Exercise Sheet 10 - Task 5

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L is the set of locations, S the set of segments, A the set of cars and T the set of times. L is the set of locations and $l \in L$ a location. $e \in S$ is a segment of a path in our instance, where S is the set of segments. For our time horizon T is $t \in T$ a time. We count the time in days. The cars are written as $i \in A$, where A is the set of all cars. The binary variables x_{iet} and y_{ilt} tell us, on which segment e or location l a car i is at time t. $K_{et} \in \mathbb{N}$ is the capacity of a segment e at a time t, which is given in the instances as planned transports and vehicle capacity. The origin ${l_i}' \in L$ and destination ${l_i}^* \in L$ with available time $a_i \in T$ and due date $d_i \in T$ for each car i are also given in the instances. Furthermore we use the binary variable $\bar{d}_i$, which takes the value 1 if a car has a due date and 0 if not. The binary variables s_i and w_i tell us for each car i, if the car is late and if the car arrives at its destination after at least $z_i \in \mathbb{N}$ days, that is double its network transportation time. We need those variables to compute the penalties given in the info sheet.
In the formulation of our IP we would like to have as few variables as possibly, but for the sake of readability we introduce four variables as short terms for certain expressions. We call

$$v_i = \bar{d}_i(b_i - d_i) \tag{1}$$

the lateness of car i with transport time

$$b_i = \sum_{t\in T}\Big(\sum_{e\in S} x_{iet} + \sum_{l\in L}(y_{ilt} - y_{il_i^*t})\Big). \tag{2}$$

At last, we write the number of segments a car is at at a time t as $X_{it} = \sum_{e\in S} x_{iet}$ and the number of locations a car is at as $Y_{it} = \sum_{l\in L} y_{ilt}$. We need this numbers to make sure, that a car is only on one transport or at one location at a time.

Here is our IP with the additional variables v_i, b_i, X_{it} and Y_{it}:

LP

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i\in A}(100\bar{d}_i s_i + 25 s_i v_i + 5 w_i(b_i - z_i) + (1-\bar{d}_i)b_i) && && \text{(3a)}\\
\text{subject to} \quad & \sum_{i\in A} x_{iet} \le K_{et} && \forall i \in A, e\in S, t\in T, && \text{(3b)}\\
& \sum_{t\in T} x_{iet} \le LTH_e && \forall i \in A, e \in S, && \text{(3c)}\\
& x_{iet} LTH_e \le \sum_{t'\in T} x_{iet'} && \forall e\in S, i\in A, t\in T, && \text{(3d)}\\
& \sum_{e\in\delta^-(l)} x_{iet} + y_{ilt} = y_{ilt+1} + \sum_{e\in\delta^+(l)} x_{iet+1} && \forall l \in L, i\in A, t\in T\setminus\{|T|\}, && \text{(3e)}\\
& X_{it} + Y_{it} = 1 && \forall i\in A, t\in T, && \text{(3f)}\\
& y_{il_i't} = 1 && \forall i \in A, t\in\{1,...,a_i\}, && \text{(3g)}\\
& v_i \le M s_i && \forall i\in A, && \text{(3h)}\\
& s_i \le v_i + |v_i| && \forall i\in A, && \text{(3i)}\\
& b_i - z_i \le M w_i && \forall i\in A, && \text{(3j)}\\
& w_i \le b_i - z_i + |b_i - z_i| && \forall i\in A, && \text{(3k)}\\
& x_{iet}, y_{ilt}, s_i, w_i \in \{0,1\} && \forall i\in A, e\in S, l\in L, t\in T, && \text{(3l)}
\end{aligned}
$$

where LTH_e is the duration of the transport (lead time hours).

Explanations

In our objective 3a we sum over the costs of all cars, consisting of all penalties for each car i.

The penalties contain a one time penalty for lateness (100euro), a penalty for each late day (25euro) and an additional penalty (5euro), if the car arrives at its destination more than double its network transportation time z_i. The penalties for lateness only make sense if car i has a due date. If not, the costs for i increase by 1(euro) per day, until the car is delivered.
Inequality 3b describes the need to maintain the capacity on each segment. The inequalities 3c and 3d describe the fact that a car must stay on a segment until the transport duration is reached, so if a car i is on a segment e at a time t, it has to be there for LTH_e-long. In equation 3e we establish the flow condition including cars staying at a location. Line 3f is needed, because a car can only be in one place at a time. The equality 3g guarantees that a car stays at its origin, until its available date is reached. Lines 3h and 3i guarantee that $s_i = 1$ if, and only if, the car i is late. Analogously the inequalities 3j and 3k describe w_i. At last, all our variables are binary.
We use two big-M-constraints, where M should be a great number with respect to time, for example $M = 2|T|$.