The rational use of drugs and the design of effective dosage regimens are facilitated by the appreciation of the central paradigm of clinical pharmacology that there is a defined relationship between the administered dose of a drug, the resulting drug concentrations in various body fluids and tissues, and the intensity of pharmacologic effects caused by these concentrations.

These dose–exposure–response relationships and thus the dose of a drug required to achieve a certain effect are determined by the drug's pharmacokinetics and pharmacodynamic properties. *Pharmacokinetics* describes the time course of the concentration of a drug in a body fluid, preferably plasma or blood that results from the administration of a certain dosage regimen. In simple words, pharmacokinetics is "what the body does to the drug". *Pharmacodynamics* describes the intensity of a drug effect in relation to its concentration in a body fluid, usually at the site of drug action. It can be simplified to "what the drug does to the body".

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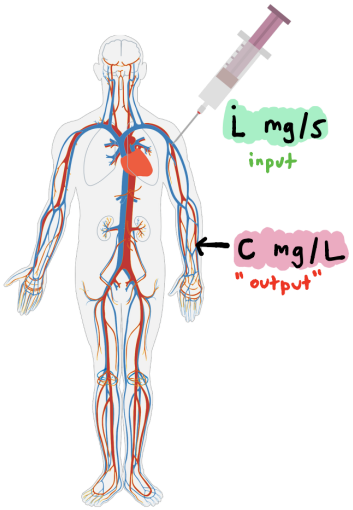
1. a patient in the hospital is administered a drug intravenously at a rate $i = i(t)$ [mg/min]. the drug clears from the blood, owing to the metabolism, with first-order kinetics and rate constant k [1/min]. let V [L] be the volume of blood in the body of the patient.

   note: intriguingly, pharmacokinetic models allow for personalized drug regimens. here, we can imagine finding the parameters k and V, which generally vary among the population, for a specific patient.



   (a) conceptualize the bloodstream in the body as a batch chemical reactor to develop a simple pharmacokinetic model for the concentration of the drug in the bloodstream, $c = c(t)$ [mg/L]. *hint:* write a mass balance, check the units of each term for consistency.

❗ check your dynamic model with a TA or LA before you proceed.

(b) *case 1:* suppose, to be effective, yet also nontoxic, the drug is required to be maintained at a steady state concentration of $\bar{c}$ [mg/L]. write an expression for the (constant) rate at which the drug should be *continuously* administered, $\bar{i}$ [mg/min], to maintain this steady state concentration $\bar{c}$.

(c) *case 2*: now instead suppose the patient is initially drug-free and is given a dose of m [mg] of the drug via an injection at time $t = 0$.

i. what is the appropriate initial condition $c(t = 0)$?

ii. what is the appropriate input $i = i(t)$?

iii. derive the response $c(t)$ to the injection, according to your model, using the Laplace transform.

iv. what is the peak concentration of the drug, $\max_t c(t)$, and when does it occur, $\arg\max_t c(t)$?

v. how much time elapses before the concentration $c(t)$ decays to 10% of its peak (maximal) value?

vi. suppose $V = 5000$ mL, $m = 2.5$ mg, and $k = 1/20$ min^{-1}. in Julia, plot the response of the concentration of the drug in the blood, $c(t)$, to the drug injection, for $t \in [-20, 100]$ min. *Hint: do dimensional analysis to ensure your units are consistent.*

(d) 💡 a critical thinking question, about the domain of applicability of a model: would this model be effective for describing the concentration, in the blood, of a drug which is *ingested* instead of injected intravenously? why/why not? think about part (c)(iv).