MLM Nested Main Section B

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Question 0: read data and process missingness

```
dat <- read_csv("classroom.csv")
```

```
## Rows: 1190 Columns: 12
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (12): sex, minority, mathkind, mathgain, ses, yearstea, mathknow, housep...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
# construct new outcome math1st
dat <- dat %>%
    mutate(math1st = mathkind + mathgain)

# remove missing data
dat <- dat %>%
    filter(complete.cases(dat))
```

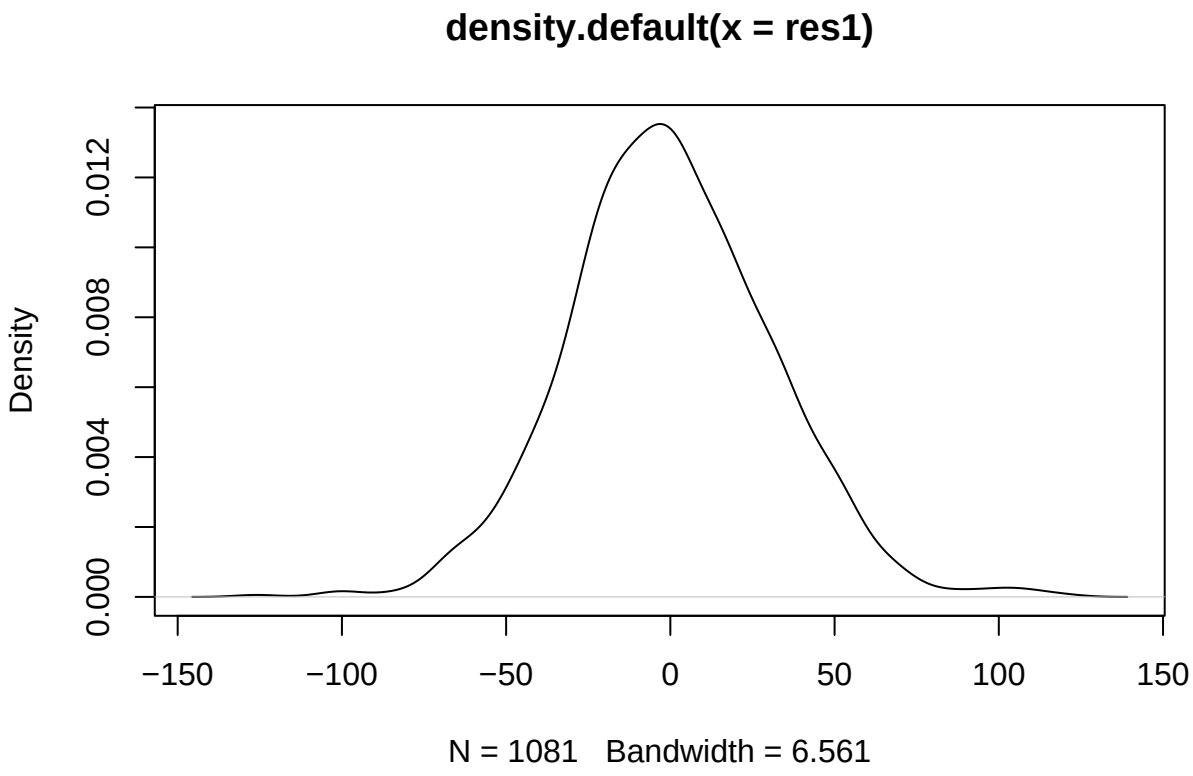
Question 1

```
# fit a model
fit1 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathprep + mathknow + ses +
    sex + minority + (1 | schoolid/classid), data = dat)
summary(fit1)
```

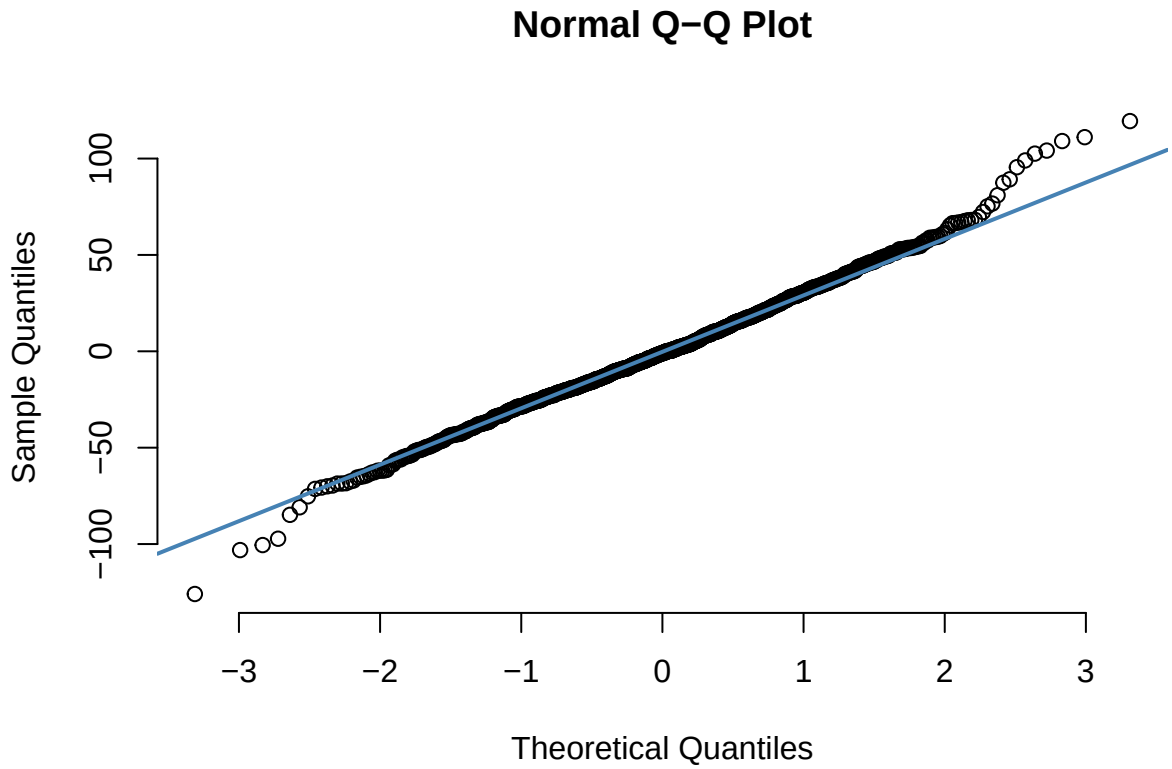
```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex +
##     minority + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
```

```
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthprp mthknw ses    sex
## housepov -0.451
## yearstea -0.259  0.071
## mathprep -0.631  0.038 -0.172
## mathknow -0.083  0.058  0.029  0.004
## ses      -0.121  0.082 -0.028  0.053 -0.007
## sex      -0.190 -0.007  0.016 -0.006  0.007  0.020
## minority -0.320 -0.178  0.024  0.001  0.115  0.162 -0.011
```

```r
# plot residuals to test normality assumption
res1 <- residuals(fit1)
# density plot
plot(density(res1))
```



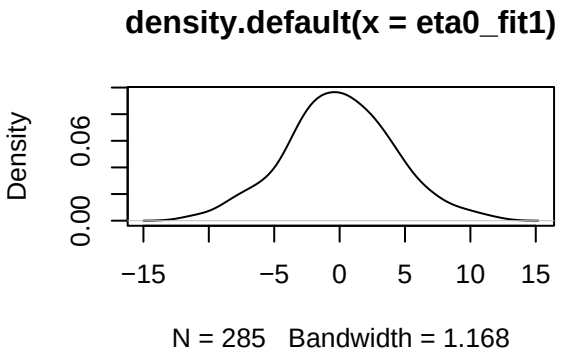
```r
# QQ plot
qqnorm(res1, pch = 1, frame = FALSE)
qqline(res1, col = "steelblue", lwd = 2)
```

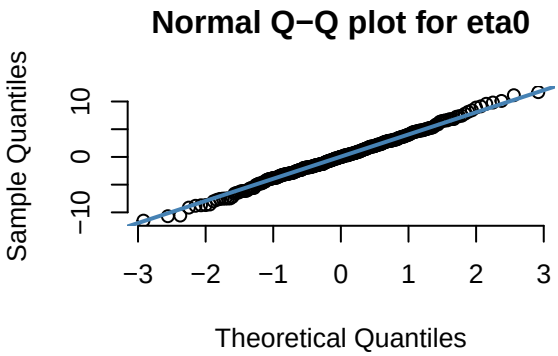


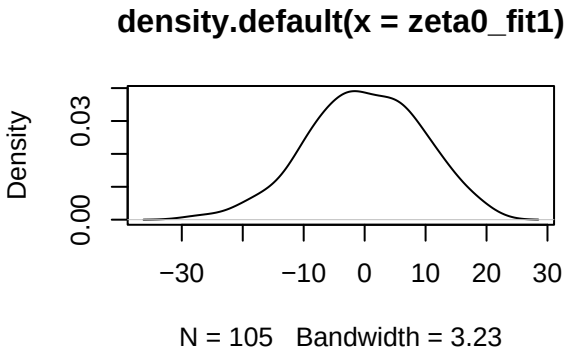
QQ plot shows that points are around the line, and thus we believe the normality assumption holds.

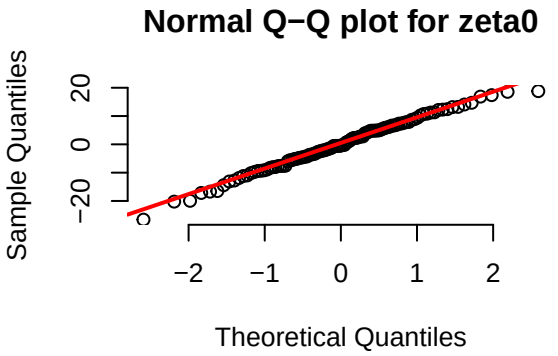
Question 2

```r
# Generate the two sets of BLUPs (for random effects zeta0 and eta0)
blups_fit1 <- ranef(fit1)

par(mfrow = c(2, 2))
# examine normality for eta0 (class-level)
eta0_fit1 <- blups_fit1$`classid:schoolid`$`(Intercept)`
# density plot
plot(density(eta0_fit1))
# QQ plot
qqnorm(eta0_fit1, pch = 1, frame = FALSE, main = "Normal Q-Q plot for eta0")
qqline(eta0_fit1, col = "steelblue", lwd = 2)

# examine normality for zeta0 (school-level)
zeta0_fit1 <- blups_fit1$schoolid$`(Intercept)`
# density plot
plot(density(zeta0_fit1))
# QQ plot
qqnorm(zeta0_fit1, pch = 1, frame = FALSE, main = "Normal Q-Q plot for zeta0")
qqline(zeta0_fit1, col = "red", lwd = 2)
```










```
par(mfrow = c(1, 1))
```

QQ plot shows that both sets of BLUPs of zeta0 and eta0 are around the line, and thus we believe the normality assumption holds.

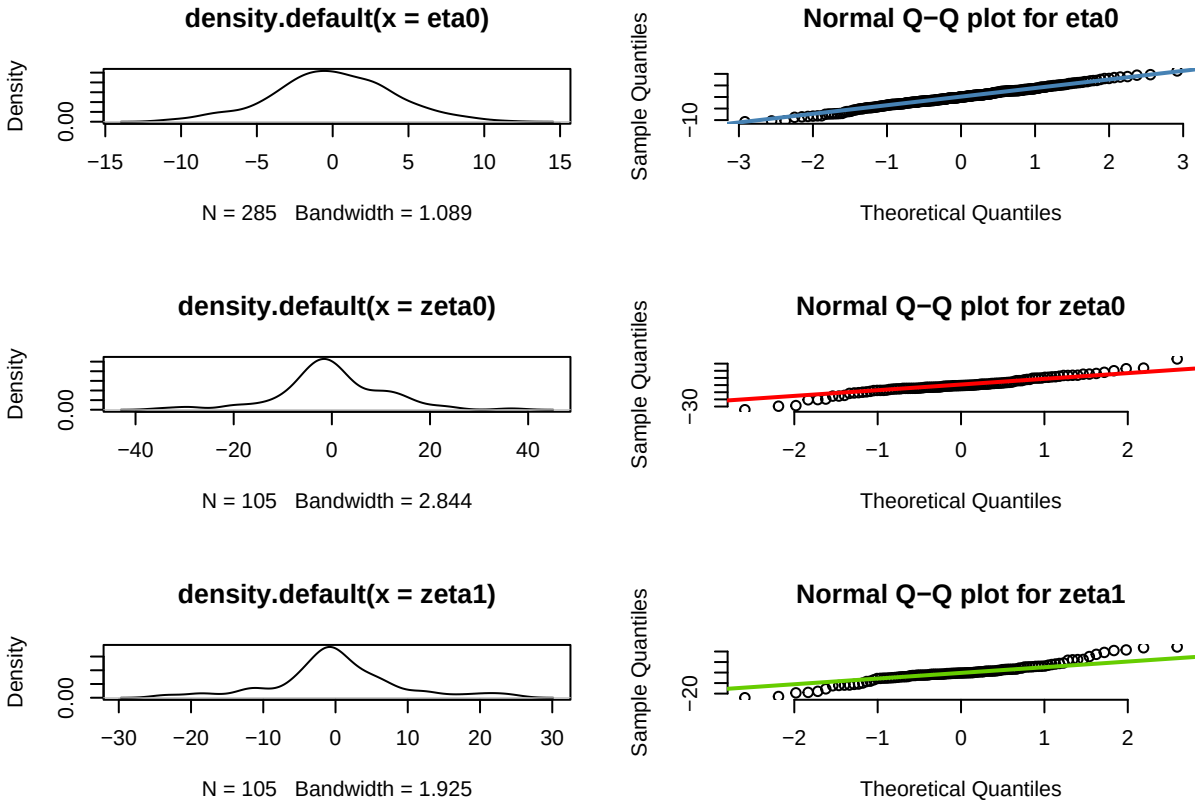
Question 3

```
# a add a random slope for minority, correlated with the random intercept, at
# the school level
fit2 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathprep + mathknow + ses +
    sex + minority + (minority | schoolid) + (1 | classid), data = dat)
print(summary(fit2))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex +
##     minority + (minority | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
```

```
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthprp mthknw ses    sex
## housepov -0.394
## yearstea -0.253  0.091
## mathprep -0.576  0.037 -0.167
## mathknow -0.078  0.061  0.024 -0.002
## ses      -0.105  0.089 -0.021  0.052 -0.005
## sex      -0.172 -0.013  0.014 -0.005  0.010  0.024
## minority -0.494 -0.157  0.027 -0.002  0.099  0.113 -0.014
```

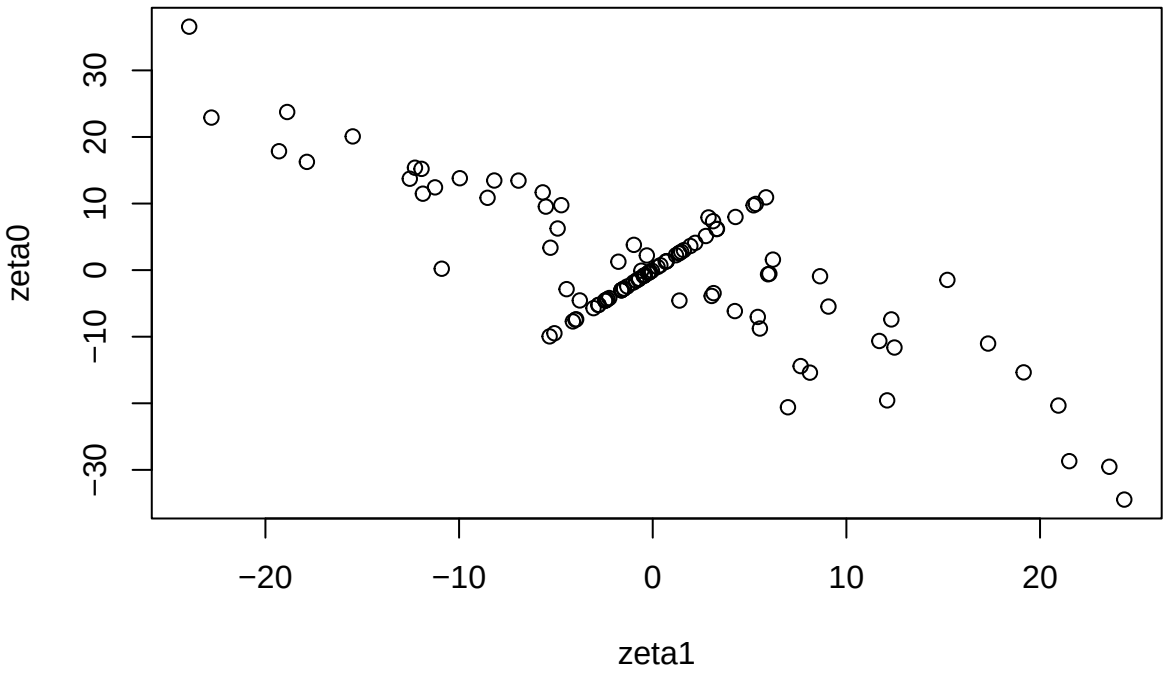
```r
# b residual
blups_fit2 <- ranef(fit2)
# BULPs
zeta0 <- blups_fit2$schoolid$`(Intercept)`
zeta1 <- blups_fit2$schoolid$minority
eta0 <- blups_fit2$classid$`(Intercept)`

# c check normality
par(mfrow = c(3, 2))
# density plot
plot(density(eta0))
# QQ plot examine normality for eta0 (class-level)
qqnorm(eta0, pch = 1, frame = FALSE, main = "Normal Q-Q plot for eta0")
qqline(eta0, col = "steelblue", lwd = 2)

# examine normality for zeta0 (school-level) density plot
plot(density(zeta0))
# QQ plot
qqnorm(zeta0, pch = 1, frame = FALSE, main = "Normal Q-Q plot for zeta0")
qqline(zeta0, col = "red", lwd = 2)

# examine normality for zeta1 (random slop) density plot
plot(density(zeta1))
# QQ plot
qqnorm(zeta1, pch = 1, frame = FALSE, main = "Normal Q-Q plot for zeta1")
qqline(zeta1, col = "chartreuse3", lwd = 2)
```




```
par(mfrow = c(1, 1))
```

QQ plot shows that BLUPs of eta0 are around the line, and thus we believe the normality assumption holds. However, BLUPs of zeta0, and zeta1 deviate from the line too much, and therefore we don't think the normality assumption holds.

```
# d
plot(zeta1, zeta0)
```



Overall, zeta0 and zeta1 are negative correlated. However, some odd points are positive correlated.

```
# e points from first quadrant
fq <- (3 - abs(blups_fit2$schoolid$minority) > 0) & (zeta1 >= 0 & zeta0 >= 0)
# points from third quadrant
sq <- (3 - abs(blups_fit2$schoolid$minority) > 0) & (zeta1 <= 0 & zeta0 <= 0)

# these schools are
unique(dat$schoolid)[fq | sq]
```

```
##  [1]   1   4   5   9  10  12  14  16  17  20  22  24  26  28  33  38  40  42  43
## [20]  45  46  47  52  53  57  61  69  73  78  79  80  84  86  89  90  96  98 100
## [39] 102 103 106
```

```
odd_point <- dat %>%
    filter(schoolid %in% unique(dat$schoolid)[3 - abs(blups_fit2$schoolid$minority) >
        0])
```

Almost all students in these school are minority.

Question 4

a

$V_S = 169.45$, $V_C = 93.89$, and $V_E = 1064.96$.

```
# fit a model
fit3 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathprep + mathknow + ses +
    sex + minority + (ses | schoolid) + (1 | classid), data = dat)
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex +
##     minority + (ses | schoolid) + (1 | classid)
##    Data: dat
```

```
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.57   9.305
##  schoolid (Intercept)  171.18  13.083
##           ses           73.36   8.565   0.19
##  Residual             1035.90  32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72222    5.27648  271.13305 102.099  < 2e-16 ***
## housepov     -15.89873   13.15396  111.71336  -1.209    0.229
## yearstea       0.03617    0.14002  220.42240   0.258    0.796
## mathprep      -0.21697    1.35642  197.10758  -0.160    0.873
## mathknow       1.26025    1.38201  230.89913   0.912    0.363
## ses            9.72646    1.82985   78.36212   5.315 9.75e-07 ***
## sex           -1.40436    2.08074 1011.40322  -0.675    0.500
## minority     -16.26698    3.03580  668.91588  -5.358 1.16e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthprp mthknw ses    sex
## housepov -0.449
## yearstea -0.259  0.073
## mathprep -0.627  0.039 -0.172
## mathknow -0.077  0.057  0.028  0.001
## ses      -0.062  0.070 -0.021  0.045  0.007
## sex      -0.188 -0.009  0.017 -0.008  0.005  0.018
## minority -0.325 -0.182  0.021  0.002  0.108  0.117 -0.011
```

c

$V_C = 86.57$, $V_{S(ses=0)} = 171.18$, and $V_E = 1035.90$.

d

$V_{S(ses=-0.50)} = 171.18 + 2 * (-0.5) * 13.083 * 8.565 * 0.19 + (-0.5)^2 * 73.36 = 168.23$

$V_{S(ses=0.50)} = 171.18 + 2 * (0.5) * 13.083 * 8.565 * 0.19 + (0.5)^2 * 73.36 = 210.81$

e

There is heteroscedasticity at school level (3) because $V_{S(ses=0.50)}$ and $V_{S(ses=-0.50)}$ are not approximate and V_S are depend on ses.