The Fresnel integrals and the Euler curve

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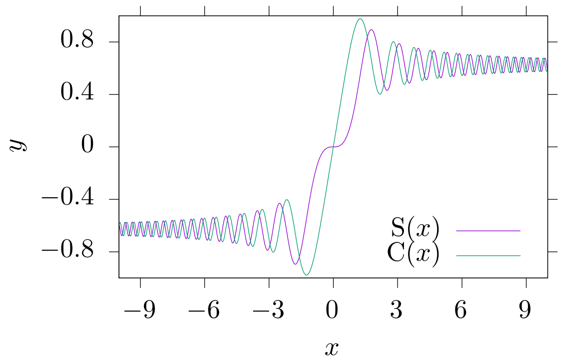

Figure 1: The $S(x)$ and $C(x)$ functions.

All information in this text is taken from Wikipedia[1].

1 The Fresnel integrals

The Frenel Integrals are two trandenceltal functions used in optics. They are defined by the following integrals:

$$S(x) = \int_0^x \sin(t^2)dt \tag{1}$$

$$C(x) = \int_0^x \cos(t^2)dt \tag{2}$$

They appear in the discription of near-field Fresnel diffraction. A plot of the integrals can be seen in figure 1.

As an alternative to equation 1 and 2, the functions can be computed by the power series expansion, which converges for all x:

$$S(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(2n+1)!(4n+3)}. \tag{3}$$

$$C(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n)!(4n+1)}. \tag{4}$$

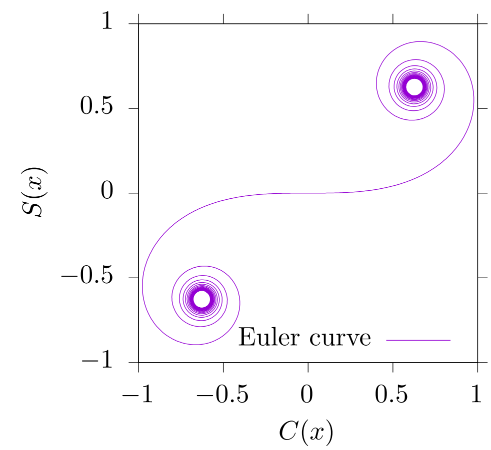

Figure 2: The euler curve for for a parameteri t in the interval $[-10, 10]$.

2 Mathematical properties

- Both of the Fresnel integrals are odd functions of their parameter x.
- Using the power series, see equation 3 and **??**, the Fresnel integrals can be extended to the complex plane.
- In the limit of x going to infinity: $C(\infty) = S(\infty) = \sqrt{\frac{\pi}{8}} \approx 0.6267$

3 The Euler Spiral

The following parametric function is called the Euler spiral, and is plottet in figure 2

$$(x, y) = (C(t), S(t)). \tag{5}$$

The euler spiral has the mathematical property that the curvature at any point is proportional to the length of the spiral, measured from the origin. Thereby, a vehicle following the spiral at a constant speed will experince a constant rate of angular acceleration.

References

[1] Wikipedia: Fresnel integral. https://en.wikipedia.org/wiki/Fresnel_integral, Retrieved 06/03/2020.