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In[1]:= ClearAll["Global`*"]
coordinateList = {r,  $\phi$ };

(* Loop over k *)
Do[
  Print["
  ----- k = ", k, " -----"];

  (* Define  $g_{\mu\nu}$  *)

$$g = \begin{pmatrix} \frac{1}{1-k \frac{r^2}{L^2}} & 0 \\ 0 & r^2 \end{pmatrix};$$


  (* Initialize  $\Gamma^\mu_{\nu\rho}$  as rank 3 tensor *)
  tmp[a_, b_, c_] := 0;
   $\Gamma$  = Array[tmp, {2, 2, 2}];

  (* Loop over indices in  $\Gamma^\mu_{\nu\rho}$  *)
  Do[
    Do[
      Do[
        Do[
          x $\mu$  = coordinateList[[ $\mu$ ]];
          x $\nu$  = coordinateList[[ $\nu$ ]];
          x $\rho$  = coordinateList[[ $\rho$ ]];

           $\Gamma[[\mu, \nu, \rho]] += \frac{1}{2} (\text{Inverse}[g]][[\sigma, \mu]] (\partial_{x\nu} g[[\rho, \sigma]] + \partial_{x\rho} g[[\nu, \sigma]] - \partial_{x\sigma} g[[\nu, \rho]]);$ ,
          { $\sigma$ , {1, 2}},
          { $\rho$ , {1, 2}},
          { $\nu$ , {1, 2}},
          { $\mu$ , {1, 2}}];

        (* Initialize  $R^\lambda_{\mu\nu\kappa}$  as a rank 4 tensor *)
        tmp[a_, b_, c_, d_] := 0;
        R = Array[tmp, {2, 2, 2, 2}];

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(* Loop over indices in  $R^{\rho}_{\mu\nu\kappa}$  *)
Do[
  Do[
    Do[
      Do[
        x $\kappa$  = coordinateList[[ $\kappa$ ]];
        x $\nu$  = coordinateList[[ $\nu$ ]];

        (*  $R^{\lambda}_{\mu\nu\kappa} = \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\nu}} + \Gamma^{\eta}_{\mu\nu} \Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa} \Gamma^{\lambda}_{\nu\eta}$  *)
        R[[ $\lambda$ ,  $\mu$ ,  $\nu$ ,  $\kappa$ ]] +=  $\partial_{x\nu} \Gamma[[\lambda, \mu, \kappa]] - \partial_{x\kappa} \Gamma[[\lambda, \mu, \nu]]$ ;
        Do[
          R[[ $\lambda$ ,  $\mu$ ,  $\nu$ ,  $\kappa$ ]] +=  $\Gamma[[\eta, \mu, \kappa]] \times \Gamma[[\lambda, \nu, \eta]] - \Gamma[[\eta, \mu, \nu]] \times \Gamma[[\lambda, \kappa, \eta]]$ ,
          { $\eta$ , {1, 2}}},

          { $\kappa$ , {1, 2}}},
          { $\nu$ , {1, 2}}},
          { $\mu$ , {1, 2}}},
          { $\lambda$ , {1, 2}}];

Print[" $R^{\theta}_{\theta\nu\kappa} =$ ", MatrixForm[FullSimplify[R[[1, 1]]]]];
Print[" $R^{\theta}_{\phi\nu\kappa} =$ ", MatrixForm[FullSimplify[R[[1, 2]]]]];
Print[" $R^{\phi}_{\theta\nu\kappa} =$ ", MatrixForm[FullSimplify[R[[2, 1]]]]];
Print[" $R^{\phi}_{\phi\nu\kappa} =$ ", MatrixForm[FullSimplify[R[[2, 2]]]]];

(* Initialize Ricci $^{\mu}_{\nu}$  as rank 2 tensor *)
tmp[a_, b_] := 0;
Ricci = Array[tmp, {2, 2}];

(* Loop over indices in Ricci $_{\mu\nu}$  *)
Do[
  Do[
    Do[
      Ricci[[ $\mu$ ,  $\nu$ ]] += R[[ $\sigma$ ,  $\mu$ ,  $\sigma$ ,  $\nu$ ]],
      { $\sigma$ , {1, 2}}},
      { $\nu$ , {1, 2}}},
      { $\mu$ , {1, 2}}];

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Print["Ricciμν = ", MatrixForm[FullSimplify[Ricci]]];
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(* Calculate curvature scalar *)
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Print["R = ", FullSimplify[Tr[Inverse[g].Ricci]]],  
{k, {-1, 0, 1}}]
```

----- k = -1 -----

$$R^{\theta}_{\theta\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\theta}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\phi}_{\theta\nu\kappa} = \begin{pmatrix} 0 & \frac{2}{L^2+r^2} \\ -\frac{2}{L^2+r^2} & 0 \end{pmatrix}$$

$$R^{\phi}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{Ricci}_{\mu\nu} = \begin{pmatrix} -\frac{2}{L^2+r^2} & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = -\frac{2}{L^2}$$

----- k = 0 -----

$$R^{\theta}_{\theta\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\theta}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\phi}_{\theta\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\phi}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{Ricci}_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = 0$$

----- k = 1 -----

$$R^{\theta}_{\theta\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\theta}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\phi}_{\theta\nu\kappa} = \begin{pmatrix} 0 & -\frac{2}{L^2-r^2} \\ \frac{2}{L^2-r^2} & 0 \end{pmatrix}$$

$$R^{\phi}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{Ricci}_{\mu\nu} = \begin{pmatrix} \frac{2}{L^2 - r^2} & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = \frac{2}{L^2}$$