat{\bar{e}}_R \end{pmatrix} m_e \ (\hat{e}_R \ \ \hat{e}_L)$$

$$= -\left(1+\frac{h}{v}\right)\hat{\bar{e}}\cdot m_e \cdot \hat{e}$$

d) $$h \longrightarrow \ell^-(p_1, s_1) + \ell^+(p_2, s_2)$$



$$i\mathcal{M} = \bar{u}_\ell^{s_1}(p_1) \quad ig_\ell \quad v_\ell^{s_2}(p_2)$$

e) $$\overline{|\mathcal{M}|^2} = \sum_{s_1}\sum_{s_2} \bar{u}_\ell^{s_1}(p_1)\, g_\ell\, v_\ell^{s_2}(p_2)\, \bar{v}_\ell^{s_2}(p_2)\, g_\ell\, u_\ell^{s_1}(p_1) \quad , \quad \ell = e, \mu, \tau$$

$$= g_\ell^2 \sum_{s_1, s_2} \bar{u}_\ell^{s_1}(p_1)\, v_\ell^{s_2}(p_2)\, \bar{v}_\ell^{s_1}(p_2)\, u_\ell^{s_1}(p_1)$$

$$= g_\ell^2 \,(\not{p}_1 + m_\ell)(\not{p}_2 - m_\ell)$$

f) $$\overline{|\mathcal{M}|^2} = g_\ell^2 \; tr\,[(\not{p}_1 + m_\ell)(\not{p}_2 - m_\ell)]$$

$$= g_\ell^2 \Big(\underbrace{tr\,[\not{p}_1 \not{p}_2]}_{= 4\, p_1 \cdot p_2} + 4 m_\ell^2\Big)$$

$$= 4 g_\ell^2 \,(p_1 \cdot p_2 + m_\ell^2)$$

g) $$p_1 \cdot p_2 = E^2 - \vec{p}\cdot(-\vec{p}) = E^2 + |\vec{p}|^2 = 2|\vec{p}|^2 + m_\ell^2$$

$$\Rightarrow \overline{|\mathcal{M}|^2} = 8 g_\ell^2 \underbrace{(|\vec{p}|^2 + m_\ell^2)}_{E^2} = 2 g_\ell^2 m_h^2$$

$$\lambda = m_h^4 + \cancel{2 m_\ell^4} - 2 m_h^2 m_\ell^2 - 2 m_h^2 m_\ell^2 - \cancel{2 m_\ell^4}$$
$$= m_h^2 \; m_h^2 - 4 m_\ell^2$$

h) $$\Gamma = \frac{1}{16\pi m_h^3}\, \lambda(m_h^2, m_\ell^2, m_\ell^2)^{\frac{1}{2}} \cdot 2 g_\ell^2 m_h^2$$

$$= g_\ell^2 \frac{1}{8\pi m_h} \sqrt{\lambda} = \frac{1}{8\pi} \frac{m_\ell^2}{v^2} \sqrt{m_h^2 - 4 m_\ell^2}$$

g) e: $\Gamma = 2.14 \cdot 10^{-2}$ eV

μ: $\Gamma = 9.23 \cdot 10^{2}$ eV

τ: $\Gamma = 2.57 \cdot 10^{5}$ eV $\longrightarrow$ most likely to happen.

All come back to $g_e = \frac{m_e}{v}$.

More massive, stroger the coupling to H.

H6.2

a)

$$(P_R + P_L)^2 = \mathbb{1}$$

$$\Leftrightarrow \quad P_R + P_L + P_R P_L + P_L P_R = \mathbb{1}$$

$$\Rightarrow \quad \{P_R, P_L\} = 0$$

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\Psi$$

$$= (\bar{\Psi}_L + \bar{\Psi}_R)(i\not{\partial} - m)(\Psi_L + \Psi_R)$$

$$= \bar{\Psi}_L i\not{\partial}\Psi_L + \bar{\Psi}_R i\not{\partial}\Psi_R + \underline{\underline{\bar{\Psi}_L i\not{\partial}\Psi_R}} + \underline{\underline{\bar{\Psi}_R i\not{\partial}\Psi_L}}$$

$$- m(\underline{\bar{\Psi}_L\Psi_L} + \bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_R + \underline{\bar{\Psi}_R\Psi_L})$$

$$\left(\begin{array}{l} \bar{\Psi} P_R i\not{\partial} P_R \Psi = \bar{\Psi} i\not{\partial} P_L P_R \Psi = -\bar{\Psi} i\not{\partial} P_R P_L \Psi = -\bar{\Psi}_R i\not{\partial}\Psi_L \\ \bar{\Psi}_L \Psi_L = \bar{\Psi} P_R P_L \Psi = -\bar{\Psi}_R \Psi \end{array} \right.$$

$$= \bar{\Psi}_L i\not{\partial}\Psi_L + \bar{\Psi}_R i\not{\partial}\Psi_R - m(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_R)$$

b) $\Psi = \Psi_L + \Psi_R$

$$C\Psi = C(\Psi_L + \Psi_R) = (\Psi_R^c + \Psi_L^c) = \Psi^c$$

$$\mathcal{L} = i\bar{\Psi}_L \not{\partial}\Psi_R^c + i\bar{\Psi}_R \not{\partial}\Psi_L^c - m(\bar{\Psi}_L\Psi_L^c + \bar{\Psi}_R\Psi_R^c)$$

c) $\nu_R = P_R \Psi_\nu$

term: $\bar{L} \Phi \nu_R$ with $Y = +1$ ($\bar{L}$), $Y = 1$ (Φ), $Y = -2$ (ν_R) but $Q = T^3_L + Y \iff Y = 0$

$\Rightarrow$ contradicting

d) $y_D \bar{L} \tilde{\Phi} \nu_R$ has hypercharge zero $\Rightarrow U(1)_Y$ invariant

$\hookrightarrow Y(\tilde{\Phi}) = Y(i\tau_2 \Phi^*) = -1$

$$y_D \bar{L} \tilde{\Phi} \nu_R = y_D (\bar{e}_L \;\; \bar{\nu}_L) \underbrace{i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ (\phi^0)^* \end{pmatrix}}_{} \nu_R$$

$$= \begin{pmatrix} -i(\phi^0)^* \\ i\phi^- \end{pmatrix} \qquad \leftarrow \tilde{\Phi} = i\tau_2 \Phi = \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix}$$

$$= y_D (\bar{e}_L \;\; \bar{\nu}_L) \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix} \nu_R$$

$$= y_D \bar{e}_L (\phi^0)^* \nu_R - \underbrace{y_D \bar{\nu}_L \phi^- \nu_R}_{\text{(dirac) mass term}}$$

e) $m_\nu \leq \mathcal{O}(0.1 eV)$

$v \sim \mathcal{O}(100 GeV)$

$$y_D = \frac{\sqrt{2} m_\nu}{v} \leq \mathcal{O}\left(\frac{1.4 \cdot 0.1 eV}{100 GeV}\right) \sim \mathcal{O}(1 \cdot 10^{-12} eV)$$

f) $\frac{y_M}{M_N} (\bar{L} \tilde{\Phi})(\tilde{\Phi}^T L^c)$

$$\left(\begin{aligned} &Y(\bar{L}) + Y(\tilde{\Phi}) + Y(\tilde{\Phi}) + Y(L^c) \\ &= +1 - 1 - 1 + 1 = 0 \end{aligned} \right. \quad \rightarrow U(1)_Y \text{ invariant}$$

$$= \frac{y_M}{M_N} (\bar{\nu}_L \;\; \bar{e}_L) \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix} ((\phi^0)^* \;\; -\phi^-) \begin{pmatrix} \nu_L^c \\ e_L^c \end{pmatrix}$$

$$= \frac{y_M}{M_N} \left(\bar{\nu}_L (\phi^0)^* - \bar{e}_L \phi^- \right) \left((\phi^0)^* \nu_L^c - \phi^- e_L^c \right)$$

$$= \frac{y_M}{M_N} \Big[\underbrace{(\phi^0)^{*2} \, \bar{\nu}_L \nu_L^c}_{\text{majoramna mass}} - (\phi^0)^* \phi^- \, \bar{\nu}_L e_L^c - \phi^- (\phi^0)^* \, \bar{e}_L \nu_L^c + (\phi^-)^2 \, \bar{e}_L e_L^c \Big]$$

g) $[\, y_M / M_N \,] = -1$

$\Rightarrow$ non-renormalisable

h) $y_M \sim 1 \,, \quad m_\nu \leq \mathcal{O}(0.1\,\text{eV})$

$$v^2 \frac{y_M}{M_N} = m_\nu \quad \rightarrow \quad M_N = \frac{y_M}{m_\nu} v^2 \gtrsim \frac{(246G)^2}{0.1} \sim 6.1 \cdot 10^{23}\,\text{eV}$$

$610\ \text{ZeV}$?

i) Majorama mass term does not conserve lepton number, since it mixes ν and $\bar{\nu}$

$\rightarrow$ second also doesn't conserver N_L, more relevant.