PHYS 512 Assignment 7
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1) $$\frac{\zeta^{2dt}-1}{2dt} = -v\zeta^{dt}\frac{exp(ikdx)) - exp(-ikdx)}{2dx}$$

$$\zeta^{2dt} - 1 = -i2v\frac{dt}{dx}sin(kdx)\zeta^{dt}$$

$$(\zeta^{dt})^2 + i2v\frac{dt}{dx}sin(kdx)\zeta^{dt} - 1 = 0$$

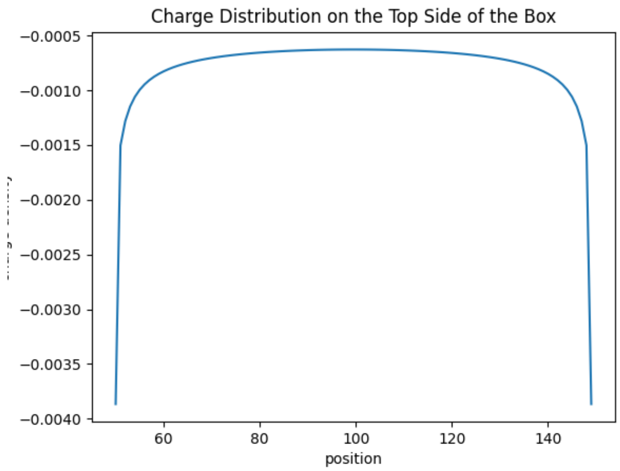
$$\zeta^{dt} = -iv\frac{dt}{dx}sin(kdx) + \sqrt{1 - (v\frac{dt}{dx}sin(kdx))^2}$$

$$\zeta = \big(-iv\frac{dt}{dx}sin(kdx) + \sqrt{1 - (v\frac{dt}{dx}sin(kdx))^2}\big)^{\frac{1}{dt}}$$

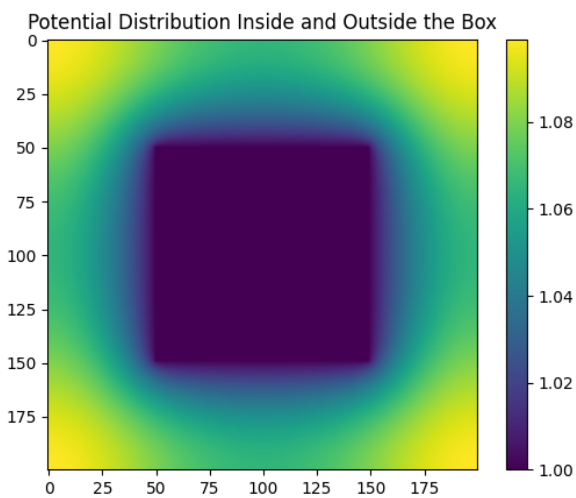
$|\zeta|^2 \leq 1$ if $vdt < dx$ which is the stability condition. Since the equation is stable, energy should be preserved.

2) a) Assuming $V(r) \;\; - ln(r)$ we can set $V[1,0] = V(1) = 0$ and it's neighbour cells $V[1,1] = V[1,-1] = -ln(\sqrt{2}) = -\frac{1}{2}ln(2)$ and $V[2,0] = -ln(2)$. for the average of the cells around (1,0) to equal zero we require $V[0,0] = 2ln(2)$. If we then scale such that $V[0,0] = 1$ we get $V[x,y] = -\frac{\sqrt{x^2+y^2}}{2ln(2)}$ as a rough approximation of the potential everywhere, giving us $V[1,0] = 0$, $V[2,0] = 0.5$, though $V[5,0] = -1.16$ which isn't quite -1.05 so this isn't a perfect model. We can make this better by subtracting off the potential from the inferred charge as described in the bonus. After 5 iterations the charge inferred charge outside of the origin goes as 1.38, 1.47, 0.80, 0.27, 0.028, 0.0033 and we get $V[1,0] = 2.5\times10^{-5}$, $V[2,0] = -0.45$ and $V[5,0] = -1.05$, which is a much more accurate approximation of the Green's function.

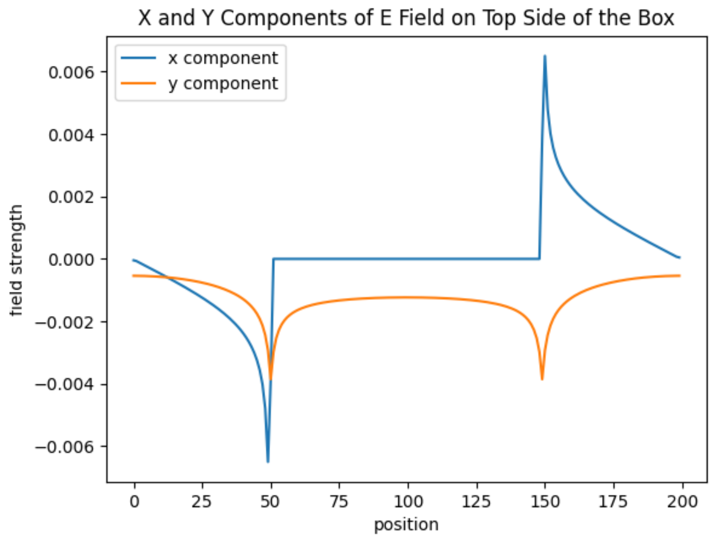
b)

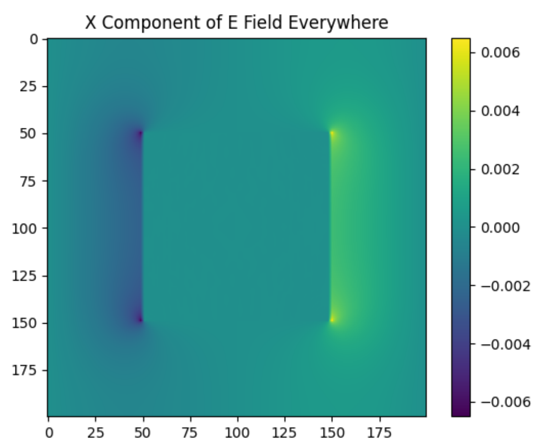


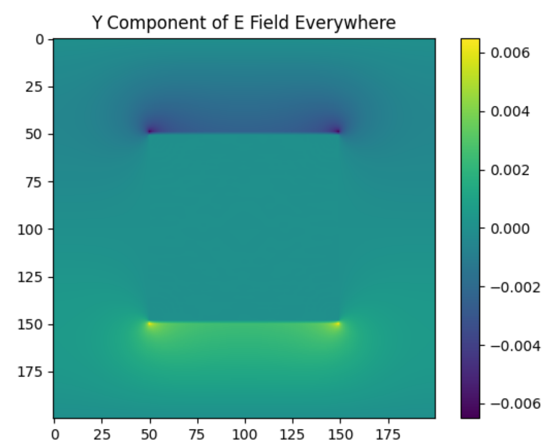
c)



The potential is 1 everywhere in the box so it is exactly constant.







The X and Y components along the top edge makes sense, the field is exactly pointed in the y direction along the top of the box since that's perpendicular to the charge distribution and is stronger near the corners where there is more charge. Looking at the field on every edge we see that the field points toward the equipotent surface everywhere, which is what we expect given that the charge distribution is negative, and is stronger near the corners where there is more charge so the field works everywhere.