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# **LMECA2300 Advanced Numerical Methods**

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# 2-D acoustic and electromagnetic waves

## 1.1 Physical laws

The expression of the force of the source applied to its surrounding is derived from Newton's law  $F = ma$ :

$$\rho_0 \frac{\partial \vec{u}}{\partial t} + \nabla p = r_v \quad (1.1)$$

where  $\rho_0$  is the average density of the surrounding,  $\vec{u}$  is the velocity field,  $p$  is the variation of pressure and  $r_v$  is the "pressure force".

The conservation law of energy is

$$\nabla \cdot \vec{u} + \chi \frac{\partial p}{\partial t} = s_v \quad (1.2)$$

where  $\chi$  is the compressibility [ $kg^{-1}ms^2$ ] and is given by the equation  $\frac{\rho}{\rho_0} = \chi p$ .  $s_v$  is the "velocity source".

## 1.2 Wave equation

The wave equation is

$$\nabla^2 p - \rho_0 \chi \frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial s_v}{\partial t} \quad (1.3)$$

In 1D, with  $s_v = 0$ , the solution is any function  $f(t - \frac{x}{v})$  or  $g(x - tv)$  with  $v = 1/\sqrt{\rho_0 \chi}$ .

## 1.3 Plane wave

In the plane, the standard wave is given by

$$p(x, t) = p_0 \cos(\omega t - kx) \quad (1.4)$$

where the phase velocity is  $v_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\chi \rho_0}}$ .

The wave impedance is  $\nu = \frac{p}{u_x} = \sqrt{\frac{\rho_0}{\chi}} [kgm^{-2}s^{-1}]$ .