

```

In[1]:= ClearAll["Global`*"]
coordinateList = {θ, ϕ};

(* Initialize and define Γμνρ as a rank 3 tensor *)
tmp[a_, b_, c_] := 0;
Γ = Array[tmp, {2, 2, 2}];
Γ[[1, 2, 2]] = - $\frac{1}{2}$  Sin[2 θ];
Γ[[2, 1, 2]] = Cot[θ];
Γ[[2, 2, 1]] = Cot[θ];

(* Initialize Rλμνκ as a rank 4 tensor *)
tmp[a_, b_, c_, d_] := 0;
R = Array[tmp, {2, 2, 2, 2}];

(* Loop over indices in Rρμνκ *)
Do[
  Do[
    Do[
      Do[
        xκ = coordinateList[[κ]];
        xv = coordinateList[[v]];

        (* Rλμνκ =  $\frac{\partial \Gamma^\lambda_{\mu\nu}}{\partial x^\kappa} - \frac{\partial \Gamma^\lambda_{\mu\kappa}}{\partial x^\nu} + \Gamma^\eta_{\mu\nu} \Gamma^\lambda_{\kappa\eta} - \Gamma^\eta_{\mu\kappa} \Gamma^\lambda_{\nu\eta}$  *)
        R[[λ, μ, ν, κ]] += ∂xν Γ[[λ, μ, κ]] - ∂xκ Γ[[λ, μ, ν]];
        Do[
          R[[λ, μ, ν, κ]] += Γ[[η, μ, κ]] × Γ[[λ, ν, η]] - Γ[[η, μ, ν]] × Γ[[λ, κ, η]],
          {η, {1, 2}}],
        {κ, {1, 2}}],
      {v, {1, 2}}],
    {μ, {1, 2}}],
  {λ, {1, 2}}]

Print["Rθθνκ = ", MatrixForm[FullSimplify[R[[1, 1]]]]]
Print["Rθϕνκ = ", MatrixForm[FullSimplify[R[[1, 2]]]]]

```

```
Print["Rϕθνκ = ", MatrixForm[FullSimplify[R[[2, 1]]]]]
```

```
Print["Rϕϕνκ = ", MatrixForm[FullSimplify[R[[2, 2]]]]]
```

$$R^{\theta}_{\theta\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$R^{\theta}_{\phi\nu\kappa} = \begin{pmatrix} 0 & \sin[\theta]^2 \\ -\sin[\theta]^2 & 0 \end{pmatrix}$$

$$R^{\phi}_{\theta\nu\kappa} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$R^{\phi}_{\phi\nu\kappa} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

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In[15]:= (* Initialize Ricciμν as rank 2 tensor *)
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```
tmp[a_, b_] := 0;
```

```
Ricci = Array[tmp, {2, 2}];
```

```
(* Loop over indices in Ricciμν *)
```

```
Do[
```

```
  Do[
```

```
    Do[
```

```
      Ricci[[μ, ν]] += R[[σ, μ, σ, ν]],
```

```
      {σ, 1, 2}],
```

```
    {ν, 1, 2}],
```

```
  {μ, 1, 2}]
```

```
Print["Ricciμν = ", MatrixForm[FullSimplify[Ricci]]]
```

$$\text{Ricci}_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \sin[\theta]^2 \end{pmatrix}$$

```
In[19]:= (* Calculate curvature scalar *)
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$$g\text{Inv} = \begin{pmatrix} 1 & 0 \\ 0 & 1/\sin[\theta]^2 \end{pmatrix};$$

```
Print["R = ", FullSimplify[Tr[gInv.Ricci]]]
```

```
R = 2
```