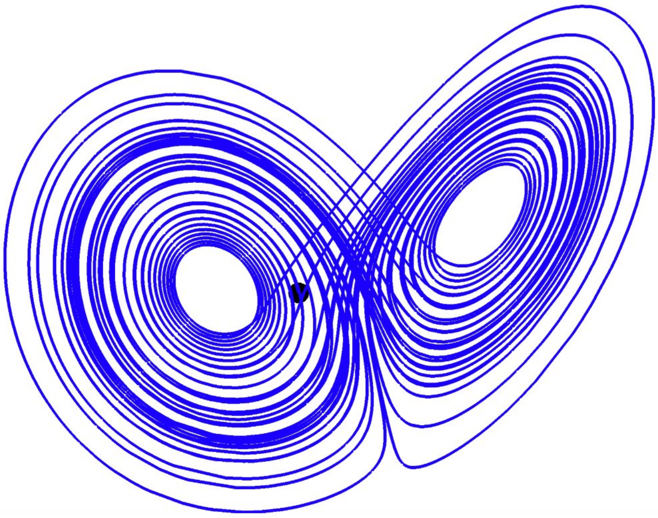

LINMA2171 Numerical Analysis

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Linear continuous-time 2D dynamical systems

1.1 Introduction

Consider the 2D dynamical system $\dot{x} = f(x)$, $x \in \mathbb{R}^n$. Let $\Omega \subseteq \mathbb{R}^n$ that is compact and positively invariant. Assume that $f \in \mathcal{C}^1$ on Ω, i.e. it is continuously differentiable on Ω. Let $x(0) \in \Omega$. Then the system has one and only one solution for all positive times.

Ω is positively invariant if $x(0) \in \Omega \implies x(t) \in \Omega \, \forall t \geq 0$.

1.2 General form

The general form of a linear dynamical system is

$$\dot{x} = Ax \qquad x \in \mathbb{R}^n \tag{1.1}$$

and its solution is of the form $x(t) = \exp(At)x(0)$. We will now study the different possibilities of stability, based on the matrix A :

1. A is diagonalizable :
   (a) $\lambda_1 > \lambda_2 > 0$: unstable (repelling) node (see LINMA2370).
   (b) $\lambda_1 > \lambda_2 = 0$:
   (c) $\lambda_1 > 0 > \lambda_2$: saddle point.
   (d) $0 = \lambda_1 > \lambda_2$:
   (e) $0 > \lambda_1 > \lambda_2$: attracting node.
   (f) $\lambda_1 = \lambda_2 > 0$: unstable star, the eigenvectors are perpendicular and the direction is repelling from the origin.
   (g) $\lambda_1 = \lambda_2 = 0$: every point is an equilibrium.
   (h) $\lambda_1 = \lambda_2 < 0$: stable star, the eigenvectors are perpendicular and the direction is going to the origin.
2. A has two equal eigenvalues with only one eigenvector, i.e. is not diagonalizable :
   (a) $\lambda < 0$: convergence to the equilibrium point.
   (b) $\lambda = 0$:
   (c) $\lambda > 0$:

3. A has complex conjugate eigenvalues :
   (a) $Re(\lambda) > 0$: diverges from the equilibrium in a spiral form.
   (b) $Re(\lambda) = 0$: the trajectory is periodic, does not converge nor diverge.
   (c) $Re(\lambda) < 0$: converges to the equilibrium in a spiral form.

Nonlinear CT, 2D systems

The general form of a nonlinear CT 2D dynamical system is

$$\dot{x} = f(x) \qquad x \in \mathbb{R}^2,\ f : \mathbb{R}^2 \to \mathbb{R}^2 \tag{2.1}$$

A nullcline is a curve such that for an element x_i of x, $\dot{x}_i = 0$. There are two in a 2D system and their intersections are the equilibrium points. The vector field has a horizontal or vertical direction on those curves., and their sense is given by the sign of $\dot{x}$ or $\dot{y}$.

$\rightarrow$ N.B. : The nullclines cross trajectories, as they are not trajectories themselves.

An equilibrium point is stable if all eigenvalues of the jacobian matrix evaluated at that point have nonpositive real part.