PHYS 512 Assignment 1

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1. a)

$$f_1' = \frac{f(x+\delta) - f(x-\delta)}{2\delta} = f' + \frac{1}{6}\delta^2 f''' + \frac{\delta^4}{120} f''''' + \cdots$$

$$f_2' = \frac{f(x+2\delta) - f(x-2\delta)}{4\delta} = f' + \frac{4}{6}\delta^2 f''' + \frac{16}{120}\delta^4 f''''' + \cdots$$

$$\Rightarrow f'_{opt} = \frac{4f_1' - f_2'}{3} = f' + \cancel{0 f'''} - \frac{1}{3}\underbrace{\frac{15}{120}}_{\frac{1}{24}}\delta^4 f'''''$$

$$\Rightarrow f' \approx f'_{opt} = \frac{8f(x+\delta) - 8f(x-\delta) - f(x+2\delta) + f(x-2\delta)}{12\delta}$$

b) rounding error $\approx \varepsilon \frac{f}{\delta}$ taylor error $\approx \frac{1}{24}\delta^4 f'''''$

total error $\approx \varepsilon \frac{f}{\delta} + \frac{1}{24}\delta^4 f'''''$

$$\frac{d}{d\delta}\text{error} = 0 \Rightarrow -\varepsilon\frac{f}{\delta^2} + \frac{4}{24}\delta^3 f''''' = 0$$

$$\Rightarrow \delta = \left(\frac{24}{4}\varepsilon\frac{f}{f'''''}\right)^{1/5}$$

$$\approx (6\varepsilon)^{1/5}$$

I calculated $(e^x)'$ and $(e^{0.01x})'$ at $x=1$ with this epsilon as well as $(6\varepsilon)^{1/4}$ and $(6\varepsilon)^{1/6}$ and this epsilon gave the least error which shows this is correct

4. All method produced a graph nearly identical to the original. Integrating the error I found that poly interpolation was best followed by rational and then spline (with errors ~ 0.002, ~ 0.003, and ~ 0.009 respectively). For the lorentzian with $n=3, m=3$ it was rational, spline, then poly with errors ~ 0, ~ 0.004, and ~ 0.02 respectively. We should expect the rational method error to be zero for the lorentzian since it's already rational however for $n=4, m=5$ the rational method had a pole near $x = \sim 0.75$ and was overall very inaccurate, though switching to linalg.pinv fixed the issue. For pinv the function we got was $\frac{1 - \frac{1}{3}x^2}{1 + \frac{2}{3}x^2 - \frac{1}{3}x^4}$ which reduces to $\frac{1}{1+x^2}$, this may explain what happened in the other case, since the ability to factor a polynomial from both sides causes us to lose uniqueness our matrix becomes singular and the inverse is nonsensical, similar to the loss of uniqueness we have when m_0 is allowed to not equal 1.