LINMA2415 Quantitative Energy Economics

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https://github.com/SimonDesmidt/Syntheses

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Reminder about optimization

1.1 Formulations

Definition 1.1. The primal problem is the original optimization problem that we want to solve. It is formulated as:

$$\begin{aligned} \max_x \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{1.1}$$

Definition 1.2. The dual problem is derived from the primal problem. It is formulated as:

$$\begin{aligned} \min_y \quad & b^T y \\ \text{s.t.} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned} \tag{1.2}$$

Another way to look at the dual problem is by using the Lagrangian function. Consider now the primal under the form:

$$\begin{aligned} p^* = \min \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \leq 0, \quad i = 1, \dots, m \\ & h_j(x) = 0, \quad j = 1, \dots, p \end{aligned} \tag{1.3}$$

Definition 1.3. The Lagrangian function is defined as:

$$L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{j=1}^{p} \nu_j h_j(x) \tag{1.4}$$

Definition 1.4. The Lagrange dual function is defined as:

$$g(\lambda, \nu) = \inf_x L(x, \lambda, \nu) \tag{1.5}$$

Definition 1.5. The Lagrange dual problem is defined as:

$$\begin{aligned} d^* = \max \quad & g(\lambda, \nu) \\ \text{s.t.} \quad & \lambda \geq 0 \end{aligned} \tag{1.6}$$

1.2 Optimality conditions

Definition 1.6. Weak duality is when $d^* \leq p^*$, it always holds and can be used to find bounds on the optimal value of the primal problem.

Definition 1.7. Strong duality is when $d^* = p^*$, does not holds in general, but usually holds for convex problems.

Definition 1.8. Complementary slackness is a condition that holds at optimality, it states that for each constraint, either the constraint is active (i.e. $f_i(x) = 0$) or the corresponding dual variable is zero (i.e. $\lambda_i = 0$).

$$0 \leq \lambda_i^* \perp f_i(x^*) \leq 0 \tag{1.7}$$

Definition 1.9. The KKT conditions are a set of necessary conditions for optimality. Consider this problem:

$$\begin{aligned} \max & f(x,y) \\ \text{s.t.} & Ax + By \leq b \qquad (\lambda) \\ & Cx + Dy = d \qquad (\mu) \\ & x \geq 0 \qquad (\lambda_2) \end{aligned} \tag{1.8}$$

The KKT conditions are:

$$\begin{aligned} Cx + & Dy - d = 0 \\ 0 \leq \lambda \perp & Ax + By - b \leq 0 \\ 0 \leq x \perp & \lambda^T A + \mu^T C - \nabla_x f(x,y)^T \geq 0 \\ \lambda^T B + & \mu^T D - \nabla_y f(x,y)^T = 0 \end{aligned} \tag{1.9}$$

And the Lagrangian function can be written as:

$$L(x,y,\lambda,\mu) = f(x,y) + \lambda^T(b - Ax - By) + \mu^T(d - Cx - Dy) + \lambda_2^T x \tag{1.10}$$