MAS AI Assignment 7

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Task 1: Definitions

All answers use the book "Artificial Intelligence - A modern approach" from Russel and Norvig as basis.

Enumeration A way to **check** a sentence in against a **model** in propositional logic. This works by *enumerating* each possible row of a truth table for a given sentence.

Validity Also a property of propositional logic. A sentence is *valid* if it is true in **all** models.

Satisfiability A sentence is *satisfiable*, if it is true in a subset of all models.

CNF and DNF

Conjunctive normal form, CNF Expressing a sentence as a conjunction of clauses. For every sentence exists as CNF. This way the sentence can be put into an resolution algorithm. The sentence $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ can be converted in following CNF: $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,2})$

Disjunctive normal form, DNF This is a form were a disjunction of conjunctions of literals is used. Example from the book: $(A \wedge B \wedge C) \vee (\neg A \wedge C) \vee (B \wedge \neg C)$

Resolution This is a specific inference rule which creates a complete inference algorithm when using any search algorithm. It uses factoring to eliminate multiple copies of literals.

Task 2: Implications

Given sentence: $[(F \implies P) \vee (D \implies P)] \implies [(F \wedge D) \implies P]$

Proof by enumeration

The truth table shows that the sentence is valid because it holds true in all models:

F	P	D	$F \implies P$	$D \implies P$	$(F \implies P) \vee (D \implies P)$	$F \wedge D$	$(F \wedge D) \implies P$	Full sentence
0	0	0	1	1	1	0	1	1
0	0	1	1	0	1	0	1	1
0	1	0	1	1	1	0	1	1
0	1	1	1	1	1	0	1	1
1	0	0	0	1	1	0	1	1
1	0	1	0	0	0	1	0	1
1	1	0	1	1	1	0	1	1
1	1	1	1	1	1	1	1	1

Convert to CNF

1. Given sentence:
   $[(F \implies P) \lor (D \implies P)] \implies [(F \land D) \implies P]$
2. Eliminate three times the inner *implications*:
   $[(\neg F \lor P) \lor (\neg D \lor P)] \implies [\neg(F \land D) \lor P]$
3. Use *De Morgan* on the right side:
   $[(\neg F \lor P) \lor (\neg D \lor P)] \implies [(\neg F \lor \neg D) \lor P]$
4. Eliminate remaining *implication*:
   $\neg[(\neg F \lor P) \lor (\neg D \lor P)] \lor [(\neg F \lor \neg D) \lor P]$
5. Use *De Morgan* on the left side:
   $[\neg(\neg F \lor P) \land \neg(\neg D \lor P)] \lor [(\neg F \lor \neg D) \lor P]$
6. Use two times *De Morgan* on the left side:
   $[(F \land \neg P) \land (D \land \neg P)] \lor [(\neg F \lor \neg D) \lor P]$
7. Remove inner brackets:
   $[F \land \neg P \land D \land \neg P] \lor [\neg F \lor \neg D \lor P]$
8. Remove double $\neg P$ get CNF:
   $[F \land D \land \neg P] \lor [\neg F \lor \neg D \lor P]$

Prove using resolution

F	D	P	!F	!D	!P	$F \land D \land \neg P$	$\neg F \lor \neg D \lor P$	$[F \land D \land \neg P] \lor [\neg F \lor \neg D \lor P]$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	1	1
0	1	0	1	0	1	0	1	1
0	1	1	1	0	0	0	1	1
1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	1	0	1
1	1	1	0	0	0	0	1	1

We see that the CNF $[F \land D \land \neg P] \lor [\neg F \lor \neg D \lor P]$ is equal to $False \lor True$ which is always $True$.

Task 3: Entailment

Given formula $(A \land B) \implies C$ prove that $(A \implies C) \lor (B \implies C)$ is equivalent.

1. Given formula:
   $(A \land B) \implies C$
2. Eliminate *implication*:
   $\neg(A \land B) \lor C$
3. Use *De Morgan*:
   $\neg A \lor \neg B \lor C$
4. Which is equivalent to:
   $\neg A \lor C \lor \neg B \lor C$
5. Recreate *implications*:
   $(A \implies C) \lor (B \implies C)$

Task 4: Proving

a)

1. Given formula:
   $\neg P \wedge \neg Q \Leftrightarrow \neg(P \vee Q)$

2. Use *De Morgan* on the right side:
   $\neg P \wedge \neg Q \Leftrightarrow \neg P \wedge \neg Q$

b)

1. Given formula:
   $\neg P \vee \neg Q \Leftrightarrow \neg(P \wedge Q)$

2. Use *De Morgan* on the right side:
   $\neg P \wedge \neg Q \Leftrightarrow \neg P \wedge \neg Q$

c)

1. Given formula:
   $P \vee (P \wedge Q) \Leftrightarrow P$

2. Left side, distribute $\vee$ over $\wedge$:
   $(P \vee Q) \wedge (P \vee P) \Leftrightarrow P$

3. Which is equivalent to:
   $P \wedge (P \vee Q) \Leftrightarrow P$

4. Because P overshadows all values of Q this is equivalient to:
   $P \Leftrightarrow P$