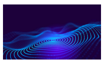

LMECA2300 Advanced Numerical Methods

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UCLouvain

Contents

2-D acoustic and electromagnetic waves

1.1 Physical laws

The expression of the force of the source applied to its surrounding is derived from Newton's law $F = ma$:

$$\rho_0 \frac{\partial \vec{u}}{\partial t} + \nabla p = r_v \tag{1.1}$$

where ρ_0 is the average density of the surrounding, $\vec{u}$ is the velocity field, p is the variation of pressure and r_v is the "pressure force".
The conservation law of energy is

$$\nabla \cdot \vec{u} + \chi \frac{\partial p}{\partial t} = s_v \tag{1.2}$$

where χ is the compressibility $[kg^{-1}ms^2]$ and is given by the equation $\frac{\rho}{\rho_0} = \chi p$. s_v is the "velocity source".

1.2 Wave equation

The wave equation is

$$\nabla^2 p - \rho_0 \chi \frac{\partial^2 p}{\partial t^2} - -\rho_0 \frac{\partial s_V}{\partial t} \tag{1.3}$$

In 1D, with $s_V = 0$, the solution is any function $f\left(t - \frac{x}{v}\right)$ or $g\left(x - tv\right)$ with $v = 1/\sqrt{\rho_0 \chi}$.

1.3 Plane wave

In the plane, the standard wave is given by

$$p(x,t) = p_0 \cos(\omega t - kx) \tag{1.4}$$

where the phase velocity is $v_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\chi \rho_0}}$.
The wave impedance is $\nu = \frac{p}{u_x} = \sqrt{\frac{\rho_0}{\chi}}$ $[kgm^{-2}s^{-1}]$.