In[1]:=
```
ClearAll["Global`*"]
$Assumptions = (M ∈ Reals && M > 0 && (*mass *)
                e ∈ Reals && (*energy*)
                x ∈ Reals && y ∈ Reals && z ∈ Reals &&
                px ∈ Reals && py ∈ Reals && pz ∈ Reals);

(*Calculate {L,H}*)
rvec = {x, y, z};
pvec = {px, py, pz};
Lvec = Cross[rvec, pvec];
V[vec_] := -k/Norm [vec];
H = Dot[pvec, pvec]/(2 M) + V[rvec];
PoissonBraket[A_, B_] := D[A, x] D[B, px] - D[B, x] D[A, px] +
                         D[A, y] D[B, py] - D[B, y] D[A, py] +
                         D[A, z] D[B, pz] - D[B, z] D[A, pz];
```

In[9]:=
```
Print["{L,H} = ", FullSimplify [PoissonBraket[Lvec, H]]]
```

```
{L,H} = {0, 0, 0}
```

In[10]:=
```
(*Check {L_i,L_m }*)
ϵ = LeviCivitaTensor[3];
δ[i_, j_] := KroneckerDelta[i, j];
poissonBraketLiLm = IdentityMatrix[3];(*Initiate*)
poissonBraketLiLm2 = IdentityMatrix[3];(*Initiate*)

For[i = 1, i ≤ 3, i++,
  For[m  = 1, m  ≤ 3, m  ++,
    poissonBraketLiLm [[i, m ]] = PoissonBraket[Lvec[[i]], Lvec[[m ]]];
    poissonBraketLiLm2 [[i, m ]] = Sum [ϵ[[q, i, m ]] Lvec[[q]], {q, 1, 3}];
    (*su(2) structure constants*)
  ]
]

Print["{L_i,L_m } = ", MatrixForm [poissonBraketLiLm ]]
Print["ϵ_qim  L_q = ", MatrixForm [poissonBraketLiLm2 ]]
```

$$\{L\_i,L\_m\ \} = \begin{pmatrix} 0 & pyx-pxy & pzx-pxz \\ -pyx+pxy & 0 & pzy-pyz \\ -pzx+pxz & -pzy+pyz & 0 \end{pmatrix}$$

$$\epsilon\_qim\ \ L\_q = \begin{pmatrix} 0 & pyx-pxy & pzx-pxz \\ -pyx+pxy & 0 & pzy-pyz \\ -pzx+pxz & -pzy+pyz & 0 \end{pmatrix}$$

In[17]:=
```
(*Check {B_i,B_m }*)
Avec = Cross[pvec, Lvec] - kMrvec / Norm [rvec];
Bvec = Avec / Sqrt[2 MAbs[e]];
Print["{A,H} = ", FullSimplify [PoissonBraket[Avec, H]]]
```

```
{A,H} = {0, 0, 0}
```

In[20]:=
```
poissonBraketBiBm = IdentityMatrix[3];(*Initiate*)
poissonBraketBiBm2 = IdentityMatrix[3];(*Initiate*)

For[i = 1, i ≤ 3, i++,
  For[m  = 1, m  ≤ 3, m  ++,
    poissonBraketBiBm [[i, m ]] = PoissonBraket[Bvec[[i]], Bvec[[m ]]];
    poissonBraketBiBm2 [[i, m ]] = Sum [ϵ[[q, i, m ]] Lvec[[q]], {q, 1, 3}] Sign[e];
  ]
]

Print["{B_i,B_m } = ", MatrixForm [FullSimplify [poissonBraketBiBm e/H]]]
(*multiply  by 1=e/H to help FullSimplify*)
Print["ϵ_qim  L_q Sgn[e] = ", MatrixForm [FullSimplify [poissonBraketBiBm2 ]]]
```

$$\{B\_i,B\_m\ \} = \begin{pmatrix} 0 & (-pyx+pxy)\,\mathrm{Sign}[e] & (-pzx+pxz)\,\mathrm{Sign}[e] \\ (pyx-pxy)\,\mathrm{Sign}[e] & 0 & (-pzy+pyz)\,\mathrm{Sign}[e] \\ (pzx-pxz)\,\mathrm{Sign}[e] & (pzy-pyz)\,\mathrm{Sign}[e] & 0 \end{pmatrix}$$

$$\epsilon\_qim\ \ L\_q\ \mathrm{Sgn}[e] = \begin{pmatrix} 0 & (pyx-pxy)\,\mathrm{Sign}[e] & (pzx-pxz)\,\mathrm{Sign}[e] \\ (-pyx+pxy)\,\mathrm{Sign}[e] & 0 & (pzy-pyz)\,\mathrm{Sign}[e] \\ (-pzx+pxz)\,\mathrm{Sign}[e] & (-pzy+pyz)\,\mathrm{Sign}[e] & 0 \end{pmatrix}$$

In[25]:=
```
(*Check {B_i,L_m }*)
poissonBraketBiLm = IdentityMatrix[3];(*Initiate*)
poissonBraketBiLm2 = IdentityMatrix[3];(*Initiate*)

For[i = 1, i ≤ 3, i++,
  For[m  = 1, m  ≤ 3, m  ++,
    poissonBraketBiLm [[i, m ]] = PoissonBraket[Bvec[[i]], Lvec[[m ]]];
    poissonBraketBiLm2 [[i, m ]] = Sum [ϵ[[q, i, m ]] Bvec[[q]], {q, 1, 3}];
  ]
]

Print["{B_i,L_m } = ", MatrixForm [FullSimplify [poissonBraketBiLm ]]]
Print["ϵ_qim  B_q = ", MatrixForm [FullSimplify [poissonBraketBiLm2 ]]]
```

$$\{B\_i,L\_m\} = \begin{pmatrix} 0 & \dfrac{-px\,pz\,x-py\,pz\,y+px^2\,z+py^2\,z-\frac{kMz}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} & \dfrac{px\,py\,x-px^2\,y-pz^2\,y+py\,pz\,z+\frac{kMy}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} \\ \dfrac{px\,pz\,x+py\,pz\,y-px^2\,z-py^2\,z+\frac{kMz}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} & 0 & \dfrac{py^2\,x+pz^2\,x-px\,py\,y-px\,pz\,z-\frac{kMx}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} \\ \dfrac{-px\,py\,x+px^2\,y+pz^2\,y-py\,pz\,z-\frac{kMy}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} & \dfrac{-py^2\,x-pz^2\,x+px\,py\,y+px\,pz\,z+\frac{kMx}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{\frac{eM}{Sign[e]}}} & 0 \end{pmatrix}$$

$$\in\_qim\ \ B\_q = \begin{pmatrix} 0 & \dfrac{-px\,pz\,x-py\,pz\,y+px^2\,z+py^2\,z-\frac{kMz}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} & \dfrac{px\,py\,x-px^2\,y-pz^2\,y+py\,pz\,z+\frac{kMy}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} \\ \dfrac{px\,pz\,x+py\,pz\,y-px^2\,z-py^2\,z+\frac{kMz}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} & 0 & \dfrac{py^2\,x+pz^2\,x-px\,py\,y-px\,pz\,z-\frac{kMx}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} \\ \dfrac{-px\,py\,x+px^2\,y+pz^2\,y-py\,pz\,z-\frac{kMy}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} & \dfrac{-py^2\,x-pz^2\,x+px\,py\,y+px\,pz\,z+\frac{kMx}{\sqrt{x^2+y^2+z^2}}}{\sqrt{2}\,\sqrt{M}\,\sqrt{Abs[e]}} & 0 \end{pmatrix}$$