Physics of Animation

Introduction:

Animation is a method of photographing successive drawings or models to create an illusion of movement in a sequence. Since our eyes can retain an image for nearly 0.1 s, when multiple images appear fast, the brain blends them into a single moving image.

In initial days animation was done by drawing or painting pictures on transparent celluloid sheets and then photographed but today most of the animation work is done with computer-generated imagery or CGI.

Animation is inherently fake. It's not real actors on real sets—it is entirely manipulated from a computer. Even though animators sometime break the laws of physics animation requires an understanding of physics and an animator should have a basic understanding of mechanics and bio-mechanics. Principles of physics are universal, they can be applied to cartoon-style drawings as well as CGI and make audiences to escape reality and enter a fantasy world!

The Taxonomy of Physics-Based Animation Methods (classifications)

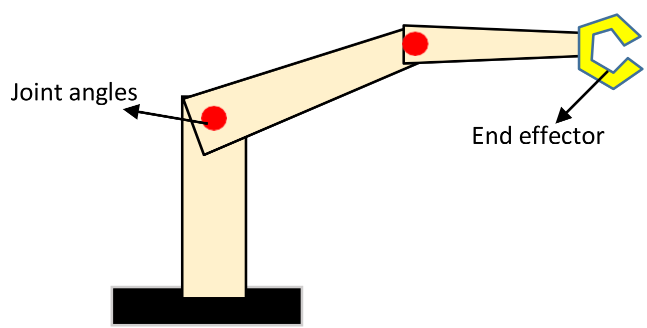
Since animation is an illusion of movement in sequence one need to consider the physics of motion. There are two categories in this. They are

- **Kinematics**

The study of motion of bodies without considering the actual cause for the motion is known as *kinematics*. Here mass and forces are not considered.

- **Dynamics**

The study of motion of bodies by considering the actual cause for the motion is known as *dynamics*. Here mass and forces are taken into consideration



In both of these we have two subgroups. They are

➢ **Inverse:** It is the study of motion when both the starting and ending points are known. Here one typically knows where to go, but needs to figure out how to do it. It is also known as *backward*. Ex, in robots the motion of its joint angles in robots is inverse or backward

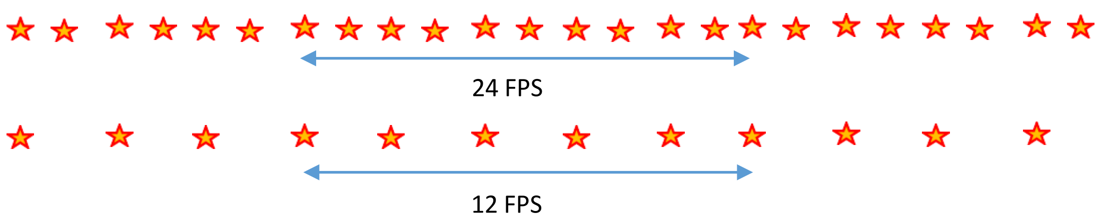
- **Forward:** It is the study of motion when only the starting point is given. Here the goal is to predict the final destination. Ex, end effector movement in the robots is forward motion. It only moves towards the destiny

Frames and Frames per second (FPS)

In animation successive drawings or pictures are made. Each picture is treated as *frame*. To create the appearance of motion from these images, frames shall be displayed.

Definition: The number of frames displayed in one second for smooth movement effect is known as *frame rate*, or *frames per second* (FPS)

The human brain can process only about 10 to 12 FPS, ie, individuals can distinguish separate still pictures in a series with a frame rate under 12 FPS. 12 frames do produce the motional effect but may look choppy (recall very old black and white movies!!-may be Charlee Chaplin movies). Hence the frame rate has to be increased to produce smooth moving effects. In cinema, a frame rate of 24 frames per second is often used (Different video formats have different FPS). For TV the frame rate is 30 FPS

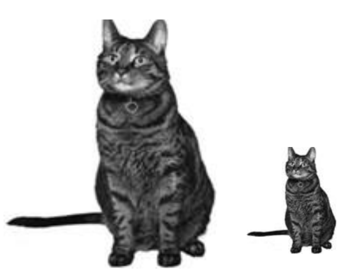


Scale and Size

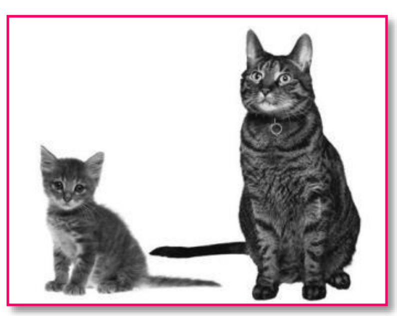
Animation of large objects (cricket field, railway stations, dams etc) is not done in their actual size. Their size must be reduced several times. This is nothing but *scaling*. Scaling is not only done for larger dimension objects. Even smaller dimensional objects (insects, body organs etc) also need scaling.

Definition: The ratio of animation size to the actual size of the object is known as **scale** or **scale of animation**. It is also defined as the ratio of pixels of the animation to the physical units of length.

Scaling is not only done for altering the sizes of the characters but also done in other aspects such as movement, energy etc. This is essential to distinguish between other parameters such as weight and strength, younger and older etc.



Improper scaling



Proper scaling

For ex: If only the size of the cat and its kitten is scaled down then the kitten will look like small adult cat only. Actually kitten will move slower than cat and has lesser strength. Hence the scaling is needed to these factors also for better effects

Proportion and Scale (weight and strength)

When object is scaled its volume and area does not change in equal proportionate. Change in the volume is more compare to area. Volume increases by *cube* times and area increases by *square* times. Hence proportionate scaling is very important. Consider the following example.

The **weight** of a man depends on his body volume and the muscle **strength** depends on cross sectional area. If we want to double the muscle strength the width must be increased $\sqrt{2}$ times. But the weight is not scaled proportionately. Hence care has to be taken while creating larger or smaller objects by means of scaling.

Motion and Timing in Animations

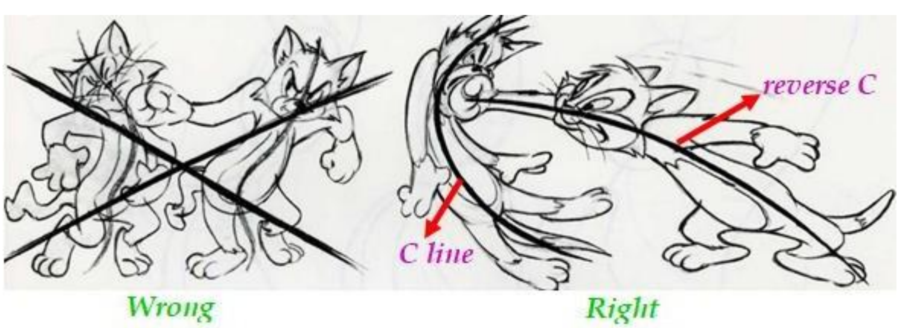
Motion is an essential part of animation. Several types of motion need to be considered while animating a scene. Common types of motion are

- Linear motion
- Parabolic motion
- Circular motion
- Wave motion or oscillatory motion

Line of action and path of action

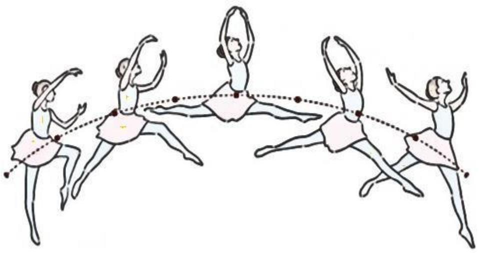
All these types are familiar to us. In connection with animation of motion, we define two parameters namely *line of action* and *path of action*.

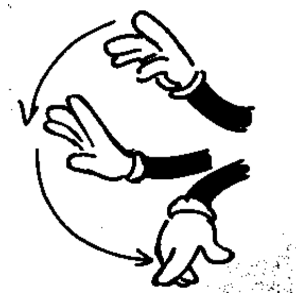
Line of action is an imaginary line that describes the direction and motion of a character's body. A good line of action improves character's poses, makes the character look more dynamic, energetic and alive. There are 3 types of lines of action, the C, reverse C and S curves. Look at the following fig. It is clear that the second pose gives better impression than first one

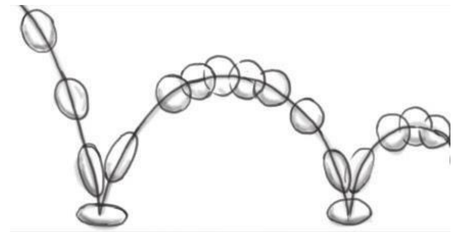


The path along which the object or character moves is known as ***path of action*** or ***arc*.** For ex, path of bouncing ball, trajectory of projectile, jumping of ballet dancer, path of moving arm etc.

Arcs are used extensively in animation because they create motion that is more expressive and less stiff than action along a straight path.







Timing

Timing refers to the time it takes for an action to complete from the starting point to the end. Timing is the amount of frames it takes for an action to take place.

Timing can have a huge effect on how we perceive a character. If a character moves with less frames rate it will appear to be heavy and likely very big. If a character has high frames rate it will appear to be light and probably small. It is explained in the following example

Consider three blocks of same size. 1st block is given 6 frames rate, 2nd block is given 12 frames rate and 3rd one is given 24 frames rate. As a result 1st block falls fast giving the illusion of being heavy

Timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion. If images are placed at same location in all the frames it won't give motion effect (fig-A). Hence we work with very small intervals of time (fig-B).

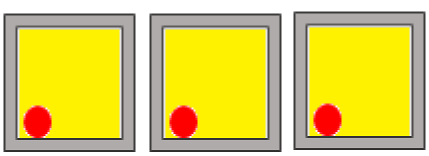


Fig-A- red dot at same place in all the frames

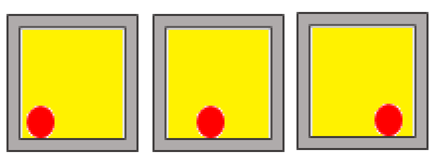
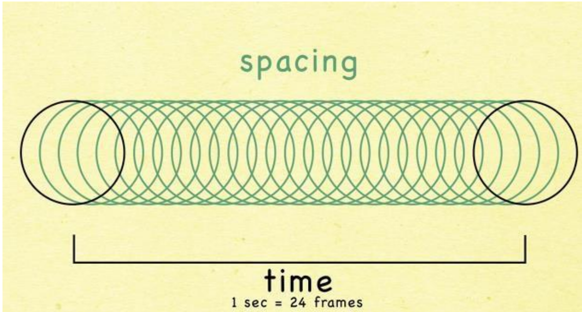


Fig-B-red dot at different places

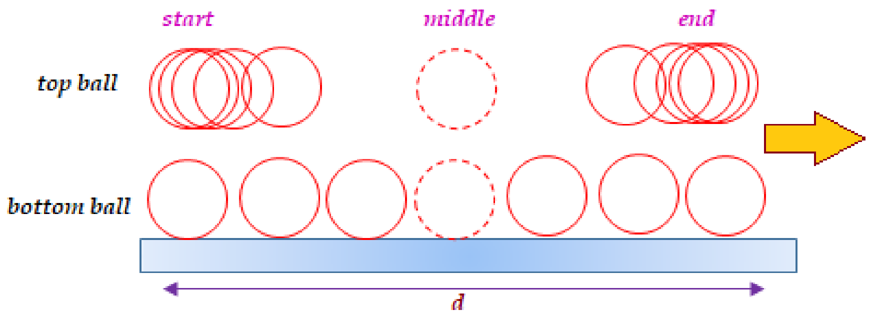
Spacing

Spacing is the distance an object moves within a defined time for that action (it is also the distance an object moves for every frame of that action)



The main difference between timing and spacing is that timing refers to everything that happens over time whereas spacing refers to how much fast something moves. It is illustrated as follows

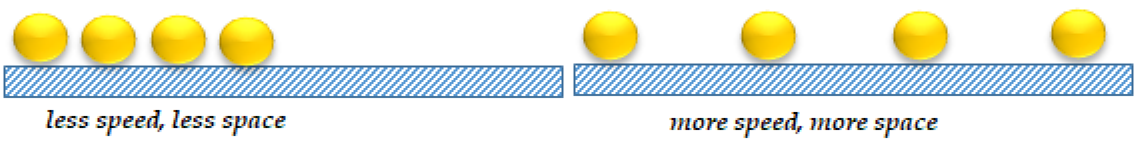
Consider two balls moving same horizontal distance in same time. Assume the top ball will be moving slowly to begin with, then speed up and then slow down again at the end. The bottom ball will just begin and keep a constant speed until the end. Both will be in the same position at the beginning, middle and end of the animation. To achieve this we are going to change the spacing. If we have more drawings near the starting pose, one or two in the middle, and more drawings near the next pose lesser drawings make the action faster and more drawings make the action slower.



Linear motion, Uniform motion and timing

A body moving in a straight line is said to be in *linear motion*. It always move along same direction. Ex: a ball rolling on the inclined plane, a ball moving on a horizontal plane, a stone falling under gravity etc

In linear motion if the body travels without acceleration (or deceleration) then it is known as *uniform motion*. Here the net force acting on the object is zero and distance travelled between consecutive frames is same. Hence timing is very easy. If the speed is less the spacing between the frames is less. If the speed is more the spacing is also more



When the net force acting on the object is not equal to zero then according to Newton's second law, there will be acceleration (even deceleration also). As a result the object will get faster and faster until the force stops acting on it. Now comes one of the Principles of Animation. It is called "slow-in and slow-out".

Slow-in and slow-out

All types of movements start with acceleration and end with deceleration. Even in animation also this principle is followed. An object or a character in animated video starts its movements more slowly, then picks up speed and finishes with deceleration. As a result the beginning and end of the movement are softened. This is called 'slow-in' (or ease in) &'slow-out'(or ease out)

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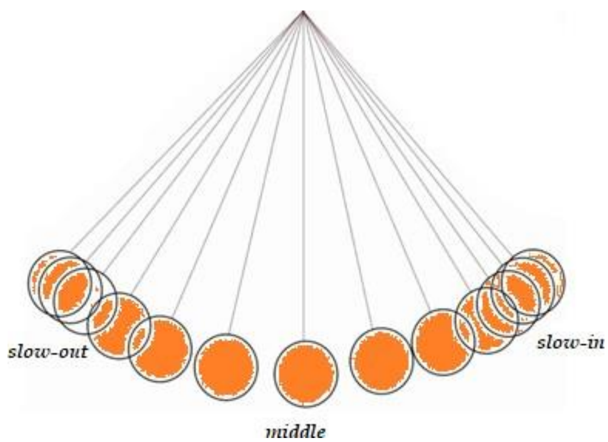
Slow-in (or ease in) is process in which the body is preparing for stopping and **Slow-out** (or ease out) is the process in which the body is speeding up from a still position.

Slow-in and slow-out is achieved by adjusting the spacing (as explained in the above section). To create a slow-out, we need to place the frames close together and then move them apart gradually. To create the slow-in we will need to gradually place the frames closer together at the end of the animation.



Consider the animation of oscillating pendulum. There are more circles at slow-in and slow-out positions and less at middle positions.

Constant force, acceleration and timing

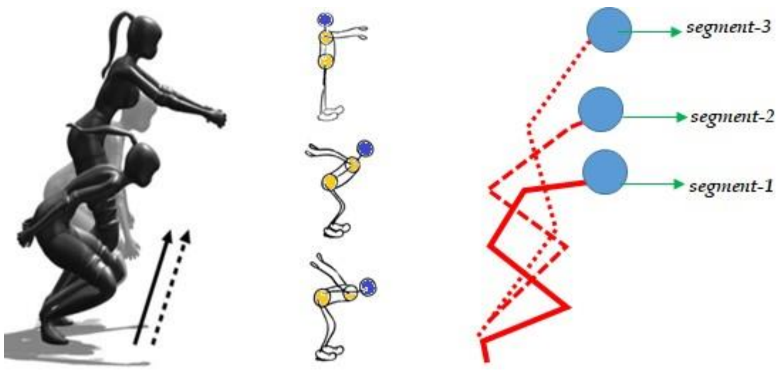
If the applied force is not varying with time then it is called constant force. Some of the examples for constant force are *gravitational force, frictional force, force on a charge in uniform electric field, force on a charge in uniform magnetic field etc.*

If the force is varying with respect to time then it is variable force. Ex, *force on a charge in variable magnetic and electric field, spring force etc.* Timing of the action is very difficult for motions due to variable forces.

Timing for variable force

Normally all the forces exerted by us in everyday life are variable. Force applied during walking, running, jumping, skating, dancing etc is a variable one. If you break the motion into smaller segments then force in each segment is treated as constant and timing becomes easier.

In the below fig, jumping (by girl) is divided into 3 segments. Force is nearly constant over each segment. The timing is very short for each of these segments



Timing for constant force and acceleration

If the applied force is constant then the body will move with constant acceleration. The acceleration depends on the direction of the applied force. There are 3 possibilities

i. If a constant force is applied on a body at rest then it accelerates
ii. If a constant force is applied in the direction of motion then the body is accelerated
iii. If a constant force is applied in the opposite direction to motion then it is decelerated.

For such type of motions timing is done with **odd rule**

Odd rule

The distance travelled by the object between two successive frames is calculated by odd rule. According to this rule

"The distance travelled between two successive frames is obtained by multiplying the base distance by odd numbers 1,3,5,7 etc during accelerated motion and by multiplying the base distance by odd numbers 7,5,3,1 etc during decelerated motion"

Base distance: Base distance is the smallest distance between the two frames. For a slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames. It is given by

$$\text{Base distance} = \frac{\text{total distance}}{(\text{last frame number} - 1)^2}$$

Explanation:

This rule is applicable to both vertical and horizontal motion. It is explained for a body falling under gravity. Consider a ball falling under gravity with zero initial velocity. Distance travelled

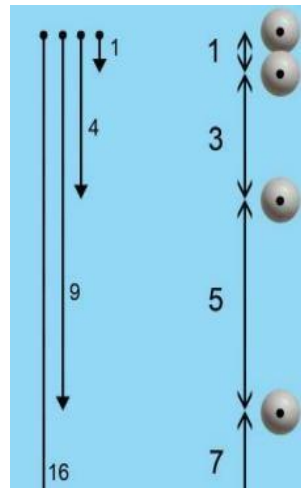
after 1 sec $\quad S_1 = \frac{1}{2} g t^2 = \frac{1}{2} g = 1 \times (\frac{1}{2} g)$

after 2 sec $\quad S_2 = \frac{1}{2} g t^2 = 4 \times (\frac{1}{2} g)$

after 3 sec $\quad S_3 = \frac{1}{2} g t^2 = 9 \times (\frac{1}{2} g)$

after 4 sec $\quad S_4 = \frac{1}{2} g t^2 = 16 \times (\frac{1}{2} g)$

and so on



Here the difference between any two successive distances is an odd number. For ex, 4-1 = 3; 9-4 =5; 16-9 = 7 and so on. Hence the rule is named as odd rule

Odd rule multipliers

The process of calculating the distance from the first frame to the current frame and use these distances to place the object on specific frames is known as **odd rule multiplier**. For slow-out, it is as follows

i. Find the distance between first two frames. It is also known as "base distance"

ii. Multiply it by 1 to get the distance between frame 1 and 2. The total distance travelled is 0+1=1

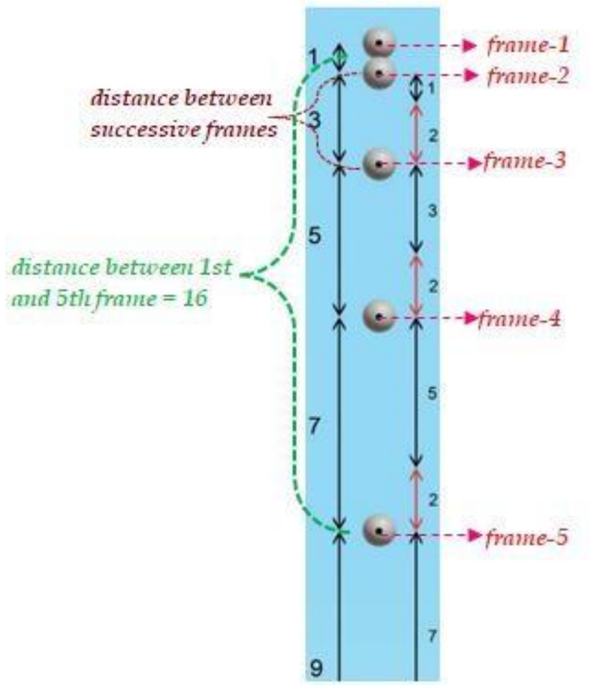
iii. Multiply by 3 to get the distance between frame 2 and 3. The total distance travelled is 0+1+3=4

iv. Multiply by 5 to get the distance between frame 3 and 4. The total distance travelled is 0+1+3+5=9

And so on…It is summarized as follows

Frame No. (A)	Distance between successive frames (B)	Distance between 1st to present frames (C)
1	--	0
2	1	$0+1=1=1^2$
3	3	$0+1+3=4=2^2$
4	5	$0+1+3+5=9=3^2$
5	7	$0+1+3+5+7=16=4^2$
6	9	$0+1+3+5+7+9=25=5^2$

The same is represented in the following fig. From fig, it may be observed that after the first increment, all the others are longer by the same amount (amount of 2- Red arrow mark).
Ie, 3 – 1 =2; 5 – 3 = 2; 7 – 5 = 2 and so on



It is possible to write the relation between A, B and C as follows

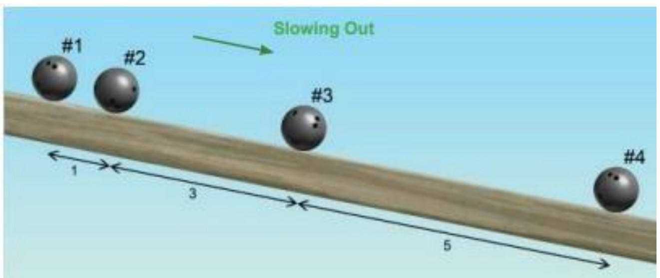
$$B = 2(A - 1) - 1$$
$$C = (A - 1)^2$$

Ex:

If A = 5 then B = 2(5-1)-1 = 7

$C = (5 - 1)^2 = 16$

This is true even for a horizontal motion also



The whole process is applicable for ‘slow-in’ process also (decelerated motion). Only difference is that we have to multiply in the reverse manner ie, 7, 5, 3, 1.

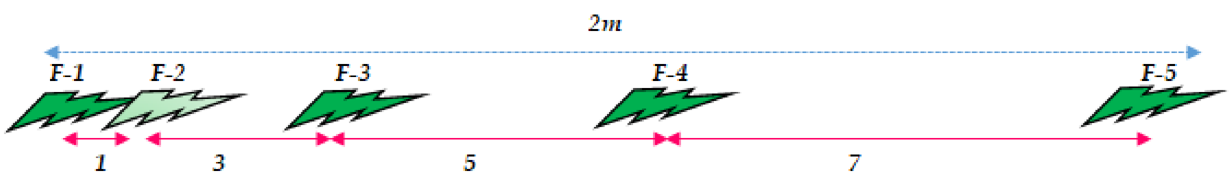
Odd Rule Scenarios

Here we summarize the odd rule for 4 different cases.

1. *Slow-out process*

First of all, find the distance between first two successive frames. It is also known as *base distance*. Then multiply it by odd numbers 1,3,5,7 etc to get the distances between consecutive frames. Finally use squares to multiply the base distance to get the total distance traveled on each frame (See the above table)
If the total distance is known then the base distance is calculated using the formula

$$\text{Base distance} = \frac{\text{total distance}}{(\text{last frame number} - 1)^2}$$



For ex: In the above fig, the total distance is 2m and number of frames is 5 then

$$\text{Base distance} = \frac{2}{(5-1)^2} = 0.125m$$

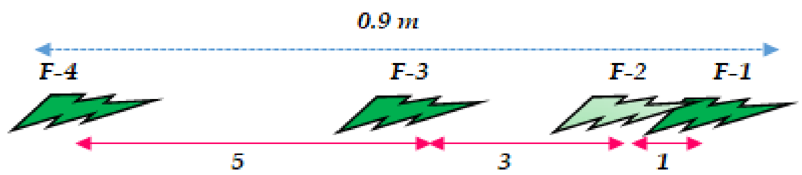
Distance between frame 1 and 2 = 1 × 0.125m = 0.125 m
Distance between frame 2 and 3 = 3 × 0.125m = 0.375 m
Distance between frame 3 and 4 = 5 × 0.125m = 0.625 m
Distance between frame 4 and 5 = 7 × 0.125m = 0.875 m

∴ Total distance = 0.125+0.375+0.625+0.875 = 2m

2. *Slow-in process*

First, find the distance between the last two frames. Then multiply it by odd numbers in reverse order, ie 7,5,3,1
If the total distance is known then the base distance is calculated using the formula

$$\text{Base distance} = \frac{\text{total distance}}{(\text{last frame number} - 1)^2}$$



In the above fig, the total distance is 0.9 m and number of frames is 4 then

$$\text{Base distance} = \frac{0.9}{(4-1)^2} = 0.1m$$

Distance between frame 4 and 3 = 5 × 0.1m = 0.5 m
Distance between frame 3 and 2 = 3 × 0.1m = 0.3 m
Distance between frame 2 and 1 = 1 × 0.1m = 0.1 m

∴ Total distance = 0.5 + 0.3 + 0.1 = 0.9 m

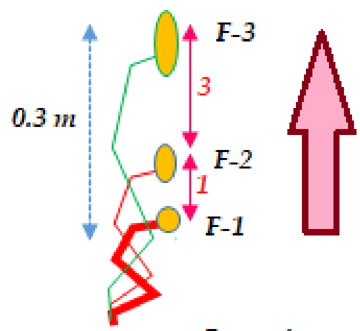
3. *Jumping up process*

Consider a girl jumping up. The whole process is divided into 3 small intervals (or 3 frames). Let the total height is 0.3 m, then

$$\text{Base distance} = \frac{0.3}{(3-1)^2} = 0.075m$$

Distance between frame1 and 2 = 1 × 0.075m = 0.075 m
Distance between frame2 and 3 = 3 × 0.075m = 0.0225 m

∴ Total distance = 0.075 + 0.225 = 0.3 m



Jumping up

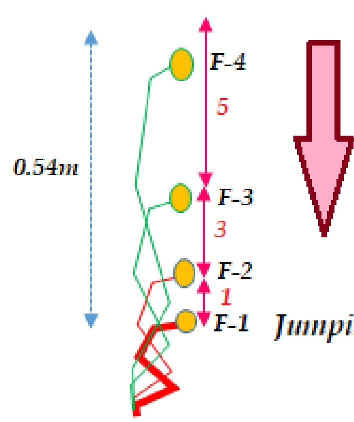
4. *Jumping down process*

Consider a girl jumping down through a distance 0.54 m. The whole process is divided into 4 frames, then

$$\text{Base distance} = \frac{0.54}{(4-1)^2} = 0.06m$$

Distance between frame 4 and 3 = 5 × 0.06m = 0.3 m
Distance between frame 3 and 2 = 3 × 0.06m = 0.18 m
Distance between frame 2 and 1 = 1 × 0.06m = 0.06 m

∴ Total distance = 0.3 + 0.18 + 0.06 = 0.54 m



Jumping down

Motion Graphs

A graph of distance versus time is known as **motion graph**. It is also called as position-time graph or *x-t* graph. It is very important for animation because the nature of the motion is ascertained from the graph. Some of the motion graphs are as follows

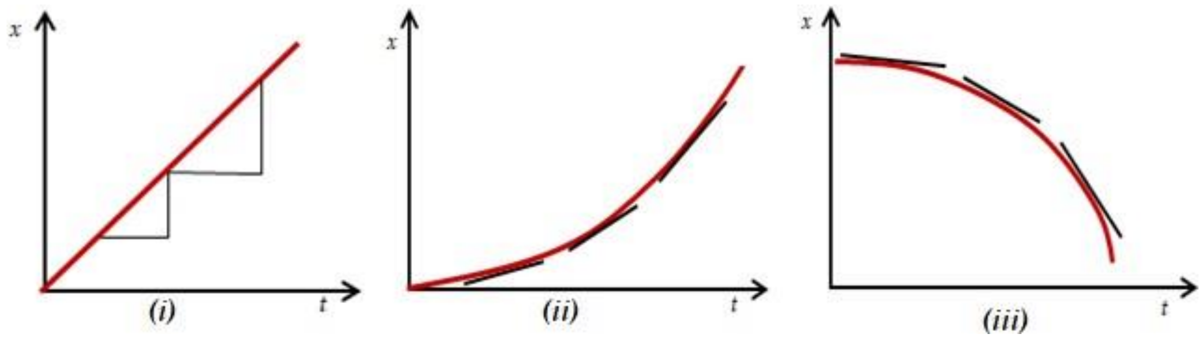


Fig. (i) represents constant velocity and constant slope. There is no acceleration
Fig. (ii) represents positively increasing slope and velocity. Hence acceleration is positive. More the acceleration more will be the curvature
Fig.(iii) represents negatively increasing slope. It represents decreasing velocity. Hence it is deceleration

Examples of Character Animation

Motion of an object is an integral part of animation. By observing and studying the real life movements one can make the animation more attractive and lively. Jumping and walking are two real life actions where the entire body is in motion. Now we shall take up their character animation by applying the knowledge of physics in animation

1. Jumping

Following are the different stages of jumping process.

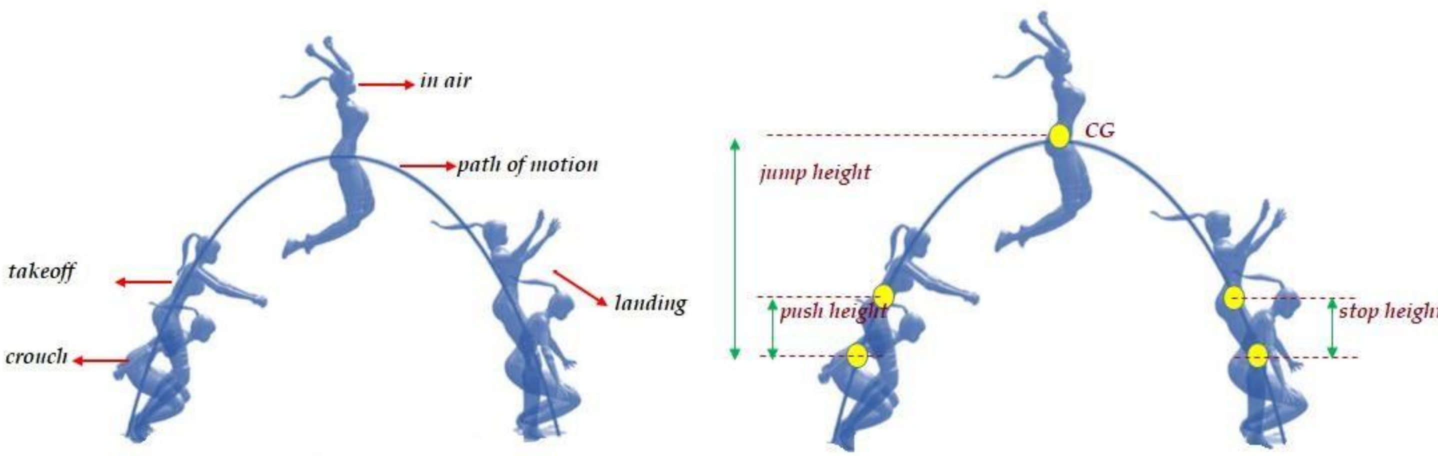
- ***Crouch***—It is the bending pose taken as preparation for jumping.

- ***Takeoff*** —Character pushes up fast and straightens legs with feet still on the ground. The distance from the character’s center of gravity (CG) in the crouch to the CG in takeoff position is called the *push height*. The amount of time (or number of frames) needed for the push is called the *push time*.

- ***In the air***—Both the feet are off the ground, and the character’s center of gravity (CG) moves in a parabolic arc. First it reaches a maximum height (apex), and then falls back to the ground. The maximum height or *jump height*, is measured from the CG at takeoff to the CG at the apex of the jump. The amount of time the character is in the air from takeoff to apex is called the *jump time*. If the takeoff pose and the landing pose are similar, then jump time is same for going up and coming down (it is exactly similar to time of ascent and time of descent in projectile motion).

- **Landing**—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the *stop height*. The stop height is not always exactly the same as the push height

- **Path of action**—It is the trajectory along which the character jumps



Calculations

i. Calculation of jump timing

Let the jump height be 1.2 m and acceleration due to gravity is 9.8 ms^{-2}. The jump time is

$$t^2 = \frac{2h}{g} = \frac{2 \times 1.2}{9.8} = 0.25$$

$$\therefore t = 0.5\ s$$

This is the time taken to go from crouch position to apex position. As we know that the frame rate is 24 (FPS), the total number of frames for this action should be $24 \times 0.5 = 12$ frames

ii. Jump magnification (JM)

The JM is the ratio of the jump height to the push height.

$$JM = \frac{\text{jump height}}{\text{push height}} - - - (1)$$

It is also defined as the ratio of jump time and push time. It is used to calculate the push timing and stop timing.

$$JM = \frac{\text{jump time}}{\text{push time}} --- (2)$$

In terms of acceleration it is given by

$$JM = \frac{\text{push acceleration}}{g} --- (3)$$

[To prove Eq (3): (Optional – not in syllabus)

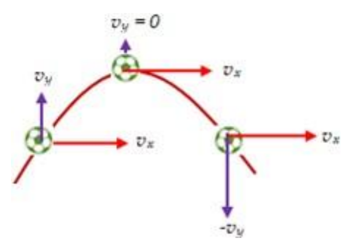
Jumping motion is equivalent to projectile motion. While going from crouch to take off position the girl will have some acceleration called *push acceleration*. After wards her body moves upward and experiences acceleration due to gravity. During downward motion she experiences gravity only.

Jump velocity = push acceleration × push time (because $v = at$)

Landing velocity = landing acceleration × jump time

But the jump velocity shall be equal to landing velocity. Hence

push acceleration × push time = landing acceleration × jump time



Since landing acceleration is same as g we can write

push acceleration × push time = g × jump time

$$\frac{\text{jump time}}{\text{push time}} = \frac{\text{push acceleration}}{g}$$

$$JM = \frac{\text{push acceleration}}{g}$$

Hence the proof]

Ex, if there are 15 frames and JM is 3 then

According to Eq (2)

$$JM = \frac{\text{jump time}}{\text{push time}} = \frac{15}{\text{push time}}$$

$$\text{push time} = \frac{15}{3} = 5 \text{ frames}$$

iii. Jump height

The distance between CG at crouch and take off position is called push height. Assume that it is 0.4 and JM is 3 then from eq (1) we get

$$JM = \frac{\text{jump height}}{\text{push height}} = \frac{\text{jump height}}{0.4}$$

$$\text{jump height} = \text{JM} \times 0.4 = 3 \times 0.4 = 1.2 \text{ m}$$

iv. Stop time

Since timing of the push and stop are same we can write

$$\frac{\text{push height}}{\text{push time}} = \frac{\text{stop height}}{\text{stop time}}$$

$$\text{stop time} = \text{stop height} \times \frac{\text{push time}}{\text{push height}}$$

It may be noted here that the stop height is little bit longer than push height (because the girl may finally lay down on the ground after landing). For ex, if push height is 0.4, stop height is 0.5, push time 5 frames then

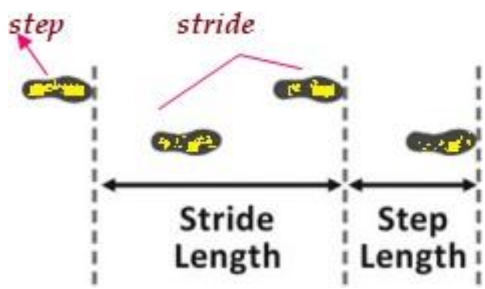
$$\text{stop time} = 0.5 \times \frac{5}{0.4} \approx 6\,frames$$

2. Walking

Walking is another event where a lot of physics is involved and the animation of walking needs the complete knowledge of mechanics. Walking is nothing but a step-by-step movement. Each step is divided into 4 poses namely **passing, step, contact,** and **lift**. Also we need the concept of Strides and steps as well as walking time

Strides and Steps

A step means one step with one foot. A stride means two steps, one with each foot. *Step length* is the distance between two successive steps and *Stride length* is the distance between two parts of the same step. Step and stride length indicate lengthwise spacing for the feet during a walk.



Walk timing

While walking each foot is in contact with ground for 60% of time (ie, single support) and both the feet will be in contact for 20 % of time (double support). In this connection we define a quantity called *Gait*. It is the timing of the motion for each foot, including how long each foot is on the ground or in the air. For normal walk it is about ½ second

To walk faster, obviously one has to increase stride length and decrease the time of double support. A fast walk has a stride rate of about 4 feet/sec. If it becomes 6-7 then walking turns into running.