$$V = -\frac{1}{2} \mu^{2} (\pi^{2} + (v + \sigma)^{2}) + \frac{\lambda}{4} (\pi^{2} + (v + \sigma)^{2})^{2};$$

V /. 
$$\left\{ v \rightarrow \frac{\mu}{\sqrt{\lambda}} \right\}$$
 // FullSimplify // Expand

Out[3]= 
$$\frac{\pi^4 \lambda}{4} - \frac{\mu^4}{4 \lambda} + \pi^2 \sqrt{\lambda} \mu \sigma + \frac{1}{2} \pi^2 \lambda \sigma^2 + \mu^2 \sigma^2 + \sqrt{\lambda} \mu \sigma^3 + \frac{\lambda \sigma^4}{4}$$