MODULE - 5

KINEMATICS

Date 23/5/17

→ Kinematics : without considering forces.

→ Kinetics : considering the forces.

→ Displacement : rate of change of position.

→ Linear Velocity

→ Angular Velocity

→ Uniform velocity

→ Non-uniform velocity

→ Average Velocity

$$V_{av} = \frac{\Delta s}{\Delta t}$$

→ Instantaneous Velocity

$$V_{in} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

→ Deaccelaration / Retardation : Acceleration with decreasing velocity.

Imp → Derivation of equations of motions:-

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2.$$

$$v^2 - u^2 = 2as.$$

→ Acceleration due to gravity (g) -

$$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

$\int a\,dx = \int v\,dv.$

$dv = a\,dt. \qquad dt\,v = dx$

$\Rightarrow ax = \frac{v^2}{2} \qquad \int_u^v dv = a\int_0^t dt \qquad dx = (u+at)\,dt$

$\int_0^s dx = \int_0^t (u+at)\,dt$

$\Rightarrow 2as = v^2(v-u) = a(t-0)$

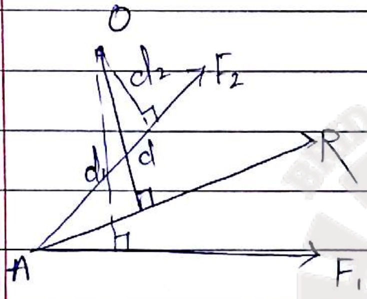
$\Rightarrow 2as = v^2 - u^2 \qquad v = u + at \qquad s - 0 = \left(ut + \frac{1}{2}at^2\right)_0^t$

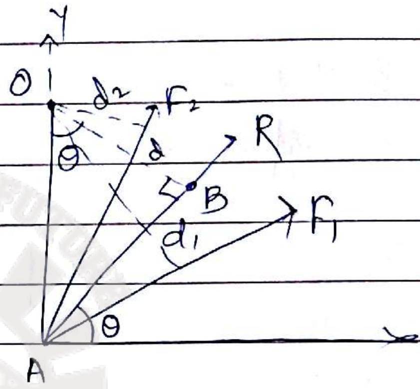
$\boxed{a = v \cdot \frac{dv}{dx}}$

$S = ut + \frac{at^2}{2}$

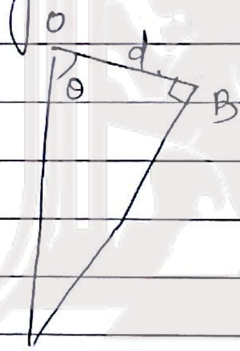
Varignon's Theorem :-

Statement :- The algebraic sum of all the moments of a system of co-planar forces about a moment centre is equal to the moment of the resultant force about the same moment centre.





$\angle AOB$ = angle made by resultant with axis $= \theta$.



from ΔAOB :-

$$\cos\theta = \frac{d}{OA}.$$

$\Rightarrow d = OA \cos\theta.$

$\Rightarrow R \times d = R \times OA \cos\theta.$

$\Rightarrow R \times d = R_x \; OA.$ — ③.i

R_x - Component of R along x-axis.

Similarly $F_1 \times d_1 = F_x \, OA.$ — ①

$F_2 \times d_2 = F_2 \times OA$ — ②.

∴ Adding eq ① and ②.

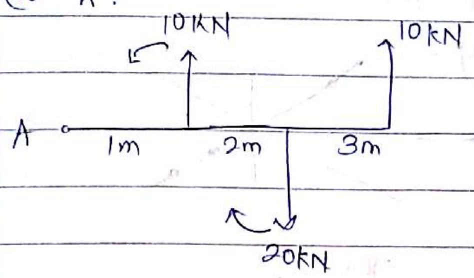
$(F_1 \times d_1) + (F_2 \times d_2) = (F_{1x} + F_{2x})\, OA.$

$(F_1 \times d_1) + (F_2 \times d_2) = (R_x)\, OA.$ ← ③(ii)

$(F_1 \times d_1) + (F_2 \times d_2) = R \times d$ //

Hence proved ☺

@ A.

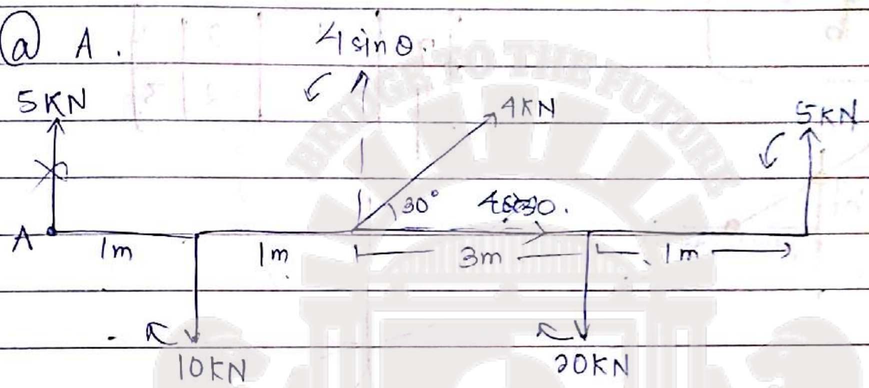


$m_A = (-10)\,1 + (20\times 3) - (10)\,5.$

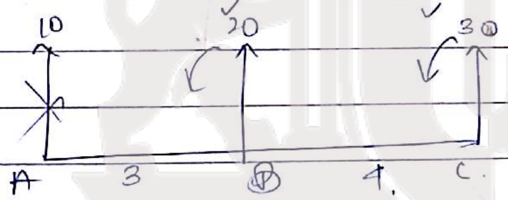
$= -10 + 60 - 50$

$= 10$

@ A.



$\therefore\; m_A = (-5)(6) + (20)(5) - (4\sin\theta)(2) + 10(1)$



@ A.

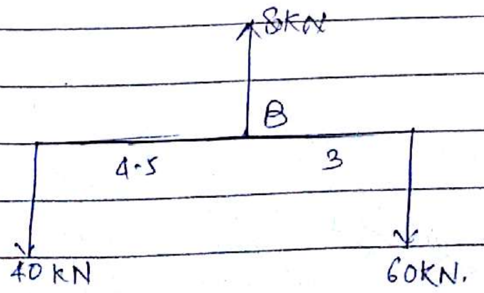
$m_A = (-20)\,3 - (30)\,7$

$= -60 - 210 = -270.$

$d = \dfrac{|m_A|}{R}$

x intercept $= \dfrac{|m_A|}{\Sigma V} = \dfrac{|M_A|}{\Sigma V} =$

@ A:-



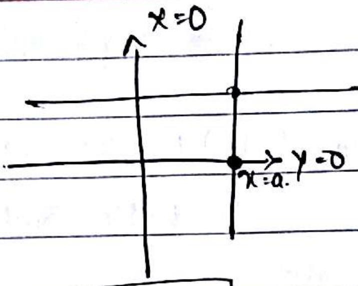
$\Sigma H = 0$

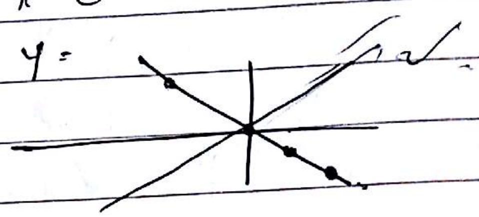
$\Sigma V = 80 - 40 - 60$

$= -20.$

$R = \sqrt{0^2 + (-20)^2} = \sqrt{400} = 20\,kN.$

$m_A =$

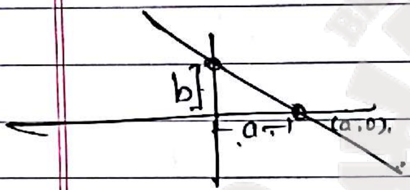


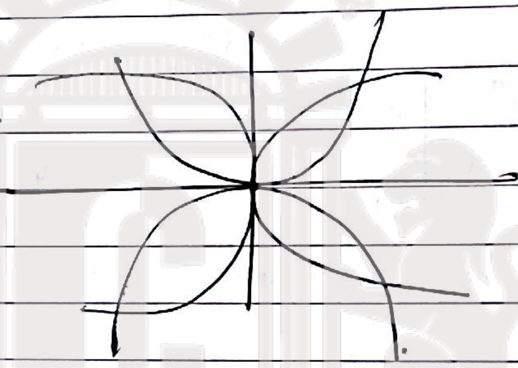


$$\boxed{\frac{x}{a} + \frac{y}{b} = 1.}$$

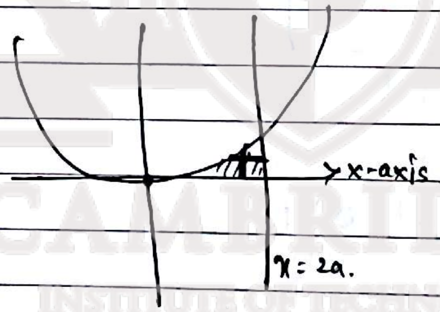
$x = -y.$

x	1	2	-3
y	-1	-2	3





$$\iint_A xy \, dx \, dy.$$



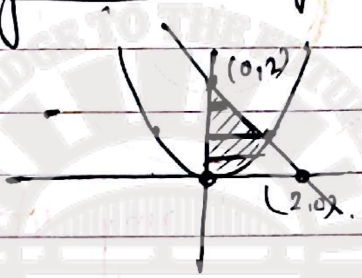
$$\iint.$$

$$\int_0^1 \int_{x^2}^{2x} x\,y\,dx\,dy.$$

$\Rightarrow$ $y = x^2$ to $2x$

$\boxed{y = x^2}$ $\boxed{y = 2x}$ $\cancel{y - 2x = 0}$. $y + x = 2$. $\frac{y}{2} + \frac{x}{2} = 1$.

x	0	1	-1
y	0	1	1



$x = r\sin\theta$.

$y = r\cos\theta$.

$r = \sqrt{}$

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$$

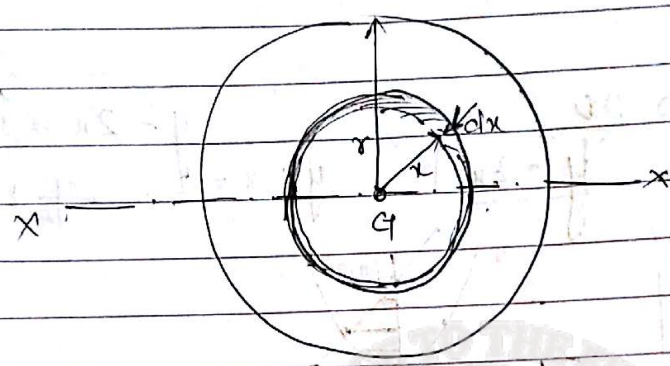
$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\,dx.$$

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$\Gamma(m) = \int_0^\infty e^{-t}\, t^{m-1}\,dt \qquad t = x^2.$$

$$= \int_0^\infty e^{-x^2} x^2$$

MI of Circle



Area of elemental ring $dA = (2\pi x)\,dx$.

MI of this element from centre $= \Sigma x^2\,dA$.

$$= (2\pi x\,dx)\,x^2$$

$$= 2\pi x^3\,dx$$

MI of whole cle from centre $I_{zz} = \int_0^r 2\pi x^3\,dx$

$$= 2\pi\left(\frac{x^4}{4}\right)_0^r$$

$$= \frac{2\pi r^4}{\cancel{4}\,2}$$

$$I_{zz} = \frac{\pi r^4}{2} \quad \text{—— (1)}$$

$I_{zz} = I_{xx} + I_{yy}$.

Circular lamina $I_{xx} = I_{yy}$.

$\therefore I_{zz} = 2 I_{xx}$ —— (2)

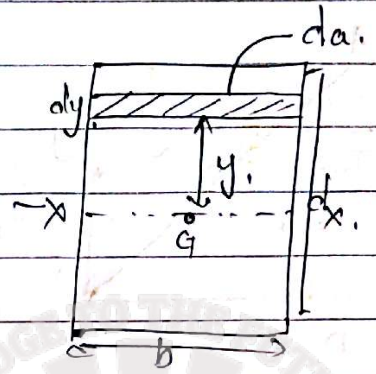
$$2 I_{xx} = \frac{\pi r^4}{2}$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi r^4}{4}}$$

If $d = 2r$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi d^4}{64}}$$

MI for rectangle :-



Area of elemental strip = da = $b \times dy$.

MI of this element area about xx = $(da y^2)$

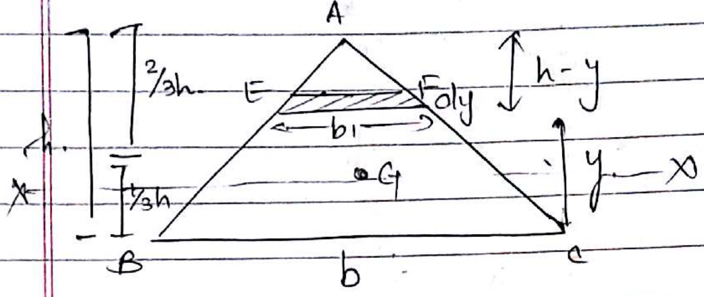
$= b \times dy \, y^2$

$= b y^2 dy$

MI of whole rectangle $I_{xx} = \int_{-d/2}^{d/2} y^2 b \, dy$

$$\boxed{I_{xx} = b\left(\frac{y^3}{3}\right) = \frac{bd^3}{12}}$$

MI of Δ :-



ΔABC and ΔAEF are similar Δ.

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = b\left(\frac{h}{h} - \frac{y}{h}\right)$$

$$b_1 = b\left(1 - \frac{y}{h}\right).$$

Area of element EF $= da = b_1 dy$.

$$da = b\left(1 - \frac{y}{h}\right)dy.$$

MI of elemental strip about base BC $= da\, y^2$.

$$= b\left(1 - \frac{y}{h}\right)dy\, y^2.$$

$$= \left(y^2 - \frac{y^3}{h}\right)b\,dy.$$

MI of whole Δ :-

$$AB = I_{BC} = b\int_0^h y^2 - \frac{y^3}{h}\,dy.$$

$$\Rightarrow \left(\frac{y^3}{3} - \frac{y^4}{4h}\right)_0^h b$$

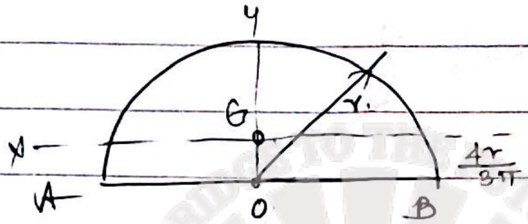
$$= b\left[\frac{h^3}{3} - \frac{h^{\cancel{4}3}}{4\cancel{h}}\right]$$

$$= \left(b\,\frac{h^3}{12}\right)$$

MI of Centroid $\Rightarrow I_{BC} = I_{xx} + A\bar{y}^2 = \frac{bh^3}{12} = I_{xx} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$

$$\boxed{I_x = \frac{bh^3}{36}}$$

MI of Semi-circle :-



. MI of the semicircle about A $= \frac{1}{2}$ MI of circle.

$$= \frac{1}{2} \times \frac{\pi r^4}{4}$$

$$= \frac{\pi r^4}{8}$$

MI about centroidal axis

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$\frac{\pi r^4}{8} = I_{xx} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$\frac{\pi r^4}{8} - \frac{\cancel{\pi} r^2}{\cancel{2}}\left(\frac{\cancel{16}^{\,8} r^2}{9\pi^{\cancel{2}}}\right) = I_{xx}$$

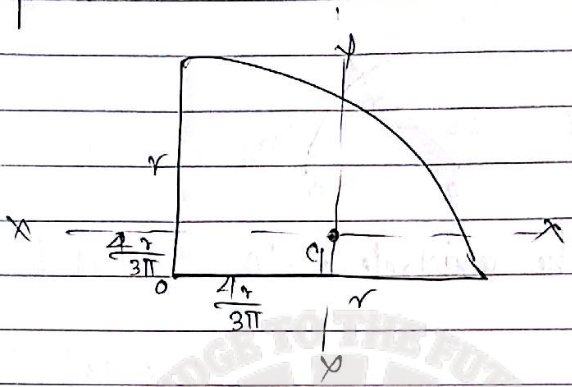
$$\frac{\pi r^4}{8} - \frac{8 r^4}{9\pi} = I_{xx}$$

$$\Rightarrow \boxed{I_{xx} = 0.11 r^4}$$

$$\boxed{I_{yy} = \frac{1}{2} \times \text{MI of circle} = \frac{\pi r^4}{8}}$$

$r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ 0.3925 − 2790

MI of Quarter Circle.



MI of quarter circle $= \frac{1}{4}$ of MI of circle.

$$= \frac{1}{4} \times \frac{\pi r^4}{4}$$

$$I_{AB} = \frac{\pi r^4}{16}$$

MI about the centroidal axis :-

$\cancel{I_{xx}} = \cancel{I_1}$

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{AB} - A\bar{y}^2$$

$$= \frac{\pi r^4}{16} - \left(\frac{\pi r^2}{4}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$= \frac{\pi r^4}{16} - \frac{\pi r^2}{4}\left(\frac{16 r^2}{9\pi^2}\right)$$

$$= \frac{\pi r^4}{16} - \frac{4r^4}{9\pi}$$

$$= \frac{9\pi r^4 - 64 r^4}{16 \times 9\pi}$$ (16, 9

$$= \frac{r^4(35.74)}{452.16}$$

$$= r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$