In[1]:=
```
ClearAll["Global`*"]
$Assumptions = (L ∈ Reals && L > 0);
```

$$g = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};\text{(* } ds^2 = -du^2 - dv^2 + dx^2 + dy^2 \text{ *)}$$

$$u = \sqrt{L^2 + r[\bullet]^2}\ \mathrm{Cos}\left[\frac{t[\bullet]}{L}\right];$$

$$v = \sqrt{L^2 + r[\bullet]^2}\ \mathrm{Sin}\left[\frac{t[\bullet]}{L}\right];$$

$$x = r[\bullet]\ \mathrm{Cos}[\phi[\bullet]];$$

$$y = r[\bullet]\ \mathrm{Sin}[\phi[\bullet]];$$

$$\mathrm{Print}\left[\text{"}ds^2 = \text{"}, \mathrm{FullSimplify}\left[(\partial_\bullet u \ \ \partial_\bullet v \ \ \partial_\bullet x \ \ \partial_\bullet y).g.\begin{pmatrix} \partial_\bullet u \\ \partial_\bullet v \\ \partial_\bullet x \\ \partial_\bullet y \end{pmatrix}\right][[1, 1]]\right]$$

$$ds^2 = \frac{L^2\ r'[\bullet]^2}{L^2 + r[\bullet]^2} - \frac{\left(L^2 + r[\bullet]^2\right) t'[\bullet]^2}{L^2} + r[\bullet]^2\ \phi'[\bullet]^2$$