

```
In[1]:= ClearAll["Global`*"]
```

```
dim = 4;
```

```
coordinateList = {u, v, x, y};
```

$$g = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -a[u]^2 & 0 \\ 0 & 0 & 0 & -b[u]^2 \end{pmatrix};$$

```
gInv = Inverse[g];
```

```
(* Initialize  $\Gamma^\mu_{\nu\rho}$  as rank 3 tensor *)
```

```
tmp[a_, b_, c_] := 0;
```

```
 $\Gamma$  = Array[tmp, {dim, dim, dim}];
```

```
(* Loop over indices in  $\Gamma^\mu_{\nu\rho}$  *)
```

```
Do[
```

```
Do[
```

```
Do[
```

```
Do[
```

```
(* Calculate  $\Gamma^\mu_{\nu\rho}$  *)
```

```
x $\mu$  = coordinateList[[ $\mu$ ];
```

```
x $\nu$  = coordinateList[[ $\nu$ ];
```

```
x $\rho$  = coordinateList[[ $\rho$ ];
```

```
x $\sigma$  = coordinateList[[ $\sigma$ ];
```

$$\Gamma[[\mu, \nu, \rho]] += \frac{1}{2} gInv[[\mu, \sigma]] (\partial_{x\nu} g[[\sigma, \rho]] + \partial_{x\rho} g[[\nu, \sigma]] - \partial_{x\sigma} g[[\rho, \nu]]),$$

```
{ $\sigma$ , 1, dim}];
```

```
(* Print nonzero values *)
```

```
If[ $\Gamma[[\mu, \nu, \rho]]$  == 0, (* Do nothing *),
```

```
Print[" $\Gamma^{x\mu}_{x\nu, x\rho}$ , " = ",  $\Gamma[[\mu, \nu, \rho]]$ ], Print[" $\Gamma^{x\mu}_{x\nu, x\rho}$ , " = ",  $\Gamma[[\mu, \nu, \rho]]$ ],
```

```
{ $\rho$ , 1, dim}],
```

```
{ $\nu$ , 1, dim}],
```

```
{ $\mu$ , 1, dim}]
```

$$\Gamma^v_{x,x} = a[u] a'[u]$$

$$\Gamma^v_{y,y} = b[u] b'[u]$$

$$\Gamma^x_{u,x} = \frac{a'[u]}{a[u]}$$

$$\Gamma^x_{x,u} = \frac{a'[u]}{a[u]}$$

$$\Gamma^y_{u,y} = \frac{b'[u]}{b[u]}$$

$$\Gamma^y_{y,u} = \frac{b'[u]}{b[u]}$$

In[9]:= (* Initialize $R^\rho_{\sigma\mu\nu}$ as rank 4 tensor *)

tmp[a_, b_, c_, d_] := 0;

R = Array[tmp, {dim, dim, dim, dim}];

(* Loop over indices in $R^\rho_{\sigma\mu\nu}$ *)

Do[

Do[

Do[

Do[

(* Calculate $R^\rho_{\sigma\mu\nu}$ *)

xμ = coordinateList[[μ]];

xν = coordinateList[[ν]];

xρ = coordinateList[[ρ]];

xσ = coordinateList[[σ]];

R[[ρ, σ, μ, ν]] = $\partial_{x\mu} \Gamma[[\rho, \nu, \sigma]] - \partial_{x\nu} \Gamma[[\rho, \sigma, \mu]] +$

Sum[$\Gamma[[\rho, \mu, \tau]] \times \Gamma[[\tau, \nu, \sigma]] - \Gamma[[\rho, \nu, \tau]] \times \Gamma[[\tau, \mu, \sigma]]$, {τ, 1, dim}];

(* Print nonzero values *)

If[R[[ρ, σ, μ, ν]] == 0, (* Do nothing *), Print["R"^{xρ}_{xσ,xμ,xν}, " = ", R[[ρ, σ, μ, ν]]],

Print["R"^{xρ}_{xσ,xμ,xν}, " = ", R[[ρ, σ, μ, ν]]],

{ν, 1, dim}],

{μ, 1, dim}],

{σ, 1, dim}],

{ρ, 1, dim}]

$$R^v_{x,u,x} = a[u] a''[u]$$

$$R^v_{x,x,u} = -a[u] a''[u]$$

$$R^v_{y,u,y} = b[u] b''[u]$$

$$R^v_{y,y,u} = -b[u] b''[u]$$

$$R^x_{u,u,x} = \frac{a''[u]}{a[u]}$$

$$R^x_{u,x,u} = -\frac{a''[u]}{a[u]}$$

$$R^y_{u,u,y} = \frac{b''[u]}{b[u]}$$

$$R^y_{u,y,u} = -\frac{b''[u]}{b[u]}$$

```
In[12]:= (* Initialize Ricci tensor Ricciμν as rank 2 tensor *)
tmp[a_, b_] := 0;
Ricci = Array[tmp, {dim, dim}];

(* Loop over indices in Ricciμν *)
Do[
  Do[
    (* Calculate Ricciμν *)
    Ricci[[μ, ν]] = Sum[R[[ρ, μ, ρ, ν]], {ρ, 1, dim}];

    (* Print nonzero values *)
    xμ = coordinateList[[μ]];
    xv = coordinateList[[ν]];
    If[Ricci[[μ, ν]] == 0, (* Do nothing *),
      Print["Ricci"xμ, xv, " = ", Ricci[[μ, ν]], Print["Ricci"xμ, xv, " = ", Ricci[[μ, ν]]],

    {ν, 1, dim}],
  {μ, 1, dim}]

Ricciu,u = -  $\frac{a''[u]}{a[u]}$  -  $\frac{b''[u]}{b[u]}$ 
```