MPRI 2.34.2: Quantum Information and Cryptography **2024 - 2025**

Exercises 4: Hamiltonian simulation

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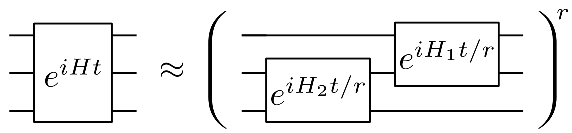
We saw how a circuit model computation can be encoded into an adiabatic computation. Here we describe how an adiabatic evolution (more precisely, evolution by a Hamiltonian) can be simulated in the circuit model.

We consider a simple Hamiltonian of the form $H = H_1 + H_2$. Assuming that we can implement $e^{iH_1 t}$ and $e^{iH_2 t}$ for various $t \in \mathbb{R}$, we wish to implement e^{iHt}. Unfortunately, if H_1 and H_2 do not commute, then we do not have that $e^{iHt} = e^{iH_1 t} e^{iH_2 t}$. However, by the Lie product formula, we do have that

$$e^{iHt} = \lim_{r\to\infty} \left(e^{iH_1 t/r} e^{iH_2 t/r}\right)^r .$$

For finite r, this corresponds to a quantum circuit with r gates of the form $e^{iH_j t/r}$:



Here we bound the error incurred for finite r.

- Assume Hermitian A_1 and A_2 with $\|A_1\|, \|A_2\| \leq 1$. Use the Taylor series $e^B = I + B + B^2/2 + O(\|B\|^3)$ for $\|B\| \leq 1$ to show that

$$e^{A_1 + A_2} = e^{A_1} e^{A_2} + E, \quad \|E\| \in O(\|A_1\| \cdot \|A_2\|). \tag{1}$$

- Assume $\|H_1\|, \|H_2\| \leq 1$. For $r \geq t$, argue that

$$e^{i(H_1+H_2)t} = \left(e^{iH_1 t/r} e^{iH_2 t/r}\right)^r + E_r, \quad \|E_r\| \in O(t^2/r).$$

Picking $r \in O(t^2/\epsilon)$, it follows that we can ϵ-approximate the Hamiltonian evolution e^{iHt} using $r \in O(t^2/\epsilon)$ calls to the simpler evolutions $e^{iH_1 t}$ and $e^{iH_2 t}$.