

Project A1+A2 - Model Selection and Notation

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5/7/2022

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A1 Chongjun Liao

0. We will use the classroom.csv data for this project.
 - a. math1st will be the outcome of interest for this first part
 - b. Recall that `math1st = mathkind + mathgain`
 - c. Read in the data (R: store as `dat`)
 - d. Fit all models using REML
 - e. It's best if you use `lmerTest::lmer` rather than `lme4::lmer` to call the MLM function. The former provides p-values for fixed effects in the summary.
 - f. There are 2 common error messages one can get from lmer calls: failed to converge (problem with hessian: negative eigenvalue; `max|grad| = ...`); and singularity. They may both be problematic in a real problem, but the latter suggests that a variance component is on the boundary of the parameter space.
1. In your discussion/writeup, consider the latter to be a “convergence problem” and ignore the former.

```
dat <- read.csv("~/Documents/GitHub/mlm_final_project/data/classroom.csv")
dat <- dat %>%
  mutate(math1st = mathkind + mathgain)
```

1. Estimate an Unconditional Means Model (UMM) with random intercepts for both schools and classrooms (nested in schools).

```
fit1 <- lmer( math1st ~ (1 | schoolid/classid), dat)
summary(fit1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
```

```
## Data: dat
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##   Groups             Name             Variance Std.Dev.
##   classid:schoolid (Intercept)      85.46   9.244
##   schoolid         (Intercept)    280.68  16.754
##   Residual                        1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)   522.540      2.037 104.407   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and the ICC for classrooms

Answer: The ICC for schools is $\frac{\sigma_{\zeta_0}^2}{\sigma_{\zeta_0}^2 + \sigma_{\eta_0}^2 + \sigma_{\varepsilon}^2} = 0.1855203$ and the ICC for classrooms is $\frac{\sigma_{\eta_0}^2}{\sigma_{\zeta_0}^2 + \sigma_{\eta_0}^2 + \sigma_{\varepsilon}^2} = 0.0564856$.

b. **WRITE OUT THIS MODEL** using your preferred notation, but use the same choice of notation for the remainder of your project

c. Be mindful and explicit about any assumptions made.

$MATH1ST_{ijk} = b_0 + \zeta_{0k} + \eta_{0jk} + \varepsilon_{ijk}$, with $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$, $\eta_{0jk} \sim N(0, \sigma_{\eta_0}^2)$ and $\varepsilon_{ijk} \sim N(0, \sigma_{\varepsilon}^2)$, independently of one another, j represents classrooms and k represents *schools*.

2. ADD ALL School level predictors

```
fit2 <- lmer( math1st ~ housepov + (1 | schoolid/classid), dat)
anova(fit1, fit2, refit = T)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: dat
## Models:
## fit1: math1st ~ (1 | schoolid/classid)
## fit2: math1st ~ housepov + (1 | schoolid/classid)
##      npar   AIC    BIC logLik deviance Chisq Df Pr(>Chisq)
## fit1     4 11956 11976 -5973.9   11948
## fit2     5 11948 11973 -5968.8   11938 10.125  1   0.001463 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
wald.test(b = fixef(fit2), Sigma = summary(fit2)$vcov, Terms = 2)
```

```
## Wald test:
```

```
## -----
##
## Chi-squared test:
## X2 = 10.3, df = 1, P(> X2) = 0.0013
```

a. Report if adding the predictors as a block is justified

Answer: There is only one school-level predictor which is `housepov`, its p-value is $0.0017029 < 0.05$, and I do a LRT on model with and without the school-level predictor, the p-value is $0.0014627 < 0.05$. So it is reasonable to add school-level predictor. I also do the wald-test, the p-value is also < 0.05 .

b. Report change in σ_{ζ}^2 .

The change in σ_{ζ}^2 is $280.6812733 - 250.9258585 = 29.7554148$.

3. ADD ALL Classroom level predictors

```
fit3 <- lmer( math1st ~ yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid),
             dat)
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## Data: dat
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)   94.36   9.714
## schoolid         (Intercept)  223.31  14.943
## Residual                        1136.43  33.711
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268 < 2e-16 ***
## yearstea    0.06193     0.14717 223.76570   0.421  0.67432
## mathknow     2.55143     1.44530 231.06560   1.765  0.07883 .
## mathprep    -0.75440     1.42809 203.20755  -0.528  0.59790
## housepov   -41.62117    14.08834 109.83230  -2.954  0.00383 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) yearst mthknw mthprp
## yearstea -0.264
## mathknow  -0.052  0.030
```

```
## mathprep -0.666 -0.175 0.004
## housepov -0.568 0.077 0.082 0.032
```

```
wald.test(b = fixef(fit3), Sigma = summary(fit3)$vcov, Terms = 2:4)
```

```
## Wald test:
## -----
##
## Chi-squared test:
## X2 = 3.5, df = 3, P(> X2) = 0.32
```

- a. Report if adding the predictors as a block is justified [must use WALD test, not LRT]

Answer: The Wald test generates a p-value = 0.32, which shows that we have no reason to add classroom-level predictors as a block. But it might be reasonable to include `mathknow` since it is significant according to the t-test.

- b. Report change in σ_η^2 and change in σ_ϵ^2 .

Answer: The change in σ_η^2 is $94.3625825 - 82.3601958 = 12.0023867$ and change in σ_ϵ^2 is $1136.4309806 - 1146.9548045 = -10.5238239$.

- c. Give a potential reason as to why σ_ϵ^2 is reduced, but not σ_η^2 ?

One potential reason is that there are only 3~4 sampled student in each classroom. Since the sample size with each classroom is small, the classroom predictors describe aggregate limited individual characteristics, which would explain student-level variation.

4. ADD (nearly) ALL student level predictors (but not `mathgain` or `mathkind`, as these are outcomes in this context).

```
fit4 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
             housepov + (1 | schoolid/classid), dat)
summary(fit4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid:schoolid (Intercept)   93.89   9.689
## schoolid         (Intercept)  169.45  13.017
## Residual                        1064.96  32.634
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
```

```
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585 < 2e-16 ***
## ses          10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex          -1.21419    2.09483 1022.42110  -0.580  0.562
## yearstea      0.01129    0.14141  226.80861   0.080  0.936
## mathknow      1.35004    1.39168  234.49768   0.970  0.333
## mathprep     -0.27705    1.37583  205.27111  -0.201  0.841
## housepov     -17.64850   13.21755  113.87814  -1.335  0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) ses    minrty sex    yearst mthknw mthprp
## ses        -0.121
## minority   -0.320  0.162
## sex        -0.190  0.020 -0.011
## yearstea   -0.259 -0.028  0.024  0.016
## mathknow   -0.083 -0.007  0.115  0.007  0.029
## mathprep   -0.631  0.053  0.001 -0.006 -0.172  0.004
## housepov   -0.451  0.082 -0.178 -0.007  0.071  0.058  0.038

wald.test(b = fixef(fit4), Sigma = summary(fit4)$vcov, Terms = 2:4)
```

```
## Wald test:
## -----
##
## Chi-squared test:
## X2 = 85.1, df = 3, P(> X2) = 0.0
```

- Report if justified statistically as a block of predictors [must use WALD test, not LRT]
Answer: The wald test gives a p-value less than 0.05, which justifies the significance of adding a block of individual predictors.
- Report change in variance components for all levels
Answer: The change in σ_η^2 is $93.8853485 - 94.3625825 = -0.477234$, increases; the change in σ_ζ^2 is $169.4480999 - 223.3059856 = -53.8578857$, decreases; and change in σ_ϵ^2 is $1064.9564422 - 1136.4309806 = -71.4745383$, decreases.
- Give a potential reason as to why the school level variance component drops from prior model
The aggregate effect of student predictors, can be seen as the school-level means and student deviation from the school mean. The school means would account for school-level variance, as a result the school-level variance component drops.
- WRITE OUT THIS MODEL** using your chosen notation (include assumptions).

$MATH1ST_{ijk} = b_0 + b_1SES_{ijk} + b_2MINORITY_{ijk} + b_3SEX_{ijk} + b_4YEARSTEA_{jk} + b_5MATHKNOW_{jk} + b_6MATHPREP_{jk} + b_7HOUSEPOV_k + \zeta_{0k} + \eta_{0jk} + \varepsilon_{ijk}$, with $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$, $\eta_{0jk} \sim N(0, \sigma_{\eta_0}^2)$ and $\varepsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$, independently of one another, j represents classrooms and k represents *schools*.

- Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately- not all together)

```
fit5.1 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
               housepov + (1 | schoolid/classid) + (0 + yearstea | schoolid),
               dat)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00805459 (tol = 0.002, component 1)
```

```
summary(fit5.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + yearstea | schoolid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid.schoolid (Intercept) 9.247e+01  9.6159
## schoolid         (Intercept) 1.684e+02 12.9758
## schoolid.1       yearstea     1.008e-02  0.1004
## Residual                1.065e+03 32.6361
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780 266.47954 101.662 < 2e-16 ***
## ses          10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## sex          -1.21060    2.09480 1022.21558  -0.578  0.563
## yearstea      0.01128    0.14192  122.87743   0.079  0.937
## mathknow      1.33106    1.39155  234.33195   0.957  0.340
## mathprep     -0.26584    1.37588  204.90504  -0.193  0.847
## housepov     -17.72082   13.21686  113.58577  -1.341  0.183
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.320  0.162
## sex      -0.191  0.020 -0.010
## yearstea -0.258 -0.027  0.023  0.015
## mathknow -0.082 -0.007  0.115  0.006  0.028
## mathprep -0.632  0.053  0.001 -0.006 -0.172  0.003
## housepov -0.450  0.082 -0.179 -0.007  0.070  0.057  0.037
## optimizer (nloptwrap) convergence code: 0 (OK)
## Model failed to converge with max|grad| = 0.00805459 (tol = 0.002, component 1)
```

```
fit5.2 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
               housepov + (1 | schoolid/classid) + (0 + mathknow | schoolid),
               dat)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(fit5.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + mathknow | schoolid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups             Name             Variance Std.Dev.
## classid.schoolid (Intercept) 9.389e+01  9.689654
## schoolid          (Intercept) 1.694e+02 13.017245
## schoolid.1         mathknow      2.323e-07  0.000482
## Residual                        1.065e+03 32.633630
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38873 101.585 < 2e-16 ***
## ses          10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## minority     -16.18678    3.02605  704.47917  -5.349 1.20e-07 ***
## sex          -1.21419    2.09483 1022.42143  -0.580  0.562
## yearstea      0.01129    0.14141  226.80898   0.080  0.936
## mathknow      1.35004    1.39169  234.49763   0.970  0.333
## mathprep     -0.27705    1.37583  205.27161  -0.201  0.841
## housepov     -17.64848   13.21759  113.87742  -1.335  0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.320  0.162
## sex      -0.190  0.020 -0.011
## yearstea -0.259 -0.028  0.024  0.016
## mathknow -0.083 -0.007  0.115  0.007  0.029
## mathprep -0.631  0.053  0.001 -0.006 -0.172  0.004
## housepov -0.451  0.082 -0.178 -0.007  0.071  0.058  0.038
## optimizer (nlptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```

```
fit5.3 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
               housepov + (1 | schoolid/classid) + (0 + mathprep | schoolid),
               dat)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(fit5.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + mathprep | schoolid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid.schoolid (Intercept) 9.388e+01 9.689e+00
## schoolid         (Intercept) 1.694e+02 1.302e+01
## schoolid.1       mathprep     2.171e-07 4.659e-04
## Residual                    1.065e+03 3.263e+01
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39223 101.586 < 2e-16 ***
## ses          10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## sex          -1.21419    2.09483 1022.42070  -0.580  0.562
## yearstea      0.01129    0.14141  226.80838   0.080  0.936
## mathknow      1.35003    1.39167  234.49786   0.970  0.333
## mathprep     -0.27705    1.37582  205.27063  -0.201  0.841
## housepov     -17.64851   13.21749  113.87941  -1.335  0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.320  0.162
## sex      -0.190  0.020 -0.011
## yearstea -0.259 -0.028  0.024  0.016
## mathknow -0.083 -0.007  0.115  0.007  0.029
## mathprep -0.631  0.053  0.001 -0.006 -0.172  0.004
## housepov -0.451  0.082 -0.178 -0.007  0.071  0.058  0.038
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```


- b. Report the model fit or lack of fit **Answer:** The model with random slope on `mathknow` and the model with random slope on `mathprep` have convergent problem. Besides, all these three random slopes have variation that is close to 0, which indicates that these models are poorly fitted.
- c. Retry the above, allowing the slopes to be correlated with the random intercepts (still one by one)

```
fit5.c.1 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
  housepov + (yearstea | schoolid) + (1 | schoolid:classid),
  dat)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00352934 (tol = 0.002, component 1)
```

```
summary(fit5.c.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##      housepov + (yearstea | schoolid) + (1 | schoolid:classid)
##      Data: dat
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7461 -0.6037 -0.0291  0.6041  3.8451
##
## Random effects:
##      Groups                Name                Variance Std.Dev. Corr
## schoolid:classid (Intercept)    37.8479    6.1521
## schoolid          (Intercept)  366.2230   19.1370
##                  yearstea        0.5527    0.7434   -0.78
## Residual                        1066.4855   32.6571
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.95171    5.48812  222.68165  98.203 < 2e-16 ***
## ses          10.15050    1.53873  1062.66116   6.597 6.62e-11 ***
## minority     -16.44545    2.99653   669.47204  -5.488 5.77e-08 ***
## sex          -1.33563    2.08775  1024.45847  -0.640  0.522
## yearstea      0.02205    0.15767   75.75723   0.140  0.889
## mathknow      1.04618    1.34371  209.64590   0.779  0.437
## mathprep      0.05077    1.34539  190.74479   0.038  0.970
## housepov     -17.14026   13.45947  119.64252  -1.273  0.205
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.119
## minority -0.305  0.168
## sex      -0.184  0.022 -0.012
```

```
## yearstea -0.370 -0.019 0.032 0.009
## mathknow -0.085 -0.001 0.122 0.008 0.012
## mathprep -0.606 0.049 -0.007 -0.004 -0.139 0.014
## housepov -0.455 0.079 -0.169 -0.004 0.084 0.049 0.050
## optimizer (nloptwrap) convergence code: 0 (OK)
## Model failed to converge with max|grad| = 0.00352934 (tol = 0.002, component 1)
```

```
fit5.c.2 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                 housepov + (mathknow | schoolid) + (1 | schoolid:classid),
                 dat)
summary(fit5.c.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (mathknow | schoolid) + (1 | schoolid:classid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
## Groups              Name                Variance Std.Dev. Corr
## schoolid:classid (Intercept) 9.393e+01  9.6915
## schoolid          (Intercept) 1.693e+02 13.0118
##                   mathknow      9.182e-04 0.0303 0.97
## Residual                    1.065e+03 32.6341
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.64041    5.31203  275.38950 101.588 < 2e-16 ***
## ses          10.04788    1.54488 1062.12269   6.504 1.20e-10 ***
## minority     -16.19378    3.02608  703.80365  -5.351 1.18e-07 ***
## sex          -1.21328    2.09485 1021.79810  -0.579 0.563
## yearstea      0.01114    0.14141  226.85275   0.079 0.937
## mathknow      1.35458    1.39201  214.62575   0.973 0.332
## mathprep     -0.27754    1.37599  201.27759  -0.202 0.840
## housepov     -17.64141   13.21242  103.98208  -1.335 0.185
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.320 0.162
## sex      -0.190 0.020 -0.011
## yearstea -0.259 -0.028 0.024 0.016
## mathknow -0.082 -0.007 0.115 0.007 0.029
## mathprep -0.631 0.053 0.001 -0.006 -0.173 0.004
## housepov -0.451 0.082 -0.178 -0.007 0.071 0.057 0.038
```

```
fit5.c.3 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
  housepov + (mathprep | schoolid) + (1 | schoolid:classid),
  dat)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(fit5.c.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##   housepov + (mathprep | schoolid) + (1 | schoolid:classid)
## Data: dat
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##   Min       1Q   Median       3Q      Max
## -3.8542 -0.6034 -0.0221  0.5914  3.6475
##
## Random effects:
##   Groups                Name                Variance Std.Dev. Corr
## schoolid:classid (Intercept)    78.46     8.858
## schoolid          (Intercept)  552.78    23.511
##                   mathprep      15.89     3.986   -1.00
## Residual                    1064.26    32.623
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.60853    5.60817  159.88504  96.040 < 2e-16 ***
## ses          10.14166    1.53961  1060.93429   6.587 7.04e-11 ***
## minority    -16.46420    2.99525   663.67458  -5.497 5.52e-08 ***
## sex         -1.16760    2.08697  1023.15165  -0.559  0.576
## yearstea     -0.02587    0.13949   223.50105  -0.185  0.853
## mathknow      1.29890    1.37194   229.68059   0.947  0.345
## mathprep      0.04076    1.34846   139.04922   0.030  0.976
## housepov     -14.01322   12.88712   116.05270  -1.087  0.279
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.275  0.161
## sex      -0.183  0.024 -0.013
## yearstea -0.260 -0.033  0.025  0.023
## mathknow -0.071 -0.001  0.107  0.002  0.049
## mathprep -0.692  0.061 -0.035 -0.008 -0.155  0.012
## housepov -0.461  0.095 -0.187  0.003  0.089  0.027  0.107
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```

Table 1: variation explained by classroom-level random intercept

	five_b	five_c
yearstea	92.466	37.848
mathknow	93.889	93.925
mathprep	93.882	78.462

Table 2: variation explained by school-level random intercept

	five_b	five_c
yearstea	168.372	366.223
mathknow	169.449	169.306
mathprep	169.446	552.775

Table 3: variation explained by school-level random slope

	five_b	five_c
yearstea	0.01	0.553
mathknow	0.00	0.001
mathprep	0.00	15.886

- d. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occurred (hint: what did you add to the model?).

Answer: For **mathknow**, the variation of school-level random slope increase while variation of school-level random intercept decrease. For **yearstea** and **mathprep**, both school-level random slope and school-level random intercept increase variation. Potential reason is that random slope on **mathknow** and random intercept are positively correlated, to explain same amount of school-level variation, the decrease in variation of school-level random intercept would be compensated by the positive covariance. Similarly for **yearstea** and **mathprep** the increase in variance of random slope and random intercept would be compensated by the negative covariance. 6. Question: a. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

Answer: Classroom-level variables does not vary within classroom, if there is no variation on variable, the slope could not be measured, so adding a random slope on classroom variable at classroom level makes no sense.

A2 Jeremy Lu

7. Question:

- a. For UMM, write down: V_S , V_C , V_E for the three variance components (simply the estimates)

Answer: We have that $V_S = 280.68$, $V_C = 85.46$, and $V_E = 1146.8$

- b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C , V_S , V_E ?

Answer: We have in this model that $V_S = 169.45$, $V_C = 93.89$, $V_E = 1064.96$

c. By what fraction did these each decrease with the new predictors in the model?

Answer: The fraction decrease for V_S, and V_E are 0.396, and 0.071, respectively. But for V_C it actually increased 0.099 fraction-wise.

8. a.

```
fit8.a.1 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                 housepov + (1 | schoolid/classid) + (0 + ses | schoolid),
                 dat)
summary(fit8.a.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + ses | schoolid)
## Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
## Groups              Name                Variance Std.Dev.
## classid.schoolid (Intercept)    88.56    9.411
## schoolid          (Intercept)  167.98   12.961
## schoolid.1        ses             72.50    8.515
## Residual                        1035.12   32.173
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.13754    5.27918  270.54292 102.125 < 2e-16 ***
## ses          9.78982     1.82217   79.01642  5.373 7.62e-07 ***
## minority    -16.52526    3.02189  700.06722 -5.469 6.32e-08 ***
## sex         -1.40185     2.08170 1011.28952 -0.673  0.501
## yearstea      0.03079    0.14052  223.94368  0.219  0.827
## mathknow      1.35576    1.38459  232.20020  0.979  0.329
## mathprep     -0.19801    1.35994  198.59489 -0.146  0.884
## housepov     -16.94561   13.21117  112.82498 -1.283  0.202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.091
## minority -0.323  0.124
## sex      -0.190  0.017 -0.010
## yearstea -0.260 -0.019  0.024  0.018
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628  0.042  0.001 -0.007 -0.172  0.002
## housepov -0.451  0.076 -0.180 -0.007  0.070  0.056  0.041
```

```
fit8.a.2 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                 housepov + (1 | schoolid/classid) + (0 + sex | schoolid),
                 dat)
summary(fit8.a.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + sex | schoolid)
## Data: dat
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid.schoolid (Intercept)   96.08   9.802
## schoolid         (Intercept)  161.63  12.713
## schoolid.1       sex           35.84   5.986
## Residual                    1054.36  32.471
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.54993 101.638 < 2e-16 ***
## ses          9.98477     1.54243 1058.27916   6.473 1.46e-10 ***
## minority    -16.16537     3.02861  704.25756  -5.338 1.27e-07 ***
## sex         -1.33535     2.18747  138.09087  -0.610  0.543
## yearstea     0.01448     0.14163  226.44539   0.102  0.919
## mathknow     1.40067     1.39464  234.45909   1.004  0.316
## mathprep    -0.27193     1.38011  205.78530  -0.197  0.844
## housepov    -16.77652    13.22879  112.39634  -1.268  0.207
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.120
## minority -0.320  0.161
## sex      -0.179  0.020 -0.015
## yearstea -0.259 -0.029  0.024  0.013
## mathknow -0.081 -0.007  0.114  0.007  0.028
## mathprep -0.633  0.052  0.001 -0.004 -0.172  0.004
## housepov -0.449  0.081 -0.178 -0.010  0.070  0.055  0.036
```

```
fit8.a.3 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                 housepov + (1 | schoolid/classid) + (0 + minority | schoolid),
                 dat)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(fit8.a.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | schoolid/classid) + (0 + minority | schoolid)
## Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid.schoolid (Intercept)   93.89   9.69
## schoolid         (Intercept)  169.45  13.02
## schoolid.1       minority       0.00   0.00
## Residual                    1064.96  32.63
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39107 101.585 < 2e-16 ***
## ses          10.05075    1.54484 1066.56217   6.506 1.18e-10 ***
## minority     -16.18677    3.02605  704.47765  -5.349 1.20e-07 ***
## sex          -1.21419    2.09483 1022.42106  -0.580  0.562
## yearstea      0.01129    0.14141  226.80889   0.080  0.936
## mathknow      1.35003    1.39168  234.49798   0.970  0.333
## mathprep     -0.27705    1.37583  205.27126  -0.201  0.841
## housepov     -17.64847   13.21752  113.87889  -1.335  0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.320  0.162
## sex      -0.190  0.020 -0.011
## yearstea -0.259 -0.028  0.024  0.016
## mathknow -0.083 -0.007  0.115  0.007  0.029
## mathprep -0.631  0.053  0.001 -0.006 -0.172  0.004
## housepov -0.451  0.082 -0.178 -0.007  0.071  0.058  0.038
## optimizer (nlptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```

b. Retry part (a), allowing the slopes to be correlated with the random intercepts.

```
fit8.b.1 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                  housepov + (1 | classid) + (ses | schoolid), dat)
summary(fit8.b.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##      housepov + (1 | classid) + (ses | schoolid)
## Data: dat
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 86.57 9.305
## schoolid (Intercept) 171.18 13.083
## ses 73.36 8.565 0.19
## Residual 1035.90 32.185
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 538.72222 5.27648 271.13305 102.099 < 2e-16 ***
## ses 9.72646 1.82985 78.36212 5.315 9.75e-07 ***
## minority -16.26698 3.03580 668.91588 -5.358 1.16e-07 ***
## sex -1.40436 2.08074 1011.40322 -0.675 0.500
## yearstea 0.03617 0.14002 220.42240 0.258 0.796
## mathknow 1.26025 1.38201 230.89913 0.912 0.363
## mathprep -0.21697 1.35642 197.10758 -0.160 0.873
## housepov -15.89873 13.15396 111.71336 -1.209 0.229
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) ses minrty sex yearst mthknw mthprp
## ses -0.062
## minority -0.325 0.117
## sex -0.188 0.018 -0.011
## yearstea -0.259 -0.021 0.021 0.017
## mathknow -0.077 0.007 0.108 0.005 0.028
## mathprep -0.627 0.045 0.002 -0.008 -0.172 0.001
## housepov -0.449 0.070 -0.182 -0.009 0.073 0.057 0.039

fit8.b.2 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                housepov + (1 | schoolid:classid) + (sex | schoolid), dat)
summary(fit8.b.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##      housepov + (1 | schoolid:classid) + (sex | schoolid)
## Data: dat
##
## REML criterion at convergence: 10727.6
```



```
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##   Groups             Name             Variance Std.Dev. Corr
## schoolid:classid (Intercept)    97.33   9.866
## schoolid          (Intercept)  206.34  14.364
##                  sex             84.08   9.169   -0.43
## Residual                    1041.76  32.276
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661 < 2e-16 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## minority     -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## sex          -1.340e+00  2.301e+00  8.742e+01  -0.582   0.562
## yearstea      6.877e-03  1.418e-01  2.277e+02   0.048   0.961
## mathknow      1.379e+00  1.396e+00  2.364e+02   0.988   0.324
## mathprep     -2.795e-01  1.378e+00  2.061e+02  -0.203   0.839
## housepov     -1.742e+01  1.326e+01  1.136e+02  -1.314   0.191
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.121
## minority -0.319  0.163
## sex      -0.222  0.018 -0.011
## yearstea -0.258 -0.028  0.024  0.014
## mathknow -0.082 -0.006  0.114  0.006  0.027
## mathprep -0.627  0.053  0.004 -0.005 -0.172  0.004
## housepov -0.449  0.083 -0.178 -0.003  0.072  0.060  0.038
```

```
fit8.b.3 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
                housepov + (1 | classid) + (minority | schoolid), dat)
summary(fit8.b.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##          housepov + (1 | classid) + (minority | schoolid)
## Data: dat
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##   Groups   Name             Variance Std.Dev. Corr
```

```
## classid (Intercept) 86.69 9.311
## schoolid (Intercept) 381.20 19.524
## minority 343.13 18.524 -0.83
## Residual 1039.39 32.240
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 5.395e+02 5.655e+00 1.731e+02 95.399 < 2e-16 ***
## ses 9.431e+00 1.543e+00 1.063e+03 6.111 1.39e-09 ***
## minority -1.638e+01 3.896e+00 5.824e+01 -4.203 9.17e-05 ***
## sex -8.628e-01 2.084e+00 1.022e+03 -0.414 0.679
## yearstea -4.368e-03 1.376e-01 2.172e+02 -0.032 0.975
## mathknow 1.632e+00 1.359e+00 2.248e+02 1.201 0.231
## mathprep -2.918e-01 1.335e+00 1.981e+02 -0.218 0.827
## housepov -1.606e+01 1.257e+01 9.999e+01 -1.277 0.204
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) ses minrty sex yearst mthknw mthprp
## ses -0.105
## minority -0.494 0.113
## sex -0.172 0.024 -0.014
## yearstea -0.253 -0.021 0.027 0.014
## mathknow -0.078 -0.005 0.099 0.010 0.024
## mathprep -0.576 0.052 -0.002 -0.005 -0.167 -0.002
## housepov -0.394 0.089 -0.157 -0.013 0.091 0.061 0.037
```

c. Report anything unusual about the variance components (changes that are unexpected)

Answer: Adding the correlation between school-level random slope on any of these student-level predictors, and the school-level random intercept, both the variations captured by the school-level random slope and the variation of random intercept increase substantially, especially for adding correlation between random slope on `minority` and random intercept.

9. a. Take the two predictors that had significant (at .05 level) random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on already including the other.

```
# check significance of random slope
anova(fit8.a.1, fit4, refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.a.1: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## npar AIC BIC logLik deviance Chisq Df Pr(>Chisq)
## fit4 11 10752 10806 -5364.8 10730
## fit8.a.1 12 10749 10809 -5362.4 10725 4.6972 1 0.03021 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(fit8.b.1,fit4,refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.b.1: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | classid)
##          npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit4          11 10752 10806 -5364.8    10730
## fit8.b.1     13 10750 10815 -5362.2    10724 5.1385  2    0.07659 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(fit8.a.2,fit4,refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.a.2: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
##          npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit4          11 10752 10806 -5364.8    10730
## fit8.a.2     12 10753 10813 -5364.4    10729 0.6137  1    0.4334
```

```
anova(fit8.b.2,fit4,refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.b.2: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
##          npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit4          11 10752 10806 -5364.8    10730
## fit8.b.2     13 10754 10818 -5363.8    10728 1.8631  2    0.394
```

```
anova(fit8.a.3,fit4,refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.a.3: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
##          npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit4          11 10752 10806 -5364.8    10730
## fit8.a.3     12 10754 10813 -5364.8    10730    0  1          1
```

```
anova(fit8.b.3,fit4,refit=F)
```

```
## Data: dat
## Models:
## fit4: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## fit8.b.3: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | classid)
##          npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit4          11 10752 10806 -5364.8    10730
## fit8.b.3     13 10744 10808 -5358.8    10718 11.967  2    0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# random slope of ses without correlation and random slope of minority with
# correlation are significant.
fit9 <- lmer( math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
             housepov + (1 | classid:schoolid) + (0 + ses | schoolid) +
             (minority | schoolid), dat)
anova(fit8.a.1, fit9, refit=F) #P = 0.00204
```

```
## Data: dat
## Models:
## fit8.a.1: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | schoolid)
## fit9: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | classid:schoolid)
##          npar    AIC    BIC  logLik deviance Chisq Df Pr(>Chisq)
## fit8.a.1   12 10749 10809 -5362.4    10725
## fit9       14 10740 10810 -5356.2    10712 12.39  2    0.00204 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(fit8.b.3, fit9, refit=F) #P = 0.02365
```

```
## Data: dat
## Models:
## fit8.b.3: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | classid:schoolid)
## fit9: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov + (1 | classid:schoolid)
##          npar    AIC    BIC  logLik deviance Chisq Df Pr(>Chisq)
## fit8.b.3   13 10744 10808 -5358.8    10718
## fit9       14 10740 10810 -5356.2    10712  5.12  1    0.02365 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b. Is the more complex model (with both random slopes in it) justified?

Answer: Yes, both random slopes are significant according to the LRT.

c. WRITE OUT THIS MODEL in your preferred notation (include assumptions)

$MATH1ST_{ijk} = b_0 + (b_1 + \zeta_{1k})SES_{ijk} + (b_2 + \zeta_{2k})MINORITY_{ijk} + b_3SEX_{ijk} + b_4YEARSTEA_{jk} + b_5MATHKNOW_{jk} + b_6MATHPREP_{jk} + b_7HOUSEPOV_k + \zeta_{0k} + \eta_{0jk} + \varepsilon_{ijk}$, with $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$, $\zeta_{1k} \sim N(0, \sigma_{\zeta_1}^2)$, $\zeta_{2k} \sim N(0, \sigma_{\zeta_2}^2)$, $\eta_{0jk} \sim N(0, \sigma_{\eta_0}^2)$ and $\varepsilon_{ijk} \sim N(0, \sigma_{\varepsilon}^2)$, $corr(\zeta_{0k}, \zeta_{1k}) = 0$, and $corr(\zeta_{0k}, \zeta_{2k}) = 0$, the other random components are independent of each others.

10. Now consider the model with a random slope *only* in minority. We will make predictions at levels of minority in the range 0 to 1 for illustrative purposes.

a. What are: V_C , V_S (minority=0), V_E ? i. We need to list 'minority=0' here, or we don't know how to use the slope variance. **Answer:** $V_C = 86.69$, $V_S = 381.20$, $V_E = 1039.39$

```
data.frame(VarCorr(fit8.b.3))
```

```
##          grp          var1          var2          vcov          sdcor
## 1  classid (Intercept)      <NA>      86.69412  9.3109677
## 2  schoolid (Intercept)      <NA>     381.20088 19.5243664
## 3  schoolid  minority      <NA>     343.12842 18.5237258
## 4  schoolid (Intercept) minority -299.26986 -0.8274803
## 5 Residual              <NA>      <NA>    1039.38897 32.2395560
```

b. What are: $V_S(\text{minority}=0.25)$, $V_S(\text{minority}=+0.50)$, $V_S(\text{minority}=+0.75)$?

```
# minority = 0.25
paste0("V_S(minority=0.25) =", data.frame(VarCorr(fit8.b.3))[2,4]+0.25^2 *
      data.frame(VarCorr(fit8.b.3))[3,4] + 0.25*2*
      data.frame(VarCorr(fit8.b.3))[4,5]*data.frame(VarCorr(fit8.b.3))[3,5]*
      data.frame(VarCorr(fit8.b.3))[2,5])
```

```
## [1] "V_S(minority=0.25) =253.011478742454"
```

```
# minority = 0.5
paste0("V_S(minority=0.5) =", data.frame(VarCorr(fit8.b.3))[2,4]+0.5^2 *
      data.frame(VarCorr(fit8.b.3))[3,4] + 0.5*2*
      data.frame(VarCorr(fit8.b.3))[4,5]*data.frame(VarCorr(fit8.b.3))[3,5]*
      data.frame(VarCorr(fit8.b.3))[2,5])
```

```
## [1] "V_S(minority=0.5) =167.713127497944"
```

```
# minority = 0.75
paste0("V_S(minority=0.75) =", data.frame(VarCorr(fit8.b.3))[2,4]+0.75^2 *
      data.frame(VarCorr(fit8.b.3))[3,4] + 0.75*2*
      data.frame(VarCorr(fit8.b.3))[4,5]*data.frame(VarCorr(fit8.b.3))[3,5]*
      data.frame(VarCorr(fit8.b.3))[2,5])
```

```
## [1] "V_S(minority=0.75) =125.305828492755"
```

Answer:

$$V_S(\text{minority}=0.25) = \sigma_{\zeta_{0k}}^2 + 2 \times 0.25 \times \rho_{\zeta_{0k}, \zeta_{2k}} + 0.25^2 \sigma_{\zeta_{2k}}^2 = 253.0114787,$$

$$V_S(\text{minority}=0.50) = \sigma_{\zeta_{0k}}^2 + 2 \times 0.5 \times \rho_{\zeta_{0k}, \zeta_{2k}} + 0.5^2 \sigma_{\zeta_{2k}}^2 = 167.7131275,$$

$$V_S(\text{minority}=0.75) = \sigma_{\zeta_{0k}}^2 + 2 \times 0.75 \times \rho_{\zeta_{0k}, \zeta_{2k}} + 0.75^2 \sigma_{\zeta_{2k}}^2 = 125.3058285$$

c. Is the variance between schools monotonically *increasing* in the value of minority?

Answer: No, it seems to be decreasing from minority 0 to 0.75 given the variance calculated.