Assignment II

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Q1

a) Result:

(13, 15, 16, 17, 19, 20, 20, 21, 22, 23, 25, 25, 26, 30, 31, 32, 35, 35, 35, 36)

(48, 50, 52, 55)

(70)

b) Result:

Use boxplot to calculute

c) Result:

Random Walk, Exponential Smoothing

Q2

a) 0 + (72000 - 12800) / (100000 - 12800) * (1.0 - 0) = 0.206

b) (72000 - 54000) / 16000 = 1.125

c) 72000 / 100000 = 0.72

Q3

```
print("Author: Simon Han YANG")
print("Student ID: 1930026144")
```

```
Author: Simon Han YANG
Student ID: 1930026144
```

```
import pandas as pd
import numpy as np
```

```
# 1. load csv
df = pd.read_csv('mnist.csv')
```

```
df
```

<style scoped> .dataframe tbody tr th:only-of-type { vertical-align: middle; }

```
.dataframe tbody tr th {
    vertical-align: top;
}

.dataframe thead th {
    text-align: right;
}
```

</style>

	label	1x1	1x2	1x3	1x4	1x5	1x6	1x7	1x8	1x9	...	2
0	5	0	0	0	0	0	0	0	0	0	...	0
1	0	0	0	0	0	0	0	0	0	0	...	0
2	4	0	0	0	0	0	0	0	0	0	...	0
3	1	0	0	0	0	0	0	0	0	0	...	0
4	9	0	0	0	0	0	0	0	0	0	...	0
...	...	...	...	...	...	...	...	...	...	...	...	...
59995	8	0	0	0	0	0	0	0	0	0	...	0
59996	3	0	0	0	0	0	0	0	0	0	...	0
59997	5	0	0	0	0	0	0	0	0	0	...	0
59998	6	0	0	0	0	0	0	0	0	0	...	0
59999	8	0	0	0	0	0	0	0	0	0	...	0

60000 rows × 785 columns

```
# 2. print the shape of the data
dfshape = df.shape
print(dfshape)
```

```
(60000, 785)
```

```
# extract first row data
firstrow = df.loc[0]

# 3.1 get first row pixel values
firstrowPixel = firstrow.drop(df.columns[[0]])
print(firstrowPixel)
```

```
1x1      0
1x2      0
1x3      0
1x4      0
1x5      0
        ..
28x24    0
28x25    0
28x26    0
28x27    0
28x28    0
Name: 0, Length: 784, dtype: int64
```

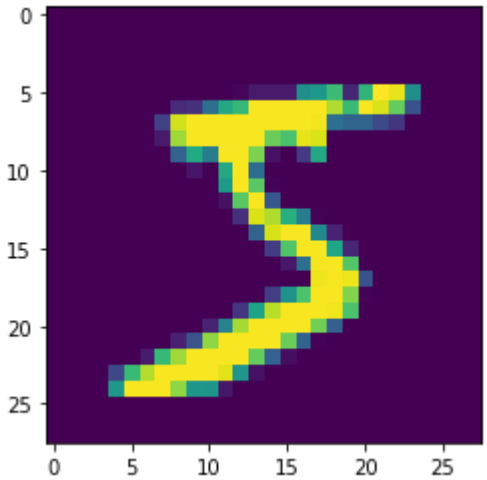
```
# 3.2 reshape the first row pixel values to (28, 28)
firstrowPixelReshape = firstrowPixel.values.reshape(28, 28)
print(firstrowPixelReshape.shape)
```

```
(28, 28)
```

```
# 4. plot the first sample
from matplotlib import pyplot as plt

plt.imshow(firstrowPixelReshape)
```

```
<matplotlib.image.AxesImage at 0x7fb1b2c13fd0>
```




```
# split the data into X(pixel values) and y(digit label)
X = df.drop(columns=['label'])
y = df['label']
```

```
# the pixel values
X
```

<style scoped> .dataframe tbody tr th:only-of-type { vertical-align: middle; }

```
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    vertical-align: top;
}

.dataframe thead th {
    text-align: right;
}
```

</style>

	1x1	1x2	1x3	1x4	1x5	1x6	1x7	1x8	1x9	1x10	...	2
0	0	0	0	0	0	0	0	0	0	0	...	0
1	0	0	0	0	0	0	0	0	0	0	...	0
2	0	0	0	0	0	0	0	0	0	0	...	0
3	0	0	0	0	0	0	0	0	0	0	...	0
4	0	0	0	0	0	0	0	0	0	0	...	0
...	...	...	...	...	...	...	...	...	...	...	...	.
59995	0	0	0	0	0	0	0	0	0	0	...	0
59996	0	0	0	0	0	0	0	0	0	0	...	0
59997	0	0	0	0	0	0	0	0	0	0	...	0
59998	0	0	0	0	0	0	0	0	0	0	...	0
59999	0	0	0	0	0	0	0	0	0	0	...	0

60000 rows × 784 columns

```
# the digit label
y
```

```
0        5
1        0
2        4
3        1
4        9
        ..
59995    8
59996    3
59997    5
59998    6
59999    8
Name: label, Length: 60000, dtype: int64
```

```
# Standardize X by dividing 255.0
X_stand = X / 255
```

```
X_stand
```

<style scoped> .dataframe tbody tr th:only-of-type { vertical-align: middle; }

```
.dataframe tbody tr th {
    vertical-align: top;
}

.dataframe thead th {
    text-align: right;
}
```

</style>

	1x1	1x2	1x3	1x4	1x5	1x6	1x7	1x8	1x9	1x10	...	2
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
...	...	...	...	...	...	...	...	...	...	...	...	.
59995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
59996	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
59997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
59998	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0
59999	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0

60000 rows × 784 columns

```
# 7. Apply PCA to X
X_mean = np.mean(X_stand, axis=0)
X_new = X_stand - X_mean

eigvalue, eigvector = np.linalg.eig(np.cov(X_stand.T))
```

```
eigvalue
```

```
array([ 5.11687301e+00,  3.74139084e+00,  3.25270845e+00,  2.84162070e+00,
        2.56711774e+00,  2.27366339e+00,  1.72515498e+00,  1.52056024e+00,
        1.45630525e+00,  1.24275009e+00,  1.11208951e+00,  1.06664054e+00,
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```
eigvector
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       [0., 0., 0., ..., 0., 0., 0.],
       ...,
       [0., 0., 0., ..., 1., 0., 0.],
       [0., 0., 0., ..., 0., 1., 0.],
       [0., 0., 0., ..., 0., 0., 1.]])
```

```
# 8. remove the imaginary part
eigvalue_f = np.real(eigvalue)
eigvector_f = np.real(eigvector)
```

```
eigvalue_f
```

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        0.00000000e+00,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00])
```

```
eigvector_f
```

```
array([[0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.],
       [0., 0., 0., ..., 0., 0., 0.],
       ...,
       [0., 0., 0., ..., 1., 0., 0.],
       [0., 0., 0., ..., 0., 1., 0.],
       [0., 0., 0., ..., 0., 0., 1.]])
```

```
'''
    9. Use the top-30 eigenvectors (with the largest thirty eigenvalues)
    to transform the training data to Z
'''
Z = sorted(eigvalue_f, reverse=True)[:30]
```

```
Z
```

```
[5.11687300955891,
 3.74139083537875,
 3.2527084514920177,
 2.8416206976115173,
 2.5671177441085247,
 2.273663386010388,
 1.7251549821112324,
 1.5205602403242982,
 1.4563052526419076,
 1.2427500889188123,
 1.1120895080287407,
 1.0666405432780142,
 0.9046808327483584,
 0.8921812228637239,
 0.832353048039762,
 0.781900323368878,
 0.6983868713000024,
 0.6732558113274747,
 0.625995239575561,
 0.6077630200674494,
 0.5621458802208237,
 0.5307988293548577,
 0.5027802547225588,
 0.48114703293427064,
 0.46578320029933734,
 0.4425386905090258,
 0.4284396063785778,
 0.4146187072926854,
 0.39266733699323514,
 0.36426179130874825]
```

```
# Reconstruct the training data from Z to X~, then multiply it by 255.0
eigval_index = np.argsort(eigvalue_f)
neigval_index = eigval_index[-1: -(30 + 1): -1]
neigvector = eigvector_f[:,neigval_index]

X_recon = X_new.dot(neigvector) * 255
```

```
X_recon
```

<style scoped> .dataframe tbody tr th:only-of-type { vertical-align: middle; }

```
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    vertical-align: top;
}

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	0	1	2	3	4	
0	-123.932589	-312.674262	-24.514052	-555.757958	27.296668	-
1	-1011.718376	-294.857038	596.339561	-460.766197	827.275222	-
2	51.849608	392.173153	-188.509749	521.011347	306.680729	-
3	799.127037	-607.197217	273.651240	105.878237	1.853226	
4	382.754942	730.542867	16.353747	-241.697519	-98.161346	
...	...	...	...	...	...	
59995	99.621760	-293.475562	135.940210	-347.124204	-304.275921	
59996	-152.757810	-687.899701	-88.932413	-650.013832	-198.407548	
59997	178.053450	160.078211	-257.613082	-714.650442	210.315152	-
59998	-130.606072	-5.591936	513.858674	342.703565	521.165818	-
59999	173.435952	-24.718802	556.018894	-120.861880	198.000309	

60000 rows × 30 columns