Derivation of the Plan Check basic calculation

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1 Preamble

This document outlines the derivation for the basic calculation (on CRA, no wedges, no inhomogeneity correction) of the dose at a point using the recorded planning data. This basic calculation can be seen in Equation 1:

$$D(d, r, SSD_p) = MU \times \frac{PDD(d, r, 100)}{100} \times \left(\frac{100 + d}{SSD_p + d}\right)^2 \times S_p(r) \times S_c(r_{iso}) \quad (1)$$

Of note at RCCC the plan check data has both S_c and S_p PDD corrected back to d_{max}. Furthermore S_c is parameterised by the equivalent square field size area defined at isocentre and S_p is parameterised by equivalent square field size area defined at the surface of the patient where SSD_p is defined by the rayline passing through the isocentre.

The geometry used for this derivation is schematically shown in Figure 1. D_1 is the dose at a point (marked $\otimes$) at depth d in a phantom with surface defined by SSD_p, on the CRA, in a field of equivalent square area of r_{iso} defined at the isocentre. Throughout, the isocentre will be referenced by the SAD (source to gantry axis distance). In the diagram D_4 is the dose at a depth of d_{max} if the surface were at SSD_p, D_3 is the dose at a depth of d_{max} if the surface were at SAD, and D_2 is defined at the same distance as D_1 but the surface is shifted so that D_2 is at a depth of d_{max}.

The various field sizes given are defined as following: r is field size at SSD_p, r_{iso} is field size at SAD, $r_{d-d_{max}}$ is the field size at $SSD_p + d - d_{max}$, and r_d is the field size at $SSD_p + d$.

2 Initial relationships

The definitions of TMR and PDD in terms of D_1, D_2, and D_4 are given in Equations 2 and 3:

$$TMR(d, r_d) = \frac{D_1}{D_2} \quad (2)$$

$$PDD(d, r, SSD_p) = 100 \times \frac{D_1}{D_4} \quad (3)$$

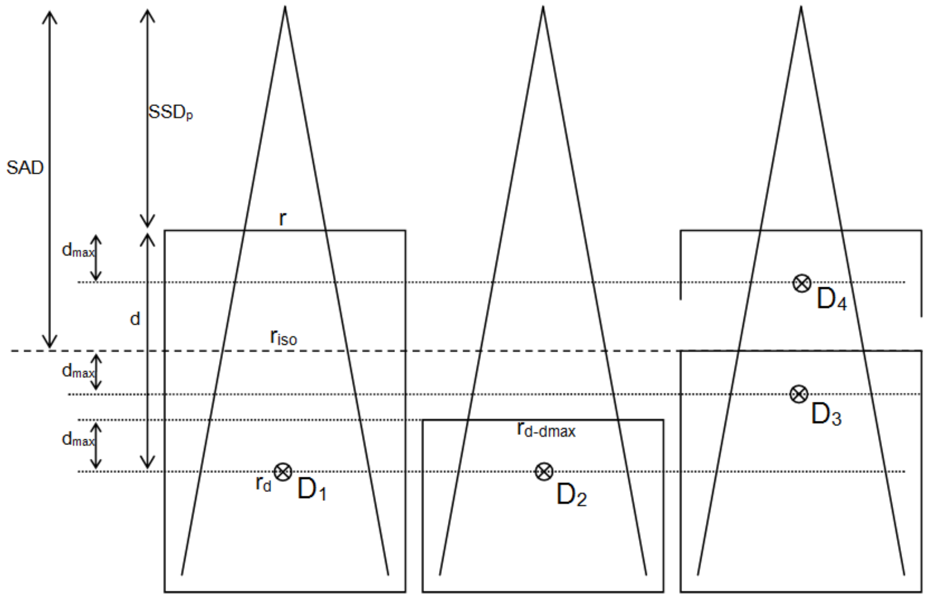

Figure 1: Diagram of the geometry used for the derivation

Assuming that the difference in S_p with SSD is negligible the following SSD corrections can be used to relate D_2 to D_3 (see Equation 4) and D_4 to D_3 (see Equation 5):

$$D_2 = D_3 \times \left(\frac{SAD + d_{max}}{SSD_p + d} \right)^2 \times \frac{S_p(r_{d-d_{max}})}{S_p(r_{iso})} \tag{4}$$

$$D_4 = D_3 \times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}} \right)^2 \times \frac{S_p(r)}{S_p(r_{iso})} \tag{5}$$

The calibration of MU is related to D_3 by Equation 6:

$$D_3 = MU \times (1.00 \ \ cGy/MU) \times S_c(r_{iso}) \times S_p(r_{iso}) \tag{6}$$

To correct PDD data from the available SAD reference to the required SSD_p the Meyneord factor from Kahn (edition 4, page 147, equation 9.10) given in Equation 7 is used.

$$\frac{PDD(d, r, SSD_p)}{PDD(d, r, SAD)} = \left(\frac{SSD_p + d_{max}}{SAD + d_{max}} \right)^2 \times \left(\frac{SAD + d}{SSD_p + d} \right)^2 \tag{7}$$

3 TMR to in terms of PDD

To make use of the PDDs measured at SAD for an arbitrary SSD set up need to convert PDDs into TMRs. The following steps are to produce this PDD to TMR conversion.

Subbing Equation 4 into Equation 2 gives TMR with respect to D_3 as so:

$$TMR(d, r_d) = \frac{D_1}{D_3} \times \left(\frac{SSD_p + d}{SAD + d_{max}} \right)^2 \times \frac{S_p(r_{iso})}{S_p(r_{d-d_{max}})} \tag{8}$$

and subbing Equation 5 into Equation 3 gives PDD with respect to D_3 as so:

$$PDD(d, r, SSD_p) = 100 \times \frac{D_1}{D_3} \times \left(\frac{SSD_p + d_{max}}{SAD + d_{max}} \right)^2 \times \frac{S_p(r_{iso})}{S_p(r)} \tag{9}$$

Rearranging Equation 9 gives:

$$\frac{D_1}{D_3} = \frac{PDD(d, r, SSD_p)}{100} \times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}} \right)^2 \times \frac{S_p(r)}{S_p(r_{iso})} \tag{10}$$

Subbing Equation 10 into the TMR Equation 8 gives:

$$\begin{aligned} TMR(d, r_d) &= \frac{PDD(d, r, SSD_p)}{100} \times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}} \right)^2 \times \frac{S_p(r)}{S_p(r_{iso})} \\ &\times \left(\frac{SSD_p + d}{SAD + d_{max}} \right)^2 \times \frac{S_p(r_{iso})}{S_p(r_{d-d_{max}})} \\ TMR(d, r_d) &= \frac{PDD(d, r, SSD_p)}{100} \times \left(\frac{SSD_p + d}{SSD_p + d_{max}} \right)^2 \times \frac{S_p(r)}{S_p(r_{d-d_{max}})} \qquad (11) \end{aligned}$$

4 Calculate the dose D_1

To calculate the dose D_1 and produce the plan check basic calculation formula given in Equation 1 the following steps are taken.

Rearrange the TMR definition in Equation 2 and subbing in to Equation 4 gives:

$$\begin{aligned} D_1 &= TMR(d, r_d) \times D_2 \\ D_1 &= TMR(d, r_d) \times D_3 \times \left(\frac{SAD + d_{max}}{SSD_p + d} \right)^2 \times \frac{S_p(r_{d-d_{max}})}{S_p(r_{iso})} \qquad (12) \end{aligned}$$

Then subbing the calibration equation (see Equation 6) into Equation 12 gives:

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times TMR(d, r_d) \times S_c(r_{iso}) \times S_p(r_{iso}) \\ &\times \left(\frac{SAD + d_{max}}{SSD_p + d} \right)^2 \times \frac{S_p(r_{d-d_{max}})}{S_p(r_{iso})} \end{aligned}$$

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times TMR(d, r_d) \times S_c(r_{iso}) \\ &\times S_p(r_{d-d_{max}}) \times \left(\frac{SAD + d_{max}}{SSD_p + d} \right)^2 \qquad (13) \end{aligned}$$

Then subbing the TMR to PDD conversion (see Equation 11) into Equation 13 gives:

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \\ &\times \frac{PDD(d, r, SSD_p)}{100} \times \left(\frac{SSD_p + d}{SSD_p + d_{max}}\right)^2 \times \frac{S_p(r)}{S_p(r_{d-d_{max}})} \\ &\times S_c(r_{iso}) \times S_p(r_{d-d_{max}}) \times \left(\frac{SAD + d_{max}}{SSD_p + d}\right)^2 \end{aligned}$$

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times \frac{PDD(d, r, SSD_p)}{100} \\ &\times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}}\right)^2 \times S_c(r_{iso}) \times S_p(r) \end{aligned} \tag{14}$$

And finally, so that PDD data collected at SAD can be used the Meyneord factor is applied from Equation 7. This gives the final basic calculation for D_1:

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times \frac{PDD(d, r, SSD_p)}{100} \\ &\times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}}\right)^2 \times S_c(r_{iso}) \times S_p(r) \\ &\times \frac{PDD(d, r, SAD)}{PDD(d, r, SSD_p)} \times \left(\frac{SSD_p + d_{max}}{SAD + d_{max}}\right)^2 \times \left(\frac{SAD + d}{SSD_p + d}\right)^2 \end{aligned}$$

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \\ &\times \frac{PDD(d, r, SAD)}{100} \times \left(\frac{SAD + d}{SSD_p + d}\right)^2 \times S_p(r) \times S_c(r_{iso}) \end{aligned}$$

And since D_1 is $D(d, r, SSD_p)$ and $SAD = 100$ cm this results in Equation 1 as required.

A Variation should data be collected at a different SSD

Should the data collected not be at SAD, but instead at some SSD_{data} then the applied Meyneord factor is different. Instead, since the PDD needs to be corrected to SSD_{data} Equation 7 becomes:

$$\frac{PDD(d,r,SSD_p)}{PDD(d,r,SSD_{data})} = \left(\frac{SSD_p + d_{max}}{SSD_{data} + d_{max}}\right)^2 \times \left(\frac{SSD_{data} + d}{SSD_p + d}\right)^2$$

And as a result the basic dose calculation equation becomes:

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times \frac{PDD(d,r,SSD_p)}{100} \\ &\times \left(\frac{SAD + d_{max}}{SSD_p + d_{max}}\right)^2 \times S_c(r_{iso}) \times S_p(r) \\ &\times \frac{PDD(d,r,SSD_{data})}{PDD(d,r,SSD_p)} \times \left(\frac{SSD_p + d_{max}}{SSD_{data} + d_{max}}\right)^2 \times \left(\frac{SSD_{data} + d}{SSD_p + d}\right)^2 \end{aligned}$$

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times \frac{PDD(d,r,SSD_{data})}{100} \\ &\times \left(\frac{SAD + d_{max}}{SSD_{data} + d_{max}}\right)^2 \times \left(\frac{SSD_{data} + d}{SSD_p + d}\right)^2 \\ &\times S_c(r_{iso}) \times S_p(r) \end{aligned}$$

B The inclusion of inhomogeneity correction

When provided with an effective depth this can be used to approximately correct for inhomogeneity. This is achieved by looking up the PDD based upon the effective depth and then correcting the for inverse square law discrepancy between the effective depth (d_{eff}) used to look up the PDD and the true depth. Therefore the dose calculation with inhomogeneity correction becomes:

$$\begin{aligned} D_1 &= MU \times (1.00\ cGy/MU) \times \frac{PDD(d_{eff},r,SSD_{data})}{100} \\ &\times \left(\frac{SSD_p + d_{eff}}{SSD_p + d}\right)^2 \times \left(\frac{SAD + d_{max}}{SSD_{data} + d_{max}}\right)^2 \times \left(\frac{SSD_{data} + d}{SSD_p + d}\right)^2 \\ &\times S_c(r_{iso}) \times S_p(r) \end{aligned}$$