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In[1]:= ClearAll["Global`*"]
```

```
coordinateList = {t, r,  $\phi$ ,  $\theta$ };
```

```
(* Define Schwarzschild metric  $g_{\mu\nu}$  *)
```

$$g = \begin{pmatrix} -\left(1 - \frac{rs}{r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{rs}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin[\theta]^2 \end{pmatrix};$$

```
(* Initialize  $\Gamma^\mu_{\nu\rho}$  as rank 3 tensor *)
```

```
tmp[a_, b_, c_] := 0;
```

```
 $\Gamma$  = Array[tmp, {4, 4, 4}];
```

```
(* Loop over indices in  $\Gamma^\mu_{\nu\rho}$  *)
```

```
Do[
```

```
Do[
```

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Do[
```

```
Do[
```

```
  x $\mu$  = coordinateList[[ $\mu$ ]];
  x $\nu$  = coordinateList[[ $\nu$ ]];
  x $\rho$  = coordinateList[[ $\rho$ ]];
  x $\sigma$  = coordinateList[[ $\sigma$ ]];

```

$$\Gamma[[\mu, \nu, \rho]] += \frac{1}{2} (\text{Inverse}[g]][[\sigma, \mu]] (\partial_{x\nu} g[[\rho, \sigma]] + \partial_{x\rho} g[[\nu, \sigma]] - \partial_{x\sigma} g[[\nu, \rho]]),$$

```
  { $\sigma$ , 1, 4}],
```

```
  { $\rho$ , 1, 4}],
```

```
  { $\nu$ , 1, 4}],
```

```
  { $\mu$ , 1, 4}];
```

```
Print[" $\Gamma^\mu_{\nu\rho}$  = ", MatrixForm[FullSimplify[ $\Gamma$ [[2]]]]];
```

$$\Gamma^\mu_{\nu\rho} = \begin{pmatrix} \frac{(r-rs)rs}{2r^3} & 0 & 0 & 0 \\ 0 & -\frac{rs}{2r^2-2r rs} & 0 & 0 \\ 0 & 0 & -r+rs & 0 \\ 0 & 0 & 0 & (-r+rs)\sin[\theta]^2 \end{pmatrix}$$

In[8]:= (\* Simplify differential equation obtained from geodesic equation \*)

$$\text{FullSimplify}\left[r p p + \frac{r s (r - r s)}{2 r^3} \left( \left( 1 - \frac{r s}{r} \right)^{-2} r p^2 + \left( 1 - \frac{r s}{r} \right)^{-1} r^2 \right) - \frac{r s}{2 r^2 - 2 r r s} r p^2 + (r s - r)\right]$$

Out[8]=  $-r + r p p + \frac{3 r s}{2}$