Problem Set 4

Standard model of particle physics
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We are considering the Lagrangian describing the interaction between Higgs and the field a,

$$\mathcal{L}_{DM} = k\Phi^\dagger \Phi aa.$$

Let

$$\Phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix},$$

i.e., Φ aqures a VEV and h is the Higgs boson. The Lagrangian becomes

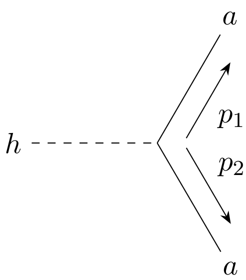
$$\mathcal{L}_{DM} = \frac{1}{2}kv^2 aa + \frac{1}{2}kvhaa + \frac{1}{2}khhaa.$$

We can read off the mass of a from the quadratic free term, thus

$$m_a^2 = kv^2.$$

Noting that v is of mass dimension $\dim(v) = 1$, and that the Lagrangian density must have mass dimension $\dim(\mathcal{L}) = 4$, it follows that our interaction strength k must have mass dimension $\dim(k) = 0$.

We now want to calculate the decay rate $\Gamma(h \to aa)$, the relevant amplitude is

$i\mathcal{M} =$  $= \frac{i}{2}kv.$

Proceeding with the calculation of the decay rate, we must keep in mind to divide by two due to the two a obeying bose statistics.

$$\begin{aligned}
\Gamma(h \to aa) &= \frac{1}{2!}\frac{1}{m_h}\int \frac{\mathrm{d}^3\mathbf{P_1}}{2E_1(2\pi)^3}\frac{\mathrm{d}^3\mathbf{P_2}}{2E_2(2\pi)^3}\frac{1}{4}k^2v^2(2\pi)^4\delta(m_h - E_1 - E_2)\delta^{(3)}(\mathbf{P_1}+\mathbf{P_2}) \\
&= \frac{k^2v^2}{4}\frac{1}{4m_h}\frac{1}{16\pi^2}\int \frac{\mathrm{d}^3\mathbf{P_1}}{E_1E_2}\delta(m_h - E_1 - E_2) \\
&= \left[\mathrm{d}^3\mathbf{P} = 4\pi|\mathbf{P}|^2\,\mathrm{d}P.E_1 = E_2 = |\mathbf{P}|^2\right] \\
&= \frac{k^2v^2}{4}\frac{1}{4m_h}\frac{1}{16\pi^2}\int \mathrm{d}P\delta(m_h - 2P)\frac{4\pi P^2}{P^2} \\
&= \frac{1}{128\pi}\frac{k^2v^2}{m_h} = \frac{1}{128\pi}\frac{m_a^4}{m_hv^2}
\end{aligned}$$

As a sanity check, we see that the mass dimension of the decay rate $\dim(\Gamma) = 1$, which is correct.

Proceeding to consider the fact that the Higgs decay width to invisible particles cannot be greater than 3.1 MeV, we can find an upper bound on m_a. To do this, we use the numerical values $v = 246$ GeV and $m_h = 125$ GeV. Solving for m_a at the critical point, we find that m_a must satisfy the inequality

$$m_a \leq \left(128\pi \cdot \left[3.1 \text{ MeV} \cdot (246 \text{ GeV})^2 \cdot 125 \text{ GeV}\right]\right) \approx 55.41 \text{ GeV}.$$