In[1]:=
```
ClearAll["Global`*"]
$Assumptions = (v ∈ Reals
     && r[x] ∈ Reals
     && ϕ[x] ∈ Reals);

Φ[x_] := (v + r[x]) Exp[I ϕ[x]];
L = -1/4 F[A] × F[A] + (∂_x Φ[x] + I e A Φ[x]) (∂_x Φ[x]* + I e A Φ[x]*) -
      m^2 Φ[x] × Φ[x]* - λ/2 (Φ[x] × Φ[x]*)^2 // ComplexExpand // FullSimplify;
Print["L = ", Expand[L]]

(* Euler-Lagrange equation of free r-field *)
ELeq = ∂_x(∂_{r'[x]} L) - ∂_{r[x]} L == 0 /. {λ → 0} // FullSimplify
(* Substitute *)
rTmp[x_] := 1;
ϕTmp[x_] := p x;
ELeq /. {r → rTmp, ϕ → ϕTmp} // FullSimplify
```

$$L\ =\ -A^2 e^2 v^2 - m^2 v^2 - \frac{v^4 \lambda}{2} - \frac{F[A]^2}{4} - 2 A^2 e^2 v\, r[x] - 2 m^2 v\, r[x] - 2 v^3 \lambda\, r[x] - A^2 e^2 r[x]^2 - m^2 r[x]^2 - 3 v^2 \lambda\, r[x]^2 - 2 v \lambda\, r[x]^3 - \frac{1}{2} \lambda\, r[x]^4 + 2 i A e v\, r'[x] + 2 i A e\, r[x]\, r'[x] + r'[x]^2 + v^2 \phi'[x]^2 + 2 v\, r[x]\, \phi'[x]^2 + r[x]^2 \phi'[x]^2$$

Out[6]= $(v + r[x]) \left(A^2 e^2 + m^2 - \phi'[x]^2\right) + r''[x] == 0$

Out[9]= $\left(A^2 e^2 + m^2 - p^2\right) (1 + v) == 0$

In[10]:=
```
(* Euler-Lagrange equation of ϕ-field *)
ELeq = ∂_x(∂_{ϕ'[x]} L) - ∂_{ϕ[x]} L == 0 // FullSimplify

(* Substitute *)
rTmp[x_] := 1;
ϕTmp[x_] := p x;
ELeq /. {r → rTmp, ϕ → ϕTmp} // FullSimplify
```

Out[10]= $(v + r[x]) (2 r'[x]\, \phi'[x] + (v + r[x])\, \phi''[x]) == 0$

Out[13]= True