Synthesising Choreographies from Local Session Types (extended version)

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Abstract. Designing and analysing multiparty distributed interactions can be achieved either by means of a global view (e.g. in choreography-based approaches) or by composing available computational entities (e.g. in service orchestration). This paper proposes a typing systems which allows, under some conditions, to synthesise a choreography (i.e. a multiparty global type) from a set of local session types which describe end-point behaviours (i.e. local types).

1 Introduction

Communication-centred applications are paramount in the design and implementation of modern distributed systems such as those in service-oriented or cloud computing. Session types [8] and their multiparty variants [7,9] offer an effective formal framework for designing, analysing, and implementing this class of applications. Those theories feature rather appealing methodologies that consists of (i) designing a global view of the interactions – aka *global type* –, (ii) effective analysis of such a global view, (iii) automatic projection of the global view to local end-points – aka *local types* –, and (iv) type checking end-point code against local types. Such theories guarantee that, when the global view enjoys suitable properties (phase (ii)), the end-points typable with local types enjoy e.g., liveness properties like progress.

A drawback of such approaches is that they cannot be applied when the local types describing the communication patterns of end-points are not obtained by an a priori designed global view. For instance, in service-oriented computing, one typically has independently developed end-points that have to be combined to form larger services. Hence, deciding if the combined service respects its specification becomes non trivial. To illustrate this, we introduce a simple example used throughout the paper.

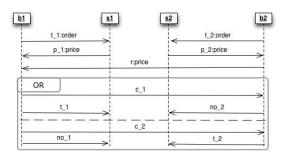
Consider a system $S_{BS} = b_1[P_1] \mid s_1[S_1] \mid b_2[P_2] \mid s_2[S_2]$ consisting of two buyers (b₁ and b₂) and two servers (s₁ and s₂) running in parallel, so that

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P_1 = t_1! \text{order.} p_1? \text{price.} r? \text{price.} (c_1!.t_1! \text{addr} \oplus c_2!.no_1!) is the behaviour of b_1 P_2 = t_2! \text{order.} p_2? \text{price.} r! \text{price.} (c_2?.t_2! \text{addr} + c_1?.no_2!) is the behaviour of b_2 S_i = t_i? \text{order.} p_i! \text{price.} (t_i? \text{addr} + no_i?), \quad i \in \{1,2\} is the behaviour of s_1
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with a!e (resp. a?e) representing the action of sending (resp. receiving) a message of type e on a channel a (we omit e when the message is immaterial), \oplus representing an internal choice, and + a choice made by the environment.

Intuitively, the overall behaviour of $S_{\rm BS}$ should be that either b_1 or b_2 purchase from their corresponding sellers. A natural question arises: is $S_{\rm BS}$ correct? Arguably, it is not immediate to decide this by considering the end-point behaviours.

We propose to construct a global view of distributed end-points like $S_{\rm BS}$ by a straightforward extension of the multiparty session types introduced in [9]. Such types formalise a global view of the behaviour which, for $S_{\rm BS}$, resembles the informal diagram below, where the choreography of the overall protocol becomes much clearer.



An advantage of our approach is that we can reuse the results of the theory of multiparty session types to prove properties of end-points e.g. safety and progress. In fact, we show that when the choreography can be constructed, its projections correspond to the initial end-points. Therefore, the well-formedness of the synthesised global choreography guarantees progress and safety properties of end-points.

The extraction of session types from programs has been studied extensively [6, 8, 9]. We assume in this work that such session types are readily available before addressing the construction of a global type.

Contributions. We introduce a theory whereby, under some conditions, it is possible to assign a global type to a set of local types. If a global type can be constructed from a set of local types, we show that it is unique (Theorem 2) and well-formed (Theorem 3). In addition, we show that typability is preserved by reduction (Theorem 4). Our theory also guarantees progress and safety properties (Theorems 5 and 6). We also show that the projections of a constructed global type are equivalent to the original system (Theorem 7). Finally, we show that for every well-formed global types, an equivalent global type can be assigned to its projections (Theorem 8).

Synopsis. In § 2, we give the syntax and semantics of the local types from which it is possible to construct a global type. In § 3, we present an extension of the global types in [9]. In § 4, we introduce a typing systems for local types, and we give our main results. Finally, in § 5 we conclude, and discuss related and future work.

2 Local Types

We use CCS-like processes (with guarded external and internal choices) to infer a global type from *local types* that correspond to the participants in the inferred choreography.

Hereafter, \mathbb{P} is a denumerable set of *participant names* (ranged over by s, r, n, ...) and \mathbb{C} is a denumerable set of *channel names* (ranged over by a, b, ...).

Syntax. The syntax of local types below is parametrised wrt basic data types such as bool, int,... (ranged over by e):

$$\begin{array}{llll} S,T & ::= & S \mid S' \mid & \mathtt{n}[P] \mid & a: \mathsf{p} \mid & \mathbf{0} \\ \\ P,Q & ::= & \bigoplus_{i \in I} a_i ! \mathtt{e_i}.P_i \mid & \sum_{i \in I} a_i ? \mathtt{e_i}.P_i \mid & \mu \mathbf{x}.P \mid & \mathbf{x} \end{array}$$

A system S consists of the parallel composition of *processes* and queues. A *process* n[P] is a behaviour P identified by $n \in \mathbb{P}$; we assume that the participant names are all different. A behaviour is either a guarded external choice, a guarded internal choice, or a recursive process. An internal choice $\bigoplus_{i \in I} a_i! e_i.P_i$ is guarded by output prefixes $a_i!e_i$ representing the sending of a value of sort e_i on channel a_i . An external choice $\sum_{i \in I} a_i?e_i.P_i$ is guarded by input prefixes $a_i?e_i$ representing the reception of a value of type e_i on channel a_i . We adopt asynchronous (order-preserving) communications and assume that the channels in the guards of choices are pairwise distinct; moreover

$$\mathbf{0} \stackrel{\text{def}}{=} \bigoplus_{i \in \varnothing} a_i! \mathbf{e}_i.P_i = \sum_{i \in \varnothing} a_i? \mathbf{e}_i.P_i$$

Finally, in a recursive behaviour $\mu x.P$, all occurrences of x in P are bound and prefixguarded; also, we consider closed behaviours only that is, behaviours with no free occurrences of recursion variables. We assume that bound variables are pairwise distinct.

A *program* is a system with no queues, while a *runtime system* is a system having exactly one queue $a : \rho$ per channel name $a \in \mathbb{C}$ in S. In the following, S, T, \ldots denote either a program or runtime system.

Semantics. The semantics of local types is given by the labelled transition system (LTS) whose labels are

$$\lambda := \alpha \mid \checkmark \mid a \cdot e \mid e \cdot a \mid n[\alpha] \mid n : \alpha$$
 where $\alpha := a!e \mid a?e$

Label α indicates either sending or reception by a process. Label \checkmark indicates termination, $a \cdot e$ and $e \cdot a$ respectively indicate push and pop operations on queues. Label $n[\alpha]$ indicates a communication action done by participant n while n:a!e and n:a?e indicate a synchronisation between n and a queue.

Assume the usual laws for commutative monoids for \mid and $\mathbf{0}$ on systems and $\mu \mathbf{x}.P \equiv P[\mu \mathbf{x}.P/\mathbf{x}]$. The LTS $\xrightarrow{\lambda}$ is the smallest relation closed under the following rules:

$$[\text{INT}] \bigoplus_{i \in I} a_i ! \text{e}_{\text{i}} . P_i \xrightarrow{a_j ! \text{e}_{\text{j}}} P_j \qquad [\text{EXT}] \sum_{i \in I} a_i ? \text{e}_{\text{i}} . P_i \xrightarrow{a_j ? \text{e}_{\text{j}}} P_j \quad j \in I$$

$$[\text{PUSH}] \ a : \rho \xrightarrow{a \cdot \text{e}} a : \rho \cdot \text{e} \qquad [\text{POP}] \ a : \text{e} \cdot \rho \xrightarrow{\text{e} \cdot a} a : \rho \qquad [\text{END}] \ \mathbf{0} \xrightarrow{\checkmark} \mathbf{0}$$

$$[\text{IN}] \ \frac{S \xrightarrow{\text{I}[a^2 \text{e}]} S' \qquad T \xrightarrow{\text{e} \cdot a} T'}{S \mid T \xrightarrow{\text{n}:a^2 \text{e}} S' \mid T'} \qquad [\text{DOX}] \ \frac{P \xrightarrow{\alpha} P'}{S \mid T \xrightarrow{\text{n}:a^2 \text{e}} S' \mid T'} \qquad [\text{BOX}] \ \frac{P \xrightarrow{\alpha} P'}{\text{n}[P] \xrightarrow{\text{n}[\alpha]} \text{n}[P']}$$

$$[\text{EQ-P}] \ \frac{P \equiv Q \xrightarrow{\alpha} Q' \equiv P'}{P \xrightarrow{\alpha} P'} \qquad [\text{EQ-S}] \ \frac{S \equiv T \xrightarrow{\lambda} T' \equiv S'}{S \xrightarrow{\lambda} S'}$$

Rules [INT] and [EXT] are trivial. By [PUSH] (resp. [POP]), a queue receives a (resp. sends the first) datum (resp. if any). Processes can synchronise with queues according to rules [IN] and [OUT]. The remaining rules are rather standard. Let $S \longrightarrow$ iff there are S' and λ s.t. $S \xrightarrow{\lambda} S'$ and λ (resp. \Longrightarrow) be the reflexive transitive closure of $\xrightarrow{\lambda}$ (resp. \Longrightarrow).

3 Global Types

A global type \mathcal{G} specifies an ordering of the interactions in a choreography. The syntax for global types in [9] is extended with a generalised sequencing in the following syntax:

$$G ::= s \rightarrow r : a\langle e \rangle.G \mid G; G' \mid G+G' \mid G \mid G' \mid \mu \chi.G \mid \chi \mid \mathbf{0}$$

The prefix $s \to r : a\langle e \rangle$ represents an interaction where $s \in \mathbb{P}$ sends a value of sort $ext{e}$ to $r \in \mathbb{P}$ on $a \in \mathbb{C}$ (we let t range over interactions $s \to r : a\langle e \rangle$ and assume that $s \neq r$). The production g : g' indicates a generalised form of sequencing, where the interactions in g' are enabled only after the ones in g. The production g + g' indicates a (exclusive) choice of interactions. Concurrent interactions are written $g \mid g'$. A global type $\mu \chi . g$, indicates a recursive type, where g is bound in g. We assume that global types are closed and that recursion is guarded. We often omit trailing occurrences of g.

Example 1. The first two interactions between b_i and s_i in the example of § 1 are

$$G_i = b_i \rightarrow s_i : t_i \langle \text{order} \rangle. s_i \rightarrow b_i : p_i \langle \text{price} \rangle \quad i \in \{1, 2\}$$
 (3.1)

The type G_i says that a participant b_i sends a message of type order to participant s_i on channel t_i , then s_i replies with a message of type price on channel p_i .

The smallest equivalence relation satisfying the laws for commutative monoids for | , +, and $\mathbf{0}$ and the axioms below is the structural congruence for global types:

$$\begin{aligned} \mathcal{G}; \mathbf{0} &\equiv \mathcal{G} & \mathbf{0}; \mathcal{G} &\equiv \mathcal{G} & (\mathcal{G}; \mathcal{G}'); \mathcal{G}'' &\equiv \mathcal{G}; (\mathcal{G}'; \mathcal{G}'') \\ \mathbf{1}.(\mathcal{G}; \mathcal{G}') &\equiv (\mathbf{1}.\mathcal{G}); \mathcal{G}' & \mu \chi.\mathcal{G} &\equiv \mathcal{G}[\mu \chi.\mathcal{G}/\chi] \end{aligned}$$

The syntax of global types may specify behaviours that are not implementable. The rest of this section borrows from [5] and [9] and adapts the requirements a global type must fulfil to ensure that the ordering relation it prescribes is indeed feasible.

3.1 Channel Usage and Linearity

It is paramount that no race occurs on the channels of a global type (i.e. a datum sent on a channel is received by its intended recipient). As in [9], we require that a global type is *linear*, that is actions on channels shared by different participants are temporally ordered. For this, we use generic environments (ranged over by C) which keep track of channel usage. Such environments are trees defined as follows:



Each node c has a label \underline{c} of the form o, $s \to r : a$, or $\mu \chi$ respectively representing the root of choice or concurrent branches, an interaction between s and r on a, and a recursive behaviour. We write $c \in C$ if c is a node in C. We use $\underline{\ }$ as a wild-card when some of the components of a label are immaterial, e.g. $\underline{\ } \to \underline{\ } : a$ matches any label representing an interaction on a. Given a tree C, we write $c_1 \prec c_2$, if $c_1, c_2 \in C$ and c_2 is a node in the sub-tree rooted at c_1 . We adapt the definitions in [9] to our framework.

Definition 1 (Dependency relations [9]) *Fix C, we define the following relations:*

$$c_1 \prec_{II} c_2 \text{ if } c_1 \prec c_2 \text{ and } c_i = s_i \rightarrow r : a_i \quad i \in \{1, 2\}$$

 $c_1 \prec_{I0} c_2 \text{ if } c_1 \prec c_2 \text{ and } c_1 = s_1 \rightarrow r : a_1 \text{ and } c_2 = r \rightarrow s_2 : a_2$
 $c_1 \prec_{00} c_2 \text{ if } c_1 \prec c_2 \text{ and } c_i = s \rightarrow r_i : a \quad i \in \{1, 2\}$

An input dependency from c_1 to c_2 is a chain of the form $c_1 \prec_{\phi_1} \ldots \prec_{\phi_k} c_2$ $(k \ge 0)$ such that $\phi_i \in \{II, I0\}$ for $1 \le i \le k-1$ and $\phi_k = II$. An output dependency from c_1 to c_2 is a chain $c_1 \prec_{\phi_1} \ldots \prec_{\phi_k} c_2$ $(k \ge 1)$ such that $\phi_i \in \{00, I0\}$.

Definition 2 (Linearity [9]) *C* is linear if and only if whenever $c_1 \prec c_2$ with $\underline{c_1} = _ \rightarrow _ : a$ and $c_2 = _ \rightarrow _ : a$ then there is both input and output dependencies from c_1 to c_2 .

We also define a function _*_ to append trees as follows

$$\begin{vmatrix}
c & c & c \\
 & \star C' &= & | \\
C_0 & C_0 \star C'
\end{vmatrix}, \quad \bullet \star C = C, \quad C_1 \quad \star C' = C_1 \star C' \quad C_2 \star C'$$

and a partial function to append a tree C' to a tree C while preserving linearity: $C \prec C' = C \star C'$ if $C \star C'$ is linear, otherwise $C \prec C' = \bot$. Also, let $\mathcal{T}(\mathcal{G})$ be the total function (cf. Appendix A.1) which returns a tree C corresponding to the use of channels in \mathcal{G} .

3.2 Well-formed Global Types

We define the conditions for a global type to be well-formed. We write $\mathcal{P}(\mathcal{G})$ (resp. $\mathcal{C}(\mathcal{G})$) for the set of participant (resp. channel) names in \mathcal{G} , and $\mathsf{fv}(\mathcal{G})$ for the set of free variables in \mathcal{G} , similarly for a system S. We give a few accessory functions. Let

$$\begin{split} R(\mathcal{G}) &\stackrel{\text{def}}{=} \left\{ \mathbf{s} \rightarrow \mathbf{r} : a \, | \, \mathcal{G} \equiv (\mathbf{s} \rightarrow \mathbf{r} : a \langle \mathbf{e} \rangle, \mathcal{G}_1 + \mathcal{G}_2 \mid \mathcal{G}_3) \, ; \mathcal{G}_4 \right\} \\ F_P(\mathcal{G}) &\stackrel{\text{def}}{=} \left\{ \begin{matrix} F_P(\mathcal{G}_1) \cup F_P(\mathcal{G}_2), & \mathcal{G} = \mathcal{G}_1 \mid \mathcal{G}_2 \\ \{\mathcal{P}(\mathcal{G})\}, & \text{otherwise} \end{matrix} \right. \\ \\ F_0(\mathcal{G}) &\stackrel{\text{def}}{=} \left\{ \begin{matrix} F_0(\{\mathbf{s}, \mathbf{r}\} \cup \mathcal{P}, \mathcal{G}_1), & \mathcal{G} = \mathbf{s} \rightarrow \mathbf{r} : a \langle \mathbf{e} \rangle, \mathcal{G}_1 \\ F_0(\mathcal{G}, \mathcal{G}_1) \cup F_0(\mathcal{G}, \mathcal{G}_2), & \mathcal{G} = \mathcal{G}_1 \mid \mathcal{G}_2 \\ F_0(\mathcal{P}, \mathcal{G}_1), & \mathcal{G} = \mathcal{G}_1 + \mathcal{G}_2 \text{ and } F_0(\mathcal{P}, \mathcal{G}_1) = F_0(\mathcal{P}, \mathcal{G}_2) \\ F_0(\mathcal{P}, \mathcal{G}_1), & \mathcal{G} = \mathcal{H}_{\mathcal{X}}, \mathcal{G}_1 \\ F_0(\mathcal{G}, \mathcal{G}_2), & \mathcal{G} = \mathcal{G}_1 \, ; \, \mathcal{G}_2 \\ \{\mathcal{P}\}, & \mathcal{G} = \mathbf{0} \text{ or } \mathcal{G} = \chi \\ \downarrow, & \text{otherwise} \end{matrix} \end{split}$$

 $R(\mathcal{G})$ is the *ready set* of \mathcal{G} . $F_P(\mathcal{G})$ is the family of sets of its participants running in different concurrent branches. That is, $N \in F_P(\mathcal{G})$ iff all $n \in N$ are in a same top-level thread of \mathcal{G} . $F_0(\mathcal{G}, \mathcal{P})$ is the family of sets of participants of \mathcal{G} , so that for all $N, M \in F_0(\mathcal{P}, \mathcal{G})$, the participants in N and those in M are in different concurrent branches in the last part of \mathcal{G} ; define $F_0(\mathcal{G}) \stackrel{\text{def}}{=} F_0(\emptyset, \mathcal{G})$. Note that $F_0(-, -)$ is a partial function.

Example 2. Let $\mathcal{G}_{i,j} = \mathbf{b_1} \to \mathbf{b_2} : c_i \langle \rangle. (\mathbf{b_i} \to \mathbf{s_i} : t_i \langle \mathtt{addr} \rangle \mid \mathbf{b_j} \to \mathbf{s_j} : no_j \langle \rangle)$ describe each of the branches of the *or* box in the example of § 1, where $i \neq j \in \{1,2\}$, then

$$\mathtt{R}(\mathcal{G}_{1,2}) = \left\{\mathtt{b_1} \to \mathtt{b_2} : c_1\right\}, \ \mathtt{F_P}(\mathcal{G}_{1,2}) = \left\{\mathtt{b_1}, \mathtt{s_1}, \mathtt{b_2}, \mathtt{s_2}\right\}, \ \mathtt{F_0}(\mathcal{G}_{1,2}) = \left\{\left\{\mathtt{b_1}, \mathtt{s_1}\right\}, \left\{\mathtt{b_2}, \mathtt{s_2}\right\}\right\}$$

The global type below corresponds to the whole protocol of § 1

$$G = (G_1 \mid G_2); b_2 \rightarrow b_1 : r\langle price \rangle. (G_{1,2} + G_{2,1})$$

hence
$$R(\mathcal{G}) = \{b_i \to s_i : t_i\}_{i=1,2}, F_P(\mathcal{G}) = F_P(\mathcal{G}_{1,2}), \text{ and } F_0(\mathcal{G}) = F_0(\mathcal{G}_{1,2}).$$
 \diamond

Well-formedness. The well-formedness of a global type \mathcal{G} depends on how it uses channels; a judgement of the form $C \vdash \mathcal{G}$ states that \mathcal{G} is well-formed according to the channel environment C (cf. § 3.1); \mathcal{G} is well-formed if $\bullet \vdash \mathcal{G}$ can be derived from the rules given in Fig. 1. We assume that each premise of the rules in Fig. 1 does not hold if any of the functions used are not defined (e.g., in $[w_{F^-};]$, if $F_0(\mathcal{G}) = \bot$ then $C \vdash \mathcal{G}$; \mathcal{G}' is not derivable). Hereafter, we assume that a node c is fresh (i.e. $c \notin C$). The environment C permits to tackle one of the main requirements for a global type to be well-formed: there should not be any race on channels. In the following, we discuss the rules of Fig. 1, which are grouped according to three other requirements: sequentiality, single threaded, and knowledge of choice.

Sequentiality [5]. Rules [wF-.], [wF-.] and [wF-.] ensure that sequentiality is preserved. In [wF-.], there must be an ordering dependency between a prefix and its continuation so that it is possible to implement each participant so that at least one action of the first prefix always happens before an action of the second prefix. More concretely, we want to avoid global types of the form, e.g.

$$\mathtt{s_1} \rightarrow \mathtt{r_1} : a\langle \mathtt{e} \rangle.\mathtt{s_2} \rightarrow \mathtt{r_2} : b\langle \mathtt{e}' \rangle$$
 X

where, evidently, it is not possible to guarantee that s_2 sends after r_1 receives on a. Since we are working in an asynchronous setting, we do not want to force both send and receive actions of the first prefix to happen before both actions of the second one. Rule $[w_{F^-}]$ requires the following for generalised sequencing. (i) For each pair of "first" participants in \mathcal{G}' , there exist two concurrent branches of \mathcal{G} such that these two participants appear in different branches. This is to avoid global types of the form, e.g.

$$(s_1 \rightarrow r_1 : a\langle e \rangle \mid s_2 \rightarrow r_2 : b\langle e \rangle); s_1 \rightarrow r_1 : c\langle e \rangle$$

since there is no possible sequencing between the prefix on b and the one on c. (ii) For all top-level concurrent branches in G, there is a participant in that branch which is also in one of the branches of G'. This requirement discards global types of the form, e.g.

$$(s_1 \rightarrow r_1 : a\langle e \rangle \mid s_2 \rightarrow r_2 : b\langle e \rangle \mid s_3 \rightarrow r_3 : c\langle e \rangle); s_1 \rightarrow r_2 : d\langle e \rangle$$

$$[\text{WF-.}] \frac{\forall \, \mathbf{s}' \to \mathbf{r}' : \, _{-} \in \mathsf{R}(\mathcal{G}) : \{ \mathbf{s}', \mathbf{r}' \} \cap \{ \mathbf{s}, \mathbf{r} \} \neq \varnothing \qquad C \, _{-} \, \mathsf{C} \, _{-} \, \mathsf{G} \qquad \underline{\mathsf{C}} = \mathbf{s} \, _{-} \, \mathsf{r} \, : \, a}{C \, \vdash \mathbf{s} \, \to \mathbf{r} : \, a \, \langle \mathbf{e} \rangle . \, \mathcal{G}}$$

$$\forall \, \mathbf{s} \, \to \mathbf{r} : \, _{-} \in \mathsf{R}(\mathcal{G}') . \, \exists N_{1} \neq N_{2} \in \mathsf{F}_{\mathsf{D}}(\mathcal{G}) . \, \mathbf{s} \, \in N_{1} \, \land \mathbf{r} \, \in N_{2}$$

$$\forall N \in \mathsf{F}_{\mathsf{P}}(\mathcal{G}) . \, \exists N' \in \mathsf{F}_{\mathsf{P}}(\mathcal{G}') . \, N \cap N' \neq \varnothing \qquad C \, \vdash \, \mathcal{G} \qquad C \, \land \, \mathcal{T}(\mathcal{G}) \, \vdash \, \mathcal{G}'$$

$$C \, \vdash \, \mathcal{G} : \, \mathcal{G}'$$

$$C \, \vdash \, \mathcal{G}' : \, \mathcal{C} : \,$$

Fig. 1. Rules for Well-formedness

since it is not possible to enforce an order between s_3 and r_3 and the others. (*iii*) \mathcal{G} and \mathcal{G}' are also well-formed. Observe that (*i*) implies that for \mathcal{G} ; \mathcal{G}' to be well-formed, \mathcal{G} is of the form $\mathcal{G}_1 \mid \mathcal{G}_2$, with $\mathcal{G}_1 \neq \mathbf{0}$ and $\mathcal{G}_2 \neq \mathbf{0}$. Both [wf-.] and [wf-.] are only applicable when linearity is preserved. Finally, rule [wf-.] is a special case of \mathcal{G} ; \mathcal{G}' .

Single threaded [9]. A participant should not appear in different concurrent branches of a global type, so that each participant is single threaded. This is also reflected in the calculus of § 2, where parallel composition is only allowed at the system level. Therefore, in $[w_F-|]$, the participant (resp. channel) names in concurrent branches must be disjoints. Rule $[w_F-\mu\chi]$ adds a new node in C to keep track of recursive usage of the channels, and requires that G is single threaded, i.e. concurrent branches cannot appear under recursion. If that was the case, a participant would appear in different concurrent branches of the unfolding of a recursive global type. Rule $[w_F-\chi]$ unfolds C at χ to ensure that the one-time unfolding of C preserves linearity (see [9] for details).

Knowledge of choice [5, 9]. Whenever a global type specifies a choice of two sets of interactions, the decision should be made by exactly one participant. For instance,

$$s_1 \rightarrow r_1 : a_1 \langle e \rangle. G_1 + s_2 \rightarrow r_2 : a_2 \langle e' \rangle. G_2$$

specifies a choice made by two participants. Indeed, s_1 is the one making a decision in the first branch, while s_2 makes a decision in the second one; this kind of choreographies cannot be implemented (without using hidden interactions). Also, we want to avoid global types where a participant n behaves differently in two choice branches without being aware of the choice made by others. For instance, in

$$s \rightarrow r : a\langle e \rangle.n \rightarrow r : c\langle e \rangle.G_1 + s \rightarrow r : b\langle e \rangle.n \rightarrow r : d\langle e \rangle.G_2$$

where n ignores the choice of s and behaves differently in each branch. On the other hand, we want global types of following form to be accepted.

$$\begin{split} \mathbf{s} \rightarrow \mathbf{r} : a \langle \mathbf{e} \rangle. \mathbf{n} \rightarrow \mathbf{s} : b \langle \mathbf{e} \rangle. \mathbf{s} \rightarrow \mathbf{n} : c \langle \mathbf{e} \rangle. \mathbf{n} \rightarrow \mathbf{r} : d \langle \mathbf{e} \rangle \\ + \\ \mathbf{s} \rightarrow \mathbf{r} : a' \langle \mathbf{e} \rangle. \mathbf{n} \rightarrow \mathbf{s} : b \langle \mathbf{e} \rangle. \mathbf{s} \rightarrow \mathbf{n} : c' \langle \mathbf{e} \rangle. \mathbf{n} \rightarrow \mathbf{r} : d' \langle \mathbf{e} \rangle \end{split}$$

Indeed, in this case n behaves differently in each branch, but only after "being informed" by s about the chosen branch.

Together with the projection map defined below, rule $[w_{F-+}]$ guarantees that "knowledge of choice" is respected. In particular, the rule requires that the participant who makes the decision is the same in every branch of a choice, while the channels guarding the choice must be distinct.

Definition 1 (__). *The* projection of a global type G wrt. $n \in \mathcal{P}(G)$ is defined as

$$\mathcal{G}|_{\mathbf{n}} = \begin{cases} a? \mathbf{e}.\mathcal{G}'|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathbf{s} \rightarrow \mathbf{n} : a \langle \mathbf{e} \rangle.\mathcal{G}' \\ a! \mathbf{e}.\mathcal{G}'|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathbf{n} \rightarrow \mathbf{r} : a \langle \mathbf{e} \rangle.\mathcal{G}' \\ \mathcal{G}'|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathbf{s} \rightarrow \mathbf{r} : a \langle \mathbf{e} \rangle.\mathcal{G}' \text{ and } \mathbf{s} \neq \mathbf{n} \neq \mathbf{r} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}} \uplus \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathcal{G}_{\mathbf{l}} + \mathcal{G}_{\mathbf{l}} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}} [\mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathcal{G}_{\mathbf{l}} \mid \mathcal{G}_{\mathbf{l}} \text{ and } \mathbf{n} \notin \mathcal{P}(\mathcal{G}_{\mathbf{j}}), i \neq j \in \{1, 2\} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}} [\mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}/\mathbf{0}], & \text{if } \mathcal{G} = \mathcal{G}_{\mathbf{l}} ; \mathcal{G}_{\mathbf{l}} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}} [\mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}/\mathbf{0}], & \text{if } \mathcal{G} = \mathcal{G}_{\mathbf{l}} ; \mathcal{G}_{\mathbf{l}} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}} [\mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}/\mathbf{0}], & \text{if } \mathcal{G} = \mathcal{G}_{\mathbf{l}} ; \mathcal{G}_{\mathbf{l}} \\ \mathcal{G}_{\mathbf{l}}|_{\mathbf{n}}, & \text{if } \mathcal{G} = \mathcal{K} \text{ or } \mathcal{G} = \mathbf{0} \\ \mathcal{L}, & \text{otherwise} \end{cases}$$

We say that a global type is projectable if $G|_n$ is defined for all $n \in \mathcal{P}(G)$.

The projection map is similar to the one given in [9], but for the generalised sequencing case and the use of $_ \uplus _$ to project choice branches. Observe that if $\mathcal{G} = \mathcal{G}_1$; \mathcal{G}_2 , we replace $\mathbf{0}$ by the projection of \mathcal{G}_2 in the projection of \mathcal{G}_1 . Function $_ \uplus _$ basically merges (if possible) the behaviour of a participant in different choice branches; $_ \uplus _$ is defined only when the behaviour is the same in all branches, or if it differs after having received enough information about the branch which was chosen. The definition of $_ \uplus _$ is given in Appendix A.2. A global type may be projected even if is not well-formed, but in that case none of the properties given below are guaranteed to hold.

4 Synthesising Global Types

We now introduce a typing systems to synthesise a global type \mathcal{G} from a system S so that S satisfies safety and progress properties (e.g. no race on channels and no participant gets stuck). Also, the set of typable systems corresponds exactly to the set of systems obtained by projecting well-formed global types. To synthesise \mathcal{G} from a system S, a careful analysis of what actions can occur at each possible state of S is necessary.

If $S \equiv n[P] \mid S'$ then S(n) denotes P (if $S \not\equiv n[P] \mid S'$ then $S(n) = \bot$). We define the *ready set* of a system as follows:

$$\mathtt{R}(S) = \begin{cases} \{a_i | i \in I\} \cup \mathtt{R}(S') \text{ if } S \equiv \mathtt{r}[\sum_{i \in I} a_i?\mathtt{e_i}.P_i] \mid S' \\ \{\overline{a_i} | i \in I\} \cup \mathtt{R}(S') \text{ if } S \equiv \mathtt{s}[\bigoplus_{i \in I} a_i!\mathtt{e_i}.P_i] \mid S' \\ \{\overline{a}\} \cup \mathtt{R}(S') \text{ if } S \equiv a : \mathtt{e} \cdot \mathtt{p} \mid S' \end{cases}$$

$$\varnothing \qquad \qquad \mathsf{if } S \equiv \mathbf{0}$$

We overload $R(_)$ on behaviours in the obvious way and define $\overline{\mathbb{R}}(S) \stackrel{\text{def}}{=} \{a \in \mathbb{C} \mid a \in \mathbb{R}(S) \text{ or } \overline{a} \in \mathbb{R}(S)\}$ and $S \updownarrow \iff \exists a \in \mathbb{C} : a \in \mathbb{R}(S) \land \overline{a} \in \mathbb{R}(S)$; we write $S \updownarrow \text{ if } S \updownarrow \text{ does not hold.}$

4.1 Validation Rules

A judgement of the form A; Γ ; $C \vdash S \triangleright G$ says that the system S forms a choreography defined by a global type G, under the environments A, Γ , and C. The environment A is a superset of the channel names used in S, and corresponds to the channels S is entitled to use. The environment Γ maps participant names and local recursion variables to global recursion variables (\circ is the empty context Γ). The channel environment C records the use of channels. Hereafter, we use \cdot for the disjoint union of environments.

Programs. A global type G can be synthesised from the program S if the judgement

$$C(S)$$
; \circ ; $\bullet \vdash S \triangleright G$

(stating that *S* is entitled to use all its channels in empty environments) is derivable from the rules in Fig. 2 (driven by the ready set of *S* and the structure of its processes).

Rule [.] validates prefixes provided that the system is entitled to linearly use the channel a, that the continuation is typable, and that no other interactions are possible in S. For instance, [.] does not apply to

$$s_1[a!e.P_1] \mid r_1[a?e.Q_1] \mid s_2[b!e.P_2] \mid r_2[b?e.Q_2]$$
 X

because there is no ordering relation between the actions on a and b; in this case either [|] or [;] should be used.

Rule [1] validates concurrent branches when they can be validated using a partition A_1 and A_2 of the channels (recall that $\mathcal{P}(S) \cap \mathcal{P}(S') = \emptyset$).

Rule [:] splits the system into two sequential parts and it relies on the function $\mathtt{split}(_)$ defined in § 4.2; for now it suffices to notice that linearity is checked for in the second part of the split by adding the channel environment corresponding to \mathcal{G}_1 to C (recall that $C \prec C'$ is undefined if $C \star C'$ is not linear).

Rule $[\oplus]$ introduces the global type choice operator, it requires that both branches are typable and that no other interactions are possible in S.

Rule [+] allows to discharge a branch of an external choice; together with the premises of [1], rule [+] discards systems such as the one on left below (due to a race on b) but

$$[.] \frac{\{a\} \cup A; \Gamma; C \prec c \vdash s[P] \mid r[Q] \mid S \blacktriangleright \mathcal{G} \quad \underline{c} = s \rightarrow r : a \quad S \cancel{\updownarrow} }{\{a\} \cup A; \Gamma; C \vdash s[a!e.P] \mid r[a?e.Q] \mid S \blacktriangleright s \rightarrow r : a \langle e \rangle. \mathcal{G} }$$

$$[] \frac{A_1; \circ; C \vdash S \blacktriangleright \mathcal{G} \quad A_2; \circ; C \vdash S' \blacktriangleright \mathcal{G}' \quad A_1 \cap A_2 = \varnothing}{A_1 \cup A_2; \Gamma; C \vdash S \mid S' \blacktriangleright \mathcal{G} \mid \mathcal{G}'}$$

$$[] \frac{A_1; \circ; C \vdash S \blacktriangleright \mathcal{G} \quad A_2; \circ; C \vdash S' \blacktriangleright \mathcal{G}' \quad A_1 \cap A_2 = \varnothing}{A_1 \cup A_2; \Gamma; C \vdash S \mid S' \blacktriangleright \mathcal{G} \mid \mathcal{G}'}$$

$$[] \frac{A; \circ; C \vdash S_1 \blacktriangleright \mathcal{G}_1 \quad \text{split}(S) = (S_1, S_2) \quad A; \circ; C \prec T(\mathcal{G}_1) \vdash S_2 \blacktriangleright \mathcal{G}_2 }{A; \Gamma; C \vdash S \blacktriangleright \mathcal{G}_1; \mathcal{G}_2}$$

$$[] \frac{A; \Gamma; C \vdash s[P] \mid S \blacktriangleright \mathcal{G} \quad A; \Gamma; C \vdash s[Q] \mid S \blacktriangleright \mathcal{G}' \quad S \cancel{\updownarrow}' }{A; \Gamma; C \vdash s[P \oplus Q] \mid S \blacktriangleright \mathcal{G} \quad S \cancel{\updownarrow}'}$$

$$[] \frac{\overline{R}(Q) \subseteq A \quad A; \Gamma; C \vdash r[P] \mid S \blacktriangleright \mathcal{G} \quad S \cancel{\updownarrow}' }{A; \Gamma; C \vdash r[P + Q] \mid S \blacktriangleright \mathcal{G}}$$

$$\frac{\exists 1 \leq i, j \leq k. (n_1[P_i] \mid n_j[P_j]) \updownarrow}{A; \Gamma; C \vdash n_1[\mu x_1.P_1] \mid \dots \mid n_k[\mu x_k.P_k] \blacktriangleright \mu \chi. \mathcal{G}}$$

$$[] \frac{A; \Gamma; C \vdash n_1[\mu x_1.P_1] \mid \dots \mid n_k[\mu x_k.P_k] \blacktriangleright \mu \chi. \mathcal{G}}{A; \Gamma; C \vdash n_1[x_1] \mid \dots \mid n_k[x_k] \blacktriangleright \chi}$$

$$[] \frac{S \equiv S' \quad A; \Gamma; C \vdash S' \blacktriangleright \mathcal{G}}{A; \Gamma; C \vdash S \blacktriangleright \mathcal{G}} \qquad [0] \frac{\forall n \in \mathcal{P}(S).S(n) = \mathbf{0} \quad C(S) = \varnothing}{A; \Gamma; C \vdash S \blacktriangleright \mathbf{0}}$$

Fig. 2. Validation Rules for Programs

permits those like the one on the right (as only the channels guarding the choice must be in A).

Rules $[\mu]$ and [x] handle recursive systems. The former rule "guesses" the participants involved in a recursive behaviour. If two of them interact, $[\mu]$ validates the recursion provided that the system can be typed when such participants are associated to the global recursion variable χ (assuming that χ is not in Γ). Rule [x] checks that all the participants in the recursion have reached a local recursion variable corresponding to the global recursion, and that the unfolding of C on $\mu\chi$ preserves linearity; for this we define $C(\mu\chi)$ to be the subtree of C rooted at the *deepest* node of C labelled by $\mu\chi$ (note that this node is unique since bound variables are distinct).

Rule [0] only applies when all the participants in S end while [eq] validates a system up to structural congruence.

Theorem 1 (**Decidability**). *Typability is decidable*.

The proofs follows from the fact that the typing is done wrt to the (finite) partitions of channels in a system, and that the number of required behaviour unfoldings is finite.

Theorem 2 (Unique typing). If
$$A$$
; Γ ; $C \vdash S \triangleright G$ and A ; Γ ; $C \vdash S \triangleright G'$ then $G \equiv G'$.

Theorem 3 (Well-formedness). *If* A; Γ ; $C \vdash S \triangleright G$ then $\bullet \vdash G$ and G is projectable.

The proofs for these two theorems are by induction on the structure of the derivation.

Runtime system. In order to have subject reduction for our typing systems, queues have to be handled effectively; we use a distinguished participant name * to denote an anonymous participant. Assume $* \notin \mathbb{P}$ and write $* \to r : a \langle e \rangle . G$ to specify that there is a message of sort e on channel a for participant r.

Example 3. Let $S = n[a!e] \mid s[b!e.a?e] \mid r[b?e] \mid a : [] \mid b : []$. Consider the judgement

$$A; \Gamma; C \vdash S \triangleright s \rightarrow r : b\langle e \rangle.n \rightarrow s : a\langle e \rangle$$

If *S* evolves to $S' = n[0] \mid s[b!e.a?e] \mid r[b?e] \mid a:e \mid b:[]$, the identity of the sender n is lost. However, the judgement

$$A; \Gamma; C \vdash S' \triangleright s \rightarrow r : b\langle e \rangle .* \rightarrow s : a\langle e \rangle$$

types S' using *.

Runtime systems can be handled by slightly extending Def. 1 so that we have 1

$$c_1 \prec_{00} c_2$$
 if $c_1 \prec c_2$ and $c_1 = * \rightarrow r : a$ and $c_2 = s \rightarrow r : a$

and by adding two rules to the validation rules for handling queues:

$$[\rho] \ \frac{\{a\} \cup A; \circ; C \prec \mathsf{c} \vdash a : \rho \mid \mathsf{r}[P] \mid S \blacktriangleright \mathcal{G} \qquad \underline{\mathsf{c}} = * \rightarrow \mathsf{r} : a \qquad S \not \downarrow }{\{a\} \cup A; \Gamma; C \vdash a : e \cdot \rho \mid \mathsf{r}[a?e.P] \mid S \blacktriangleright * \rightarrow \mathsf{r} : a \langle e \rangle. \mathcal{G} } \qquad [[]] \ \frac{A; \Gamma; C \vdash S \blacktriangleright \mathcal{G}}{A; \Gamma; C \vdash a : [] \mid S \blacktriangleright \mathcal{G}}$$

Rule $[\rho]$ is similar to rule $[\cdot]$, except that a non-empty queue replaces the sender, and Γ is emptied. Rule $[\cdot]$ simply allows to remove empty queues from the system.

Theorem 4. If
$$A : \circ : C \vdash S \triangleright G$$
, $S \xrightarrow{\lambda} S'$, and $C(\lambda) \notin C$ then $A : \circ : C \vdash S' \triangleright G'$

The proof is by case analysis on the different types of transitions a system can make. The recursive case follows from the fact that reduction preserves closeness of behaviours.

4.2 Splitting Systems

The purpose of systems' splitting is to group participants according to their interactions. For this we use judgements of the form

$$\Psi; \Theta \vdash S \Leftrightarrow \Omega \tag{4.1}$$

 \Diamond

which reads as "S splits as Ω under Ψ and Θ ". The environment Ψ is a set of (pairwise disjoint) *ensembles* that is disjoint sets $N \subseteq \mathcal{P}(S)$ containing participants that interact with each other for a while; and then some of them may interact with participants in other ensembles in Ψ . The environment Θ is a set of (pairwise disjoint) *duos*, that is

¹ This extension makes sense since the order of messages is preserved in the calculus.

two-element sets of participants $\{s, r \in \mathcal{P}(S) : r \neq s\}$ representing the first participants able to interact once the first part of the split is finished. Under suitable conditions, one could identify when $n \in N$ has to interact with a participant of another ensemble. In other words, one can divide S(n) as $n[P_1 \cdot \varepsilon \cdot P_2]$: the interactions in P_1 happen with participants in the ensemble of n, while P_2 starts interacting with a participant in another ensemble. Finally, the environment Ω assigns behaviours augmented with a separator ε to participant names, and lists of sorts to queues a.

Given a judgement as (4.1), we say that $N, M \in \Psi$ are Θ -linked ($N \stackrel{\Theta}{\frown} M$ in symbols) iff $\exists D \in \Theta : N \cap D \cap M \neq \emptyset$; also, we say that $n, m \in \mathcal{P}(S)$ are Ω -linked ($n \stackrel{\Omega}{\frown} m$ in symbols) iff $C(\Omega(n)) \cap C(\Omega(m)) \neq \emptyset$. We define $S[N] \stackrel{\text{def}}{=} \prod_{n \in N} n[S(n)] \mid \prod_{a \in C(S)} a : S(a)$.

Definition 2. The judgement Ψ ; $\Theta \vdash S \Leftrightarrow \Omega$ is coherent if it can be derived from the rules in Fig. 3, $\Theta \neq \emptyset$, and for all $N \in \Psi$, $S[N] \uparrow$ and the following conditions hold

$$\exists ! \mathbf{n} \in N : ((\exists ! \mathbf{m} \in N \setminus \{\mathbf{n}\} : S[N \setminus \{\mathbf{n}\}] \updownarrow) \circ r(S[N \setminus \{\mathbf{n}\}] \updownarrow))$$
 (4.2)

$$\circledast$$
 is total on N and \leftrightarrow_{Θ} is total on Ψ (4.3)

where $\leftrightarrow_{\Theta} \stackrel{\text{def }\Theta}{=} \stackrel{*}{\frown}$ is the reflexive and transitive closure of $\stackrel{\Theta}{\frown}$ and $\stackrel{\text{def }\Omega}{=} \stackrel{+}{\frown}$ is the transitive closure of $\stackrel{\Omega}{\frown}$.

Essentially, Def. 2 ensures that rule [;] is the only rule of Fig. 2 applicable when the system can be split. Condition (4.2) ensures that, in each ensemble N, there is a unique pair of synchronising participants or there is a unique participant that can synchronise with a queue a. Condition (4.3) is the local counterpart of the well-formedness rule for global types of the form G; G'. The totality of \circledast on N guarantees that the participants in an ensemble share channels. The totality of \leftrightarrow_{Θ} on Ψ guarantees that each ensemble in Ψ has one "representative" which is one of the first participants to interact in the second part of the split. Together with condition $\Theta \neq \emptyset$, the condition on \leftrightarrow_{Θ} ensures that there are (at least) two ensembles of participants in Ψ . Note that (4.3) also ensures that all the set of participants in Ψ are interdependent (i.e. one cannot divide them into independent systems, in which case rule [1] should be used).

A judgement (4.1) is to be derived with the rules of Fig.3 (we omit rules for commutativity and associativity of systems). The derivation is driven by the structure of up to two processes in S, and whether they are in the same ensemble and/or form a duo.

Rule [e] marks two processes m and n as "to be split" when m and n form a duo in Θ and are in different ensembles of Ψ . The continuation of the system is to be split as well, with m and n removed from the system and from the environments.

Rule [sync] records in Ω the interactions of participants in a same ensemble of Ψ .

Rule [+] discharges the branch of an external choice for participants in a same ensemble while $[\oplus]$ deals with internal choice. The premise $\Omega \simeq \Omega'$ holds only when Ω and Ω' have the same domain and differ only up to external choice, i.e. for each n either its split is the same in both branches, or its split is an external choice (guarded by different channels); $\Omega \sqcup \Omega'$ merges Ω and Ω' accordingly (cf. Appendix A.3). The additional premise $s[P \oplus P'] \mid r[Q] \uparrow$ is required so that the split is done *before* a branching if a participant cannot interact with one of its peer in N after the branching.

$$[\epsilon] \frac{\mathbf{n} \in N, \mathbf{m} \in M \quad (\mathbf{n}[P] \mid \mathbf{m}[Q]) \updownarrow \quad \Psi \cdot N \backslash \{\mathbf{n}\} \cdot M \backslash \{\mathbf{m}\}; \Theta \vdash S \Leftrightarrow \Omega}{\Psi \cdot N \cdot M; \Theta \cdot \{\mathbf{n}, \mathbf{m}\} \vdash \mathbf{n}[P] \mid \mathbf{m}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \epsilon \cdot \mathbf{m} : \epsilon} \\ \frac{\mathbf{s}, \mathbf{r} \in N \quad \Psi \cdot N; \Theta \vdash \mathbf{s}[P] \mid \mathbf{r}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \epsilon \cdot \mathbf{m} : \epsilon}{\Psi \cdot N; \Theta \vdash \mathbf{s}[a!e.P] \mid \mathbf{r}[a?e.Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{s} : a!e.\pi \cdot \mathbf{r} : a?e.\Phi} \\ \frac{\mathbf{m}, \mathbf{n} \in N \quad (\mathbf{m}[P] \mid \mathbf{n}[Q]) \updownarrow \quad \Psi \cdot N; \Theta \vdash \mathbf{m}[P] \mid \mathbf{n}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{m} : \pi}{\Psi \cdot N; \Theta \vdash \mathbf{m}[P+P'] \mid \mathbf{n}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{m} : \pi} \\ \frac{\mathbf{n}, \mathbf{m} \in N \quad (\mathbf{n}[P \oplus P'] \mid \mathbf{m}[Q]) \updownarrow \quad \Omega \times \Omega'}{\Psi \cdot N; \Theta \vdash \mathbf{n}[P] \mid \mathbf{m}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \pi} \\ \frac{\Psi \cdot N; \Theta \vdash \mathbf{n}[P] \mid \mathbf{m}[Q] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \pi}{\Psi \cdot N; \Theta \vdash \mathbf{n}[P'] \mid \mathbf{m}[Q] \mid S \Leftrightarrow \Omega' \cdot \mathbf{n} : \Phi} \\ \frac{[ax]}{\{\varnothing\}; \varnothing \vdash \mathbf{0} \Leftrightarrow \varnothing} \qquad [0] \quad \frac{\Psi \backslash \mathbf{n}; \Theta \vdash S \Leftrightarrow \Omega}{\Psi; \Theta \vdash \mathbf{n}[0] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \Phi} \\ \frac{[ax]}{\{\varnothing\}; \varnothing \vdash \mathbf{0} \Leftrightarrow \varnothing} \qquad \frac{[ax]}{\Psi; \Theta \vdash \mathbf{n}[P] \mid S \Leftrightarrow \Omega \cdot \mathbf{n} : \epsilon} \\ \frac{\mathbf{r} \in N \quad \Psi \cdot N; \Theta \vdash \mathbf{r}[P] \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : \pi \cdot \mathbf{a} : \rho}{\Psi \cdot N; \Theta \vdash \mathbf{r}[a?e.P] \mid a : \mathbf{e} \cdot \rho \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : a?e.\pi \cdot \mathbf{a} : \mathbf{e} \cdot \rho} \\ \frac{[ax]}{\Psi \cdot N; \Theta \vdash \mathbf{r}[a?e.P] \mid a : \mathbf{e} \cdot \rho \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : a?e.\pi \cdot \mathbf{a} : \mathbf{e} \cdot \rho} \\ \frac{[ax]}{\Psi \cdot N; \Theta \vdash \mathbf{r}[a?e.P] \mid a : \mathbf{e} \cdot \rho \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : a?e.\pi \cdot \mathbf{a} : \mathbf{e} \cdot \rho} \\ \frac{[ax]}{\Psi \cdot N; \Theta \vdash \mathbf{r}[a?e.P] \mid a : \mathbf{e} \cdot \rho \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : a?e.\pi \cdot \mathbf{a} : \mathbf{e} \cdot \rho} \\ \frac{[ax]}{\Psi \cdot N; \Theta \vdash \mathbf{r}[a?e.P] \mid a : \mathbf{e} \cdot \rho \mid S \Leftrightarrow \Omega \cdot \mathbf{r} : a?e.\pi \cdot \mathbf{a} : \mathbf{e} \cdot \rho}$$

Fig. 3. Splitting Systems.

Rule [ax] terminates a derivation (all environments emptied) while [0] completes the split of a process (abusing notation, $\Psi \setminus n$ denotes the removal of n from any $N \in \Psi$).

Rule [rem] marks a process to be split when it cannot interact with anyone in S. The premise $P \not\equiv \mathbf{0}$ allows to differentiates a process which terminates after the split, from others which terminate before. In the latter case, rule $[\mathbf{0}]$ is to be used.

Rule [q] records in Ω interactions with non-empty queues.

We now define a (partial) function split which splits a system into two parts.

Definition 3 (split($_$)). Let Ψ ; $\Theta \vdash S \Leftrightarrow \Omega$ be a coherent judgement. Define split(S) = (S_1, S_2) where

```
- \forall n \in \mathcal{P}(S). S_1(n) = S(n) - \Omega(n) \text{ and } S_2(n) = S(n) \% \Omega(n)
- \forall a \in \mathcal{C}(S). S_1(a) = \Omega(a) \text{ and } S_2(a) = S(a) \setminus \Omega(a)
```

if
$$S(n) \% \Omega(n) \neq \bot$$
 for all $n \in \mathcal{P}(S)$, *and* $split(S) = \bot$ *otherwise.*

The auxiliay funtions $_-$ and $_\%$ used in Def. 3 are defined in Appendix A.3; we give here their intuitive description. Let $n \in \mathcal{P}(S)$, and Ψ ; $\Theta \vdash S \approx \Omega$ be a coherent judgement. Function $S(n) - \Omega(n)$ returns the "first part" of the split of n, that is the longest common prefix of S(n) and S(n), while S(n) S(n) are partial and returns the the remaining part of the behaviour of S(n) after S(n).

Example 4. Taking S_{BS} as in § 1, we have $split(S_{BS}) = (S_1, S_2)$ such that

$$\begin{array}{ll} S_1(\mathbf{b_1}) = t_1! \texttt{order}.p_1? \texttt{price} & S_2(\mathbf{b_1}) = r? \texttt{price}.(c_1!.t_1! \texttt{addr} \oplus c_2!.no_1!) \\ S_1(\mathbf{s_i}) = t_i? \texttt{order}.p_i! \texttt{price} & S_2(\mathbf{s_i}) = t_i? \texttt{addr} + no_i? \end{array}$$

 \Diamond

Note that $\{\{b_1, s_1\}, \{b_2, s_2\}\}; \{\{b_1, b_2\}\} \vdash S_{BS} \approx \Omega$ is coherent.

4.3 Properties of Synthesised Global Type

Progress and safety. If a system is typable, then it will either terminate or be able to make further transitions (e.g. if there are recursive processes).

Theorem 5. If $A : \circ : C \vdash S \triangleright G$ then $S \longrightarrow S'$, or $\forall n \in \mathcal{P}(S) . S(n) = \mathbf{0}$, or $S \stackrel{\checkmark}{\longrightarrow} .$

Let us add the rule [ERROR] below to the semantics given in § 2.

$$[ERROR] \xrightarrow{S} \xrightarrow{a'e'} S' \qquad T \xrightarrow{e \cdot a} T'$$

$$S \mid T \longrightarrow error$$

$$e \neq e'$$

Theorem 6. If A; \circ ; $C \vdash S \triangleright G$, then S is race free and $S \Longrightarrow \longrightarrow \text{error}$ is not possible. The proofs of Theorems 5 and 6 are by contradiction, using Theorem 4.

Behavioural equivalences. We show that there is a correspondence between the original system and the projections of its global type. First, let us introduce two relations.

Definition 3 (\lesssim and \approx) $P \lesssim Q$ if and only if $Q \xrightarrow{\alpha} Q'$ then there is P' such that $P \xrightarrow{\alpha} P'$. Also, $S \approx T$ iff whenever $S \xrightarrow{\alpha} S'$ then $T \xrightarrow{\alpha} T'$ and $S' \approx T'$; and whenever $T \xrightarrow{\alpha} T'$ then $S \xrightarrow{\alpha} S'$ and $S' \approx T'$ where $\alpha \in \{n : a!e, n : a?e, \sqrt{}\}$.

The behaviour of a participant in S is a simulation of the projection of a synthesised global type from S onto this participant. Intuitively, the other direction is lost due to rule [+], indeed external choice branches which are never chosen are not "recorded" in the synthesised global type.

Lemma 1. If
$$A : \circ : C \vdash S \triangleright \mathcal{G}$$
 then $\forall n \in S . \mathcal{G} \mid_n \lesssim S(n)$.

The proof is by case analysis on the transitions of *S*, using Theorem 4.

Since the branches that are not recorded in a synthesised global type are only those which are never chosen, we have the following result.

Theorem 7. If
$$A : \circ : C \vdash S \triangleright G$$
 then $\prod_{n \in \mathcal{P}(S)} n[\mathcal{G}|_n] \approx S$.

The proof is by case analysis on the transitions of S, using Theorem 4 and Lemma 1.

Our completeness result shows that every well-formed and projectable global type is inhabited by the system consisting of the parallel composition of all its projections.

Theorem 8. If $\bullet \vdash \mathcal{G}$ and \mathcal{G} is projectable, then there is $\mathcal{G}' \equiv \mathcal{G}$ such that $A; \Gamma; C \vdash \prod_{\mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{n}[\mathcal{G}|_{\mathbf{n}}] \blacktriangleright \mathcal{G}'$.

The proof is by induction on the structure of (well-formed) G.

5 Concluding Remarks and Related Work

We presented a typing system that, under some conditions, permits to synthesise a choreography (represented as global type) from a set of end-point types (represented as local types). The synthesised global type is unique and well-formed; moreover, its projections are equivalent to the original local session types. We have shown safety and progress properties for the local session types. Finally, the derivatives of local types which form a choreography can also be assigned a global type (subject reduction).

Related work. A bottom-up approach to build choreographies is studied in [11]; this work relies on global and local types, but uses local and global graphs. A local graph is similar to a local type while a global graph is a disjoint union of family of local graphs. We contend that global types are more suitable than global graphs to represent choregraphies; in fact, differently from the approach in [11], our work allows us to reuse most of the theories and techniques based on multiparty global types.

Our work lies on the boundary between theories based on *global types* (e.g. [1, 5, 7, 9]) and the ones based on the *conversation types* [3]. Our work relies on the formalism of global types, but uses it the other way around. We start from local types and construct a global type. We have discussed the key elements of the global types in § 3.

Conversation types [3] abandon global views of distributed interactions in favour of a more flexible type structure allowing participants to dynamically join and leave sessions. The approach in [6] fills the gap between the theories based on session types and those based on behavioural contracts [4] (where the behaviour of a program is approximated by some term in a process algebra). We are also inspired from [12], where session types are viewed as CCS-like "projections" of process behaviours. The approach of considering local types as processes is similar to ours. However, the theory of [12] is based on a testing approach. The *connectedness* conditions for a choreography given in [2] is similar to our notion of *well-formed* global type.

Future work. We aim to extend the framework so that global types can be constructed from session types which features name passing and restriction. We also plan to refine the theory and use it in a methodology so that if a choreography cannot be synthesised, the designers are given indications on why this has failed. Finally, we are considering implementing an algorithm from the rules of Fig. 2 and Fig. 3, and integrate it in an existing tool [10] implementing the framework from [1].

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A Additional Definitions

In this section, we give the definitions of the accessory functions used in the main sections of the paper.

A.1 Linearity

Definition 4 $(\mathcal{T}(_{-}))$.

The function $\mathcal{T}(\cdot)$ returns a channel environment corresponding to a global type.

A.2 Projections

Definition 5 ($_ \uplus _$).

$$P \uplus Q = \begin{cases} P + Q, & \text{if } P = \sum_{i \in I} a_i ? \mathbf{e_i}.P_i' \text{ and } Q = \sum_{j \in J} a_j ? \mathbf{e_i}.Q_j' \\ & \text{and } \forall i \in I. \forall j \in J. a_i \neq a_j \text{ and } I, J \neq \varnothing \\ P \oplus Q, & \text{if } P = \bigoplus_{i \in I} a_i ! \mathbf{e_i}.P_i' \text{ and } Q = \bigoplus_{j \in J} a_j ! \mathbf{e_i}.Q_j' \\ & \text{and } \forall i \in I. \forall j \in J. a_i \neq a_j \text{ and } I, J \neq \varnothing \\ a? \langle \mathbf{e} \rangle.(P' \uplus Q'), & \text{if } P = a? \langle \mathbf{e} \rangle.P' \text{ and } Q = a? \langle \mathbf{e} \rangle.Q' & ? \in \{!,?\} \\ P, & \text{if } P \equiv Q \\ \bot, & \text{otherwise} \end{cases}$$

The function merges the behaviour of a participant in different choice branches. In the first two cases, it merges two guarded internal (resp. external) choices, if their sets of guard channels are disjoint. In the third case, the function merges the continuation of both processes, if both are guarded by the same prefix. Note that it is a partial function, e.g. it might be the case that a participant behaves differently in two branches without being aware of which branch was chosen, in which case the projection of that participant is undefined.

Definition 6 (Substitution). The substitution P[Q/R], where R is 0.

$$P[Q/R] = \begin{cases} a!e.(P'[Q/R]) & \text{if } P = a!e.P' \\ a?e.(P'[Q/R]) & \text{if } P = a?e.P' \\ P_1[Q/R] \oplus P_2[Q/R] & \text{if } P = P_1 \oplus P_2 \\ P_1[Q/R] + P_2[Q/R] & \text{if } P = P_1 + P_2 \\ \mu x.(P'[Q/R]) & \text{if } P = \mu x.P' \\ Q & \text{if } P = R = \mathbf{0} \\ P & \text{otherwise} \end{cases}$$

Substitution is used in the projection map.

A.3 Splitting Systems

Omitted rules in Fig. 3:

$$[com] \frac{\Psi; \Theta \vdash S' \mid S \Leftrightarrow \Omega}{\Psi; \Theta \vdash S \mid S' \Leftrightarrow \Omega} \qquad [com] \frac{\Psi; \Theta \vdash (S \mid T) \mid T' \Leftrightarrow \Omega}{\Psi; \Theta \vdash S \mid (T \mid T') \Leftrightarrow \Omega}$$

Definition 7 ($_ \simeq _$). $\Omega \simeq \Omega'$ holds if and only if $\forall n \in \mathcal{P}(\Omega) \cup \mathcal{P}(\Omega')$ either

$$\Omega(\mathbf{n}) \equiv \Omega'(\mathbf{n}) \quad or \quad (\Omega(\mathbf{n}) \equiv \sum_{i \in I} a_i ? \mathbf{e_i}.P_j \ and \ \Omega'(\mathbf{n}) \equiv \sum_{j \in j} a_j ? \mathbf{e_j}.P_j)$$

where $\forall i \in I. \forall j \in J. a_i \neq a_j$.

The boolean function $_ \times _$ holds only if two maps Ω differ wrt external choice.

Definition 8 ($_{\perp} \sqcup _{-}$).

$$\Omega = \Omega_0 \sqcup \Omega_1$$
 is defined only if $\Omega_0 \simeq \Omega_1$ holds, in which case

$$\forall \mathtt{n} \in \mathscr{P}(\Omega_0) \cup \mathscr{P}(\Omega_1)$$
 . $\Omega(\mathtt{n}) = \mathtt{merge}(\Omega_0(\mathtt{n}), \Omega_1(\mathtt{n}))$

where

$$\mathtt{merge}(P,Q) egin{cases} P & \textit{if } P \equiv Q \\ P+Q & \textit{if } P \equiv \sum_{i \in I} a_i ? \mathtt{e_i}.P_j \textit{ and } Q \equiv \sum_{j \in j} a_j ? \mathtt{e_j}.P_j \end{cases}$$

The function $_ \sqcup _$ merges two Ω maps, if $_ \asymp _$ holds.

Definition 9 $(_- _-)$.

$$P-Q = \begin{cases} \bigoplus_{(k,k') \in K} a_k ! \mathbf{e_k}.(P_k - Q_{k'}) & \text{if } P \equiv \bigoplus_{i \in I} a_i ! \mathbf{e_i}.P_i \text{ and } Q \equiv \bigoplus_{j \in J} a_j ! \mathbf{e_j}.Q_j \\ \sum_{(k,k') \in K} a_k ? \mathbf{e_k}.(P_k - Q_{k'}) + \sum_{n \in N} a_n ? \mathbf{e_n}.P_n & \text{if } P \equiv \sum_{i \in I} a_i ? \mathbf{e_i}.P_i \text{ and } Q \equiv \sum_{j \in J} a_j ? \mathbf{e_j}.Q_j \\ \mathbf{0}, & \text{if } P \equiv \mathbf{0} \text{ and } Q \equiv \mathbf{0}, \text{ or } Q = \mathbf{\varepsilon}, \end{cases}$$

where
$$K = \{(i, j) \in I \times J \mid a_i = a_j\}$$
 and $N = \{n \in I \mid \forall j \in J. a_n \neq a_j\}.$

The function $_-$ computes the first part of a split behaviour, given the original behaviour, e.g. S(n) and its prefix in $\Omega(n)$. The case for external choices keeps the branches from the original behaviour which do not appear in $\Omega(n)$. The rationale is that even if some branches are never "chosen" in the system, they might still induce e.g. races and therefore they need to be taken into account in the main system.

Definition 10 (-%).

$$P\%Q = \begin{cases} P_0 & \text{if } P \equiv \bigoplus_{i \in I} a_i! \mathbf{e_i}.P_i \text{ and } Q \equiv \bigoplus_{j \in J} a_j! \mathbf{e_j}.Q_j \\ P_0 & \text{if } P \equiv \sum_{i \in I} a_i? \mathbf{e_i}.P_i \text{ and } Q \equiv \sum_{j \in J} a_j? \mathbf{e_j}.Q_j \\ \mathbf{0}, & \text{if } P \equiv \mathbf{0} \text{ and } Q \equiv \mathbf{0}, \\ P, & \text{if } Q = \varepsilon, \\ \bot, & \text{otherwise} \end{cases}$$

with P_0 defined as follows

$$P_{0} = \begin{cases} P_{i} \% Q_{j} \text{ with } (i, j) \in K & \text{if } \forall (i, j)(k, l) \in K. P_{i} \% Q_{j} \equiv P_{k} \% Q_{l} \\ \bot & \text{otherwise} \end{cases}$$

where
$$K = \{(i, j) \in I \times J \mid a_i = a_j\}$$

The function $_{\mathcal{N}_{-}}$ computes the second part of a split behaviour. Essentially, it returns the "rest" of a behaviour after $\Omega(n)$. Note that if $\Omega(n)$ is a branching behaviour, then the rest must be the same in all branches (since only *one* behaviour can be returned), e.g. $S(n) \% \Omega(n) = \bot$, if

$$S(n) = a!e.b!e \oplus c!e.d!e$$
 and $\Omega(n) = a!e.\varepsilon \oplus c!e.\varepsilon$

A.4 Results

Definition 11. There is a race in S if and only if there is $S \Longrightarrow S'$ such that $\exists a \in \overline{\mathbb{R}}(S')$ such that

$$\exists \{n,m\} \in \mathcal{P}(S) : a \in R(S(n)) \text{ and } a \in R(S(m)) \text{ or } \overline{a} \in R(S(n)) \text{ and } \overline{a} \in R(S(m))$$

B Definitions Used in the Proofs

Definition 12 (Connected - R).

- Two participants are connected in a system S if $(n,m) \in R_S$

$$(\mathtt{n},\mathtt{m}) \in R_S \iff \mathcal{C}(S(\mathtt{n})) \cap \mathcal{C}(S(\mathtt{m})) \neq \varnothing \ or \ \exists n'. \ (\mathtt{n},\mathtt{n}') \in R_S \land (\mathtt{n}',\mathtt{m}) \in R_S$$

- Two participants are connected in a global type G if $(n,m) \in R_G$

$$(\mathtt{n},\mathtt{m}) \in \mathit{R}_{\mathit{G}} \iff \mathcal{C}(\mathit{G} \, |_{\mathtt{m}}) \cap \mathcal{C}(\mathit{G} \, |_{\mathtt{m}}) \neq \varnothing \; \mathit{or} \; \exists \mathit{n}'. \, (\mathtt{n},\mathtt{n}') \in \mathit{R}_{\mathit{G}} \land (\mathtt{n}',\mathtt{m}) \in \mathit{R}_{\mathit{G}}$$

Definition 13 (Projections with queues).

$$\mathcal{G} \mid_{a} = \begin{cases} \mathbf{e} \cdot \mathcal{G}' \mid_{a}, & \text{if } \mathcal{G} = * \to \mathtt{a} : r \langle \mathtt{e} \rangle.\mathcal{G}' \\ \mathcal{G}' \mid_{a}, & \text{if } \mathcal{G} = \mathtt{s} \to \mathtt{r} : a \langle \mathtt{e} \rangle.\mathcal{G}' \\ \mathcal{G}_{1} \mid_{a} \uplus \mathcal{G}_{2} \mid_{a}, & \text{if } \mathcal{G} = \mathcal{G}_{1} + \mathcal{G}_{2} \\ \mathcal{G}_{i} \mid_{a}, & \text{if } \mathcal{G} = \mathcal{G}_{1} \mid \mathcal{G}_{2} \text{ and } a \notin \mathcal{C}(\mathcal{G}_{j}), i \neq j \in \{1, 2\} \\ \mathcal{G}_{1} \mid_{a} \cdot \mathcal{G}_{2} \mid_{a}, & \text{if } \mathcal{G} = \mathcal{G}_{1} ; \mathcal{G}_{2} \\ \parallel, & \text{if } * \to \mathtt{r} : a \langle \mathtt{e} \rangle \notin \mathcal{G} \\ \perp, & \text{otherwise} \end{cases}$$

Definition 14 (_\etu_).

$$\rho_1 \uplus \rho_2 = \begin{cases} \rho_1, & \textit{if } \rho_1 = \rho_2 \\ \bot, & \textit{otherwise} \end{cases}$$

C Proofs for Theorem 1 (Decidability)

Typability is decidable.

Proof. The typing systems is decidable from the fact that the ready set of a system, the number of participants, and their behaviours are finite. Here, we show that the number of behaviour unfoldings needed to type a system is also finite.

Let (non-recursive) behaviour context $C[_]$ defined as follows

$$C[_] ::= \bigoplus_{i \in I} a_i ! e_i . C_i[_] \mid \sum_{i \in I} a_i ? e_i . C_i[_] \mid [_] \mid \mu \mathbf{x}.P \mid \mathbf{x}$$

and (possibly recursive) behaviour context $C'[\cdot]$ defined as follows

$$C'[_] ::= \bigoplus_{i \in I} a_i ! \mathbf{e_i} . C'_i[_] \mid \sum_{i \in I} a_i ? \mathbf{e_i} . C'_i[_] \mid [_] \mid \mu \mathbf{x} . C'[_] \mid \mathbf{x}$$

Let $unfold_i(P)$ be the i^{th} unfolding of P, defined as follows

$$\begin{split} \text{unfold}_i(P) &= \text{unfold}_1(\text{unfold}_{i-1}(P)) \quad i > 1 \\ \text{unfold}_1(P) &= \begin{cases} \bigoplus_{i \in I} a_i ! \texttt{e}_{\mathtt{i}}. \texttt{unfold}_1(P_i) & \text{if } P \equiv \bigoplus_{i \in I} a_i ! \texttt{e}_{\mathtt{i}}. P_i \\ \sum_{i \in I} a_i ? \texttt{e}_{\mathtt{i}}. \texttt{unfold}_1(P_i) & \text{if } P \equiv \sum_{i \in I} a_i ? \texttt{e}_{\mathtt{i}}. P_i \\ P[\mu \mathbf{x}. P/\mathbf{x}] & \text{if } P \equiv \mu \mathbf{x}. P \end{cases}$$

The need for unfolding occurs whenever a recursive participants interact with another participant, while not all the participants feature directly a recursive behaviour. In this case, we need to unfold some participants (rule [eq]), then use rules $[\oplus]$, [+], $[\cdot]$, and/or [0] until rule $[\mu]$ is applicable. Note that rules $[\cdot;]$, $[\cdot]$ and $[\rho]$ cannot be used under recursion.

Consider the following system

$$S = S_0 \mid S_1$$

where

$$S_0 = \mathbf{n}_1 \left[C_1 \left[\mu \mathbf{x} \cdot C_1' \left[\mathbf{x} \right] \right] \right] \mid \dots \mid \mathbf{n}_i \left[C_i \left[\mu \mathbf{x} \cdot C_i' \left[\mathbf{x} \right] \right] \right]$$
 (C.1)

$$S_1 = |\mathbf{n}_{j+1}[\mu \mathbf{x}.C'_{j+1}[\mathbf{x}]] | \dots | \mathbf{n}_{k}[\mu \mathbf{x}.C'_{j+k}[\mathbf{x}]]$$
 (C.2)

 $S \updownarrow$, $S_0 \updownarrow$, and there is exactly one $n \in \mathcal{P}(S)$ such that

$$S \equiv \mathbf{n}[S(\mathbf{n})] \mid T$$
 and $T \updownarrow$

Let C_i for $j \le i \le j + k$ be the empty context, we can rewrite S such that

$$S \equiv \prod_{i \in I} \mathbf{n_i} \left[C_i \left[\mu \mathbf{x} . C_i' \left[\mathbf{x} \right] \right] \right] \qquad I = \{ i \mid 1 \le i \le j + k \}$$

Given S as above, we define $|i|_{\mathbf{x}}$ to be the smallest k such that $C_i[\mathbf{x}]$ is a sub-tree of $\mathrm{unfold}_k(C_i'[\mathbf{x}])$, $|i|_{\mathbf{x}} = \bot$ if there is no such k. Note that $|i|_{\mathbf{x}}$ must be smaller than the length of $C_i[\mathbf{0}]$ (since recursion is guarded). If one $|i|_{\mathbf{x}}$ is not defined, then S is not typable.

We also defined $M = \max\{|i|_{\mathbf{x}} \mid i \in I\}$, and $K(i) = M - |i|_{\mathbf{x}}$. We can unfold each behaviour so that all of them are unfolded to the same extent, let

$$S^* \equiv \prod_{i \in I} \mathtt{n_i} \left[\mathtt{unfold}_{K(i)} \left(C_i \left[\mu \mathbf{x}.C_i' \left[\mathbf{x}
ight]
ight)
ight]$$

We show that

$$A; \Gamma; C \vdash S^* \blacktriangleright G \Rightarrow A; \Gamma; C \vdash S \blacktriangleright G$$

By definition of $unfold(_-)$, and since $C[_-]$ does not contain recursive definition, we have

$$S^* \equiv \prod_{i \in I} \mathbf{n_i} \left[C_i \left[\mathsf{unfold}_{K(i)}(\mu \mathbf{x}.C_i'[\mathbf{x}]) \right] \right] \tag{C.3}$$

$$\equiv \prod_{i \in I} \mathbf{n_i} \left[C_i \left[C_i' \left[C_i' \left[\dots C_i' \left[\mu \mathbf{x} . C_i' \left[\mathbf{x} \right] \right] \right] \dots \right] \right] \right]$$
 (C.4)

Where in (C.4), $C_i'[\cdot]$ has been unfolded K(i) times. It is easy to see that S^* is typable if

$$\prod_{i \in I} \mathbf{n_i} \left[C_i \left[C_i' \left[C_i' \left[\dots C_i' \left[\mathbf{0} \right] \right] \dots \right] \right] \right] \quad \text{and} \quad \prod_{i \in I} \mathbf{n_i} \left[C_i' \left[\mathbf{0} \right] \right]$$

are typable themselves, note that rule [eq] does not need to be used to unfold the left-hand side system, since it is recursion free; and there is exactly one recursion less in the right hand side.

In fact, if we would unfold (C.3) once more, we would not get more chances to type S^* since it would amount to add the sub-derivation of the right-hand side to one of the left-hand side.

D Proofs for Lemma 2 (Uniqueness)

If
$$A$$
; Γ ; $C \vdash S \triangleright G$ and A ; Γ ; $C \vdash S \triangleright G'$ then $G \equiv G'$.

Proof. The proof is by case analysis. We show that every time one rule from Fig.2 is applicable, either no other rule is applicable, or the derivation produces an equivalent global type.

Due to their syntactic restriction and the condition $S\mathcal{J}$, the cases for rules [.], [\oplus], and [μ] are straightforward. In addition, the cases for rule [+] is easy since it does not affect G. The cases for rules [x] and [θ] are trivial.

The case for rule [eq] follows from the fact that associativity and commutativity in S do not affect G. In addition, if one unfold behaviour once more, we have the result since $\mu\chi$. $G \equiv G[\mu\chi$. $G/\chi]$.

The interesting part of the proof is to show that $[\cdot]$ and $[\cdot]$ are mutually exclusive. In fact, if $[\cdot]$ is applicable, $[\cdot]$ cannot be used because \circledast and \leftrightarrow_{Θ} must be total on Ψ and $\Theta \neq \varnothing$. If a system S could be separated in two sub-system by $[\cdot]$, these two conditions could not hold. If $[\cdot]$ is applicable, it means that it is not possible to split the participant in two totally independent sub-systems, and therefore $[\cdot]$ is not applicable. Finally, observe that by Lemma 16 the split is unique.

E Proofs for Theorem 3 (Well-formedness)

If A; Γ ; $C \vdash S \triangleright G$ then $\bullet \vdash G$ and G is projectable.

Proof. The proof is by induction on the derivation A; Γ ; $C \vdash S \triangleright G$. We make a case analysis on the last rule used.

Case [.]. We have

$$G = s \rightarrow r : a \langle e \rangle . G'$$
 and $S = s[a!e.P] \mid r[a?e.Q] \mid S'$

- WF. We show that we have

$$\forall n_1 \to n_2 : = R(G').\{s,r\} \cap \{n_1,n_2\} \neq \emptyset$$

by contradiction. By IH, we know that

$$\{a\} \cup A; \Gamma; C \prec s \rightarrow r : a \vdash s[P] \mid r[Q] \mid S' \triangleright G'$$

If we had $\mathcal{G}' \equiv \mathbf{n_1} \to \mathbf{n_2}$: $b\langle e' \rangle \cdot \mathcal{G}_0 + \mathcal{G}_1$, with $\mathbf{n_i} \neq \mathbf{s}$ and $\mathbf{n_i} \neq \mathbf{r}$ with $i \in \{1, 2\}$, then we would have

$$S' \equiv n_1[b!e'.P'_0 \oplus P'_1]n_2[b?e'.Q'_0 + Q'_1] \mid S''$$

By Lemma 25, the result above and since $C \prec s \rightarrow r : a$ is defined, we have $\bullet \vdash G$.

- *Projection*. By Def. 1, we have that $G \mid_{s} = a!e.G' \mid_{s}$, $G \mid_{r} = a?e.G' \mid_{r}$, and $G \mid_{n} = G' \mid_{n}$, for $s \neq n \neq r$.

Case [⊕]. We have

$$G = G_0 + G_1$$
 and $S = s[P \oplus Q] \mid S'$

 WF. Observe that we have that all the guard channels are disjoint by definition of processes. We have to show that

$$\forall (\mathtt{n}_1,\mathtt{n}_2) \in \mathtt{R}(\mathcal{G}_0). \forall (\mathtt{n}_1',\mathtt{n}_2') \in \mathtt{R}(\mathcal{G}_1).\, \mathtt{n}_1 = \mathtt{n}_1'$$

i.e. that for all prefixes in G_0 and G_1 , s is the sender. In other words, the participant who makes the internal choice must be the same in all the branches. If that was not the case, we would have

$$S' \equiv n_1[b!e.Q_0 \oplus Q_1] \mid n_2[b?e.Q_2 + Q_3] \mid S''$$

By Lemma 25, the result above and $C \vdash G_0$ and $C \vdash G_1$ by IH, we have $\bullet \vdash G$.

- Projection. We have to prove that

$$\forall n \in \mathcal{P}(\mathcal{G}_0 + \mathcal{G}_1). \mathcal{G}_0 \mid_n \uplus \mathcal{G}_1 \mid_n \neq \bot$$

By IH, we have that both $\mathcal{G}_0 \mid_n$ and $\mathcal{G}_1 \mid_n$ exist. There are two cases where the projection does not exist $(i \neq j \in \{0,1\})$ in the following:

- $\mathcal{G}_i|_{\mathbf{n}} = a! \mathbf{e}.P$ and $\mathcal{G}_j|_{\mathbf{n}} = b? \mathbf{e}'.Q$. This cannot happen by definition of behaviours, i.e. such projections could only come from behaviours of the form $(a! \mathbf{e}.P) \oplus (b? \mathbf{e}'.Q)$, which is not syntactically correct.
- $G_i \mid_{n} = \sum_{k \in K} a_k ?e_k.P_k$ and $G_j \mid_{n} = \sum_{k' \in K'} a_{k'} ?e_{k'}.P_{k'}$, with $a_k = a_{k'}$ for some k and k'. This cannot happen by definition of behaviours, i.e. such projections could only come from behaviours of the form $a?e.P_0 + a?e.P_1$ which is not a syntactically correct behaviour. The same reasoning applies for internal choice.

Note that we have $G|_{s} = G_0|_{s} \oplus G_1|_{s}$.

Case [+]

- WF. By induction hypothesis, we have $C \vdash G$ and by Lemma 25, we have $\bullet \vdash G$.
- Projection. By induction hypothesis.

Case [1] We have

$$G = G_0 \mid G_1$$
 and $S = S_0 \mid S_1$

- WF. We have to show that

$$\mathcal{P}(\mathcal{G}_0) \cap \mathcal{P}(\mathcal{G}_1) = \varnothing$$
 and $\mathcal{C}(\mathcal{G}_0) \cap \mathcal{C}(\mathcal{G}_1) = \varnothing$

By definition of systems, we know that there cannot be two participants of the same name in a same system, since $S \mid S'$ is a system we have that

$$\mathcal{P}(S_0) \cap \mathcal{P}(S_1) = \emptyset$$

and by Lemma 27, we have $\mathcal{P}(\mathcal{G}_0) \cap \mathcal{P}(\mathcal{G}_1) = \emptyset$. By Lemma 29, the premise $A_0 \cap A_1$, we have $\mathcal{C}(\mathcal{G}_0) \cap \mathcal{C}(\mathcal{G}_1) = \emptyset$. We have $C \vdash \mathcal{G}_0$ and $C \vdash \mathcal{G}_1$ by assumption, with the result above and Lemma 25, we have $\bullet \vdash \mathcal{G}$.

- *Projection.* For all $n \in \mathcal{P}(\mathcal{G})$, $\mathcal{G}|_{n}$ is defined by IH and since $\mathcal{P}(\mathcal{G}_{0}) \cap \mathcal{P}(\mathcal{G}_{1}) = \emptyset$.

Case [:]. We have

$$G = G_0$$
; G_1 and $C \prec T(G_0)$ and $split(S) = (S_0, S_1)$

- WF. We have to show that

$$\forall n_1 \to n_2 : \ = \mathbb{R}(G_1) . \exists N_1 \neq N_2 \in \mathbb{F}_0(G_0) . n_1 \in N_1 \land n_2 \in N_2$$
 (E.1)

and

$$\forall N \in F_{P}(G_{0}) . \exists N' \in F_{P}(G_{1}) . N \cap N' \neq \emptyset$$
(E.2)

We know that $\mathtt{split}(S) = (S_0, S_1)$ therefore there is $\Psi; \Theta \vdash S \Leftrightarrow \Omega$ coherent. We show (E.1) first. We first show that

$$s \rightarrow r : a \in R(G_1) \iff \{s,r\} \in \Theta$$

We start with

$$n_{\mathtt{i}} \rightarrow n_{\mathtt{j}} : \underline{\ } \in \mathtt{R}(\mathcal{G}_{1}) \iff S_{1}(n_{\mathtt{i}}) \mid S_{1}(n_{\mathtt{j}}) \updownarrow$$

(\Rightarrow) If $n_i \to n_j : _- \in R(\mathcal{G}_1)$ then we must have $\mathcal{G}_1 = ((n_i \to n_j : a \langle e \rangle. \mathcal{G}_2 + \mathcal{G}_3) \mid \mathcal{G}_4)$; \mathcal{G}_5 (by definition of R). And by Lemma 26, this implies that $S_1(n_i) = a!e.P \oplus P'$ and $S_1(n_j) = a?e.Q + Q'$. Thus we have $S_1(n_i) \mid S_1(n_j) \updownarrow$.

 (\Leftarrow) If $S_1(n_i) \mid S_1(n_j) \updownarrow$, we must have $S_1(n_i) = a!e.P \oplus P'$ and $S_1(n_j) = a?e.Q + Q'$ and since G_1 is well-formed by IH, we have the required result by Lemma 26 and the definition of R.

Now, let us show that

$$S_1(\mathbf{n}_i) \mid S_1(\mathbf{n}_i) \uparrow \iff \{\mathbf{n}_i, \mathbf{n}_i\} \in \Theta$$
 (E.3)

(\Rightarrow) Since the processes interact, we know that they are \neq **0**. Moreover, since they appear in the second part of the split, the following rule must be applied so that $S_1(\mathbf{n_i}) = S(\mathbf{n_i}) \% \Omega(\mathbf{n_i})$ and $S_1(\mathbf{n_j}) = S(\mathbf{n_j}) \% \Omega(\mathbf{n_j})$ ($\neq \perp$ by assumption) with $\Omega(\mathbf{n_i})$ and $\Omega(\mathbf{n_j})$ ending with ϵ .

$$\begin{array}{c} \mathbf{n_i} \in N, \mathbf{n_j} \in M \\ \mathbf{n_i}[S_1(\mathbf{n_i})] \mid \mathbf{n_j}[S_1(\mathbf{n_j})] \updownarrow \qquad \Psi \cdot N \backslash \{\mathbf{n}\} \cdot M \backslash \{\mathbf{m}\}; \Theta \, \vdash \, S \, \Leftrightarrow \, \Omega \\ \hline \Psi \cdot N \cdot M; \, \Theta \cdot \{\mathbf{n_i}, \mathbf{n_j}\} \, \vdash \mathbf{n_i}[S_1(\mathbf{n_i})] \mid \mathbf{n_j}[S_1(\mathbf{n_j})] \mid S \, \Leftrightarrow \, \Omega \cdot \mathbf{n_i} : \epsilon \cdot \mathbf{n_j} : \epsilon \end{array}$$

which gives us the expected result. Note that the rule [rem] cannot be applied because the processes interact with each other we do not have $n[P] \mid S \updownarrow$.

 (\Leftarrow) If $\{n_i,n_j\}\in\Theta$ then, the rule $[\epsilon]$ must also be applied (the axiom cannot be reach with $\Theta\neq\varnothing$), since the rule requires that $n_i[S_1(n_i)]\mid n_j[S_1(n_j)]\updownarrow$, we have the expected result.

Let us now show that

$$\forall \{\mathbf{n}_i, \mathbf{n}_i\} \in \Theta . \exists N_i \neq N_i \in F_0(G_0) . \mathbf{n}_i \in N_i \land \mathbf{n}_i \in N_i$$
 (E.4)

First we show that

$$N \in \mathcal{F}_0(\mathcal{G}_0) \Rightarrow \exists ! N' \in \Psi : N \cap N' \neq \emptyset$$
 (E.5)

By Lemma 20, we have that $\forall N \in F_0(\mathcal{G}_0)$. $R_{\mathcal{G}_0}$ is total on N. By Lemma 21, we have that $(n,m) \in R_{\mathcal{G}_0} \Rightarrow (n,m) \in \mathfrak{B}$. By Lemma 12, we have that $(n,m) \in \mathfrak{B} \Rightarrow \{n,m\} \in N \in \Psi$. Therefore we have

$$N \in \mathbb{F}_0(\mathcal{G}_0) \Rightarrow \exists N' \in \Psi : N \cap N' \neq \emptyset$$

and N' is unique since Ψ is a set of pairwise disjoint sets.

We finalise the proof of (E.1), by showing (E.4) by contradiction. If we had

$$\exists \{n_1, n_2\} \in \Theta . \exists N \in F_0(G_0) . \{n_1, n_2\} \in N$$

by Lemma 11, we have that $\exists N_1 \neq N_2 \in \Psi : n_1 \in N_1 \in \Psi \text{ and } n_2 \in N_2 \in \Psi \text{ which is contradiction with (E.5).}$

Let us now show (E.2), i.e.

$$\forall N \in \mathbb{F}_{\mathbb{P}}(\mathcal{G}_0) . \exists N' \in \mathbb{F}_{\mathbb{P}}(\mathcal{G}_1) : N \cap N' \neq \emptyset$$

We first show that

$$F_P(G) = \Psi$$

By definition of F_P, we have that

$$\mathcal{G} = \prod_{N \in \mathtt{F}_{\mathtt{P}}(\mathcal{G})} \mathcal{G}_{N} \quad \text{with } \mathcal{G}_{N} \not\equiv \mathcal{G}_{N}' \ | \ \mathcal{G}_{N}''.$$

Therefore, by Lemma 20, $R_{\mathcal{G}_N}$ is total on $\mathcal{P}(\mathcal{G}_N)$ for each N. By Lemma 21, we have that \circledast is total on each $\mathcal{P}(\mathcal{G}_N)$ as well, and by Lemma 12, we have

$$N \in \Psi \iff \mathcal{P}(\mathcal{G}_N) = N$$

Now, we show that

$$\forall N \in \Psi. \exists N' \in F_P(G_1). N \cap N' \neq \varnothing$$

by contradiction, if we had

$$\exists N \in \Psi.N \cap \mathcal{P}(\mathcal{G}_1) = \emptyset$$

by Lemma 11, we must have $N \neq M \in \Psi$ such that $\{n,m\} \in \Theta$ and $n \in N$ and $m \in M$. Since $n \in N$ is also in Θ , we must have $S_1(n) \mid S_1(m) \updownarrow$ which implies that $S_1(n) \equiv \mathbf{0}$ and therefore $n \in \mathcal{P}(\mathcal{G}_1)$, by Lemma 27. Thus $N \cap \mathcal{P}(\mathcal{G}_1) \neq \emptyset$

From (E.1), (E.2), the fact that by assumption $C \prec T(G_0)$ is defined, and Lemma 25 we have $\bullet \vdash G$.

- *Projection*. By IH, we have that for all $n \mathcal{G}_0 \mid_n$ and $\mathcal{G}_1 \mid_n$ exist, thus we have that $\mathcal{G}_0 \mid_n [\mathcal{G}_1 \mid_n / \mathbf{0}]$ is defined for all n. Note that since $\bullet \vdash \mathcal{G}_0$, we have $\#(F_0(\mathcal{G}_0)) > 1$ (cf. E.1). Therefore, by Lemma 23, $fv(\mathcal{G}_1) = \emptyset$. In addition, by Lemma 22, we have $bv(S) = \emptyset$ (since the split(S) by assumption), and by Lemma 24, $bv(\mathcal{G}_0) = \emptyset$. Thus, every branch in \mathcal{G}_0 ends with $\mathbf{0}$, and all the projections of \mathcal{G}_0 end with $\mathbf{0}$.

Case [µ] We have

$$G = \mu \chi . G_0$$
 and $S = n_1[\mu \mathbf{x}_1 . P_1] \mid \dots \mid n_k[\mu \mathbf{x}_k . P_k]$ and

- WF. We have to show that

$$\chi \in fv(\mathcal{G}') \Rightarrow \#F_0(\mathcal{G}') = 1$$

we can apply Lemma 23 and we have the result directly. Observe that the recursion is prefix guarded since $\exists i, j : n_i[P_i] \mid m_i[P_j] \updownarrow$ holds.

- Projection. By induction hypothesis.

Case [x] We have

$$G = \chi$$
 and $\forall n \in \mathcal{P}(S).S(n) = \mathbf{x}_n$

We have to show that $C \prec C(\chi)$, which follows directly from the premises of rule [x]. **Case** [0] Trivial.

Case [p] We have

$$G = * \rightarrow r : a \langle e \rangle.G'$$
 and $S = r[a?e.Q] \mid a : e \cdot \rho \mid S'$

- WF. We show that we have

$$\forall n_1 \rightarrow n_2 : \underline{\ } \in R(\mathcal{G}') . n_i = r \text{ with } i \in \{1, 2\}$$

by contradiction. By IH, we know that

$$\{a\} \cup A; \Gamma; C \prec s \rightarrow r : a \vdash r[Q] \mid a : \rho \mid S' \blacktriangleright G'$$

If we had $\mathcal{G}' \equiv \mathbf{n_1} \to \mathbf{n_2}$: $b\langle e' \rangle$. $\mathcal{G}_0 + \mathcal{G}_1$, with $\mathbf{n_i} \neq \mathbf{r}$ with $i \in \{1, 2\}$, then we would have

$$S' \equiv n_1[b!e'.P'_0 \oplus P'_1]n_2[b?e'.Q'_0 + Q'_1] \mid S''$$

which is in contradiction with the premise $S' \not\downarrow \!\! 1$.

By Lemma 25, the result above, and since $C \prec * \rightarrow r : a$ is defined, we have $\bullet \vdash G$.

- *Projection*. By Def. 1, we have that $G|_{r} = a$?e. $G'|_{r}$, and $G|_{n} = G'|_{n}$, for $n \neq r$.

F Proofs for Theorem 4 (Subject Reduction)

If
$$A$$
; \circ ; $C \vdash S \triangleright G$, $S \xrightarrow{\lambda} S'$, and $C(\lambda) \notin C$ then A ; \circ ; $C \vdash S' \triangleright G'$

Proof. There are three cases to consider.

Case 1. $S \equiv S_1 \mid S_2 \text{ with } S_1 \xrightarrow{\operatorname{n}[a!e]} S_1' \text{ and } S_2 \xrightarrow{a \cdot e} S_2'$

In this case, S must have the following form

$$S \equiv \mathbf{n}[a!\mathbf{e}.P \oplus P'] \mid a: \mathbf{p} \mid T$$
 and $S' \equiv \mathbf{n}[P] \mid a: \mathbf{p} \cdot \mathbf{e} \mid T$

and by Lemma 2 we have that A; \circ ; $C \vdash S' \triangleright G'$.

Case 2. $S \equiv S_1 \mid S_2 \text{ with } S_1 \xrightarrow{n[a?e]} S_1' \text{ and } S_2 \xrightarrow{e \cdot a} S_2'$ In this case, S must have the following form

$$S = n[a?e.P + P'] \mid a:e \cdot p \mid T$$
 and $S' = n[P] \mid a:p \mid T$

and by Lemma 2 we have that A; \circ ; $C \vdash S' \triangleright G'$

Case 3. $S \xrightarrow{\checkmark} S$ in this case, we have $A ; \circ ; C \vdash S' \blacktriangleright \mathcal{G}'$ trivially since $S' \equiv S$.

Case 4 (unfolding). If $S \equiv n[\mu \mathbf{x}.P] \mid S_0$ and $S' \equiv n[P[\mu \mathbf{x}.P/\mathbf{x}]] \mid S_0$ then $G = \mu \chi . G_0$, and $\forall \mathbf{m} \in \mathcal{P}(S_0) . S_0(\mathbf{m}) \equiv \mu \mathbf{x}.Q$. We have the result by unfolding all the participants in S_0 with rule [eq], so to have $G' = G_0[\mu \chi . G_0/\chi]$.

In the other direction, we have the result by folding all the participants in $\mathcal{P}(S_0)$.

Case 5 (commutativity and associativity). If $S \equiv S'$, then we have the result with rule [eq].

Lemma 2. The following holds:

1.
$$A; \circ; C \vdash s[a!e.P \oplus P'] \mid a: \rho \mid T \blacktriangleright \mathcal{G} \Rightarrow A; \circ; C \vdash s[P] \mid a: \rho \cdot e \mid T \blacktriangleright \mathcal{G}'$$

2. $A; \circ; C \vdash r[a?e.P + P'] \mid a: e \cdot \rho \mid T \blacktriangleright \mathcal{G} \Rightarrow A; \circ; C \vdash r[P] \mid a: \rho \mid T \blacktriangleright \mathcal{G}'$

with $_ \rightarrow _ : a \notin C$ in 1 and 2.

Corollary 1.

$$G \mid_{\mathtt{n}} = (a \mid \mathtt{e}. G_0 \mid_{\mathtt{n}}) \oplus G_1 \mid_{\mathtt{n}} \Rightarrow G' \mid_{\mathtt{n}} = G_0 \mid_{\mathtt{n}}$$

and

$$G \mid_{n} = a$$
?e. $G_0 \mid_{n} \Rightarrow G' \mid_{n} = G_0 \mid_{n}$

Proof. We show that 1 and 2 hold by contradiction.

Case 1. Assume

$$A; \circ; C \vdash s[a!e.P \oplus P'] \mid a : \rho \mid S \blacktriangleright G$$
 is derivable.

$$A; \circ; C \vdash s[P] \mid a : \rho \cdot e \mid S \blacktriangleright G'$$
 is not.

Take $S \equiv r[a?e.Q] \mid S', \rho = []$, and $S' \updownarrow$. We must have the following sub-derivation for G

$$\frac{\vdots}{A; \circ; C \prec s \rightarrow r : a \vdash s[P] \mid a : \rho \mid r[a?e.Q] \mid S' \blacktriangleright G_0} \\
A; \circ; C \vdash s[a!e.P] \mid a : \rho \mid r[a?e.Q] \mid S' \blacktriangleright s \rightarrow r : a\langle e \rangle \cdot G_0}$$
(F.1)

Consider the non-derivable judgement for G'

$$\frac{\bot}{A; \circ; C \prec * \to \mathbf{r} : a \vdash \mathbf{s}[P] \mid a : \rho \mid \mathbf{r}[a?e.Q] \mid S' \blacktriangleright \mathcal{G}'_{0}}
A; \circ; C \vdash \mathbf{s}[P] \mid a : e \mid \mathbf{r}[a?e.Q] \mid S' \blacktriangleright * \to \mathbf{r} : a \langle e \rangle. \mathcal{G}'_{0}}$$
(F.2)

Where the rule $[\rho]$ must be applicable here, since the only difference with the above system is $C \prec * \rightarrow r : a$ which is defined since $_ \rightarrow _ : a \notin C$.

We have a contradiction here since

$$s[P] \mid a : \rho \mid r[a?e.Q] \mid S'$$
 is derivable in (F.1).

while

$$s[P] \mid a : \rho \mid r[a?e.Q] \mid S'$$
 is not derivable in (F.2).

Case 2. Assume

$$A$$
; \circ ; $C \vdash \mathbf{r}[a?e.P+P'] \mid a:e \cdot \rho \mid S \blacktriangleright G$ is derivable.

$$A; \circ; C \vdash r[P] \mid a : \rho \mid S \blacktriangleright G'$$
 is not.

Take S such that $S \updownarrow$. We must have the following sub-derivation for \mathcal{G}

$$\frac{\vdots}{A; \circ; C \prec * \to \mathbf{r} : a \vdash \mathbf{r}[P] \mid a : \rho \mid S \blacktriangleright \mathcal{G}_{0}} \\
\stackrel{[\rho]}{A; \circ; C \vdash \mathbf{r}[a?e.P] \mid a : e \cdot \rho \mid S \blacktriangleright * \to \mathbf{r} : a \langle e \rangle. \mathcal{G}_{0}}$$
(F.3)

This induces a contradiction since we would have the following for G'

$$\frac{\bot}{A; \circ; C \vdash \mathbf{r}[P] \mid a: \rho \mid S \blacktriangleright \mathcal{G}'_{0}}$$
(F.4)

Note that the lack of $* \rightarrow r : a$ in C does not affect since $_ \rightarrow _ : a \notin C$.

G Proofs for Theorem 5 (Progress)

If
$$A$$
; \circ ; $C \vdash S \triangleright G$ then $S \longrightarrow S'$, or $\forall n \in \mathcal{P}(S) . S(n) = \mathbf{0}$, or $S \stackrel{\checkmark}{\longrightarrow}$.

Proof. The proof is by contradiction. If we had $A : \Gamma : C \vdash S \blacktriangleright \mathcal{G}$ and $S \Longrightarrow S'$, with $S' \not \updownarrow$, $S' \neq \mathbf{0}$ and $\exists n \in \mathcal{P}(S') : S'(n) \neq \mathbf{0}$. By Theorem 4, we should have $A : \Gamma : C \vdash S' \blacktriangleright \mathcal{G}'$. Let us take $S' \equiv n[a!e.P] \mid S''$ (with $S' \not \updownarrow$). No rule from Fig. 2 is applicable for this process, and therefore S' is not typable.

H Proofs for Theorem 6 (Safety)

If A; \circ ; $C \vdash S \triangleright G$, then S is race free and $S \Longrightarrow \longrightarrow$ error is not possible.

Proof. No error. The proof is by contradiction. Assume we have A; Γ ; $C \vdash S \triangleright G$ and $S \Longrightarrow S'$ with

$$S' \equiv a : e \cdot \rho \mid r[a?e'.Q] \mid S''$$
 and $e \neq e'$

so that $S \longrightarrow \text{error}$. By Theorem 4, we should have $A ; \Gamma ; C \vdash S' \blacktriangleright \mathcal{G}'$, however, no rule is applicable for S'. Indeed we have $S \updownarrow$, but $e \neq e'$.

No Race. Straightforward by contradiction with Theorem 4, the following is not typable

$$S' \equiv n[a?e.P] \mid m[a?e.Q] \mid S''$$

due to the condition on C, the premise $S \not\downarrow in$ [.], and the fact that the set of channels must be disjoint in concurrent branches. The other case (i.e. two sends) is similar.

Lemma 3. *If*
$$A$$
; Γ ; $C \vdash S \triangleright G$, then, $\forall s \rightarrow r : a \in F_P(G) :$, either

$$\exists ! \{ \mathtt{s}, \mathtt{r} \} \in \mathcal{P}(S) : a \in \mathtt{R}(S(\mathtt{s})) \ and \ \overline{a} \in \mathtt{R}(S(\mathtt{r})) \quad or \quad \exists ! (\mathtt{r}, a) \in \mathcal{P}(S) \times \mathcal{C}(S) : a \in \mathtt{R}(S(\mathtt{s}))$$

Proof. By straightforward induction on the derivation. Each case follows by definition of $F_P(G)$, and the premise $S \updownarrow$.

I Proofs for Lemma 1 and Theorem 7 (Equivalences)

If A; \circ ; $C \vdash S \triangleright G$ then $\forall n \in S . G \mid_n \lesssim S(n)$.

Proof. Let B be a binary relation on processes defined as follows

$$(P,Q) \in B \iff A; \Gamma; C \vdash S \triangleright G \text{ and } \exists n \in \mathcal{P}(S). \ G|_n = P \text{ and } S(n) = Q$$

Let us show that *B* is a simulation.

- If $G|_{\mathbf{n}} \xrightarrow{a!e} P_1$ then $G|_{\mathbf{n}} \equiv a!e.P_1 \oplus P_2$ and by Lemmas 30 and 32 we have that $S(\mathbf{n}) = a!e.Q_1 \oplus Q_2$ and thus $S(\mathbf{n}) \xrightarrow{a!e} Q_1$ Now we have to show that $(P_1, Q_1) \in B$, i.e.

$$A; \Gamma; C \vdash S' \triangleright G'$$
 with $G'|_{n} = P_1$ and $S'(n) = Q_1$

Pose

$$\mathit{S}' \equiv \mathtt{n}[\mathit{Q}_1] \ | \ \mathit{a} : \mathtt{e} \ | \prod_{\mathtt{m} \neq \mathtt{n} \in \mathit{P}(\mathit{S})} \mathit{S}(\mathtt{m})$$

by Lemma 2, A; Γ ; $C \vdash S' \triangleright \mathcal{G}'$, and by Corollary 1, we have $\mathcal{G}'|_{n} = P_{1}$, as required.

- If $G \mid_{\mathbf{n}} \xrightarrow{a?\mathbf{e}} P_1$ then $G \mid_{\mathbf{n}} \equiv a?\mathbf{e}.P_1 + P_2$ and by Lemmas 30 and 32 we have that $S(\mathbf{n}) = a?\mathbf{e}.Q_1 + Q_2$ and thus $S(\mathbf{n}) \xrightarrow{a?\mathbf{e}} Q_1$ Now we have to show that $(P_1, Q_1) \in B$, i.e.

$$A; \Gamma; C \vdash S' \triangleright G'$$
 with $G'|_{n} = P_1$ and $S'(n) = Q_1$

Pose

$$S \equiv \mathbf{n}[a ? \mathbf{e}.Q_1 + Q_2] \mid a : \mathbf{e} \cdot \mathbf{p} \mid \prod_{\mathbf{m}
eq \mathbf{n} \in \mathcal{P}(S)} S(\mathbf{m})$$

and

$$\mathit{S}' \equiv \mathtt{n}[\mathit{Q}_1] \ | \ \mathit{a} : \mathsf{p} \ | \ \prod_{\mathtt{m} \neq \mathtt{n} \in \mathit{P}(\mathit{S})} \! \mathit{S}(\mathtt{m})$$

by Lemma 2, A; Γ ; $C \vdash S' \triangleright G'$, and by Corollary 1, we have $G'|_{n} = P_{1}$, as required.

- If $G \mid_{\mathbf{n}} \xrightarrow{\checkmark} P_1$ then $G \mid_{\mathbf{n}} \equiv \mathbf{0}$ and $P_1 = \mathbf{0}$, thus $\mathbf{n} \notin \mathcal{P}(G)$ and by Lemma 27 this means that $S(\mathbf{n}) = \mathbf{0}$, we then have $S(\mathbf{n}) \xrightarrow{\checkmark} \mathbf{0}$.

Lemma 4. The following holds:

1. If $A; \Gamma; C \vdash n[a!e.P \oplus P'] \mid S \blacktriangleright G \text{ then } G \mid_n \equiv a!e.Q \oplus Q'$

2. If
$$A; \Gamma; C \vdash n[a?e.P+P'] \mid S \blacktriangleright G$$
 then either $G \mid_n \equiv a?e.Q+Q'$, or $A; \Gamma; C \vdash n[P'] \mid S \blacktriangleright G$

Proof. The proof of 1 is by Lemma 26, and the proof of 2 is by Lemma 26 and rule [+].

If
$$A$$
; \circ ; $C \vdash S \triangleright \mathcal{G}$ then $\prod_{\mathbf{n} \in \mathcal{P}(S)} \mathbf{n}[\mathcal{G}|_{\mathbf{n}}] \approx S$.

Proof. For this proof we pose $T = \prod_{m \in \mathcal{P}(\mathcal{G})} m[\mathcal{G}|_m] \mid \prod_{b \in \mathcal{C}(\mathcal{G})} b : \mathcal{G}|_b$.

S sends. Assume $S \equiv S_0 \mid S_1, S_0 \stackrel{\text{n[a!e]}}{\longrightarrow} \text{and } S_1 \stackrel{a \cdot \text{e}}{\longrightarrow}$. We have

$$S \equiv n[a!e.P \oplus P'] \mid a: \rho \mid S''$$
 (I.1)

and

$$S' \equiv n[P] \mid a : \rho \cdot e \mid S''$$
 (I.2)

By Lemma 4, $\mathcal{G}|_{\mathbf{n}} \equiv a! \mathbf{e}. Q \oplus Q'$, thus $\mathcal{G}|_{\mathbf{n}} \xrightarrow{\mathbf{n}[a!\mathbf{e}]}$, and by Lemma 34 $\mathcal{G}|_a = \rho$. We then have (note that $a \in \mathcal{C}(\mathcal{G})$)

$$T \equiv \mathbf{n}[a! \mathbf{e}.Q \oplus Q'] \mid a: \mathbf{p} \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{m}[\mathcal{G}|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(\mathcal{G})} b: \mathcal{G}|_{b} \tag{I.3}$$

and by definition of \longrightarrow ,

$$T' \equiv \mathbf{n}[Q] \mid a : \rho \cdot \mathbf{e} \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(G)} \mathbf{m}[G \mid_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(G)} b : G \mid_{b}$$
 (I.4)

Let us now show that

$$A; \Gamma; C \vdash S' \blacktriangleright \mathcal{G}' \text{ with } T' \equiv \prod_{\mathbf{m} \in \mathcal{P}(\mathcal{G}')} \mathbf{m}[\mathcal{G}'|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \in \mathcal{C}(\mathcal{G}')} b : \mathcal{G}'|_{b}$$
 (I.5)

By Lemma 2 we know that A; Γ ; $C \vdash S' \triangleright G'$, we have that $G'|_{n} \equiv Q$, by Corollary 1, and $G'|_{a} \equiv \rho \cdot e$ by Lemma 34.

T sends. Assume $T \equiv T_0 \mid T_1, T_0 \stackrel{\text{n}[a!e]}{\longrightarrow} \text{ and } T_1 \stackrel{a \cdot \text{e}}{\longrightarrow}$. We have

$$T \equiv \mathtt{n}[a!\mathtt{e}.Q \oplus Q'] \mid a:\mathsf{p} \mid \prod_{\mathtt{m} \neq \mathtt{n} \in \mathcal{P}(\mathcal{G})} \mathtt{m}[\mathcal{G}|_{\mathtt{m}}] \mid \prod_{\mathtt{b} \neq a \in \mathcal{C}(\mathcal{G})} b:\mathcal{G}|_{b} \tag{I.6}$$

and

$$T' \equiv \mathbf{n}[Q] \mid a : \rho \cdot \mathbf{e} \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{m}[\mathcal{G}|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(\mathcal{G})} b : \mathcal{G}|_{b}$$
 (I.7)

By Lemma 1, we have $S(n) \xrightarrow{n[a!e]}$, and since, by Lemma 28, $a \in \mathcal{C}(\mathcal{G}) \Rightarrow a \in \mathcal{C}(S)$, there is a queue a in S. Note that a queue a can always make a transition $a : \rho' \xrightarrow{a \cdot e}$ (regardless of ρ'). By Lemma 34, $S(a) = \rho$.

Therefore, we must have

$$S \equiv n[a!e.P \oplus P'] \mid a: \rho \mid S''$$
 (I.8)

And by definition of \longrightarrow , we have

$$S' \equiv \mathbf{n}[P] \mid a : \mathbf{p} \cdot \mathbf{e} \mid S'' \tag{I.9}$$

Finally, we have

$$A; \Gamma; C \vdash S' \blacktriangleright \mathcal{G}' \text{ with } T' \equiv \prod_{\mathtt{m} \in \mathcal{P}(\mathcal{G}')} \mathtt{m}[\mathcal{G}' |_{\mathtt{m}}] \mid \prod_{\mathtt{b} \in \mathcal{C}(\mathcal{G}')} b : \mathcal{G}' |_{b} \tag{I.10}$$

by Lemma 2.

S **receives.** Assume $S \equiv S_0 \mid S_1, S_0 \xrightarrow{n[a?e]}$ and $S_1 \xrightarrow{e \cdot a}$. We have

$$S \equiv n[a?e.P + P'] \mid a:e \cdot \rho \mid S''$$
(I.11)

and

$$S' \equiv n[P] \mid a : \rho \mid S'' \tag{I.12}$$

By Lemma 4 and (I.11), we have either

$$G \mid_{\mathbf{n}} \equiv a ? \mathbf{e} . Q + Q' \tag{I.13}$$

or

$$A; \Gamma; C \vdash n[P'] \mid a : e \cdot \rho \mid S'' \blacktriangleright G$$
 (I.14)

However, by assumption we have A; Γ ; $C \vdash S \triangleright G$, with S as in (I.11), therefore (I.14) cannot hold by Lemma 5.

By Lemma 34, we have that $\mathcal{G}|_a \xrightarrow{e \cdot a}$ and by (I.13), $\mathcal{G}|_n \xrightarrow{a?e}$. By definition of \longrightarrow , we have

$$T \equiv \mathbf{n}[a? \mathbf{e}.Q \oplus Q'] \mid a: \mathbf{e} \cdot \mathbf{p} \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{m}[\mathcal{G}|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(\mathcal{G})} b: \mathcal{G}|_{b}$$
 (I.15)

Let us now show that

$$A; \Gamma; C \vdash S' \blacktriangleright \mathcal{G}' \text{ with } T' \equiv \prod_{\mathbf{m} \in \mathcal{P}(\mathcal{G}')} \mathbf{m}[\mathcal{G}'|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \in \mathcal{C}(\mathcal{G}')} b : \mathcal{G}'|_{b}$$
 (I.16)

By Lemma 2 we know that A; Γ ; $C \vdash S' \triangleright G'$, we have that $G' \mid_n \equiv Q$, by Corollary 1, and $G' \mid_a \equiv e \cdot \rho$ by Lemma 34.

T **receives.** Assume $T \equiv T_0 \mid T_1, T_0 \stackrel{\text{n}[a?e]}{\longrightarrow}$ and $T_1 \stackrel{\text{e} \cdot a}{\longrightarrow}$. We have

$$T \equiv \mathbf{n}[a? \mathbf{e}.Q \oplus Q'] \mid a: \mathbf{e} \cdot \mathbf{p} \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{m}[\mathcal{G} \mid_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(\mathcal{G})} b: \mathcal{G} \mid_{b}$$
(I.17)

and

$$T' \equiv \mathbf{n}[Q] \mid a : \rho \mid \prod_{\mathbf{m} \neq \mathbf{n} \in \mathcal{P}(G)} \mathbf{m}[G|_{\mathbf{m}}] \mid \prod_{\mathbf{b} \neq a \in \mathcal{C}(G)} b : G|_{b}$$
 (I.18)

By Lemma 1, we have $S(n) \xrightarrow{a/e}$, and by Lemma 34, we have $S(a) = \rho$, therefore

$$S \equiv n[a?e.P + P'] \mid a:e \cdot p \mid S''$$
(I.19)

and

$$S \equiv \mathbf{n}[P] \mid a : \mathbf{p} \mid S'' \tag{I.20}$$

We now have to show that

$$A; \Gamma; C \vdash S' \blacktriangleright \mathcal{G}' \text{ with } T' \equiv \prod_{\mathtt{m} \in \mathcal{P}(\mathcal{G}')} \mathtt{m}[\mathcal{G}'|_{\mathtt{m}}] \mid \prod_{\mathtt{b} \in \mathcal{C}(\mathcal{G}')} b : \mathcal{G}'|_{b}$$
 (I.21)

as before, we have $\mathcal{G}'|_{\mathbf{n}} = P$ by Corollary 1 and $\mathcal{G}'|_a = \rho$ by Lemma 34.

End. If $S \xrightarrow{\checkmark} S$, then $S \equiv \mathbf{0}$ and $G \equiv \mathbf{0}$, and vice versa if $T \xrightarrow{\checkmark} T$.

Lemma 5. If

$$A; \Gamma; C \vdash r[a?e.P+P'] \mid a:e \cdot \rho \mid S \triangleright G$$

is derivable then

$$A; \Gamma; C \vdash r[P'] \mid a : e \cdot \rho \mid S \blacktriangleright G'$$

is not derivable.

Proof. Assume $P' \equiv b$?e.P''. We show that we must have

$$\mathcal{G} \equiv (* \rightarrow \mathtt{r} : a \langle \mathtt{e} \rangle. \mathcal{G}_0 \mid \mathcal{G}_1); \mathcal{G}_2$$

and the derivation of G must have the following form

$$\vdots \\ \overline{A_0; \circ; C \vdash \mathbf{r}[P_0] \mid a : \rho_0 \mid S_{00} \blacktriangleright \mathcal{G}_0} \\ [+] \frac{\overline{A_0; \circ; C \vdash \mathbf{r}[a?e.P_0] \mid a : e \cdot \rho_0 \mid S_{00} \blacktriangleright \ast \rightarrow \mathbf{r} : a \langle e \rangle \cdot \mathcal{G}_0}}{\overline{A_0; \circ; C \vdash \mathbf{r}[a?e.P_0 + P'_0] \mid a : e \cdot \rho_0 \mid S_{00} \blacktriangleright \ast \rightarrow \mathbf{r} : a \langle e \rangle \cdot \mathcal{G}_0}} \\ \vdots \\ \overline{A_1; \circ; C \vdash S_{01} \blacktriangleright \mathcal{G}_1} \\ [+] \frac{\overline{A_1; \circ; C \vdash S_{01} \blacktriangleright \mathcal{G}_1}}{\overline{A_1; \circ; C \vdash \mathbf{r}[a?e.P_0 + P'_0] \mid a : e \cdot \rho_0 \mid S_0 \blacktriangleright \ast \rightarrow \mathbf{r} : a \langle e \rangle \cdot \mathcal{G}_0 \mid \mathcal{G}_1}}{\overline{\mathcal{G}_2}}$$

where we must have

- $S \updownarrow$ or $G_1 = G_2 = \mathbf{0}$ and $S_0 \updownarrow$ or $S_{01} \equiv \mathbf{0}$.
- $\operatorname{split}(S) = (\operatorname{r}[a?e.P_0 + P'_0] \mid a:e \cdot \rho_0 \mid S_0, _)$ (the second part of the split does not matter)
- $-\overline{b} \notin R(S)$ (by Lemma 7)
- A_0 ∩ A_1 = \emptyset and $b \notin A_2$ because of rules [1] and [+]

Now let us discuss a derivation for G'. Since we have $\overline{b} \notin R(S)$, we must have $\mathtt{split}(\mathtt{r}[P'] \mid a : e \cdot \rho \mid S) = (S_1, S_2)$ such that $S_1(\mathtt{r}) = P'$, and $S_1(a) = e \cdot \rho$, if the split does exist. If it does, we have Ψ ; $\Theta \vdash \mathtt{r}[P'] \mid a : e \cdot \rho \mid S \Leftrightarrow \Omega$ such that $\forall N \in \Psi : \mathtt{r} \notin N$. Therefore, the split for the rest of the system is the same as in the other derivation.

Again, we can divide the system using [1] if need be such that we get

$$A_0$$
; \circ ; $C \vdash \mathbf{r}[P'_0] \mid a : \mathbf{e} \cdot \mathbf{p}_0 \mid S_{00} \blacktriangleright * \rightarrow \mathbf{r} : a\langle \mathbf{e} \rangle. G_0$

with $S_{00} \not \downarrow$ therefore no rule is applicable for this judgement, and the derivation does not exist.

Lemma 6. Let S a system such $S \not\supset$, and $a, b \not\in R(S)$ the following is not derivable

$$T \equiv s_1[a!e.P_1] \mid s_2[b!e'.P_2] \mid r[a?e.Q+b?e'.Q'] \mid S$$

Proof. We show this by contradiction. Given T as above, the only rule applicable is [+] on \mathbf{r} either selecting the branch on a or on b. Therefore, the following should be derivable

$$\vdots$$

$$\overline{A;\Gamma;C\vdash \mathtt{s_1}[P_1]\mid \mathtt{s_2}[b!e'.P_2]\mid \mathtt{r}[Q]\mid S\blacktriangleright \mathcal{G}}$$

$$\overset{[.]}{\underbrace{A;\Gamma;C\vdash \mathtt{s_1}[a!e.P_1]\mid \mathtt{s_2}[b!e'.P_2]\mid \mathtt{r}[a?e.Q]\mid S\blacktriangleright \mathtt{s_1}\rightarrow \mathtt{r}:a\langle e\rangle.\mathcal{G}}}$$

$$A;\Gamma;C\vdash T\blacktriangleright \mathtt{s_1}\rightarrow \mathtt{r}:a\langle e\rangle.\mathcal{G}$$

and, we must have $a, b \in A$ and $s_1[P_1] \mid r[Q] \mid S \updownarrow$. And the other derivation as the form:

$$\vdots \\ \overline{A; \Gamma; C \vdash \mathbf{s}_1[a!e.P_1] \mid \mathbf{s}_2[P_2] \mid \mathbf{r}[Q] \mid S \blacktriangleright \mathcal{G}} \\ \xrightarrow{[+]} \overline{A; \Gamma; C \vdash \mathbf{s}_1[a!e.P_1] \mid \mathbf{s}_2[b!e'.P_2] \mid \mathbf{r}[b?e'.Q'] \mid S \blacktriangleright \mathbf{s}_2 \rightarrow \mathbf{r} : b\langle e' \rangle . \mathcal{G}'} } \\ A; \Gamma; C \vdash T \blacktriangleright \mathbf{s}_2 \rightarrow \mathbf{r} : b\langle e' \rangle . \mathcal{G}' }$$

where we have $a, b \in A$ and $s_2[P_2] \mid r[Q] \mid S \updownarrow$. However, this is clearly in contradiction with Theorem 2, i.e.

$$s_1 \rightarrow r : a\langle e \rangle. G \not\equiv s_2 \rightarrow r : b\langle e' \rangle. G'$$

Lemma 7. Let S a system such $S \updownarrow$, and $a, b \notin R(S)$ the following is not derivable

$$T \equiv s_1[a!e.P_1] \mid b:e' \cdot \rho \mid r[a?e.Q+b?e'.Q'] \mid S$$

Proof. The proof is similar to the one of Lemma 7 where $b : e' \cdot \rho$ replaces $s_2[b!e'.P_2]$.

J Proofs for Theorem 8 (Completeness wrt G)

If $\bullet \vdash \mathcal{G}$ and \mathcal{G} is projectable, then there is $\mathcal{G}' \equiv \mathcal{G}$ such that $A ; \Gamma ; C \vdash \prod_{\mathbf{n} \in \mathcal{P}(\mathcal{G})} \mathbf{n} [\mathcal{G} \mid_{\mathbf{n}}] \blacktriangleright \mathcal{G}'$.

Proof. By Lemma 8, with $P = \mathcal{P}(\mathcal{G})$ and $\Gamma = 0$ since \mathcal{G} is closed by assumption.

Lemma 8. Let $Proj(G, P) = \prod_{n \in P} n[G|_n]$ with $P(G) \subseteq P$, and if $G|_n = 0$, then n is not in P.

If $C \vdash \mathcal{G}$, \mathcal{G} *is projectable, and* $\forall \chi \in \mathtt{fv}(\mathcal{G}) . \forall n \in P . \exists (n, \chi) : \chi \in \Gamma$ *then*

$$C(G)$$
; Γ ; $C \vdash Proj(G,P) \triangleright G'$ with $G \equiv G'$

Proof. We show this by induction on the structure of G. Let

$$S \equiv \prod_{\mathbf{n} \in P} \mathcal{G}|_{\mathbf{n}}$$
 $S_{\mathcal{Q}} \equiv \prod_{\mathbf{n} \in P \setminus \mathcal{Q}} \mathcal{G}|_{\mathbf{n}}$ $A = \mathcal{C}(\mathcal{G})$

 $G \equiv s \rightarrow r : a\langle e \rangle . G_0 + G_1$.

By definition of projection, we have

$$S \equiv \mathtt{s}[a!\mathtt{e}.\mathcal{G}_0 \! \mid_{\mathtt{s}} \oplus \mathcal{G}_1 \! \mid_{\mathtt{s}}] \mid \mathtt{r}[a?\mathtt{e}.\mathcal{G}_0 \! \mid_{\mathtt{s}} + \mathcal{G}_1 \! \mid_{\mathtt{s}}] \mid S_{\{\mathtt{s},\mathtt{r}\}}$$

We can apply rules $[\oplus]$, [+] (twice) and [.] in order to have the result, i.e.

$$[.] \frac{ \frac{\text{by IH}}{A; \Gamma; C_a \vdash \mathtt{s}[\mathcal{G}_0 \mid_\mathtt{s}] \mid \mathtt{r}[\mathcal{G}_0 \mid_\mathtt{s}] \mid S_{\{\mathtt{s},\mathtt{r}\}} \blacktriangleright \mathcal{G}_0}{ A; \Gamma; C \vdash \mathtt{s}[a! \texttt{e}.\mathcal{G}_0 \mid_\mathtt{s}] \mid \mathtt{r}[a? \texttt{e}.\mathcal{G}_0 \mid_\mathtt{s}] \mid S_{\{\mathtt{s},\mathtt{r}\}} \blacktriangleright \mathcal{G}'} \qquad \frac{\text{by IH}}{A; \Gamma; C \vdash \mathsf{Proj}(\mathcal{G}_1, P) \blacktriangleright \mathcal{G}_1}$$

with $\mathcal{G}' = \mathbf{s} \to \mathbf{r} : a \langle \mathbf{e} \rangle$. \mathcal{G}_0 , and $C_a = C \prec \mathbf{s} \to \mathbf{r} : a$ note that the later is defined by $C \vdash \mathcal{G}$. Observe that $S_{\{\mathbf{s},\mathbf{r}\}} \not\supset$ otherwise that would mean that $\exists \mathbf{s}' \to \mathbf{r}' : b \in \mathbb{R}(\mathcal{G}_0)$ such that $\{\mathbf{s},\mathbf{r}\} \cap \{\mathbf{s}',\mathbf{r}'\} = \emptyset$ which is in contradiction with $C \vdash \mathcal{G}$. Finally, it is obvious that

$$a \in A$$
 and $\overline{\mathbb{R}}(\mathcal{G}_i|_r) \subseteq A$ $i \in \{0,1\}$

since $A = \mathcal{C}(G)$.

 $\mathcal{G} \equiv \mathcal{G}_0 \mid \mathcal{G}_1$.

We have S of the form, by definition of projections (and well-formedness)

$$S \equiv \text{Proj}(G_0, P_0) \mid \text{Proj}(G_1, P_1) \text{ with } P_0 \cap P_1 = \emptyset$$

Note that since $\#(F_0(G)) > 1$, we have $fv(G) = \emptyset$, therefore, by IH, we have

$$A_i$$
; \circ ; $C \vdash \text{Proj}(G_i, P) \blacktriangleright G'_i$ and $G_i \equiv G'_i$ and $A_i = C(G_i)$

We have the result by applying rule [|]:

$$\frac{\text{by IH}}{A_0; \circ; C \vdash \texttt{Proj}(\mathcal{G}_0, P_0) \blacktriangleright \mathcal{G}_1} \qquad \frac{\text{by IH}}{A_1; \circ; C \vdash \texttt{Proj}(\mathcal{G}_1, P_1) \blacktriangleright \mathcal{G}_1}$$
$$A_1 \cup A_2; \Gamma; C \vdash S \blacktriangleright \mathcal{G}$$

By well-formedness we have, $A_0 \cap A_1 = \emptyset$, and Lemmas 28 and 28 guarantee that each A_i is large enough.

$$G \equiv \mu \chi . G'$$
.

By definition of projections, we have

$$S \equiv \prod_{n \in P} n[\mu \chi. \mathcal{G}' |_{n}]$$

Since \mathcal{G}' is prefix-guarded, there must be $s, r \in \mathcal{P}(\mathcal{G}')$ such that

$$s[\mu\chi.G'|_{s}] \mid r[\mu\chi.G'|_{r}] \updownarrow$$

Therefore, rule $[\mu]$ is applicable here

$$\frac{A;\Gamma';C\vdash \mathtt{s}[\mathcal{G}'|_{\mathtt{s}}]\mid \mathtt{r}[\mathcal{G}'|_{\mathtt{r}}]\mid \prod_{\mathtt{n}\in P\backslash \{\mathtt{s},\mathtt{r}\}}\mathtt{n}[\mathcal{G}'|_{\mathtt{n}}]\blacktriangleright \mathcal{G}'}{A;\Gamma;C\vdash \mathtt{s}[\mu\chi.\mathcal{G}'|_{\mathtt{s}}]\mid \mathtt{r}[\mu\chi.\mathcal{G}'|_{\mathtt{r}}]\mid S_{\{\mathtt{s},\mathtt{r}\}}\blacktriangleright \mathcal{G}}$$

where

$$\Gamma' = \Gamma \cdot (\mathtt{s}, \chi) : \chi), (\mathtt{r}, \chi) : \chi \cdot \Gamma_S \quad \text{and} \quad \Gamma_S \text{ such that } \forall \mathtt{n} \in \mathscr{P}(S_{\{\mathtt{s},\mathtt{r}\}}).\Gamma(\mathtt{n}, \chi) = \chi$$

The rest follows by induction hypothesis.

 $G = \chi$. Then, we have

$$S \equiv \prod_{n \in P} n[\chi]$$

By assumption, we have $\forall n \in P . \exists (n,\chi) : \chi \in \Gamma$, hence we can apply rule [x] and we are done.

 $G \equiv G_0$; G_1 . Then we have

$$S \equiv \prod_{\mathbf{n} \in P} \mathcal{G}_0 \, |_{\mathbf{n}} \, [\, \mathcal{G}_1 \, |_{\mathbf{n}} / \mathbf{0}]$$

Let us show that $split(S) = (S_0, S_1)$ and

$$S_0 \equiv \prod_{\mathbf{n} \in P} \mathcal{G}_0 \!\mid_{\mathbf{n}} = \mathtt{Proj}(\mathcal{G}_0, P) \quad \text{and} \quad S_1 \equiv \prod_{\mathbf{n} \in P} \mathcal{G}_1 \!\mid_{\mathbf{n}} = \mathtt{Proj}(\mathcal{G}_1, P)$$

Since G is well-formed, we have Ψ ; $\Theta \vdash S \Leftrightarrow \Omega$ coherent if we pose

$$\Psi = \{N | N \in \mathbb{F}_{\mathbb{P}}(\mathcal{G}_0)\}$$
 and $\Theta = \{\{s, r\} | s \to r : a \in \mathbb{R}(\mathcal{G}_1)\}$

For each G_i a top level concurrent branch of G_0 , we have that

$$\{\mathcal{P}(\mathcal{G}_i)\}; \varnothing \vdash \mathtt{Proj}(\mathcal{G}_i, \mathcal{P}(\mathcal{G}_i)) \mathrel{@} \Omega_i$$

is derivable by Lemma 15, since $Proj(G_i, \mathcal{P}(G_i))$ is typable by IH. In addition, we have $G_i|_{\mathbf{n}} = \Omega_i(\mathbf{n})[\mathbf{0}/\epsilon]$.

By construction, we have

$$\Psi';\Theta \, \vdash \, \mathtt{Proj}(\,\mathcal{G}_0,P) \, \Leftrightarrow \, \Omega' \quad \text{with} \, \, \forall \mathtt{n} \in \mathit{P} \, . \, \Omega'(\mathtt{n}) = \epsilon$$

with $\Psi' = \{N' | \exists N \in \Psi : N' \subseteq N\}.$

Finally, we have $\forall n \in P. S(n) \% \Omega(n) \neq \bot$ since the same suffix $\mathcal{G}_1 \mid_n$ is added to each branch of a behaviour, and $S(n) \% \Omega(n) = \mathcal{G}_1 \mid_n$. We have the required result by IH and rule [:].

$$\mathcal{G} \equiv \mathbf{0}$$
. We have

$$S \equiv \prod_{\mathbf{n} \in P} \mathbf{n}[\mathbf{0}]$$

and the results holds by rule [0].

K Accessory Results

K.1 Linearity

Lemma 9. *If* $C \prec s \rightarrow r$: *a is defined, then either*

1.
$$a \rightarrow a : a \notin C$$
,

2.
$$C(_\rightarrow_:a) = \begin{bmatrix} s \rightarrow r : a \\ | C' \end{bmatrix}$$

3.
$$C(_\rightarrow_:a) = \begin{pmatrix} * \rightarrow \texttt{r} : a \\ | & \text{, or } \\ C' \end{pmatrix}$$

4.
$$\exists \neg \rightarrow \mathbf{r} : \neg and \rightarrow \mathbf{s} : \neg \in C$$

Proof. The proofs of 1 and 2 follow directly from Def. 2 and the definition of $C(_-)$. The proof of 3 follows by definition of $C(_-)$ and the fact that, by Def. 1, there cannot be an output dependency from $s' \to r'$: b and $* \to r$: a since

$$s \rightarrow r: b \not\prec_{00} * \rightarrow r: a$$
 and $s \rightarrow r': b \not\prec_{10} * \rightarrow r: a$ since $s \neq * \neq r$

Therefore, $* \to r : a$ must be the first prefix with label $_ \to _ : a$ in C. The proof of 4 follows from the Def. 2. In fact, since whenever the sender/receiver are different on two nodes with common channel there must be two dependency relation we have the following cases. In the following, we assume that there is no prefix on a in the ellipsis. If only the senders are different, i.e. the following appears on a path in C

$$s' \rightarrow r : a \dots s \rightarrow r : a$$

then we have $s' \to r : a \prec_{II} s \to r : a$ and we must have at least a node between the two such that, e.g. $s' \to r : a \prec_{00} s' \to s : b \prec_{I0} s \to r : a$ and we have the result. If only the receivers are different, we have

$$s \rightarrow r' : a \dots s \rightarrow r : a$$

and we have $s \to r' : a \prec_{00} s \to r : a$ and we must have at least one node between the two such that $s \to r' : a \prec_{10} r' \to r : b \prec_{II} s \to r : a$, and we have the result. If both sender and receiver are different, i.e.

$$s' \rightarrow r' : a \dots s \rightarrow r : a$$

then we need two nodes c_1 and c_2 such that, (i) $\underline{c_1} = \underline{\ } \rightarrow \mathbf{r} : b$, otherwise there would be a \prec_{II} relation in the input dependency, (ii) $\underline{c_2} = \underline{\ } \rightarrow \mathbf{s} : c$ otherwise there would be a \prec_{IO} relation in the output dependency (note that \prec_{OO} is only defined if the channels are the same). In fact, there must an input dependency, e.g.

$$s' \rightarrow r' : a \prec_{I0} r' \rightarrow s_1 : b_1 \prec_{I0} s_1 \rightarrow s_2 : b_2 \prec_{I0} s_2 \rightarrow r : b_3 \prec_{II} s \rightarrow r : a$$

and an output dependency, e.g.

$$s' \rightarrow r' : a \prec_{IO} r' \rightarrow s : c \prec_{IO} s \rightarrow r : a$$

Observe that, in the first chain, we have $s_2 \to r : b_3$, and $r' \to s : c$ in the second. Actually, the shortest (input) chain when both pair of participants are different is

$$s' \rightarrow r' : a \prec_{I0} r' \rightarrow s : c_1 \prec_{I0} s \rightarrow r : c_2 \prec_{II} s \rightarrow r : a$$

where we also have $r' \to s : c_1 \prec_{I0} s \to r : a$, for the output chain. Notice that in this case we have $r' \to s : c_1$ and $s \to r : c_2$ in C.

Corollary 2. *If linearity holds on C and* $* \rightarrow n : a \in C$, *then*

$$C(_{-} \rightarrow _{-} : a) = \begin{vmatrix} * \rightarrow n : a \\ | \\ C' \end{vmatrix}$$

Lemma 10. *If* $C \prec * \rightarrow r : a \vdash G$ *then* $C \vdash G$ *and if* $\Gamma; C \prec * \rightarrow r : a; S \vdash G \triangleright$ *then* $\Gamma; C; S \vdash G \triangleright$.

Proof. We have to show that if $* \to r : a$ is involved in a input or output dependency, then there is another dependency between the same two nodes without $* \to r : a$. Note first that by Lemma 9, we know that $* \to r : a \notin C$, therefore, if there is a need to have a chain of the form

$$* \rightarrow r : a \prec_{\underline{}} \dots \prec_{\underline{}} s' \rightarrow r' : a$$

this need disappears with $*\to r: a$ (dependencies are needed only between nodes with a common channel). Thus, if $*\to r: a$ is the first node in a dependency chain, then the result holds trivially. Observe that $*\to r: a \not\prec_{00} s \to r': b$ and $s \to r': b \not\prec_{00} *\to r: a$ for any a, b since $*\neq s$, for the same reason, we have $s \to r: b \not\prec_{10} *\to r: a$.

Finally, we have the following cases, where the left hand side describes the dependency involving a and the right hand side shows that the dependency between the two external nodes still exists without the node on a.

$$s \rightarrow r : b \prec_{II} * \rightarrow r : a \prec_{II} s' \rightarrow r : c \Rightarrow s \rightarrow r : b \prec_{II} s' \rightarrow r : c$$

$$s \rightarrow r : b \prec_{II} * \rightarrow r : a \prec_{IO} r \rightarrow s' : c \Rightarrow s \rightarrow r : b \prec_{IO} r \rightarrow s' : c$$

K.2 Splitting systems

Lemma 11.

$$\{n,m\} \subseteq \Theta \Rightarrow \exists N \neq M \in \Psi : n \in N \land m \in M$$

Proof. Direct from rules $[\varepsilon]$ and [ax].

Lemma 12. If $A : \Gamma : C \vdash S \triangleright G$ and $\Psi : \Theta \vdash S \Leftrightarrow \Omega$ is derivable and coherent, then

$$(n,m) \in \circledast \iff \{n,m\} \subseteq N \text{ with } N \in \Psi$$

Proof. (\Leftarrow) follows directly from the fact that the judgement is coherent. (\Rightarrow) The proof is by contradiction. Assume that there is $(n,m) \in \mathscr{B}$ with $n \in N$ and $m \in M$, where $N \neq M \in \Psi$ (assuming $C(\Omega(n)) \cap C(\Omega(m)) \neq \varnothing$, without loss of generality).

Let us consider the following judgement:

$$\Psi' \cdot N \cdot M$$
; $\Theta \vdash \mathbf{n}[a!e.P_1] \mid \mathbf{m}[a?e.P_2] \mid S' \Leftrightarrow \Omega' \cdot \mathbf{n} : a!e.\pi \cdot \mathbf{m} : a?e.\phi$

Note that rule $[\varepsilon]$ is not applicable here since we do not have $n : \varepsilon$ and $m : \varepsilon$. Looking at the rules we see that only two applications of [sync] would introduce $n : a!e.\pi$ and $m : a?e.\varphi$.

For this rule to be applicable we must have $S' \equiv n'[a?e.P_3] \mid m'[a!e.P_4] \mid S''$, and $n' \in N$, $m' \in M$.

By definition of split and since rule; must be applied for A; Γ ; $C \vdash S \triangleright \mathcal{G}$ to hold, we should be able to derive

$$A; \Gamma; C \vdash n[a!e.Q_1] \mid m[a?e.Q_2] \mid n'[a?e.Q_3] \mid m'[a!e.Q_4] \mid S'' \triangleright G$$

However, this is not derivable due to the obvious race on channel a!

Lemma 13. If

$$\operatorname{split}(\mathbf{r}[P] \mid a : \rho \mid T) = (S'_0, S'_1)$$
 such that $S'_0(\mathbf{r}) = P_0$ and $S'_0(a) = \rho_0$

then

$$\begin{split} & \mathsf{split}(\mathtt{r}[a?\mathtt{e}.P] \mid a:\mathtt{e} \cdot \mathsf{p} \mid T) = (S_0,S_1) \quad \textit{such that} \quad S_0(\mathtt{r}) = a?\mathtt{e}.P_0 \quad \textit{and} \quad S_0(a) = \mathtt{e} \cdot \mathsf{p}_o \\ & \textit{In addition, } \forall \mathtt{n} \in \mathcal{P}(T) : S_i(\mathtt{n}) = S_i'(\mathtt{n}), \textit{for } i \in \{0,1\}. \end{split}$$

Proof. Straightforward (runtime rule)

Lemma 14. If A; Γ ; $C \vdash s[a!e.P \oplus P'] \mid a : \rho \mid T \blacktriangleright G$ and

$$split(s[a!e.P \oplus P'] \mid a: \rho \mid T) = (S_1, S_2)$$

such that

$$S_1(s) = a!e.P_0 \oplus P'_0$$
 and $S_1(a) = \rho$

then we have

$$\operatorname{split}(\operatorname{s}[P] \mid a : \rho \cdot \operatorname{e} \mid T) = (S_1', S_2') \quad \textit{such that} \quad S_1'(\operatorname{s}) = P_0 \quad \textit{and} \quad S_1'(a) = \rho \cdot \operatorname{e}$$

$$\textit{and} \ \forall \operatorname{n} \neq \operatorname{s} \in \mathcal{P}(S).S_1(\operatorname{n}) = S_1'(\operatorname{n}) \ \textit{and} \ S_2(\operatorname{n}) = S_2'(\operatorname{n}), \ \textit{ditto} \ \forall a \in \mathcal{C}(S).$$

Proof. We must have a judgement of the form

$$\Psi; \Theta \vdash s[a!e.P \oplus P'] \mid a: \rho \mid T \Leftrightarrow \Omega \cdot s: a!e.P_0 \oplus P'_0 \cdot r: a?e.Q_0 \oplus Q'_0 \cdot a: \rho$$

Since the system is derivable, we have to use [q] until the queue is empty (otherwise linearity would not be preserved). Then only use [sync] to remove the send on a, therefore both ρ and the send should be on the same part of the split. After reduction, we have the following judgement

$$\Psi$$
; $\Theta \vdash s[P] \mid a : \rho \cdot e \mid T \Leftrightarrow \Omega \cdot s : P_0 \cdot r : a?e.Q_0 \oplus Q'_0 \cdot a : \rho \cdot e$

Since r was able to receive e from s before, it must be able to receive it from the queue as well (note that there is not restriction on wrt Ψ with action on queues).

Lemma 15. If
$$A : \Gamma : C \vdash S \triangleright G$$
 then $\{\mathcal{P}(S)\} : \varnothing \vdash S \Leftrightarrow \Omega$ and $\Omega(n)[\mathbf{0}/\epsilon] \equiv S(n)$.

Proof. Straightforward induction on the derivation of $\{\mathcal{P}(S)\}$; $\Theta \vdash S \simeq \Omega$. Note that for each rule of the split, there is a rule in the inference system, whose premises are always *weaker*. Also, $[\varepsilon]$ is not applicable (since Θ is empty) and Ω keeps track of everything that happens (module the branches which are not taken).

Lemma 16. If $A : \Gamma : C \vdash S \triangleright G$ and $\Psi : \Theta \vdash S \Rightarrow \Omega$ is coherent, then $\forall \Psi', \Theta', \Omega'$ such that $\Psi' : \Theta' \vdash S \Rightarrow \Omega'$ is coherent: $\Psi = \Psi', \Theta = \Theta'$ and $\Omega = \Omega'$.

Proof. First, recall that by Lemma 12:

$$(n,m) \in \circledast \iff \{n,m\} \subseteq N \text{ with } N \in \Psi$$

We first show that Ψ (such that the split is coherent) is unique. Note that because of condition (4.2) and the fact that A; Γ ; $C \vdash S \blacktriangleright \mathcal{G}$, it is not possible to have a coherent judgement with Ψ' like Ψ except for two sets in Ψ being merged (subdivided) in Ψ' . Indeed, the number of interacting pairs of participants is fixed in S. The only changes one can do in Ψ are as follows

- 1. Add n in $N \in \Psi$ (with n not in Ψ).
- 2. Remove n in $N \in \Psi$ (with n in Ψ).
- 3. Permute $n \in N \in \Psi$ with $m \in M \in \Psi$.

We now show that any of these changes makes the judgement not coherent.

Case 1. Assume $\Psi = N \cdot \Psi_0$ and $\Psi' = \{n\} \cup N \cdot \Psi_0$. This means that $\Omega'(n) \neq \mathbf{0}$ and $\Omega'(n) \neq \epsilon$ (otherwise \circledast would not be total on $\{n\} \cup N$), and $\Omega(n) = \mathbf{0}$, or $\Omega(n) = \epsilon$, or n only received from queues. This means that n interact with participants in Ω' but not in Ω , which means that at some point in the derivation of Ω we have $n[.] \mid S_0 \updownarrow$ while we have $n[.] \mid S_0 \updownarrow$ in the derivation for Ω' . However, since G is typable, there cannot be races in the systems and therefore, the interaction between pair of participants must be the same in both derivations of Ω and Ω' .

Case 2. Assume that $\Psi = \{n\} \cup N \cdot \Psi_0$ and $\Psi' = N \cdot \Psi_0$, the case is similar to the previous one, we have that at some point in the derivation of Ω' we have $n[_] \mid S_0 \updownarrow$ while we have $n[_] \mid S_0 \updownarrow$ in the derivation for Ω .

Case 3. Assume that $\Psi = \{m\} \cup N \cdot \{n\} \cup M \cdot \Psi_0$ and $\Psi' = \{n\} \cup N \cdot \{m\} \cup M \cdot \Psi_0$. We know that

$$\forall n' \in N . C(\Omega(n')) \cap C(\Omega(m)) = \emptyset$$
 and $\forall m' \in M . C(\Omega(m')) \cap C(\Omega(n)) = \emptyset$

since the original judgement is coherent and the system typable. This implies that we cannot have \circledast total neither on $\{n\} \cup N$ nor on $\{m\} \cup M$. Therefore the judgement is not coherent.

We now consider the changes that one can make on Ω . Note that changes on Ω must be done on pairs of participants since they always interact by pair (except for those which interact only with queues but that has no effect on Ψ or Θ).

- 1. There is n_1, n_2 such that $\Omega(n_i)$ is a prefix of $\Omega'(n_i)$. A pair of elements in Ω' can only be longer if one merges two in Ψ , which is possible (see above).
- 2. There is n_1, n_2 such that $\Omega'(n_i)$ is a prefix of $\Omega(n_i)$. A pair of elements in Ω' can only be shorter if two a set in Ψ is subdivided into two sets which is not allowed either.

We consider changes that can be made on Θ :

- 1. Add $\{n,m\}$ to Θ , then, that pair will not allow the derivation to reach the axiom, unless sets in Ψ are sub-divided, which is not possible.
- 2. Remove $\{n,m\}$ from Θ , then, there will be a pair missing to reach the axiom (plus \leftrightarrow_{Θ} might not be total any more); unless two sets in Ψ are merged, but this is not possible.
- 3. Permutation in Θ would only possible if one could permute participants in sets of Ψ which, by above, is not possible.

K.3 Others

Lemma 17. If $G \not\equiv G_0 \mid G_1 \text{ and } \bullet \vdash G \text{ then } R_G \text{ is total on } \mathcal{P}(G)$.

Proof. We show this by induction on the structure of G.

 $\mathcal{G} \equiv \mathbf{s} \to \mathbf{r} : a\langle \mathbf{e} \rangle . \mathcal{G}'$. By definition $(\mathbf{s}, \mathbf{r}) \in R_{\mathcal{G}}$ and by IH $R_{\mathcal{G}'}$ is total on $\mathcal{P}(\mathcal{G}')$. Since \mathcal{G} is well-formed we have

$$\forall n_1 \to n_2 : {}_{-} \in R(G'). \{n_1, n_2\} \cap \{s, r\}$$

Thus there is $(s, n_i) \in R_G$ or $(r, n_i) \in R_G$ and we have the required result by definition of R_G and Def. 1.

 $\mathcal{G} \equiv \mathcal{G}_0 + \mathcal{G}_1$. By IH, $R_{\mathcal{G}_i}$ is total on $\mathcal{P}(\mathcal{G}_i)$ for $i \in \{0,1\}$. Since \mathcal{G} is well-formed we have

$$\forall s \rightarrow r : a \in R(G). \forall s' \rightarrow r' : b \in R(G'). s = s' \land a \neq b$$

i.e. $s = s' \in G_i$, and we have the required result by definition of R_G and Def. 1.

 $\mathcal{G} \equiv \mathbf{s} \to \mathbf{r} : a\langle \mathbf{e} \rangle. (\mathcal{G}_0 \mid \mathcal{G}_1)$. By IH $R_{\mathcal{G}_i}$ is total on $\mathcal{P}(\mathcal{G}_i)$. Since \mathcal{G} is well-formed we must have $\mathbf{s} \in \mathcal{P}(\mathcal{G}_i)$ and $\mathbf{r} \in \mathcal{P}(\mathcal{G}_j)$ with $i \neq j \in \{0,1\}$. We have the required result since we have $(\mathbf{s},\mathbf{r}) \in R_{\mathcal{G}}$ by definition of $R_{\mathcal{G}}$ and Def. 1.

 $\mathcal{G} \equiv \mathcal{G}_0$; \mathcal{G}_1 . Observe that by IH and by definition of F_P , we have that $\forall N \in F_P(\mathcal{G}_i) . R_{\mathcal{G}_i}$ is total on N, with $i \in \{0,1\}$.

Since the projection is defined as $G|_{n} = G_{0}|_{n} [G_{1}|_{n}/\mathbf{0}]$, we have that

$$\forall\, \textit{N}_0 \times \textit{N}_1 \subseteq \texttt{F}_{\texttt{P}}(\textit{G}_0) \times \texttt{F}_{\texttt{P}}(\textit{G}_1) \,:\, \textit{R}_{\textit{G}} \text{ is total on } \textit{N}_0 \cup \textit{N}_1 \text{ if there is } \texttt{n} \in \textit{N}_0 \cap \textit{N}_1$$

Since R_G is a transitive relation, let us define a transitive relation on the intersection of sets of participants from G_0 and G_1 :

$$(N_0,N_1)\in \mathbb{W}\\ \iff\\ N_0\cap N_1\neq\varnothing\quad\text{or}\quad\exists\,(M_0,M_1):\,(N_0,M_1)\in\mathbb{W}\text{ and }(M_0,N_1)\in\mathbb{W}$$

It is easy to see that R_G is total on any $N_0 \cup N_1$ whenever $(N_0, N_1) \in \mathbb{V}$, thus

$$R_{\mathcal{G}}$$
 is total on $\bigcup_{(N_0,N_1)\in \mathbb{W}} N_0 \cup N_1$

Let us show that

$$(N_0, N_1) \in \mathbb{F}_{\mathbb{P}}(\mathcal{G}_0) \times \mathbb{F}_{\mathbb{P}}(\mathcal{G}_1) \Rightarrow (N_0, N_1) \in \mathbb{W}$$

Since G is well-formed we have

$$\forall \mathbf{s} \to \mathbf{r} : \underline{\ } \in \mathbb{R}(\mathcal{G}_1) . \exists N_1 \neq N_2 \in \mathbb{F}_0(\mathcal{G}_0) . \mathbf{s} \in N_1 \land \mathbf{r} \in N_2$$
 (K.1)

and

$$\forall N \in F_{P}(G_{0}) . \exists N' \in F_{P}(G_{1}) . N \cap N' \neq \emptyset$$
(K.2)

Therefore, by (K.1) and the fact that $s \to r : \ _- \in R(\mathcal{G}_1) \Rightarrow \{s,r\} \subseteq N \in F_P(\mathcal{G}_1)$, we have

$$\forall N_1 \in F_P(G_1) . \exists N_0 \neq N_0' \in F_P(G_0) : N_0 \cap N_1 \neq \emptyset \text{ and } N_0' \cap N_1 \neq \emptyset$$
 (K.3)

By (K.2), we have

$$\forall N_0 \in F_P(G_0) . \exists N_1 \in F_P(G_1) : N_0 \cap N_1 \neq \emptyset$$
 (K.4)

Now assume that there is $(N_0, N_1) \in F_P(\mathcal{G}_0) \times F_P(\mathcal{G}_1)$ such that $(N_0, N_1) \notin W$, this is contradiction with (K.3) and (K.4).

$$G \equiv \mu \chi$$
. G. By IH

Other cases. The cases where G = 0 or $G = \chi$ are trivial.

Lemma 18. If $A; \Gamma; C \vdash S \triangleright G$ and $\bullet \vdash G$ then the following holds

$$\forall n \in \mathcal{P}(S) : \mathcal{C}(S(n)) \subseteq \mathcal{C}(\mathcal{G}|_n)$$

Proof. Straightforward induction on the validation rules.

Lemma 19. If $A : \Gamma : C \vdash S \triangleright G$ and R_G is total on $\mathcal{P}(G)$ then R_S is total on $\mathcal{P}(S)$.

Proof. By Lemmas 17 and 18 and the definition of R_S .

Lemma 20. *If* $\bullet \vdash G$ *then* $\forall N \in \mathbb{F}_0(G)$. R_G *is total on* N.

Proof. By Lemma 17 and the definition of F₀.

Lemma 21. *If* A; Γ ; $C \vdash S \triangleright \mathcal{G}_0$; \mathcal{G}_1 , $C \vdash \mathcal{G}_0$, Ψ ; $\Theta \vdash S \Leftrightarrow \Omega$ *is coherent, and* $split(S) \neq \bot$ *then*

$$\forall \mathtt{n},\mathtt{m} \in \mathcal{P}(S) : (\mathtt{n},\mathtt{m}) \in R_{G_0} \Rightarrow (\mathtt{n},\mathtt{m}) \in \circledast$$

Proof. Straightforward by definitions of $R_{\mathcal{G}_0}$ and \circledast . Note that external choice branches which do not appear in Ω do not appear in \mathcal{G}_0 either.

Lemma 22. *If* $bv(S) \neq \emptyset$ *then* $split(S) = \bot$.

Proof. If $bv(S) \neq \emptyset$, we must have

$$S \equiv n[P] \mid S'$$
 where $\mu x \cdot P'$ is a suffix of P

The result follows from the fact that there is no rule in Fig. 3 which "removes" recursive definition. Therefore, it not possible to derive a split whenever there is a recursion definition in the system to be split.

Lemma 23. If
$$A$$
; Γ ; $C \vdash S \triangleright \mu \chi$. G and $\chi \in fv(G)$ then $\#(F_0(G)) = 1$.

Proof. The proof follows from the fact that the context Γ is emptied each time the rule $[\]$ is used in the derivation (this rule is the only one introducing concurrent branches). In addition, for the axiom [x] to be used in the derivation one must have $(_,_):\chi\in\Gamma$. Therefore, the only way one could have $\#(F_0(\mathcal{G}))>1$ (i.e. at least two concurrent branches in \mathcal{G}) is if χ does not appear in \mathcal{G} .

Lemma 24. *If*
$$A$$
; Γ ; $C \vdash S \triangleright G$ *and* $bv(S) = \emptyset$ *then* $bv(G) = \emptyset$

Proof. By straightforward induction on the rules of Fig. 2.

Lemma 25. *If* $C \vdash G$ *then* $\bullet \vdash G$

Proof. This follows from the fact that $\bullet \prec C$ is always defined.

Lemma 26. If A; Γ ; $C \vdash S \triangleright G$ and G is well-formed

$$G \equiv ((\mathtt{n} \rightarrow \mathtt{m} : a \langle \mathtt{e} \rangle . G_1 + G_2) \mid G_3); G_4 \iff S \equiv \mathtt{n}[a ! \mathtt{e} . P \oplus P'] \mid \mathtt{m}[a ? \mathtt{e} . Q + Q'] \mid S'$$

Proof. (\Rightarrow) Assume that

$$A; \Gamma; C \vdash S \blacktriangleright ((n \rightarrow m : a \langle e \rangle . G_1 + G_2) \mid G_3); G_4$$

is derivable. We show that either a rule introducing the corresponding operator is applicable or that an equivalent G can be inferred.

- If *A*; Γ; $C \vdash S$ ▶ G'; G_4 is derivable then we have either split(S) = \bot then G_4 = $\mathbf{0}$,

$$A; \Gamma; C \vdash S \triangleright G'$$

or $split(S) \neq \bot$, in this case, we must have

$$\mathtt{split}(S) = (S_1, S_1')$$

with

$$A; \circ; C \vdash S_1 \blacktriangleright \mathcal{G}'$$
 and $A; \circ; C \prec \mathcal{T}(\mathcal{G}') \vdash S_1' \blacktriangleright \mathcal{G}_4$

- Assume $G' = G'' \mid G_3$, if A; Γ ; $C_1 \vdash S_1 \triangleright G'$ is derivable then we must have either $S_1 \equiv S_2 \mid S_3, A = A_1 \cup A_2$, and $A_1 \cap A_2 = \emptyset$ such that

$$A_1; \circ; C \vdash S_2 \triangleright G''$$
 and $A_2; \circ; C \vdash S_3 \triangleright G_3$

are derivable, or $S_3 = \mathbf{0}$ and $G_3 \equiv \mathbf{0}$.

- Assume $G'' = G''' + G_2$, we must have either

$$S_2 = \mathbf{n}[P_0 \oplus P_0'] \mid S_4 \tag{K.5}$$

and

$$A_1$$
; \circ ; $C \vdash n[P_0] \mid S_4 \triangleright \mathcal{G}'''$ and A_1 ; \circ ; $C \vdash n[P'_0] \mid S_4 \triangleright \mathcal{G}_2$

are derivable, or $P_0' \equiv \mathbf{0}$ and $\mathcal{G}_2 \equiv \mathbf{0}$. - Assume $\mathcal{G}''' = \mathbf{n} \to \mathbf{m} : a \langle \mathbf{e} \rangle \cdot \mathcal{G}_1$, we must have

$$P_0 \equiv a! e.P$$
 and $S_4 \equiv m[a? e.Q + Q'] \mid S'$ (K.6)

and

$$A_1$$
; \circ ; $C \vdash n[P'] \mid m[a?e.Q+Q'] \mid S' \triangleright G_1$

derivable.

Putting (K.5) and (K.6) together, we have that

$$S \equiv n[a!e.P \oplus P'] \mid m[a?e.Q + Q'] \mid S'$$

(⇐) Assume

$$A; \Gamma; C \vdash \mathbf{n}[a!e.P \oplus P'] \mid \mathbf{m}[a?e.Q + Q'] \mid S' \triangleright G$$
(K.7)

We show that either a rule introducing the corresponding operator is applicable or that an equivalent G can be inferred.

- Either $\mathrm{split}(S) = \bot$ in which case $G \equiv G'$; **0** or

$$split(S) = (S_1, S_1')$$

and we must have $G_4 \not\equiv \mathbf{0}$ and

$$A; \Gamma; C \vdash S_1 \triangleright G$$

with

$$S_1(\mathbf{n})=a!$$
e. $P_0\oplus P_0'$ and $S_1(\mathbf{m})=a?$ e. Q_0+Q_0' and $S_1'(\mathbf{n})=P_1$ and $S_1'(\mathbf{m})=Q_1$ such that

$$a!e.P \oplus P' \equiv (a!e.P_0 \oplus P_0')[P_1/\mathbf{0}]$$
 and $a?e.Q + Q' \equiv (a?e.Q_0 + Q_0')[Q_1/\mathbf{0}]$

- Either there is

$$S = n[a!e.P_0 \oplus P'_0] \mid m[a?e.Q_0 + Q'_0] \mid S_1 \mid S_2$$

$$G' = G'' \mid G_3 \text{ and } A = A_1 \cup A_2, \text{ and } A_1 \cap A_2 = \emptyset$$

$$A_1; \circ; C \vdash n[a!e.P_0 \oplus P_0'] \mid m[a?e.Q_0 + Q_0'] \mid S_1 \blacktriangleright G''$$
 and $A_2; \circ; C \vdash S_2 \blacktriangleright G_3$

where $S_1 \not\supset$ (this is a sound assumption, since one could apply [;] and [1] as many times as necessary to obtain this), or $G_3 \equiv 0$.

- Either there is $G'' \equiv G''' + G_2$ such that

$$A; \circ; C_1 \vdash n[a!e.P_0] \mid m[a?e.Q_0 + Q'_0] \mid S_1 \triangleright G'''$$

and

$$A; \circ; C_1 \vdash n[P'] \mid m[a?e.Q_0 + Q'_0] \mid S_1 \blacktriangleright G_2$$

or
$$G_2 = \mathbf{0}$$
.

- For

$$A; \circ; C_1 \vdash n[a!e.P] \mid m[a?e.Q+Q'] \mid S_1 \triangleright G'''$$

to be derivable, we must have $G''' \equiv n \rightarrow m : a\langle e \rangle G_1$.

Putting all the pieces together, we have the required result.

Lemma 27. *If* A; Γ ; $C \vdash S \triangleright G$ *then*

$$\forall \mathtt{n} \in \mathcal{P}(S).\, S(\mathtt{n}) \neq \mathbf{0} \,:\, \mathtt{n} \in \mathcal{P}(S) \iff \mathtt{n} \in \mathcal{P}(\mathcal{G})$$

and

$$\forall n \in \mathcal{P}(S). S(n) = \mathbf{0} \iff n \notin \mathcal{P}(G)$$

Proof. Straightforward.

Lemma 28. *If* A; Γ ; $C \vdash S \triangleright G$ *then* $C(G) \subseteq C(S)$.

Proof. Straightforward induction on the derivation.

Lemma 29. *If* A; Γ ; $C \vdash S \triangleright G$ *then* $C(S) \subseteq A$.

Proof. Straightforward induction on the derivation.

Lemma 30. If G is well-formed and projectable then

- if $G \mid_{n} \equiv a!e.P \oplus Q$ then there is a branch of G such that the first prefix on $n \to m: a e$.
- if $G \mid_{n} \equiv a$?e.P+Q then there is a branch of G such that the first prefix on n is $m \to n : a \langle e \rangle$.

Proof. By definition of Projection.

Lemma 31. If $A; \Gamma; C \vdash S \mid a : e \cdot \rho \blacktriangleright G$ is derivable then for each branch in G the first prefix on a is $* \rightarrow n : a \langle e \rangle$.

Proof. Follows from the fact that for a common channel, $[\rho]$ must be used before [.] (see Lemma 2).

Lemma 32. If $A : \Gamma : C \vdash S \triangleright G$ and there is a branch in G such that $n \to m : a \langle e \rangle$ is the first prefix on n (resp. m), then $S(n) = a!e.P \oplus P'$ (resp. S(m) = a?e.Q + Q).

Proof. Straightforward.

Lemma 33. If $fv(S) = \emptyset$, and $S \longrightarrow S'$, then $fv(S') = \emptyset$, i.e. reduction of systems preserves closeness.

Proof. Follows directly from the semantics of the calculus.

Lemma 34. *If* $a \in C(G)$ *then*

$$A; \Gamma; C \vdash a : \rho \mid S \triangleright G \iff then G \mid_a = \rho$$

In addition, $a \notin C(G) \Rightarrow \rho = []$.

Proof. Follows from rules $[\rho]$ and [0].