In[1]:= ClearAll["Global`*"]

(*Solve for vL by considering centripetal acceleration*)

$$\text{Solve}\left[\frac{v^2}{RL} == \frac{G\,M}{r}, v\right]$$

Out[2]= $$\left\{\left\{v \to -\frac{\sqrt{G}\,\sqrt{M}\,\sqrt{RL}}{\sqrt{r}}\right\}, \left\{v \to \frac{\sqrt{G}\,\sqrt{M}\,\sqrt{RL}}{\sqrt{r}}\right\}\right\}$$

In[3]:= $$vL = \sqrt{\frac{G\,M}{RL}};$$

(*Solve for RL*)

$$\gamma[v\_] := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$\phi_g[r\_] := -\frac{G\,M}{c^2\,r};$$

$$\text{sol1} = \text{Solve}\left[\sqrt{1 + 2\,\phi_g[RL]} \Big/ \sqrt{1 + 2\,\phi_g[R0]} == \gamma[vL], RL\right]$$

$$\text{sol2} = \text{Solve}\left[\sqrt{1 + 2\,\phi_g[R0] - \frac{v0^2}{c^2}} == \sqrt{1 + 2\,\phi_g[RL] - \frac{vL^2}{c^2}}, RL\right]$$

Out[6]= $$\left\{\left\{RL \to \frac{1}{4}\left(3\,R0 - \frac{\sqrt{R0}\,\sqrt{-16\,G\,M + 9\,c^2\,R0}}{c}\right)\right\}, \left\{RL \to \frac{1}{4}\left(3\,R0 + \frac{\sqrt{R0}\,\sqrt{-16\,G\,M + 9\,c^2\,R0}}{c}\right)\right\}\right\}$$

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[7]= $$\left\{\left\{RL \to \frac{3\,G\,M\,R0}{2\,G\,M + R0\,v0^2}\right\}\right\}$$

In[8]:=
```
(*Evaluate solution in SI units*)
c = Quantity["SpeedOfLight"];
G = Quantity["GravitationalConstant"];
M = Quantity["EarthMass"];
R0 = Quantity["EarthMeanRadius"];
v0 = 2 π R0 Cos[0.995] / (24 Quantity["Hours"]);
(*speed of earth's surface at Gothenburg*)

Print["R_L = ", UnitConvert[RL /. sol1]]
Print["R_L = ", UnitConvert[RL /. sol2]]
```

$R_L = \{0.0029567\,\text{m},\ 9.55651\times10^{6}\,\text{m}\}$

$R_L = \{9.55166\times10^{6}\,\text{m}\}$