HAND IN 4
COMPLEX ANALYSIS IN SEVERAL VARIABLES, 2020, MMA150, GU

Deadline: 26/5

1. Determine whether $\mathbb{C}^2 \setminus \{0\}$ is holomorphically convex (by using the definition of holomorphic convexity, not that this is equivalent to other properties).

2. L 2.6.19

3. Let $U \subseteq \mathbb{C}^n$ be a domain and assume $f_1, f_2 \in \mathcal{O}(U)$ have no common zeros.

(a) Show that there are smooth functions g_1, g_2 on U such that

$$f_1 g_1 + f_2 g_2 = 1.$$

Hint: There are smooth functions $\chi_i : U \to [0,1]$ such that χ_i is identically 0 in a neighborhood of $\{f_i = 0\}$ for i=1,2, and such that $\chi_1 + \chi_2 = 1$, and you may take this for granted.

(b) Assume that for any smooth $(0,1)$-form α on U such that $\bar{\partial}\alpha = 0$, one can find a smooth function β on U such that $\bar{\partial}\beta = \alpha$. Show that one may choose g_1, g_2 in a) to be holomorphic.
Hint: Let $\tilde{g}_1, \tilde{g}_2$ be smooth solutions from a). One can then take $g_1 = \tilde{g}_1 - f_2\gamma$, $g_2 = \tilde{g}_2 + f_1\gamma$ for an appropriate choice of smooth function γ.

4. Assume that g is a smooth $(0,1)$-form and that ψ is a smooth solution to $\bar{\partial}\psi = g$. Explain why one cannot expect that the support of ψ is contained in the support of g, e.g., by finding an example where the support of ψ is strictly larger than the support of g.

5. L Exercise 4.3.7