MANGALORE INSTITUTE OF TECHNOLOGY & ENGINEERING
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Affiliated to V.T.U., Belagavi, Approved by AICTE, New Delhi.

DEPARTMENT OF MATHEMATICS

MODULE -5: LINEAR ALGEBRA

CONTENTS:

Introduction to Linear Algebra related to EC & EE Engineering applications.

- Elementary row transformation of a matrix
- Rank of a matrix
- Consistency and Solution of system of linear equations
  - Gauss-elimination method, Gauss-Jordan method and
  - approximate solution by Gauss-Seidel method.
- Eigenvalues and Eigenvectors
- Rayleigh's power method to find the dominant Eigenvalue and Eigenvector

SELF-STUDY: Solution of system of equations by Gauss-Jacobi iterative method. Inverse of a square matrix by Cayley- Hamilton theorem.

APPLICATIONS OF LINEAR ALGEBRA: Network Analysis, Markov Analysis, Critical point of a network system. Optimum solution.

(RBT LEVELS: L1, L2 AND L3)

LEARNING OBJECTIVES:

After Completion of this module, student will be able to:

- Apply concepts of row transformation to find rank of a matrix
- Solve system of Linear equations using Gauss-elimination method, Gauss-Jordan method and approximate solution by Gauss-Seidel method.
- Compute dominant Eigen values and Eigen Vectors by Rayleigh's power method

BASIC CONCEPTS AND DEFINITIONS:

DEFINITION: A matrix is a rectangular arrangement of numbers in rows and columns represented by

$$\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}.$$

If a matrix has m rows and n columns, then it is said to be of order $m \times n$ (read as "m by n" matrix). The elements a_{ij} of a matrix are identified by double subscript notation $i\,j$, where i denotes the row and j denotes the column.

ELEMENTARY ROW TRANSFORMATIONS:

These are operations that are carried out on the rows of a given matrix. The following operations constitute the three row transformations.

- Interchange of i^{th} and j^{th} rows: $R_i \leftrightarrow R_j$
- Multiplying each element of the i^{th} row by a non-zero constant k: $R_i' \to kR_i$
- Adding a constant k multiple of j^{th} row to i^{th} row: $R_i' \to R_i + kR_j$.

EQUIVALENT MATRICES:

Two matrices A and B are said to be **equivalent** if one can be obtained from the other by a sequence of Elementary transformation. Equivalent matrices are denoted by $A \sim B$.

NOTE: All the above operations can also be performed on columns

ECHELON FORM OR ROW ECHELON FORM:

A non-zero matrix A is said to be in echelon form, if

- The leading entry(non-zero element) of each non-zero row after the first row occurs to the right of the leading entry of the previous row
- All the entries of a column below a leading entry are zero
- All zero rows(all elements are zero)are at the bottom of the matrix

  OR

Note: Echelon matrix is almost similar to upper diagonal matrix

OR

A non-zero matrix A is an echelon matrix, if the number of zeros preceding the first non-zero entry of a row increases row by row until zero rows remaining.

RANK OF A MATRIX:

Rank of a matrix is the number of non-zero rows in the row Echelon Form and is denoted by ρ

Example:

$$B = \begin{bmatrix} 1 & 3 & 1 & 5 & 0 \\ 0 & 1 & 5 & 1 & 5 \\ 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ is in row-echelon form.

The rank of an echelon matrix is the number of non-zero rows in it. i.e., $\rho(B) = 3$

EXAMPLES

1. Determine the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$

Solution: The rank of the matrix can be obtained by reducing it to row echelon form.

Given matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Perform $R_1 \leftrightarrow R_2$ i.e., interchanging row 1 and row 2 we get

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2' \to R_2 - 2R_1,\ R_3' \to R_3 - 3R_1,\ R_4' \to R_4 - 6R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3' \to 5R_3 - 4R_2,\ R_4' \to 5R_4 - 9R_2$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4' \to R_4 - R_3$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

As there are no elements below the fourth diagonal element the process is complete.

$\rho(A) =$ Rank of $A =$ number of non-zero rows $= 3$.

2. Reduce the following matrix to echelon form and hence find its rank.

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 1\\ 2 & 3 & 5\\ 3 & 5 & 6\end{bmatrix}$$

Solution: Given matrix is

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 1\\ 2 & 3 & 5\\ 3 & 5 & 6\end{bmatrix}$$

interchanging row 1 and row 2 we get

Perform $R_3^{'} \to R_3 - 2R_1,\ R_4^{'} \to R_4 - 3R_1$

$$A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ 0 & -1 & 3\\ 0 & -1 & 3\end{bmatrix}$$

Perform $R_3^{'} \to R_3 + R_2,\ R_4^{'} \to R_4 + R_2$

$$A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ 0 & 0 & 5\\ 0 & 0 & 5\end{bmatrix}$$

Perform $R_4^{'} \to R_4 - R_3$

$$A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ 0 & 0 & 5\\ 0 & 0 & 0\end{bmatrix}$$

Above matrix is in the echelon from, therefore rank of matrix $A = 3$ (no. of non-zero rows).

3. Using the elementary transformations find the rank of the matrix

$$B=\begin{bmatrix}-1 & 2 & 3 & -2\\ 2 & -5 & 1 & 2\\ 3 & -8 & 5 & 2\\ 5 & -12 & -1 & 6\end{bmatrix}$$

Solution: Given matrix is

$$B=\begin{bmatrix}-1 & 2 & 3 & -2\\ 2 & -5 & 1 & 2\\ 3 & -8 & 5 & 2\\ 5 & -12 & -1 & 6\end{bmatrix}$$

$R_2' \to R_2+2R_1,\ R_3' \to R_3+3R_1,\ R_4' \to R_4+5R_1$

$$B=\begin{bmatrix}-1 & 2 & 3 & -2\\ 0 & -1 & 7 & -2\\ 0 & -2 & 14 & -4\\ 0 & -2 & 14 & -4\end{bmatrix}$$

$R_3' \to R_3-2R_2,\ R_4' \to R_4-2R_2$, we get

$$B=\begin{bmatrix}-1 & 2 & 3 & -2\\ 0 & -1 & 7 & -2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Rank of matrix $B=2$.

EXERCISE

Find the rank of the following matrices by reducing it to echelon form:

1. $\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$

2. $\begin{bmatrix}2 & 1 & 3 & 5\\ 4 & 2 & 1 & 3\\ 8 & 4 & 7 & 13\\ 8 & 4 & -3 & -1\end{bmatrix}$

3. $\begin{bmatrix}4 & 0 & 2 & 1\\ 2 & 1 & 3 & 4\\ 2 & 3 & 4 & 7\\ 2 & 3 & 1 & 4\end{bmatrix}$

4. $\begin{bmatrix}2 & -1 & 3 & 4\\ 0 & 3 & 4 & 1\\ 2 & 3 & 7 & 5\\ 2 & 5 & 11 & 6\end{bmatrix}$

5 $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

6. $\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$

7. $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

12. $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$

13. $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$

14. $\begin{bmatrix} 221 & 22 & 23 & 24 \\ 22 & 23 & 24 & 25 \\ 23 & 24 & 25 & 26 \\ 24 & 25 & 26 & 27 \end{bmatrix}$

APPLICATIONS:

- One useful application of calculating the rank of a matrix is the computation of the number of solutions of a system of linear equations.
- In the area of source enumeration.
- In the classification of an image.
- If we view a square matrix as specifying a transformation, the rank tells you about the dimension of the image.

- In control theory, the rank of a matrix can be used to determine whether a linear system is controllable, or observable.
- In the field of communication complexity, the rank of the communication matrix of a function gives bounds on the amount of communication needed for two parties to compute the function.

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS:

A Linear system of simultaneous equations of m equations in n unknowns can be expressed as

$$a_{21}x_1 + a_{22}x_2 + \dots\dots\dots + a_{2n}x_n = b_2, \qquad (1)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots\dots\dots + a_{mn}x_n = b_m.$$

The above system in the matrix equivalent form can be expressed as $AX = B$, where

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}_{m\times n}$$ is called the coefficient matrix

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$ is called the matrix of unknowns and $$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$$ is column matrix of constants.

If all b_i's for $i = 1 \dots m$ are zero i.e., $b_1 = b_2 = \dots = b_m = 0$, then the system is said to be homogenous and is said to be non-homogeneous if at least one b_i is non-zero.

AUGMENTED MATRIX:

Suppose we form a matrix of the form $[A\!:\!B]$ by appending to A an extra column whose elements are columns of B i.e.,

$$[A:B] = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} & : b_1 \\ a_{21} & a_{22} & . & . & . & a_{2n} & : b_2 \\ . & . & . & . & . & . & \\ a_{m1} & a_{m2} & . & . & . & a_{mn} & : b_m \end{bmatrix}$$

is called the augmented matrix associated with the system and is denoted by $[A|B]$ or $[A:B]$.

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS:

A system of linear equations such as (1) may or may not have a solution. However, existence of solution is guaranteed only if the system is homogeneous.

SOLUTION OF NON-HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS:

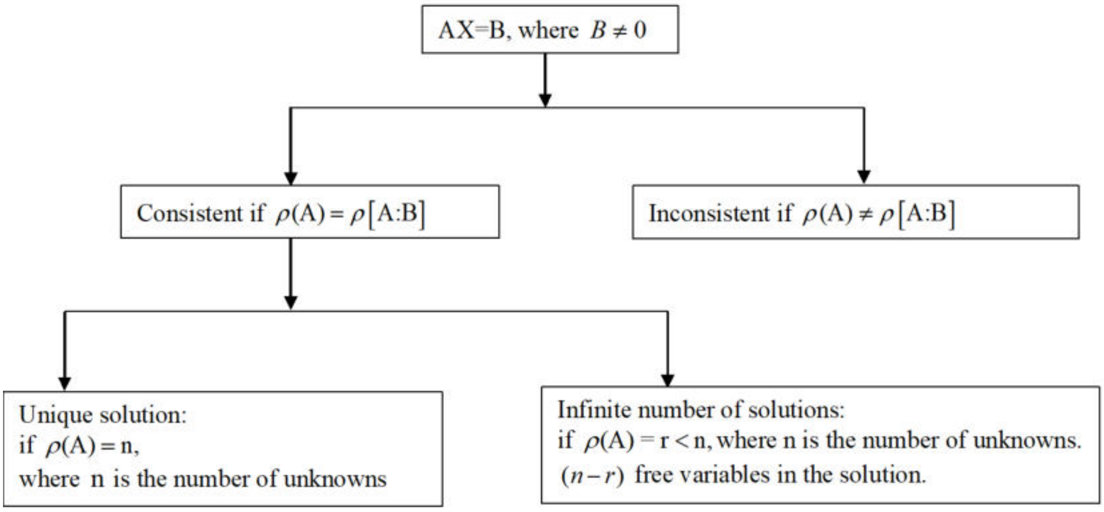
A non-homogeneous system of equations $\mathbf{AX} = \mathbf{B}$ is consistent if $\mathbf{r}$, the rank of coefficient matrix A is equal to $\mathbf{r}'$, the rank of the augmented matrix $[A:B]$ and has unique solution if $\mathbf{r} = \mathbf{r}' = \mathbf{n}$, the number of unknowns. If $\mathbf{r} = \mathbf{r}' < \mathbf{n}$ then the system possess infinite number of solutions. The system is inconsistent if $\mathbf{r} \neq \mathbf{r}'$.

SOLUTION OF HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS:

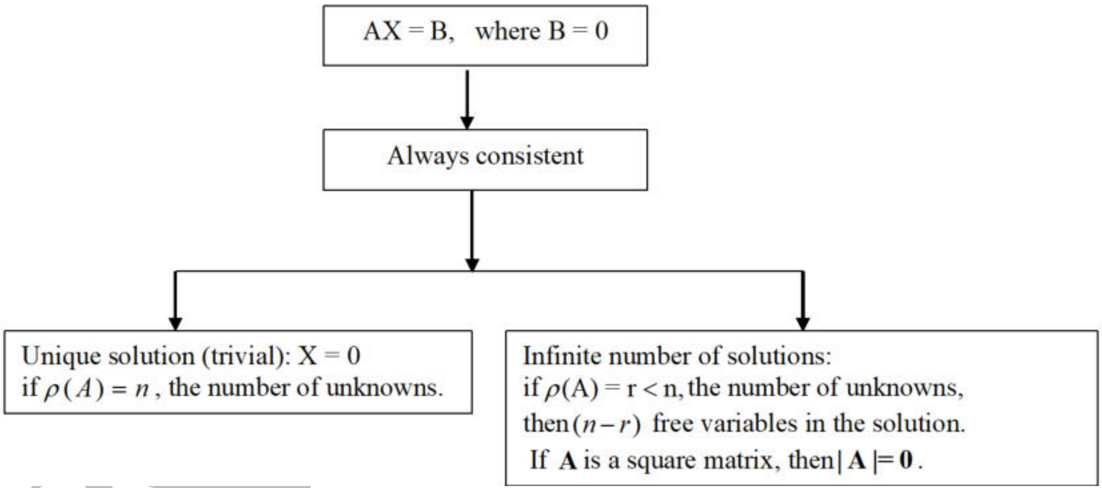
A homogeneous system of linear equations $\mathbf{AX} = \mathbf{0}$ is always consistent as for such a system, $A = [A:0]$ and hence rank of coefficient matrix is equal to the rank of the augmented matrix. If rank of A is equal to the number of unknowns $\mathbf{n}$, the system has trivial solution i.e., all unknowns $x_1, x_2, \cdots, x_n$ are zero. A non-trivial solution exists to a system if and only if $|\mathbf{A}| = \mathbf{0}$ and hence the system has infinite number of solution.

The following block diagram illustrates connection between rank of a matrix and consistence of that system.

NON-HOMOGENOUS SYSTEM



HOMOGENOUS SYSTEM



EXAMPLES

1. Test for consistency and solve

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 1 \\ -3x_1 + 4x_2 - 5x_3 &= 0 \\ x_1 + 3x_2 - 6x_3 &= 0 \end{aligned}$$

Solution: Consider the augmented matrix

$$[A:B]=\begin{bmatrix} 2 & -1 & 3 & :1 \\ -3 & 4 & -5 & :0 \\ 1 & 3 & -6 & :0 \end{bmatrix}$$

$R_2' \to R_2 + (3/2)R_1,\ R_3' \to R_3 - (1/2)R_1$

$$[A:B]\sim\begin{bmatrix} 2 & -1 & 3 & :1 \\ 0 & 5 & -1 & :3 \\ 0 & 7 & -15 & :-1 \end{bmatrix}$$

$R_3' \to R_3 - (7/5)R_2$

$$[A:B]\sim\begin{bmatrix} 2 & -1 & 3 & :1 \\ 0 & 5 & -1 & :3 \\ 0 & 0 & -68 & :-26 \end{bmatrix}$$

$\rho(A) = \rho([A:B]) = 3 =$ number of unknowns.

Thus the system of linear equations is consistent and possesses a unique solution.

To find the unknowns, consider the rows of $[A:B]$ in the last step in terms of its equivalent equations ,

$$2x_1 - x_2 + 3x_3 = 1$$
$$5x_2 - x_3 = 3$$
$$-68x_3 = -26$$

Here we make use of **back substitution** in order to find the unknowns by considering, last equation to find x_3, next second to find x_2 and finally first equation to find x_1.

Therefore, from last equation we obtain x_3

i.e., $-68x_3 = -26 \Rightarrow x_3 = \dfrac{13}{34}$.

Next, from second equation we find x_2, i.e., $5x_2 - x_3 = 3 \Rightarrow x_2 = \dfrac{3+x_3}{5} = \dfrac{3+\frac{13}{34}}{5} \Rightarrow x_2 = \dfrac{23}{34}.$

Finally, to find the x_1 we make use first equation

i.e., $2x_1 - x_2 + 3x_3 = 1 \Rightarrow x_1 = \dfrac{1}{2}(1 + x_2 - 3x_3) = \dfrac{1}{2}\left(1 + \dfrac{23}{34} - 3\dfrac{13}{34}\right) \Rightarrow x_1 = \dfrac{9}{34}.$

There the solution is given by

$x_1 = \dfrac{9}{34}, x_2 = \dfrac{23}{34}, x_3 = \dfrac{13}{34}.$

2. Check the following system of equations for consistency and solve, if consistent. $x+2y+2z = 1,\ \ 2x+y+z = 2,\ \ 3x+2y+2z = 3,\ y+z = 0$

Solution : The augmented matrix is given by

$$[A:B] = \begin{bmatrix} 1 & 2 & 2 & :1 \\ 2 & 1 & 1 & :2 \\ 3 & 2 & 2 & :3 \\ 0 & 1 & 1 & :0 \end{bmatrix}$$

$R_2' \to R_2 - 2R_1, R_3' \to R_3 - 3R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 2 & :1 \\ 0 & -3 & -3 & :0 \\ 0 & -4 & -4 & :0 \\ 0 & 1 & 1 & :0 \end{bmatrix}$$

$R_2' \to (-1/3)\,R_2, R_3' \to (-1/4)\,R_3$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 2 & :1 \\ 0 & 1 & 1 & :0 \\ 0 & 1 & 1 & :0 \\ 0 & 1 & 1 & :0 \end{bmatrix}$$

$R_3' \to R_3 - R_2, R_4' \to R_4 - R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 2 & :1 \\ 0 & 1 & 1 & :0 \\ 0 & 0 & 0 & :0 \\ 0 & 0 & 0 & :0 \end{bmatrix}$$

$\rho(A) = \rho([A:B]) = 2 < 3$ number of unknowns.

Thus the given system is consistent and possesses infinite number of solutions by assigning arbitrary values to $(n-r) = 3-2 = 1$ free variable.

$$\Rightarrow \quad \begin{aligned} x + 2y + 2z &= 0 \\ y + z &= 0. \end{aligned}$$

Here there are three unknowns, we should take z as the free variable and let $z = k$ (arbitrarily value).

From second equation, $y + z = 0 \Rightarrow y = -z = -k$.

Finally from first equation,

$$x + 2y + 2z = 1 \Rightarrow x = 1 - 2y - 2z = 1 - 2(-k) - 2k$$
$$\Rightarrow x = 1.$$

Therefore the solution are given by

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -k \\ k \end{bmatrix}.$$

3. Show that the following system of equations is not consistent.

$x + 2y + 3z = 6,\ 3x - y + z = 4,\ 2x + 2y - z = -3,\ -x + y + 2z = 5$

Solution: Consider the augmented matrix

$$[A:B] = \begin{bmatrix} 1 & 2 & 3 & :6 \\ 3 & -1 & 1 & :4 \\ 2 & 2 & -1 & :-3 \\ -1 & 1 & 2 & :5 \end{bmatrix}$$

$R_2' \to R_2 - 3R_1,\ R_3' \to R_3 - 2R_1,\ R_4' \to R_4 + R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & :6 \\ 0 & -7 & -8 & :-14 \\ 0 & -2 & -7 & :-15 \\ 0 & 3 & 5 & :11 \end{bmatrix}$$

$R_3' \to R_3 - (2/7)R_2,\ R_4' \to R_4 + (3/7)R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & :6 \\ 0 & -7 & -8 & :-14 \\ 0 & 0 & -33 & :-77 \\ 0 & 0 & 11 & :35 \end{bmatrix}$$

$R_4' \to R_4 + R_3$

$$[A:B] \sim \begin{bmatrix} 1 & 2 & 3 & :6 \\ 0 & -7 & -8 & :-14 \\ 0 & 0 & -33 & :-77 \\ 0 & 0 & 0 & :28 \end{bmatrix}$$

$\rho(A) = 3$ and $\rho([A:B]) = 4$

$\rho(A) \neq \rho([A:B])$.

Therefore, the given system is inconsistent and it has no solution.

4. Check the following system of equations for consistency and solve, if consistent.

$$x + y - 2z = 3,\ 2x - 3y + z = -4,\ 3x - 2y - z = -1,\ y - z = 2.$$

Solution: Consider the augmented matrix,

$$[A:B] = \begin{bmatrix} 1 & 1 & -2 & :3 \\ 2 & -3 & 1 & :-4 \\ 3 & -2 & -1 & :-1 \\ 0 & 1 & -1 & :2 \end{bmatrix}$$

$R_2' \to R_2 - 2R_1,\ R_3' \to R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 1 & -2 & :3 \\ 0 & -5 & 5 & :-10 \\ 0 & -5 & 5 & :-10 \\ 0 & 1 & -1 & :2 \end{bmatrix}$$

$R_3' \to R_3 - R_2,\quad R_4' \to R_4 + (1/5)R_2$

$$\sim\begin{bmatrix}1 & 1 & -2 & : & 3\\ 0 & -5 & 5 & : & -10\\ 0 & 0 & 0 & : & 0\\ 0 & 0 & 0 & : & 0\end{bmatrix}$$

We see that $\rho(\mathrm{A})=\rho([\mathrm{A}:\mathrm{B}])=2<3$ number of unknowns.

Thus the equations are consistent and posses infinite number of solutions with $(n-r)=3-2=1$ free variable.

The corresponding equations are

$$x+y-2z=3$$
$$-5y+5z=-10.$$

Let us choose $z=k$ (arbitrary constant).

Then from second equation

i.e., $-5y+5z=-10 \Rightarrow y=-\frac{1}{5}(-10-5z)=-\frac{1}{5}(-10-5k)=2+k$.

From first equation $x+y-2z=3 \Rightarrow x=3-y+2z=3-(2+k)+2k \Rightarrow x=1+k.$

Therefore the solution is given by

$$\mathrm{X}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1+k\\ 2+k\\ k\end{bmatrix}.$$

5. Find the values of λ for which the system

$x+y+z=1,\ x+2y+4z=\lambda,\ x+4y+10z=\lambda^2$ has a solution. Solve it in each case.

Solution: The augmented matrix is given by

$$[\mathrm{A}:\mathrm{B}]=\begin{bmatrix}1 & 1 & 1 & : 1\\ 1 & 2 & 4 & : \lambda\\ 1 & 4 & 10 & : \lambda^2\end{bmatrix}$$

$\mathrm{R}_2' \to \mathrm{R}_2-\mathrm{R}_1,\ \mathrm{R}_3' \to \mathrm{R}_3-\mathrm{R}_1$

$$\sim\begin{bmatrix}1 & 1 & 1 & : & 1\\ 0 & 1 & 3 & : & \lambda-1\\ 0 & 3 & 9 & : & \lambda^2-1\end{bmatrix}$$

$R_3^{'} \to R_3 - 3R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \lambda-1 \\ 0 & 0 & 0 & : & \lambda^2-3\lambda+2 \end{bmatrix}$$

We observe that $\rho(A)=2$ and $\rho([A:B])$ will be equal to 2 iff $\lambda^2-3\lambda+2=0$,

i.e., for $\lambda=1$ or $\lambda=2$.

$\Rightarrow$ System will possess a solution if $\lambda=1$ or 2 and in both the cases the system will have infinite number of solution as $\rho(A)=\rho([A:B])=2<3$ number of unknowns and hence 1 free variable.

Let us consider these cases one by one.

Case 1 When $\lambda=1$, the reduced system gives

$$x+y+z=1$$
$$y+3z=1-1=0.$$

Let $z=k$ be arbitrary and from second equation we have

$y=-3z=-3k.$

From first equation, we have

$x=1-y-z=1-(-3k)-k=1+2k$.

Case 2: When $\lambda=2$, the reduced system gives,

$$x+y+z=1$$
$$y+3z=2-1=1$$

Let z=k, then y=1-3k and x=1-y-z=1-1+3k-k=2k where k is an arbitrary constant.

6. Find the values of λ and μ for which the system $x+y+z=6,\ x+2y+3z=10,\ x+2y+z=\mu$ has (i) a unique solution (ii) infinitely many solutions (iii) no solution.

Solution: Consider the augmented matrix

$$[A:B]=\begin{bmatrix} 1 & 1 & 1 & :6 \\ 1 & 2 & 3 & :10 \\ 1 & 2 & \lambda & :\mu \end{bmatrix}$$

$R_2^{'} \to R_2 - R_1,\ R_3^{'} \to R_3 - R_1$

$$\sim\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3' \to R_3 - R_2$

$$\sim\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}.$$

Here we observe that

a) If $\lambda-3=0$ and $\mu-10\neq 0$ i.e., $\lambda=3$ and $\mu\neq 10$, then the system will be inconsistent and possesses no solution.

b) If $\lambda-3=0$ and $\mu-10=0$ i.e., $\lambda=3$ and $\mu=10$ the system will reduce to

In this case the system possesses infinite solutions.

c) If $\lambda-3\neq 0$ i.e., $\lambda\neq 3$, the system will possess a unique solution, irrespective of the value of μ.

EXERCISE

1. Show that the system $x+y+z=4;\ 2x+y-z=1;\ x-2y+2z=2$ is consistent and solve the system.
2. Find the value of $\boldsymbol{\lambda}$ for which the system has solution. Solve the system in each possible case: $x+y+z=1;\ \ x+2y+4z=\lambda;\ \ x+4y+10z=\lambda^2$.
3. Test for consistency and solve:
   $a)\ x+2y+2z=5, 2x+y+3z=6, 3x-y+2z=4, x+y+z=-1$
   $b)\ 5x+3y+7z=4,\ \ 3x+26y+2z=9,\ \ 7x+2y+10z=5.$
   $c)\ 5x+y+3z=20,\ \ 2x+5y+2z=18,\ \ 3x+2y+z=14.$
4. Investigate the value of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ so that the equations
   $2x+3y+5z=9,\qquad 7x+3y-2z=8,\qquad 2x+y+\lambda z=\mu,$
   have (i) no solution (ii) unique solution (iii) infinite solutions.
5. Find the values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ such that the system

$x + 2y + 3z = 6, \quad x + 3y + 5z = 9, \quad 2x + 5y + \lambda z = \mu,$

have (i) no solution (ii) unique solution (iii) infinite solutions.

6. For what values of **λ** and **μ** do the system of equations:

$x + y + z = 6, \quad x + 2y + 5z = 10, \quad x + 3y + \lambda z = \mu$

have (i) no solution (ii) unique solution (iii) infinite solutions.

GAUSS ELIMINATION METHOD:

In this method the unknowns are eliminated successively and the system is reduced to upper triangular system from which the unknowns are found by back substitution.

PROCEDURE:

- Write the augmented matrix of the given system of equations
- Reduce the augmented matrix to Echelon form
- Write the linear equations (starting from last) associated with the echelon form of matrix

EXERCISE

1. Solve the following system by Gauss elimination method

$x + y - z = 0, \; 2x - 3y + z = -1, \; x + y + 3z = 12, \; y + z = 5$

Solution : The augmented matrix is given by

$$[A:B] = \begin{bmatrix} 1 & 1 & -1 & :0 \\ 2 & -3 & 1 & :-1 \\ 1 & 1 & 3 & :12 \\ 0 & 1 & 1 & :5 \end{bmatrix}$$

$R_2' \to R_2 - 2R_1, \; R_3' \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & -1 & :0 \\ 0 & -5 & 3 & :-1 \\ 0 & 0 & 4 & :12 \\ 0 & 1 & 1 & :1 \end{bmatrix}$$

$R_4' \to R_4 + (1/5)R_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 & :0 \\ 0 & -5 & 3 & :-1 \\ 0 & 0 & 4 & :12 \\ 0 & 0 & 8 & :24 \end{bmatrix}$$

$R_4' \to R_4 - 2R_3$

$$\sim \begin{bmatrix} 1 & 1 & -1 & :0 \\ 0 & -5 & 3 & :-1 \\ 0 & 0 & 4 & :12 \\ 0 & 0 & 0 & :0 \end{bmatrix}.$$

By back substitution

$4z = 12 \Rightarrow z = 3,$

$5y + 3z = -1 \Rightarrow y = 2,$

$x + y - z = 0 \Rightarrow x = 2.$

2. Solve the following system by Gauss elimination method

$$2x_1 - x_2 + 2x_3 = 1$$
$$-3x_1 + 4x_2 - 5x_3 = 0$$
$$x_1 + 3x_2 - 6x_3 = 0.$$

Solution: Consider the augmented matrix

$$[A:B] = \begin{bmatrix} 2 & -1 & 3 & :1 \\ -3 & 4 & -5 & :0 \\ 1 & 3 & -6 & :0 \end{bmatrix}$$

$R_2' \to R_2 + (3/2)R_1,\ R_3' \to R_3 - (1/2)R_1$

$$\sim \begin{bmatrix} 2 & -1 & 3 & :1 \\ 0 & 5 & -1 & :3 \\ 0 & 7 & -15 & :-1 \end{bmatrix}$$

$R_3' \rightarrow R_3 - (7/5)R_2$

$$\sim \begin{bmatrix} 2 & -1 & 3 & : & 1 \\ 0 & 5 & -1 & : & 3 \\ 0 & 0 & -68 & : & -26 \end{bmatrix}$$

$$\begin{aligned} & 2x_1 - x_2 + 3x_3 = 1 \\ \Rightarrow\ & 5x_2 - x_3 = 3 \\ & -68x_3 = -26. \end{aligned}$$

By back substitution the solution is given by

$$x_3 = \frac{13}{34}, x_2 = \frac{23}{34}, x_1 = \frac{9}{34}.$$

EXERCISE

Solve the following system of equations by Gauss Elimination method:

1. $x + 2y + z = 3,\ \ 2x + 3y + 2z = 5,\ \ 3x - 5y + 5z = 2$
2. $x + y + z = 9,\qquad x - 2y + 3z = 8, 2x + y - z = 3.$
3. $2x - 3y + 4z = 7,\ 5x - 2y + 2z = 7,\ \ 6x - 3y + 10z = 23.$
4. $2x + y + 4z = 12,\ 4x + 11y - z = 33,\ 8x - 3y + 2z = 20.$
5. $4x + y + z = 4,\ \ x + 4y - 2z = 4,\ \ 3x + 2y - 4z = 6.$
6. $3x - y + 2z = 12,\ \ x + 2y + 3z = 11, 2x - 2y - z = 2$
7. $2x - y + 3z = 1,\ -3x + 4y - 5z = 0,\ \ x + 3y + 6z = 0.$
8. $2x + 5y + 7z = 52,\ \ 2x + y - z = 0,\ \ x + y + z = 9.$
9. $x - 2y + 3z = 2,\ \ 3x - y + 4z = 4,\ \ 2x + y - 2z = 5.$
10. $5x_1 + x_2 + x_3 + x_4 = 4, x_1 + 7x_2 + x_3 + x_4 = 12,\ \ x_1 + x_2 + 6x_3 + x_4 = -5,$

    $x_1 + x_2 + x_3 + 4x_4 = -6.$

GAUSS-JORDON METHOD:

This method is for finding the solution of system of linear equations by reducing the augmented matrix to diagonal matrix

EXAMPLES

Solve the following system of equations by Gauss-Jordan Method

1. $2y - 3z = 2,\ x + z = 3,\ x - y + 3z = 1.$

Solution: Let the augmented matrix of the given system is

$$[A:B]=\begin{bmatrix} 0 & 2 & -3: & 2 \\ 1 & 0 & 1: & 3 \\ 1 & -1 & 3: & 1 \end{bmatrix}$$

Interchange first and second row (to make top left entry non-zero)

$$\sim\begin{bmatrix} 1 & 0 & 1: & 3 \\ 0 & 2 & -3: & 2 \\ 1 & -1 & 3: & 1 \end{bmatrix}$$

$R_3' \to R_3 - R_1$

$$\sim\begin{bmatrix} 1 & 0 & 1: & 3 \\ 0 & 2 & -3: & 2 \\ 0 & -1 & 2: & -2 \end{bmatrix}$$

$R_3' \to R_3 + (1/2)R_2$

$$\sim\begin{bmatrix} 1 & 0 & 1: & 3 \\ 0 & 2 & -3: & 2 \\ 0 & 0 & 1/2: & -1 \end{bmatrix}$$

$R_3' \to 2R_3$

$$\sim \begin{bmatrix} 1 & 0 & 1: 3 \\ 0 & 2 & -3: 2 \\ 0 & 0 & 1: -2 \end{bmatrix}$$

$R_1^{'} \to R_1 - R_3, R_2^{'} \to R_2 + 3R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0: \ 5 \\ 0 & 2 & 0: \ -4 \\ 0 & 0 & 1: \ -2 \end{bmatrix}$$

$R_2^{'} \to (1/2)R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0: 5 \\ 0 & 1 & 0:-2 \\ 0 & 0 & 1:-2 \end{bmatrix}$$

This is now in reduced row-echelon form, so we get $x = 5$, $y = -2$, and $z = -2$.

EXERCISE

Solve the following system of equations by Gauss Jordan method:

1. $x + y + z = 9, \quad x - 2y + 3z = 8, \quad 2x + y - z = 3$
2. $2x + 5y + 7z = 52, \ 2x + y - z = 0, \ x + y + z = 9.$
3. $2x + y + z = 10, \ 3x + 2y + 3z = 18, \ x + 4y + 9z = 16.$
4. $x + y + z = 8, \ -x - y + 2z = -4, \ 3x + 5y - 7z = 14.$
5. $2x - 3y + z = 1, \ x + 4y + 5z = 25, \ 3x - 4y + z = 2.$
6. $x + y + z = 1, \ 4x + 3y - z = 6, \ 3x + 5y + 3z = 4.$
7. $x + 2y + z = 3, \ 2x + 3y + 3z = 10, \ 3x - y + 2z = 13.$
8. $2x_1 + x_2 + 3x_3 = 1, \ 4x_1 + 4x_2 + 7x_3 = 1, \ 2x_1 + 5x_2 + 9x_3 = 3.$

DIAGONALLY DOMINANT FORM:

A system of 'n' linear equations in 'n' unknowns given by

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots\dots\dots + a_{1n}x_n = b_1 \qquad (1)$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots\dots\dots + a_{2n}x_n = b_2 \qquad (2)$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots\dots\dots + a_{nn}x_n = b_n \qquad (n)$$

is said to be in diagonally dominant form if in equation (1), $|a_{11}|$ is greater the sum of the absolute values of the remaining coefficients; in (2), $|a_{22}|$ is greater than the sum of the absolute of the remaining coefficients and so on.

$$|a_{11}| > |a_{12}| + |a_{13}| + \dots\dots |a_{1n}|$$

$$|a_{22}| > |a_{21}| + |a_{23}| + \dots\dots |a_{2n}|$$

$$|a_{nn}| > |a_{n1}| + |a_{n2}| + \dots\dots |a_{n(n-1)}|.$$

GAUSS-SEIDEL METHOD:

The Gauss-Seidel method is an iterative method that can be used to solve a system of 'n' linear equations in 'n' unknowns. A starting or an initial solution is first assumed, which is then improved through successive iteration. A convergence to the actual solution is ensured if the given system of equations is arranged in the diagonally dominant form. The following example illustrates the working procedure of this method.

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EXAMPLES

1. Solve the following system of equations using Gauss-Seidel method.

$$6x+15y+2z=72,\ x+y+54z=110,\ 27x+6y-z=85.$$

Solution: In the above equations, we have $|15|>|6|+|2|, |54|>|1|+|1|\ \&\ |27|>|6|+|-1|$.

Hence the equations are arranged in the diagonally dominant form as:

$$27x+6y-z=85,\ 6x+15y+2z=72,\ x+y+54z=110.$$

The first equation is used to determine *x* and is therefore rewritten as

$$x=\frac{85-6y+z}{27}. \qquad (1)$$

The second equation is used to determine *y* and is rewritten as

$$y=\frac{72-6x-2z}{15}. \qquad (2)$$

The third equation used to determine *z* is rearranged as

$$z=\frac{110-x-y}{54}. \qquad (3)$$

Equations (1), (2), (3) are used to find sequentially *x*, *y* and *z* in each of the iterations.

Starting solution: Let us choose $[x,y,z]=[0,0,0]$ as the starting solution.

First iteration:

$$x^{(1)}=\frac{1}{27}[85-0+0]=3.148$$

$$y^{(1)}=\frac{1}{15}[72-6(3.1481)-0]=3.5407$$

$$z^{(1)}=\frac{1}{54}[110-3.1481-3.5407]=1.9132$$

Note that in finding $y^{(1)}$ the latest value $x^{(1)}=3.1481$ is used and not x=0. Similarly in finding $z^{(1)}$, the latest values $y^{(1)}=3.5407$.

Second iteration:

$$x^{(2)} = \frac{1}{27}[85 - 6(3.5407) + 1.9132] = 2.4322$$

$$y^{(2)} = \frac{1}{15}[72 - 6(2.4322) - 2(1.9132] = 3.5720$$

$$z^{(2)} = \frac{1}{54}[110 - 2.4322 - 3.5720] = 1.9258.$$

and $x^{(1)} = 3.1481$ are used. The same procedure is applied in subsequent iterations also.

Third iteration:

$$x^{(3)} = \frac{1}{27}[85 - 6(3.5720) + 1.9258] = 2.4257$$

$$y^{(3)} = \frac{1}{15}[72 - 6(2.4257) - 2(1.9258)] = 3.5729$$

$$z^{(3)} = \frac{1}{54}[110 - 2.4257 - 3.5729] = 1.9259.$$

Therefore $[x, y, z] = [2.4257, 3.5729, 1.9259]$.

Fourth iteration:

$$x^{(4)} = \frac{1}{27}[85 - 6(3.5729) + 1.9259] = 2.4255$$

$$y^{(4)} = \frac{1}{15}[72 - 6(2.4255) - 2(1.9259)] = 3.5730$$

$$z^{(4)} = \frac{1}{54}[110 - 2.4255 - 3.5730] = 1.9259.$$

Since the solutions in 3^{rd} and 4^{th} iterations agree upto 3 places of decimals, the solution can be taken as

$[x, y, z] = [2.4255, 3.5730, 1.9259]$.

EXERCISE

Solve the following system of equations by Gauss Seidel Method performing 3 iterations:

1. $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.
2. $3x + 8y + 29z = 71$, $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$.

3. $10x + 2y + z = 9,\ x + 10y - z = -22, -2x + 3y + 10z = 22.$

4. $5x - y = 9, x - 5y + z = -4, y - 5z = 6$ taking $\left(\frac{9}{5}, \frac{4}{5}, \frac{6}{5}\right)$ as first approximation.

5. $x + y + 54z = 110,\ 27x + 6y - z = 85,\ 6x + 15y + 2z = 72.$

6. $10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$

7. $12x + y + z = 31, 2x + 8y - z = 24,\ 3x + 4y + 10z = 58.$

8. $5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20.$

9. $5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$ taking initial approximation as (1,0,3).

10. $10x + 2y + z = 9,\ 2x + 20y - 2z = -44, -2x + 3y + 10z = 22$ by taking (0, 0, 0) as initial approximation root (carry out 3 iterations).

EIGEN VALUE & EIGEN VECTOR:

Let A be an $n \times n$ matrix. A number λ is said to be an eigenvalue of A if there exists a non-zero solution vector X of the system of equations.

$AX = \lambda X$ or $AX - \lambda I X = 0$, I being an identity matrix of order n.

The non-zero solution vector X is said to be an eigenvector corresponding to the eigenvalue λ. The word "eigenvalue" is a combination of German and English terms. Eigenwert (Proper value) Eigenvalues and eigenvectors are also called characteristic values and characteristic vectors, respectively.

The characteristic equation of the matrix A is defined to be $\det(A - \lambda I) = 0$.

EXAMPLE:

Let $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ and $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Consider the product

$$AX = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -5+4 \\ 2-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -2 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 2 \end{bmatrix} = (-1)X$$

The above is of the form $AX = \lambda X$, where $\lambda = -1$ and $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Hence $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is the Eigen vector corresponding to the Eigen value -1.

RAYLEIGH'S POWER METHOD:

In many problems we need to calculate only the Largest eigenvalue, this method is very useful to find the Largest eigenvalue and its corresponding eigenvectors. Note inverse power method is used to find the smallest eigenvalue and its corresponding eigenvector.

The following steps are followed while finding the Largest eigenvalue:

- We start with a column vector X_0(initial vector) which is as near the solution as possible and evaluate $AX_0$and express in the form $\lambda_1 X_1$ i.e., $AX_0 = \lambda_1 X_1$. Here λ_1 is the numerically largest value of the product vector AX_0, which is called first approximate Eigen value and X_1 is the corresponding Eigen vector of the given square matrix A.
- Similarly second approximation is given by $AX_1 = \lambda_2 X_2$
- Third approximation is given by $AX_2 = \lambda_3 X_3$
- Continuing the iterations till two successive iterations having the same values up to the desired degree of accuracy.

NOTE: Assume X_0 as a column vector with 1 as the first element and remaining as zero (if not given)

Examples:

1. Using power method find an approximate value of Eigen value and the corresponding Eigen vector of the matrix

$$A=\begin{bmatrix}4 & 3 & 0\\ 0 & 3 & 0\\ 1 & 1 & 2\end{bmatrix}.$$

Solution:

Let $X_0=\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ be the initial approximation.

$$AX_0=\begin{bmatrix}7\\3\\4\end{bmatrix}=7\begin{bmatrix}1\\3/7\\4/7\end{bmatrix}=\lambda_1 X_1$$

$$AX_1=5.28571\begin{bmatrix}1\\0.24324\\0.48649\end{bmatrix}=\lambda_2 X_2$$

$$A\,X_2=4.72972\begin{bmatrix}1\\0.15428\\0.46857\end{bmatrix}=\lambda_3 X_3$$

$$A\,X_3=4.46284\begin{bmatrix}1\\0.10371\\0.46863\end{bmatrix}=\lambda_4 X_4$$

$$A\,X_4=4.31113\begin{bmatrix}1\\0.07217\\0.47342\end{bmatrix}=\lambda_5 X_5$$

$$A\,X_5=4.02433\begin{bmatrix}1\\0.00605\\0.4970\end{bmatrix}=\lambda_6 X_6$$

$$A\,X_6=4.01815\begin{bmatrix}1\\0.0045\\0.49775\end{bmatrix}=\lambda_7 X_7$$

Therefore The largest eigenvalue is 4.02.

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The corresponding eigenvector is $\begin{bmatrix} 1 \\ 0.0045 \\ 0.49775 \end{bmatrix}$.

Find the largest Eigen value and the corresponding Eigen vector of the following matrices using power method:

1. A= $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Take (1,0,0) as initial vector. Carry out six iterations.

2. A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Take (1,1,1) as initial vector. Carry out five iterations.

3. A= $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. Take (1,0,0) as initial vector. Carry out four iterations.

4. A= $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Take (1,1,0) as initial vector. Carry out six iterations.

5. A= $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$. Take (0,0,1) as initial vector. Carry out five iterations.

6. A= $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$. Take (1,0,0) as initial vector. Carry out six iterations

7. A= $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$. Take (1,0,0) as initial vector. Carry out seven iterations.

8. A= $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$. Take (1,0,0) as initial vector. Carry out seven iterations.

9. A= $\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix}$. Take (1,1,0) as initial vector. Carry out six iterations.

10. A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$. Take (1,1,1) as initial vector. Carry out five iterations.

APPLICATIONS:

1. It allows people to find important subsystems or patterns inside noisy data sets.
2. Eigenvalues and eigenvectors have widespread practical application in multivariate statistics.
3. Powers of a Diagonal Matrix, Matrix Factorization.
4. *Eigenvalue* analysis is also used in the design of the car stereo systems, where it helps to reproduce the vibration of the car due to the music.
5. Electrical Engineering: The *application of eigenvalues and eigenvectors* is useful for decoupling three-phase systems through symmetrical component transformation.
6. Model population growth using an age transition matrix and an age distribution vector, and find a stable age distribution vector