CBCS SCHEME

USN 1 A R 2 2 C S 0 4 2

BMATS201

Second Semester B.E./B.Tech. Degree Examination, June/July 2023

Mathematics – II for CSE Stream

Ramyashree

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$.	7	L2	CO1
	b.	Evaluate by changing the order of integration $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.	7	L3	CO1
	c.	Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (2-x)\,dy\,dx$ by changing into polar coordinates. 4π	7	L3	CO1
	b.	A pyramid is bounded by three coordinate planes and the plane $x+2y+3z=6$. Compute the volume by double integration. 22	7	L3	CO1
	c.	Using Mathematical tools, write the code to find the area of the cardioids $r = a(1+\cos\theta)$ by double integration.	6	L3	CO5
		Module – 2			
Q.3	a.	Show that the two surfaces $xz + y + z^2 = 9$ and $z = 4 - 4xy$ at (1, -1, 2) are orthogonal.	7	L3	CO2
	b.	If $F = \text{grad}(xy^3z^2)$, find divF and curlF at the point (1, -1, 1).	7	L2	CO2
	c.	Prove that the cylindrical coordinate system is orthogonal.	6	L3	CO2
		OR			
Q.4	a.	Find the directional derivative of $\phi = x\log z - y^2 + 4$ at (-1, 2, 1) in the direction of the vector $2i - j - 2k$. 6	7	L2	CO2
	b.	Find the constants a, b and c such that $F = (axy - z^3)i + (bx^2 + z)j + (bxz^2 + cy)k$ is irrotational. -6 -3 1	7	L2	CO2
	c.	Using the Mathematical tools, write the codes to find the gradient of $\phi = xy^2z^3$.	6	L3	CO5

Module – 3

Q.5	**a.**	Let $W = \{(x,y,z) \mid lx + my + nz = 0\}$, then prove that W is a subspace of R^3.	7	**L2**	**CO3**
	b.	Find the basis and the dimension of the subspace spanned by the vectors $\{(2, 4, 2),(1, -1, 0),(1, 2, 1),(0, 3, 1)\}$ in $V_3(R)$.	7	**L2**	**CO3**
	c.	Prove that $T : R^3 \to R^3$ be defined by $T(x, y, z) = (2x-3y, x+4, 5z)$ is not a linear transformation.	6	**L3**	**CO3**

OR

Q.6	**a.**	Show that the matrix $E = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ lies in the sub space span $\{A, B, C\}$ of vector space M_{22} of 2×2 matrices, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$. 2 $3/2$ 2	7	**L2**	**CO3**
	b.	Verify the Rank-nullity theorem for the linear transformation $T : R^3 \to R^3$ defined by $T(x,y,z) = (x + 2y - z, y + z, x + y - 2z)$.	7	**L3**	**CO3**
	c.	Define an Inner product space. Consider $f(t) = 4t + 3$, $g(t) = t^2$, the inner product $<f, t> = \int_0^1 f(t)g(t)dt$. Find $<f, g>$ and $\lVert g \rVert$. 2 $1/\sqrt{5}$	6	**L2**	**CO3**

Module – 4

Q.7	**a.**	Find the real root of the equation $x \log_{10} x - 1.2$ by the Regula-Falsi method between 2 and 3. (Carryout three iterations).	7	**L2**	**CO4**
	b.	From the following table, estimate the number of students who have obtained the marks between 40 and 45.	7	**L2**	**CO4**

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

	c.	Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$ using Simpson's $\frac{3}{8}^{th}$ rule taking six parts.	6	**L3**	**CO4**

OR

Q.8	**a.**	Using Newton-Raphson method compute the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$, correct to four decimal places. 2.7984	7	**L2**	**CO4**
	b.	If y(0) = -12, y(1) = 0, y(3) = 6 and y(4) = 12, find the Lagrange's interpolation polynomial and estimate y(2). 4	7	**L2**	**CO4**
	c.	Evaluate $\int_0^3 \frac{dx}{4x+5}$ using Trapezoidal rule by taking 7 ordinates. 0.33825	6	**L3**	**CO4**

Module – 5

Q.9	**a.**	Employ Taylor's series method to obtain y(0.1) for $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ considering upto 4th degree terms. 0.3486	7	**L2**	**CO4**
	b.	Using Runge-Kutta method of fourth order, solve $y' = \log_{10}\left[\frac{y}{1-x}\right]$ given y(0) = 1 at x = 0.2 0.4467	7	**L3**	**CO4**

c. Solve $\frac{dy}{dx} = 2e^{x} - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$, find $y(0.4)$ using Milne's method. **6** **L2** **CO4**

OR

Q.10 **a.** Given $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, $y(0) = 1$. Compute $y(0.4)$ with $h = 0.2$ using Euler's modified method. Perform two modifications in each stage. **7** **L2** **CO4**

b. Apply Milne's predictor-corrector formulae to compute $y(4.5)$, given that $5x\frac{dy}{dx} = 2 - y^{2}$ and **7** **L2** **CO4**

x	4.1	4.2	4.3	4.4
y	1.0049	1.0097	1.0143	1.0187

c. Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = x - y^{2}$ at $y(0.2)$. Given that $y(0) = 1$ by Runge-Kutta 4th order method. (Take $h = 0.2$) **6** **L3** **CO5**

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