Problem Set 3

Standard model of particle physics
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1 Solution Q1

Peskin Shroeder p. 704 provides us with the kinetic energy for the fermions

$$\mathcal{L} = \bar{E}_L \left(i\not{D}\right) E_L + \bar{e}_R \left(i\not{D}\right) e_R + \bar{Q}_L \left(i\not{D}\right) Q_L + \bar{u}_R \left(i\not{D}\right) u_R + \bar{d}_R \left(i\not{D}\right) d_R$$

where

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+\tau^+ + W_\mu^-\tau^-\right) - i\frac{g}{\cos\theta_W} Z_\mu \left(\tau^3 - \sin^2\theta_W Q\right) - ieA_\mu Q$$

and

$$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L.$$

Inserting the covariant derivative into the Lagrangian and gathering the terms with Z_μ (note that we need not care about terms not containing Z, making calculations a lot easier), we find that this gives us precisely the current which we were looking fore if we read off J_Z^μ from $gJ_Z^\mu Z_\mu$.

$$\begin{aligned}
\mathcal{L} &= \sum_\psi \bar{\psi}\left(i\not{D}\right)\psi \\
&= -i^2 \frac{g}{\cos\theta_W} Z_\mu \sum_\psi \bar{\psi}\gamma^\mu \left(\tau^3 - \sin^2\theta_W Q\right)\psi + \text{Terms not of interest} \\
&= gZ_\mu J_Z^\mu + \text{Terms not of interest} \\
&\implies J_Z^\mu = \frac{1}{\cos\theta_W}\sum_\psi \bar{\psi}\gamma^\mu\left(\tau^3 - \sin^2\theta_W Q\right)\psi
\end{aligned}$$

2 Solution Q2

From Peskin p. 704, we also have that $\tau^3 = 0$ on right handed fields, and $\pm\frac{1}{2}$ on left handed ones, with sign chosen such that the electric charge is correct. We have that the decay rates are proportional to the squares of their factors in the current, $\left(\tau^3 - \sin\theta_W Q\right)$ in the current. Thus, for the left handed electron $\tau^3 = -\frac{1}{2}$ and $Q = -1$, and for the right handed, $\tau^3 = 0$ and $Q = -1$. For the neutrinos, $\tau^3 = \frac{1}{2}$. Since we are interested in the fraction between the decay rates, we can now write this fraction based on what we have established

$$\frac{\Gamma\left(Z \to e^+e^-\right)}{\Gamma\left(Z \to \bar{\nu}_e\nu_e\right)} = \frac{\left(-\frac{1}{2} + \sin^2\theta_W\right)^2 + \left(\sin^2\theta_W\right)^2}{\left(\frac{1}{2}\right)^2} \overset{\sin^2\theta_W \approx .223}{\approx} .51$$

3 Solution Q3

Lepton universality tells us that decay rates to the different neutrino antineutrino pairs must be equal.

$$\Gamma\left(Z \to \bar{\nu}_e\nu_e\right) = \Gamma\left(Z \to \bar{\nu}_\mu\nu_\mu\right) = \Gamma\left(Z \to \bar{\nu}_\tau\nu_\tau\right)$$

Possible decay modes of the Z boson are

$$\begin{aligned}
&Z \to e^+e^- \text{ or } \mu^+\mu^- \text{ or } \tau^+\tau^- \\
&Z \to \bar{\nu}_e\nu_e \text{ or } \bar{\nu}_\mu\nu_\mu \text{ or } \bar{\nu}_\tau\nu_\tau \\
&Z \to \bar{u}u \text{ or } \bar{b}b \text{ or } \bar{c}c \text{ or } \bar{s}s \text{ or } \bar{b}b
\end{aligned}$$

For the leptons we may use lepton universality and just slap on a factor of three. However for the quarks, we need a factor of 9 on the d decay rate due to the color and family quantum numbers. For the u this factor becomes 6 since the Z boson does not decay to the top quark due to its mass.

Having established this, we proceed with the same procediure as in Q2 to deduce

$$\begin{aligned}\Gamma\left(Z\to e^+e^-\right) &= .51\Gamma\left(Z\to\bar{\nu}\nu\right)\\ \Gamma\left(Z\to\bar{u}u\right) &= .58\Gamma\left(Z\to\bar{\nu}\nu\right)\\ \Gamma\left(Z\to\bar{d}d\right) &= .747\Gamma\left(Z\to\bar{\nu}\nu\right).\end{aligned}$$

Thus, the fraction $\Gamma_{\text{visible}}/\Gamma_{\text{total}}$, where Γ_{visible} is Γ_{total} without neutrino decay, becomes the following.

$$\frac{\Gamma_{\text{visible}}}{\Gamma_{\text{total}}} = \frac{(.51\cdot 3+.747\cdot 9+.58\cdot 6)}{(.51\cdot 3+.747\cdot 9+.58\cdot 6+3)}\frac{\Gamma\left(Z\to\bar{\nu}\nu\right)}{\Gamma\left(Z\to\bar{\nu}\nu\right)}\approx .796$$

Checking with the numbers given in the exercise, where $\Gamma_{\text{visible}} = 1992$ MeV and $\Gamma_{\text{total}} = 2490$ MeV, we find the fraction to be 0.8, which agrees with our result.