$$
\begin{aligned}
|\psi_1\rangle &= \frac{1}{2}|0\rangle \otimes |0\rangle + \frac{\sqrt{3}}{2}|1\rangle \otimes |1\rangle \\
|\psi_2\rangle &= \frac{1}{4}|0\rangle \otimes |0\rangle + \frac{3}{4}|0\rangle \otimes |1\rangle + \frac{\sqrt{3}}{4}|1\rangle \otimes |0\rangle - \frac{\sqrt{3}}{4}|1\rangle \otimes |1\rangle .
\end{aligned}
$$

$$
U \otimes I\,|\psi_1\rangle = \frac{1}{2}(U|0\rangle) \otimes |0\rangle + \frac{\sqrt{3}}{2}(U|1\rangle) \otimes |1\rangle ,
$$

so $U \otimes I\,|\psi_1\rangle = |\psi_2\rangle$ implies

$$
\begin{aligned}
\frac{1}{2}U|0\rangle &= \frac{1}{4}|0\rangle + \frac{\sqrt{3}}{4}|1\rangle \\
\frac{\sqrt{3}}{2}U|1\rangle &= \frac{3}{4}|0\rangle - \frac{\sqrt{3}}{4}|1\rangle
\end{aligned}
$$

and hence, in the $\{|0\rangle, |1\rangle\}$ basis,

$$
\begin{aligned}
U &= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \\
U^{-1} &= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} .
\end{aligned}
$$