$$\begin{aligned} & \ln[13] \coloneqq \ dx = dr \, Sin[\theta] + r \, Cos[\theta] \, d\theta \, ; (*x=r \, cos \, \theta*) \\ & dy = dr \, Cos[\theta] - r \, Sin[\theta] \, d\theta \, ; (*y=r \, sin \, \theta*) \end{aligned}$$

$$& dz = \frac{r}{\sqrt{r^2 + R^2}} dr \, ; (*x^2 + y^2 - z^2 = -R^2 \, ==> \, dz = \frac{r}{z} dr*) \\ & Print["ds^2 = ", FullSimplify[dx^2 + dy^2 - dz^2]]$$

$$& ds^2 = d\theta^2 \, r^2 + \frac{dr^2 \, R^2}{r^2 + R^2} \end{aligned}$$