In[1]:=
```
ClearAll["Global`*"]
(*Write down basis elements *)
LLket = {{1}, {0}, {0}, {0}}; LLbra = Transpose[LLket];
LRket = {{0}, {1}, {0}, {0}}; LRbra = Transpose[LRket];
RLket = {{0}, {0}, {1}, {0}}; RLbra = Transpose[RLket];
RRket = {{0}, {0}, {0}, {1}}; RRbra = Transpose[RRket];

(*Write down destiny operators*)
ρ1 = 1/2 (LLket.LLbra + RRket.RRbra);
ρ2 = 1/2 (LLket + RRket).(LLbra + RRbra);
Print["ρ1 = 1/2", MatrixForm [2 ρ1]];
Print["ρ2 = 1/2", MatrixForm [2 ρ2]];
```

$$\rho 1 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho 2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

In[9]:=
```
(*Check if ρ1 is a mixed  state*)
ρ1.ρ1 == ρ1
```

Out[9]= False

In[10]:=
```
(*Check if ρ2 is a mixed  state*)
ρ2.ρ2 == ρ2
```

Out[10]= True

In[11]:=
```
A = LLket.RRbra + RRket.LLbra;
MatrixForm [A]
Tr[ρ1.A]
Tr[ρ2.A]
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Out[13]= 0

Out[14]= 1

In[15]:=
```
MatrixForm [Eigensystem [A]]
```

Out[15]//MatrixForm=

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ \{-1, 0, 0, 1\} & \{-1, 0, 0, -1\} & \{0, 0, 1, 0\} & \{0, 1, 0, 0\} \end{pmatrix}$$