AI 619: AI for Medical Imaging and Signals

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3D X-ray Reconstruction

1. Introduction to 3D X-ray Reconstruction

Three-dimensional (3D) X-ray reconstruction is a critical method in medical imaging, allowing detailed visualization of internal structures from multiple projections. Two primary approaches to reconstruction are Fan Beam (2D) and Circular Cone Beam (3D) methods, each serving distinct purposes in medical imaging.

Fan Beam reconstruction is typically used for 2D cross-sectional imaging. In this setup, the X-ray source emits a fan-shaped beam across a plane, which is detected by an array of sensors positioned in an arc. This method is effective for generating high-resolution images of specific slices of an object. However, it cannot provide volumetric data, making it less suitable for applications where depth information is crucial.

Circular Cone Beam reconstruction extends Fan Beam methods into 3D by using a cone-shaped beam that captures data from multiple angles as the X-ray source and detectors rotate around the object. This allows for the collection of volumetric data, which is reconstructed into a 3D representation. Circular Cone Beam is widely used in dental imaging, cranial scans, and other applications that require detailed 3D views. The transition to 3D introduces additional complexity in data processing and image reconstruction, requiring advanced algorithms to manage the volume of data and address common imaging artifacts.

2. Artifacts in X-ray CT Imaging

Artifacts are distortions or inaccuracies that appear in CT images (see Figure 1), resulting from various issues during data acquisition or reconstruction. These anomalies do not represent actual structures and can significantly impact the diagnostic quality of images. Common types of artifacts include streaking, blurring, and missing frequency artifacts.

Missing frequency artifacts occur when certain spatial frequencies are not captured during scanning, leading to incomplete reconstructions. This results in areas of the image that may appear blurred or less defined, particularly around complex structures. Artifacts can also result from patient movement during scanning or from high-density materials like metal implants, which cause X-rays to scatter unpredictably.

Addressing artifacts is crucial for ensuring the accuracy of 3D reconstructions. Techniques such as differentiated back projection (DBP) and Fourier domain analysis help mitigate these issues by providing a deeper understanding of how data is transformed during reconstruction. By applying corrections based on these insights, it is possible to reduce the impact of artifacts and produce clearer, more accurate images.

3. Polar Coordinates and Fourier Transform

Transforming data between polar and Cartesian coordinates is essential in X-ray CT. During scanning, X-ray projections are often collected in a circular or polar format because of the

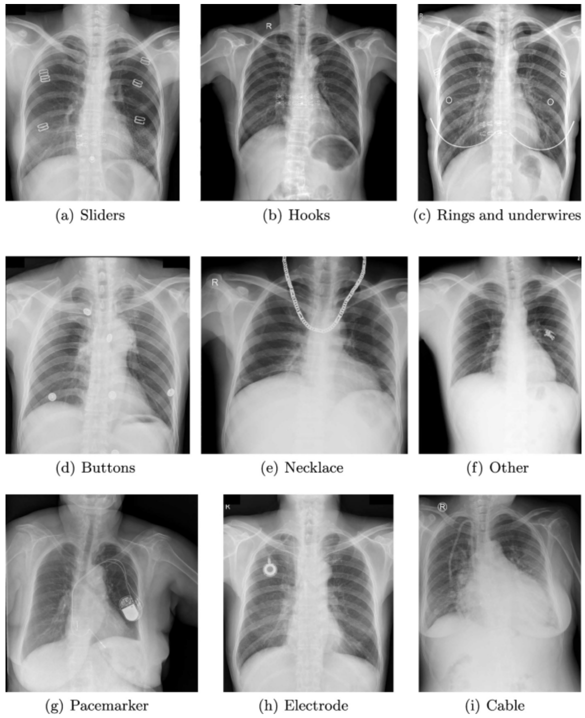

Figure 1: Examples of artifacts from X-rays.[1]

rotational movement of the source and detectors. However, the final image is typically represented in Cartesian coordinates, necessitating a transformation.

The Fourier transform plays a critical role in this process. By converting X-ray projection data into the frequency domain, it becomes easier to analyze and manipulate the data, especially for noise reduction and frequency filtering. The Fourier transform helps to identify the frequency components that make up the image, allowing for targeted processing to enhance image quality. After processing, the inverse Fourier transform is used to convert the frequency data back into spatial domain, resulting in a detailed image.

A key method in this transformation is the pixel-driven back projection algorithm. This algorithm maps each pixel in the final image back to corresponding positions in the detector space across multiple angles. The contributions from all angles are then summed, resulting in a complete reconstruction of the scanned object. The precision of this method is essential for accurate image reconstruction but can be computationally demanding due to the need for interpolation between angles.

4. Fan Beam Reconstruction Algorithm

The Fan Beam reconstruction algorithm is a fundamental method in X-ray Computed Tomography (CT) for reconstructing 2D cross-sectional images from fan-shaped X-ray projections. The algorithm converts data captured by a fan-shaped beam into an image that represents the scanned slice of the object. The process consists of two main steps: cosine weighting with ramp filtering, and distance-weighted backprojection.

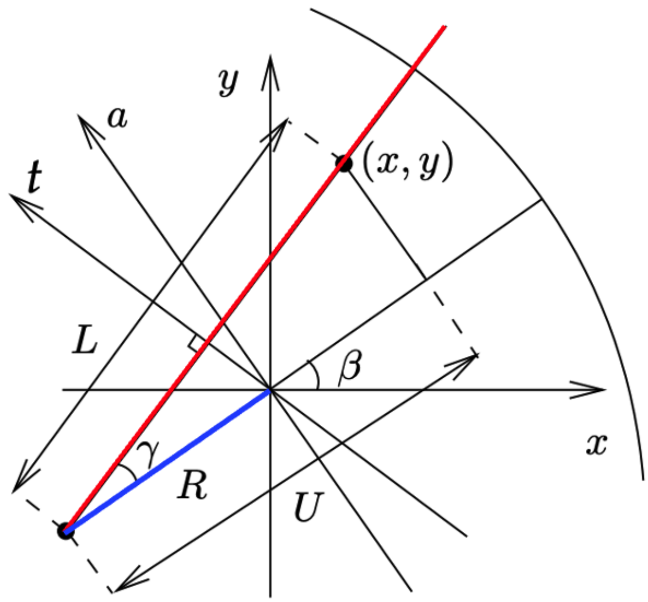

Figure 2: Fan-beam Geometry

Step 1: Cosine Weighting and Ramp Filtering

In the first step, the algorithm applies cosine weighting to the raw projections. Cosine weighting adjusts the contributions of the projections based on the angle of the X-ray source relative to each detector. This ensures that the projections are scaled properly, accounting for the varying distances at which the X-rays intersect with different parts of the scanned object. Cosine weighting helps in reducing distortions that may arise from differences in path lengths and angles of incidence across the fan beam.

Following cosine weighting, a ramp filter is applied to the projections. The ramp filter enhances high-frequency components in the data, which is crucial for accurately capturing sharp edges and fine details in the reconstructed image. Without ramp filtering, the image may appear blurred, losing important structural information. The ramp filter thus plays a key role in improving the spatial resolution of the final reconstruction.

$$\tilde{p}(\beta, \gamma) = (p(\beta, \gamma)\cos(\gamma)) * g_\infty(\gamma)$$

$$g_\infty(\gamma) = \left(\frac{\gamma}{\sin\gamma}\right)^2 g_\infty^P(\gamma)$$

Figure 3: Step 1 of the Fan Beam Reconstruction Algorithm: Cosine Weighting and Ramp Filtering applied to the X-ray projections.

Step 2: Distance-Weighted Backprojection

The second step involves distance-weighted backprojection, where the filtered projections from Step 1 are back-projected onto a 2D image grid. Distance weighting adjusts the contribution of each projection based on the source-to-detector distance, ensuring that each projection is properly aligned with the spatial geometry of the scanned object.

During backprojection, the X-ray data is redistributed across the 2D plane, mapping each projection onto the corresponding positions in the reconstructed image. Distance-weighted backprojection helps to prevent artifacts that can occur when the contributions of projections from different angles are not balanced, ensuring a uniform and accurate image. This process is essential for translating the data from the fan-shaped beam into a coherent image that accurately represents the internal structure of the object.

$$f_{\text{FBP}}(x, y) = \int_0^{2\pi} \frac{R^2}{L(x, y, \beta)^2} \tilde{p}(\beta, \gamma(x, y, \beta))\, d\beta$$

Figure 4: Step 2 of the Fan Beam Reconstruction Algorithm: Distance-Weighted Backprojection for reconstructing the 2D cross-sectional image.

Together, these two steps—cosine weighting with ramp filtering and distance-weighted backprojection—enable the Fan Beam reconstruction algorithm to produce detailed cross-sectional images. The accuracy of this method is critical for applications like CT scans, where clear and precise images of internal structures are required for effective diagnosis.

5. Three-Dimensional Reconstruction with the FDK Algorithm

The Feldkamp-Davis-Kress (FDK) algorithm is a method for extending fan beam reconstruction into the three-dimensional domain, particularly suited for cone beam geometries. The FDK algorithm consists of two main steps, each contributing to the accurate reconstruction of a 3D image from X-ray projections.

Step 1: Cosine Weighting and Ramp Filtering

The first step in the FDK algorithm involves applying cosine weighting to the X-ray projections, followed by a ramp filter. Cosine weighting corrects for the varying distances between the X-ray source and different regions of the object. This step ensures that the projections are uniformly scaled, accounting for the angle at which each X-ray beam intersects with the object.

After applying cosine weighting, a ramp filter is used to enhance the high-frequency components of the projections. This filter is essential for sharpening edges and details in the reconstructed image, ensuring that fine structures are accurately represented. The ramp filter compensates for the inherent blurring that occurs during the back projection process.

Step 2: Distance-Weighted Backprojection

The second step involves distance-weighted backprojection. In this step, the weighted projections from Step 1 are back-projected onto a 3D grid, taking into account the distance

$$\tilde{p}(\beta, \gamma, q) = \left(\cos\gamma \frac{R}{\sqrt{R^2 + q^2}} p(\beta, \gamma, q) \right) * g(a)$$

Figure 5: Step 1 of the FDK Algorithm: Cosine Weighting and Ramp Filtering applied to X-ray projections.

from the X-ray source to each voxel in the reconstructed volume. This distance weighting adjusts the contributions of each projection, ensuring that the 3D image is accurately constructed based on the varying paths that X-rays travel through the object.

Distance weighting is particularly important for maintaining the uniformity of the reconstructed image, as it prevents distortion that can occur when different regions of the object are further from the X-ray source. By adjusting the contributions of each voxel based on its relative distance, this step helps produce a final 3D reconstruction that is both accurate and consistent.

$$f_{\text{C-FDK}}(x, y, z) = \int_0^{2\pi} \frac{R^2}{L(x, y, \beta)^2} \tilde{p}(\beta, \gamma(x, y, \beta), q(x, y, z, \beta))\, d\beta$$

Figure 6: Step 2 of the FDK Algorithm: Distance-Weighted Backprojection for reconstructing the 3D image.

The combination of these two steps enables the FDK algorithm to produce high-resolution 3D images suitable for medical applications. The cosine weighting and ramp filtering refine the projections, while distance-weighted backprojection ensures accurate spatial reconstruction, making the FDK method an effective solution for complex imaging tasks in cone beam CT.

6. Circular Cone Beam Geometry

Circular Cone Beam geometry (see Figure 7) is a common setup in 3D imaging, where the X-ray source and detectors rotate around the object in a circular path. This motion captures projections from many angles, providing the data needed for a detailed 3D reconstruction.

The circular motion requires careful adjustment of cosine weighting to ensure that projections from different positions are balanced correctly. This adjustment is necessary because the angle between the X-ray source and each detector changes continuously during the scan, which can affect the intensity and quality of the detected signal. Distance corrections further refine the image by accounting for the varying distances that X-rays travel through different parts of the object.

This approach is particularly valuable for imaging small or complex anatomical regions, such as the jaw or cranial cavity, where a high degree of detail is required. By capturing a complete set of projections from multiple angles, Circular Cone Beam geometry provides a comprehensive view that is not achievable with simpler 2D methods.

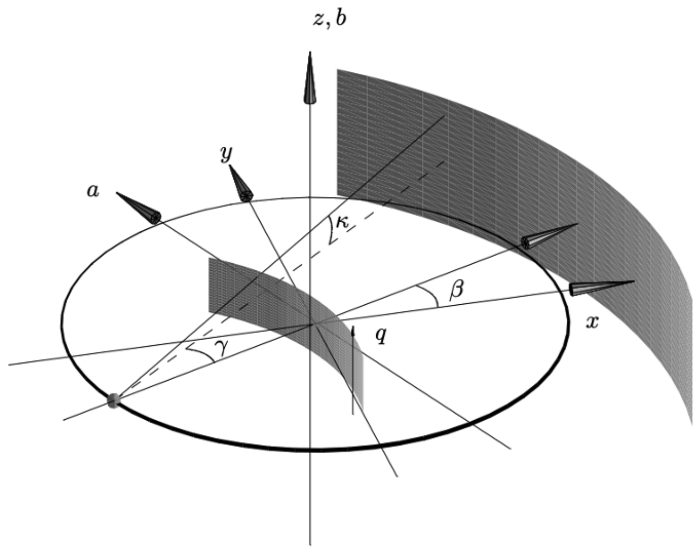

Figure 7: Circular Conebeam Geometry

7. General Reconstruction Formula and Differentiated Back Projection (DBP)

The general reconstruction formula for X-ray Computed Tomography (CT) is rooted in the Radon transform, which provides a framework for reconstructing images from X-ray projections. Differentiated Back Projection (DBP) refines this approach by incorporating differentiation into the reconstruction process. This method enhances the accuracy of the final image by focusing on the rate of change in projection data.

The key equation for DBP is as follows:

$$g(\mathbf{x}) = \int_{\lambda^-}^{\lambda^+} d\lambda \; \frac{1}{\|\mathbf{x} - \mathbf{a}(\lambda)\|} \; \frac{\partial}{\partial \mu} D_f(\mathbf{a}(\mu), \boldsymbol{\theta}) \bigg|_{\mu=\lambda}$$

Figure 8: The Differentiated Back Projection (DBP) equation. The differentiation enhances high-frequency details before backprojection onto the image grid.

In this equation:

- $\mathbf{x}$ represents the coordinates of a point in the reconstructed image.
- λ is a parameter along the path of the X-ray source, ranging from λ^- to λ^+, which defines the limits of integration.
- $\|\mathbf{x} - \mathbf{a}(\lambda)\|$ denotes the distance between the point $\mathbf{x}$ and the position of the X-ray source $\mathbf{a}(\lambda)$.
- $\frac{\partial}{\partial \mu} D_f(\mathbf{a}(\mu), \theta)|_{\mu=\lambda}$ represents the derivative of the projection data D_f with respect to μ, evaluated at $\mu = \lambda$.

The DBP process involves differentiating the projection data D_f along the path of the X-ray source before backprojecting it onto the image space. This differentiation step enhances the high-frequency details in the data, making the edges and fine structures more prominent in the reconstructed image. By focusing on changes in the projection data, DBP helps to better capture subtle variations that might be lost during standard backprojection methods.

After differentiating the data, the backprojection step integrates the adjusted projection data across the paths defined by the X-ray source movement. The term $\frac{1}{\|\mathbf{x}-\mathbf{a}(\lambda)\|}$ in the formula serves as a weighting factor, adjusting the contributions of each projection based on the distance from the X-ray source to the point $\mathbf{x}$ in the image. This ensures that the reconstructed image maintains spatial accuracy and reduces potential distortions caused by varying distances.

8. Fourier Transform and Managing Artifacts

Artifacts often stem from missing frequency components during data acquisition, which can result in incomplete reconstructions. The Fourier transform provides a way to analyze the frequency content of X-ray projections, making it possible to identify which frequencies are missing and how they affect the image.

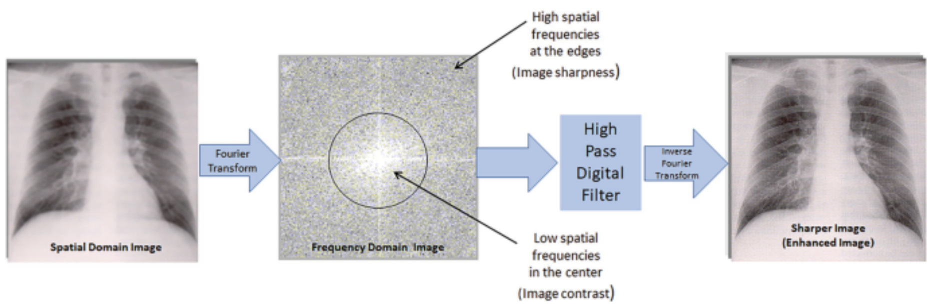

Figure 9: An example pipeline of using a fourier transform to remove X-ray artifacts [2].

By understanding these frequency gaps, adjustments can be made to the reconstruction process to reduce their impact. Techniques such as filtering and interpolation in the frequency domain can help fill in missing information, leading to clearer images. This process is essential for managing artifacts like blurring and ensuring that the reconstructed image accurately represents the internal structure of the scanned object.

References

[1] Alicja Moskal, Magdalena Jasionowska-Skop, Grzegorz Ostrek, and Artur Przelaskowski. Artifact detection on x-ray of lung with covid-19 symptoms. In Ewa Pietka, Pawel Badura, Jacek Kawa, and Wojciech Wieclawek, editors, *Information Technology in Biomedicine*, pages 234–245, Cham, 2022. Springer International Publishing.

[2] Euclid Seeram. *Computed Radiography Imaging: Physical Principles and System Components*, pages 31–45. Springer International Publishing, Cham, 2023.