Assignment 3, PHYS 512
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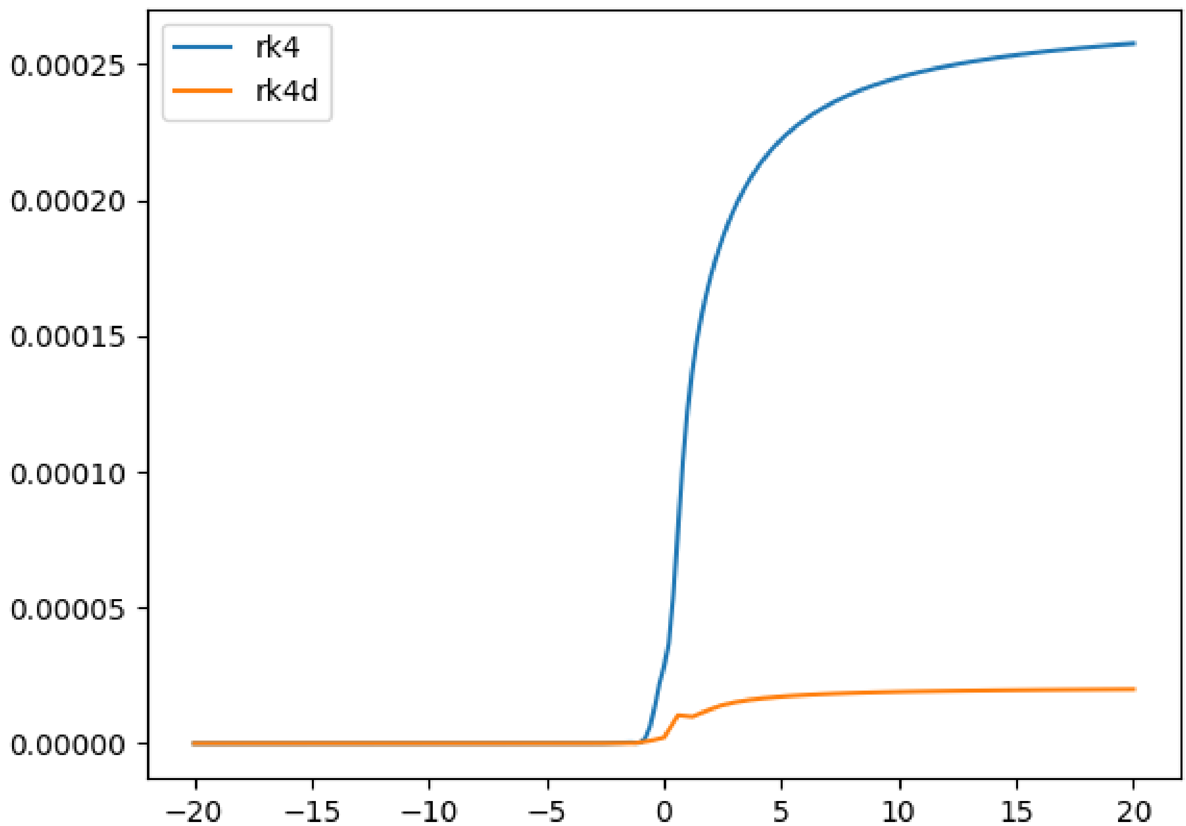
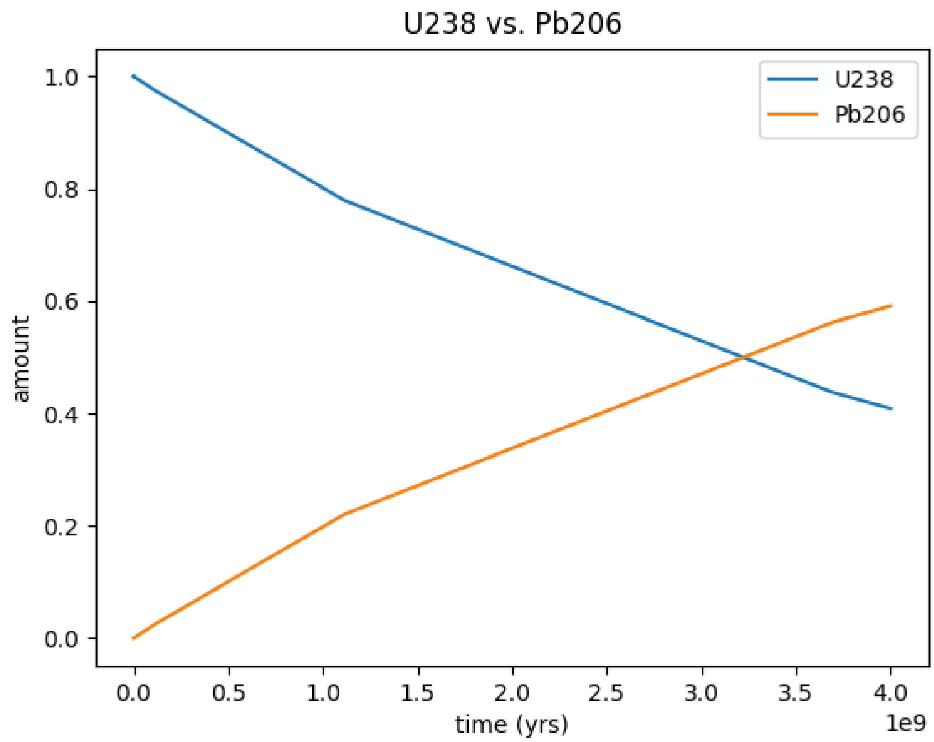
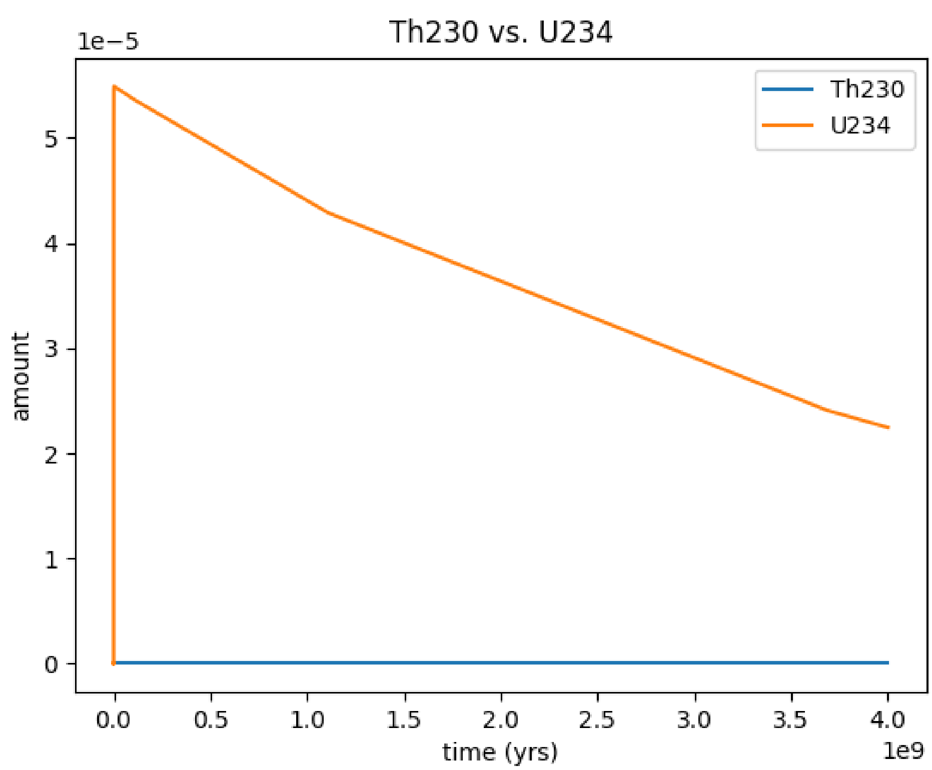
1. The rk4_stepd function was about 10 times more accurate than rk4_step going by the residual graphs shown below.



2. I used the Radau solver because the U238 amount changed much slower than the others so we were safe to use the faster solver. Half of the U238 becomes Pb206 at around ~3.4 billion years and the U238 curve is roughly inverse exponential, both of which make sense analytically given that the half life of U238 is ~4.5 billion years and the amount is expected to drop off exponentially. The Radau method seems to have introduced some imprecision given the U238 dropped by half about a billion years too early but in testing the Runge-Kutta method didn't finish executing for over a minute (given the span and the number of variables it's reasonable to assume it would have taken a very long time to finish) so it's still the best choice. Graphs shown below:


U238 vs. Pb206
U238
Pb206
amount
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
time (yrs)
1e9


Th230 vs. U234
1e−5
Th230
U234
amount
5
4
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
time (yrs)
1e9

3. $z - z_0 = a((x - x_0)^2 + (y - y_0)^2)$
$z - z_0 = a(x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2)$
$z = a(x^2 + y^2) - 2ax_0(x) - 2ay_0(y) + (ax_0^2 + ay_0^2 + z_0)$
$z = a(x^2 + y^2) + bx + cy + d$
With $a = a$, $b = -2ax_0$, $c = -2ay_0$, and $d = ax_0^2 + ay_0^2 + z_0$. We can get x0, y0, and z0 back by going $x_0 = -\frac{b}{2a}$, $y_0 = -\frac{c}{2a}$, and then $z_0 = d - a(x_0^2 + y_0^2)$. Calculating we get a: 0.0001667044547740137 , x0: -1.3604886221979768 , y0: 58.221476081578594 , z0: -1512.8772100367887. The estimate of the noise (which assuming the points have the same distribution is the variance) is 14.32. a had an error of 6.48e-08 and we calculated the focal length to be 1499.7mm with an error of 1.17mm which is consistent with the focal length being 1.5m.