

## MODULE 4

### ELECTRICAL PROPERTIES OF MATERIALS AND APPLICATIONS

#### Assumptions of classical free electron theory:

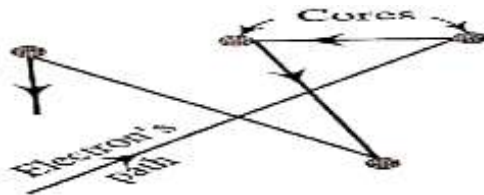
- A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons
- The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature  $T$  is given by  $3/2 kT$ , where  $k$  is a Boltzmann constant. It is related to the kinetic energy.  
$$3/2 kT = \frac{1}{2} m v_{th}^2$$
Where  $v_{th}$  is the thermal velocity same as root mean square velocity.
- The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant.
- The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

#### Drift velocity ( $v_d$ ):

The average velocity of occupied by the electrons in the steady state in an applied electric field is called drift velocity.

The drift velocity  $v_d = \frac{eE\tau}{m}$

#### Thermal velocity( $V_{th}$ ):



The velocity of electrons in random motion due to thermal agitation called thermal velocity.

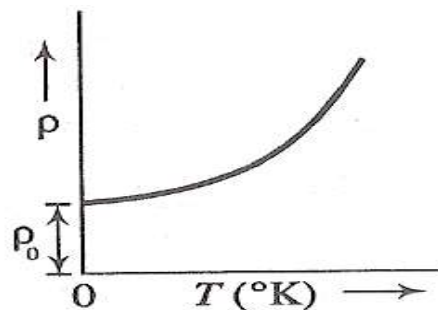
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**Mean free path ( $\lambda$ ):**

The average distance travelled by the conduction electrons between any two successive collisions with lattice ions.

**Temperature dependence of resistivity of a metal:**

All metals are good conductors of electricity. The electrical conductivity of metal varies with the temperature. The electrical resistance of a metal, to the flow of current, is due to scattering of conduction electrons by lattice vibrations. When the temperature increases the amplitude of lattice vibrations also increases, thereby increasing the resistance. The dependence of resistance of metal (non-superconducting state) is shown in figure. The resistance decreases with temperature and reaches a minimum value at  $T = 0\text{K}$ . The residual resistance at  $T = 0\text{K}$  is due to impurities in the metal.



DEPENDENCE OF  $\rho$  ON  $T$

By Matthiessen's rule

$$\rho = \rho_0 + \rho(T)$$

Where ' $\rho$ ' is the resistivity of the given material, ' $\rho_0$ ' is the residual resistivity and ' $\rho(T)$ ' is the temperature dependent part of resistivity.

**“The total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to scattering by impurities which is temperature dependent”**

**Expression for electrical conductivity of conductor according to classical free electron theory**

According to classical free electron theory the expression for electrical conductivity is given by

$$\sigma_{\text{CFET}} = \frac{ne^2 \tau}{m}$$

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Where  $\sigma$  - Electrical conductivity  
 $n$  - Electron density  
 $\tau$  - mean collision time  
 $m$  - mass of electron

**Failures of classical free electron theory:**

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

**i) Specific heat:** The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was  $C_v = 10^{-4} RT$

which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

**ii) Temperature dependence of electrical conductivity:**

Experimentally, electrical conductivity  $\sigma$  is inversely proportional to the temperature  $T$ .

i.e.  $\sigma_{\text{exp}} \propto \frac{1}{T} \rightarrow (1)$

According to the assumptions of classical free electron theory

Since  $V_{\text{th}} \propto \sqrt{T}$

But  $\tau \propto \frac{1}{V_{\text{th}}}$ ,  $\tau \propto \frac{1}{\sqrt{T}}$ ,

substituting in conductivity equation we get

$$\sigma_{\text{CFET}} = \frac{ne^2 \tau}{m} = \frac{ne^2}{m \sqrt{T}}$$

—=

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$$\text{Or } \sigma_{\text{CFET}} \propto \frac{1}{\sqrt{T}} \rightarrow (2)$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

### iii) Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$$\sigma = \frac{ne^2\tau}{m} \quad \text{i.e., } \sigma \propto n, \quad \text{where } n \text{ is the electron concentration,}$$

Consider copper and aluminum. Their electrical conductivities are  $5.88 \times 10^7 / \Omega\text{m}$  and  $3.65 \times 10^7 / \Omega\text{m}$ . The electron concentrations for copper and aluminum are  $8.45 \times 10^{28} / \text{m}^3$  and  $18.06 \times 10^{28} / \text{m}^3$ . Hence the classical free electron theory fails to explain the dependence of  $\sigma$  on electron concentration.

Experimental results:

Metals	Electron concentration(n)	conductivity ( $\sigma$ )
Copper	$8.45 \times 10^{28} / \text{m}^3$	$5.88 \times 10^7 / \Omega\text{m}$
Aluminium	$18.06 \times 10^{28} / \text{m}^3$	$3.65 \times 10^7 / \Omega\text{m}$

### Quantum free electron theory:

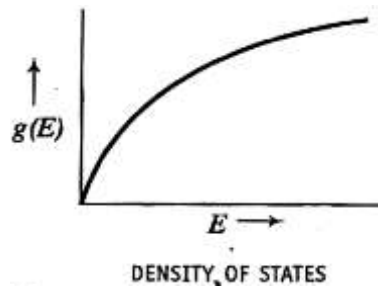
#### Assumptions of quantum free electron theory:

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

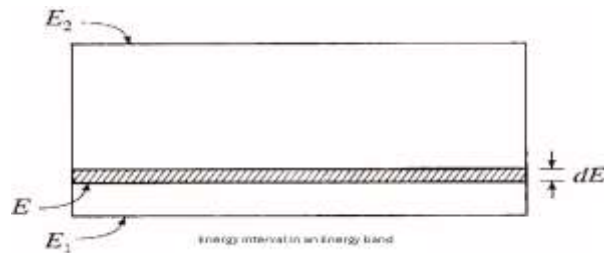
**Density of states g(E):**

Density of states is defined as the number of allowed energy states per unit energy range per unit volume in the valance band of a material. It is denoted as g(E).

A graph of g(E) verses E is shown below.



Consider an energy band spread in an energy interval between  $E_1$  and  $E_2$ . Below  $E_1$  and above  $E_2$  there are energy gaps. g(E) represents the density of states at E. As dE is small, it is assumed that g(E) is constant between E and E+dE. The density of states in range E and (E+dE) is denoted by g(E)dE.

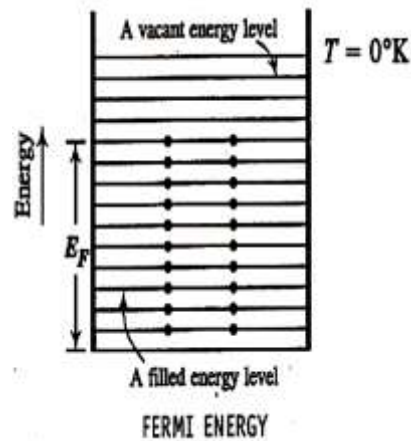


$$\text{i.e. } g(E)dE = \left[ \frac{8\sqrt{2\pi m^2}}{h^3} \right] E^{\frac{1}{2}} dE$$

It is clear g(E) is proportional to  $\sqrt{E}$  in the interval dE

**Fermi energy and Fermi level:**

The energy of electrons corresponding to the highest occupied energy level at absolute 0°K is called Fermi energy and the energy level is called Fermi level.



### **Fermi factor:**

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability  $f(E)$  that a given energy state with energy  $E$  is occupied by the electrons at a steady temperature  $T$  is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$f(E)$  is called the Fermi factor.

### **Dependence of Fermi factor with temperature and energy:**

The dependence of Fermi factor on temperature and energy is as shown in the figure.

#### **i) Probability of occupation for $E < E_F$ at $T=0K$ :**

When  $T=0K$  and  $E < E_F$

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

The probability of occupation of energy state is 100%

$$f(E)=1 \text{ for } E < E_F.$$

$f(E)=1$  means the energy level is certainly occupied and  $E < E_F$  applies to all energy levels below  $E_F$ . Therefore at  $T=0K$  all the energy levels below the Fermi level are occupied.

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**ii) Probability of occupation for  $E > E_F$  at  $T=0K$ :**

When  $T=0K$  and  $E > E_F$

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

$$f(E)=0 \text{ for } E > E_F$$

$\therefore$  At  $T=0K$ , all the energy levels above Fermi levels are unoccupied. Hence at  $T=0K$  the variation of  $f(E)$  for different energy values, becomes a step function as shown in the above figure.

**iii) The probability of occupation at ordinary temperature( for  $E \approx E_F$  at  $T > 0K$ )**

At ordinary temperatures though the value of probability remains 1, for  $E < E_F$  it starts reducing from 1 for values of  $E$  close to but lesser than  $E_F$  as in the figure.

The values of  $f(E)$  becomes  $\frac{1}{2}$  at  $E=E_F$

This is because for  $E=E_F$

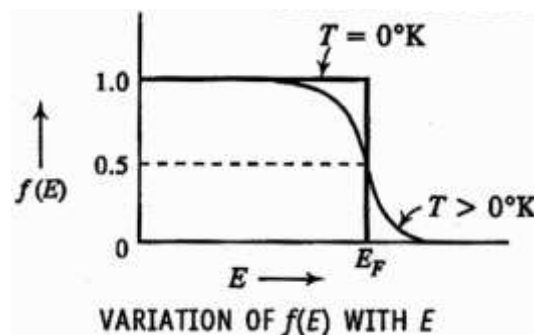
$$e^{(E-E_F)/kT} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

Further for  $E > E_F$  the probability value falls off to zero rapidly.

Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.



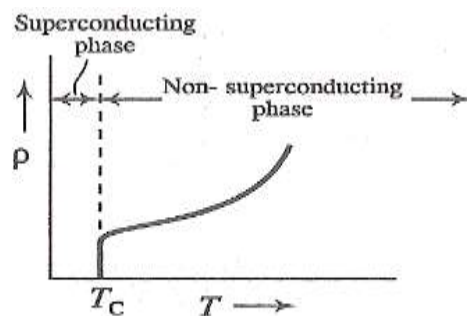
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**Super Conductivity:**

Super conductivity is the phenomenon observed in some metals and materials. Kammerlingh Onnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K. This state is called super conducting state. The material is called superconductor. The temperature at which they attain superconductivity is called critical temperature  $T_c$ .

**Temperature dependence of resistivity of a superconductor:**

One of the most interesting properties of solid at low temperature is that electrical resistivity of metals and alloys vanish entirely below a certain temperature. This zero resistivity or infinite conductivity is known as superconductivity. Temperature at which transition takes place is known as transition temperature or critical temperature ( $T_c$ ). Above the transition temperature, the substance is in the normal state and below it will be in superconducting state.  $T_c$  value is different for different materials.



DEPENDENCE OF  $\rho$  ON  $T$

“The resistance offered by certain materials to the flow of electric current abruptly drop to zero below a threshold temperature. This phenomenon is called superconductivity and threshold temperature is called “critical temperature”.

**Meissner effect:**

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.



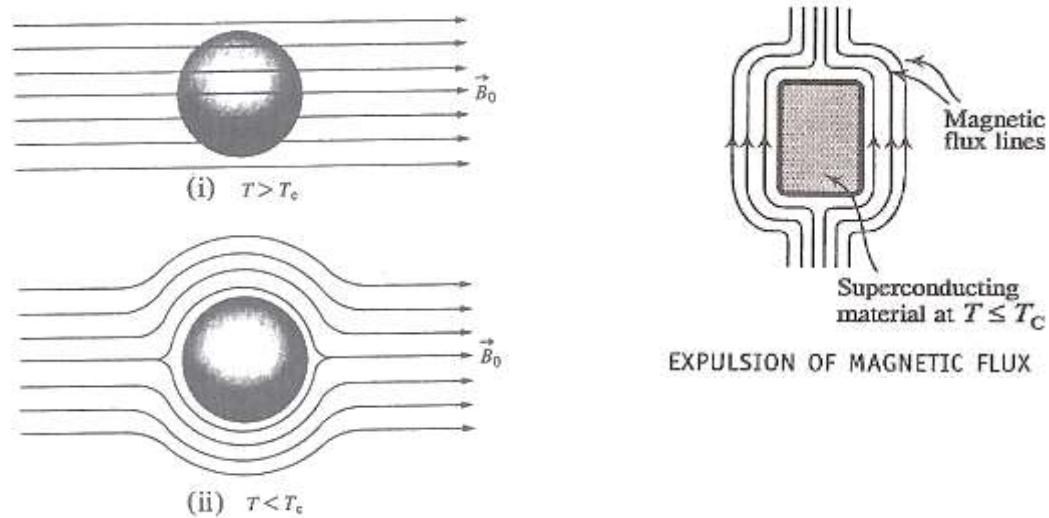


Fig: Superconductor sample subjected to an applied magnetic field with temperature (i) above and (ii) below  $T_C$ . The flux expulsion below  $T_C$  is called Meissner effect

When the temperature is lowered to  $T_C$ , the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches  $T_C$ . Then the substance will be in the normal state.

The magnetic induction inside the specimen

$$B = \mu_0 (H + M)$$

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

$$\text{For } T < T_C, \quad B = 0$$

$$\mu_0 (H + M) = 0$$

$$M = -H$$

$$M/H = -1 = \chi$$

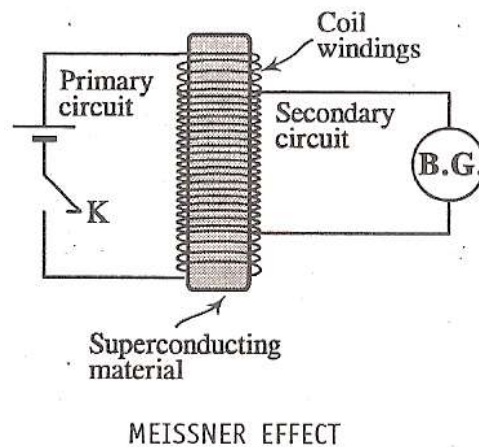
Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

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**Experimental demonstration of Meissner effect:**

Consider a primary coil and a secondary coil, wound on a superconducting material. The primary coil is connected to a battery and a key. The secondary coil is connected to ballistic galvanometer (BG). When the key is closed the current flows through the primary coil and the magnetic field is produced. This flux is linked with the secondary coil and the current flows through the secondary coil which makes a deflection in the galvanometer. If the primary current is steady the magnetic flux and the flux linked with the coil will become steady. As the temperature of the specimen is decreased below the critical temperature, BG suddenly shows a deflection indicating that the flux linked with the secondary coil is changed. This is due to the expulsion of the magnetic flux from the specimen.



**Effect of magnetic field:**

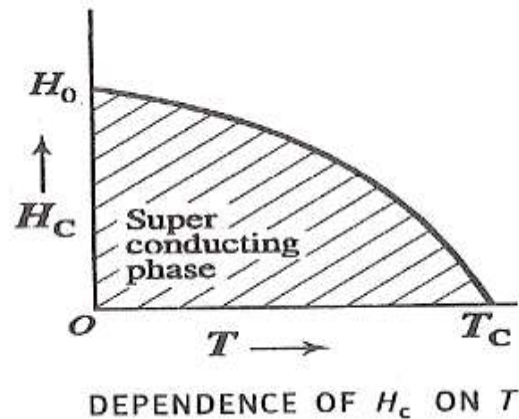
Superconductivity can be destroyed by applying magnetic field. The strength of the magnetic field required to destroy the superconductivity below the  $T_c$  is called critical field. It is denoted by  $H_c(T)$ .

If ' $T$ ' is the temperature of the superconducting material, ' $T_c$ ' is the critical temperature, ' $H_c$ ' is the critical field and ' $H_0$ ' is the critical field at  $0^\circ\text{K}$ .

They are related by

$$H_c = H_0[1 - (T/T_c)^2]$$

By applying magnetic field greater than  $H_0$ , the material can never become superconductor whatever may be the low temperature. The critical field need not be external but large current flowing in superconducting ring produce critical field and destroys superconductivity.

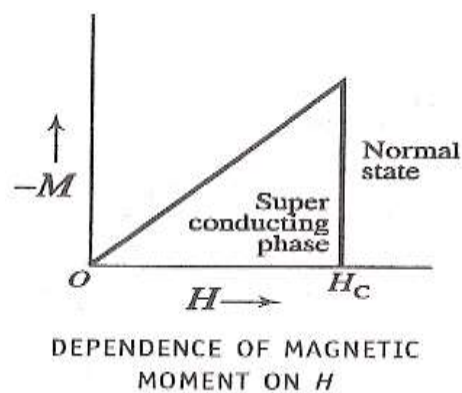


### Types of superconductors:

There are two types of superconductors. They are type-I superconductors and type-II superconductors.

#### **i) Type-I superconductors:**

Type-I superconductors exhibit complete Meissner effect. Below the critical field it behaves as perfect diamagnetic. If the external magnetic field increases beyond  $H_c$  the superconducting specimen gets converted to normal state. The magnetic flux penetrates and resistance increase from zero to some value. As the critical field is very low for type-I superconductors, they are not used in construction of solenoids and superconducting magnets.



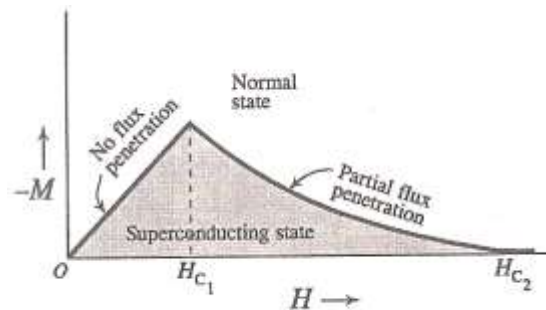
#### **ii) Type-II superconductors**

Type-II superconductors are hard superconductors. They exist in three states

- i) Superconducting state
- ii) Mixed state

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## iii) Normal state

DEPENDENCE OF MAGNETIC MOMENT ON  $H$ 

They are having two critical fields  $H_{c1}$  and  $H_{c2}$ . For the field less than  $H_{c1}$ , it expels the magnetic field completely and becomes a perfect diamagnetic. Between  $H_{c1}$  and  $H_{c2}$  the flux starts penetrating throughout the specimen. This state is called vortex state.  $H_{c2}$  is 100 times higher than  $H_{c1}$ . At  $H_{c2}$  the flux penetrates completely and becomes normal conductor. Type-II superconductors are used in the manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.

**BCS theory superconductivity:**

- Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of Cooper pairs. It is called BCS theory. It is a quantum mechanical concept.
- When a current flows in a superconductor, electrons come near a positive ion core of the lattice, due to attractive force. The ion core also gets displaced from its position, which is called lattice distortion. The lattice vibrations are quantized in a term called Phonons.
- Now an electron which comes near that place will interact with the distorted lattice. This tends to reduce the energy of the electron. It is equivalent to interaction between the two electrons through the lattice. This leads to the formation of Cooper pairs.
- "Cooper pairs are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momentum in a phonon field".
- When the electrons flow in the form of Cooper pairs in materials, they do not encounter any scattering and the resistance factor vanishes or in other words conductivity becomes infinity which is called as superconductivity.
- In the superconducting state, electron-phonon interaction is stronger than the Coulomb force of attraction of electrons. Cooper pairs are not scattered by the lattice points. They travel freely.

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without slow down as their energy is not transferred. Due to this they do not possess any electrical resistivity.

**High temperature superconductors:**

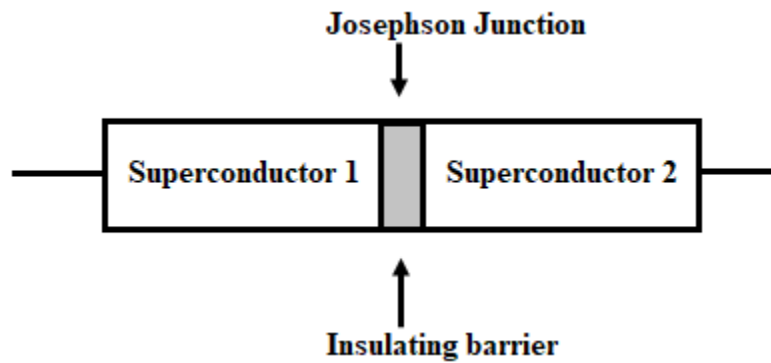
The term high-temperature superconductor was first used to designate the new family of cuprate-perovskite ceramic materials discovered by Bednorz and Müller in 1986. The first high-temperature superconductor, LaBaCuO, with a transition temperature of 30 K and in the same year LSCO ( $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ ) discovered with  $T_C$  of 40K. In 1987 it was shown that superconductors with  $T_c$  greater than 77K could be prepared, this temperature is greater than the liquid helium temperature.  $\text{YBa}_2\text{Cu}_3\text{O}_7$  was discovered to have a  $T_c$  of 92 K. Bismuth/lead strontium Calcium Copper  $(\text{Bi Pb})_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_x$  ( $x < 0.1$ ) with  $T_c = 105\text{K}$ . Thallium barium Calcium copper oxide  $(\text{Tl Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_4)$  of  $T_c = 115\text{K}$ . Mercury barium calcium copper oxide  $(\text{Hg Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_4)$  with  $T_c = 135\text{K}$ .

All high temperature superconductors are different types of oxides of copper, and bear a particular type of crystal structure called Perovskite crystal structure. The number of copper layers increases the  $T_c$  value increases. The current in the high  $T_C$  materials is direction dependent. It is strong in parallel to copper-oxygen planes and weak in perpendicular to copper-oxygen planes.

High  $T_c$  materials are Type-II superconductors and they are brittle and don't carry enough current. The formation of electron pairs is not due to interaction of electron lattice as in the BCS theory. Still it is not clear what does cause the formation of pairs. Research is being conducting in this direction. The high temperature superconductors are useful in high field applications. It can carry high currents of  $10^5$  to  $10^6$  amps in moderate magnetic fields. They are used in military applications, Josephson junction in SQUIDS, under sea communication, submarines.

**Quantum Tunneling:**

Consider two superconductors separated by insulating barrier of thickness less than 10-20 Å, then the cooper pairs tunneling through the insulating barrier is known as **Josephson superconducting quantum tunneling**. The junction between the two superconductors with insulating barrier is known as **Josephson junction**.



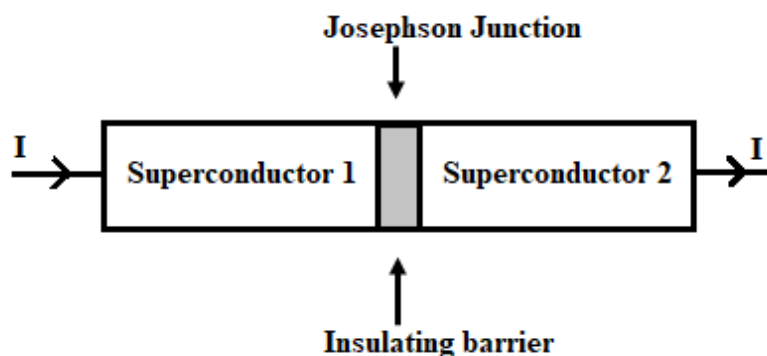
**Josephson junction** is an arrangement of two superconductors separated by an insulating barrier. When the barrier is thin enough, cooper pairs from one superconductor can tunnel through the barrier and reach the other superconductor.

Josephson proposed that this kind of tunneling leads to three kinds of effect, namely

1. dc Josephson effect
2. ac Josephson effect
3. quantum interference

### 1. dc Josephson effect

As per dc Josephson Effect, the tunneling of cooper pairs through the junction occurs without any resistance, which results in a steady dc current without any application of voltage between the two superconductors.



The super current through the junction is given by,

$$I_S = I_C \sin \Phi_0$$

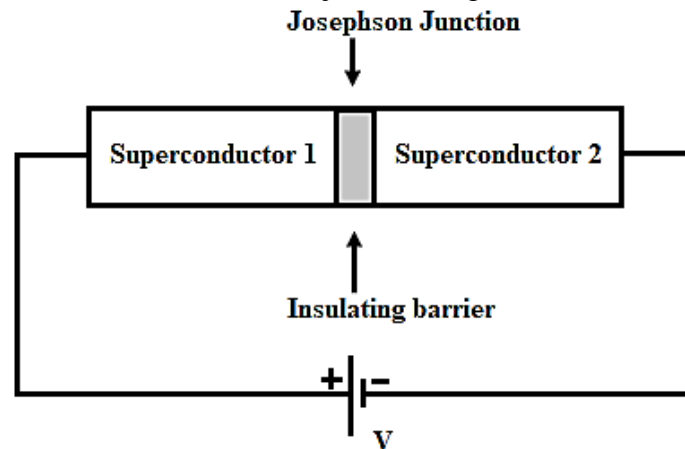
Where,  $\Phi_0$  = phase difference between the wave functions describing the cooper pairs on both the sides of the junction.

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$I_c$  = critical current at zero voltage which depends on the thickness and width of the insulating layer.

**2. ac Josephson effect**

When a dc voltage is applied across the junction, the tunneling of cooper pairs occur in such a way that an ac current would develop in the junction and this effect is called as ac Josephson Effect. When a potential difference of 'V' is applied between the two sides of the junction then a radio frequency (RF) current oscillations across the junction is generated.



$$I = I_c \sin(\Phi_0 + \Delta\Phi)$$

The energies of the cooper pairs on both sides of the barrier difference is  $E = \hbar\omega = 2eV$  (Calculated using quantum mechanical concept).

Therefore it can be shown that,  $\Delta\Phi = \omega t = 2\pi t \left( \frac{2eV}{h} \right)$

Hence,  $I = I_c \sin \left( \Phi_0 + 2\pi t \left( \frac{2eV}{h} \right) \right)$

$I$  = Alternating current of frequency  $\nu = \frac{2eV}{h}$

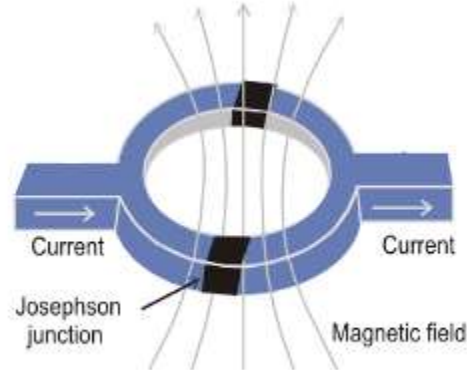
It shows that a photon of frequency  $\nu$  is emitted or absorbed when a cooper pair crosses the junction. Thus when a voltage is applied across the junction, an ac current gets generated. This is known as ac Josephson Effect.

**SQUID**

- SQUID is an acronym for Superconducting Quantum Interference Device.
- It is an Ultra-sensitive measuring instrument used for the detection of very weak magnetic fields of the order of  $10^{-14}$  T.

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- A SQUID is formed by incorporating two Josephson's junction in the loop of a superconducting material.
- When a magnetic field is applied to this superconducting circuit, it induces a circulating current which produces just that much opposing magnetic field so as to exclude the flux from the loop.

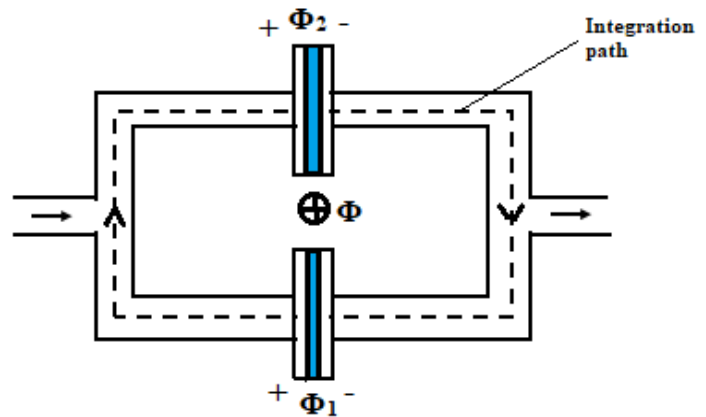
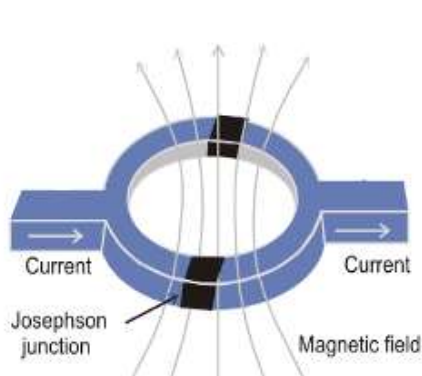


[Note: The flux remains excluded as long as the junction current do not exceed a critical value. But the circuit switches to resistive phase and thereby the flux passes into the loop once the current in either of the junction or in both the junction exceed the critical value. Thus the loop acts like a gate to allow or exclude the flux]

**DC Squids**

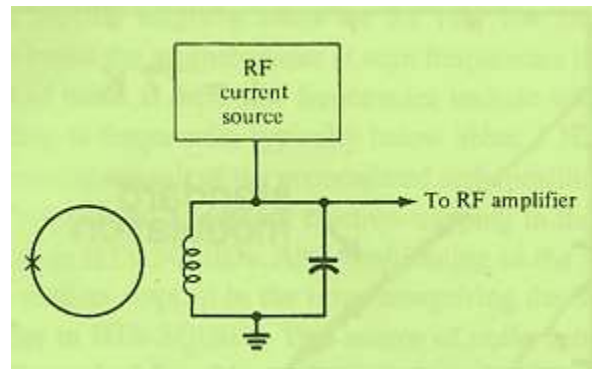
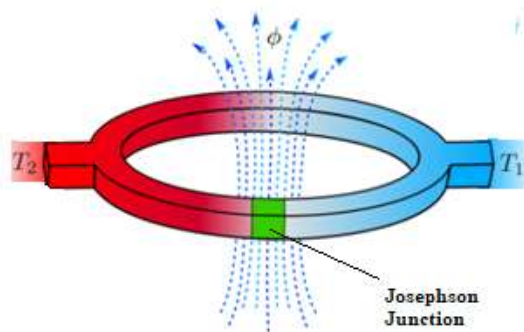
- DC Squids are nothing but a double junction SQUIDS.
- A DC SQUID consists of two Josephson junctions connected in parallel on a closed superconducting loop as shown in the Fig.
- In this device the two weak links are not shorted by superconductor path; therefore the dc current-voltage characteristics can be observed.
- This device applies current slightly greater than the critical current and one can monitor the voltage drop across the device.





### RF SQUIDS

- The single junction SQUIDS are also known as RF SQUIDS.
- The junction is shorted by superconductor path; therefore the voltage response is obtained by coupling the loop to a RF bias tank circuit.
- The RF (Radio Frequency) SQUID is a **one-junction SQUID loop that can be used as a magnetic field detector.**
- In this configuration, **the RF SQUID is inductively coupled to the inductance  $L_T$  of an Lc tank circuit.** The tank circuit is driven by an rf current, and the resultant rf voltage is periodic in the flux applied to the SQUID loop with period  $\Phi_0$ .



## **MODULE - 04**

### **ELECTRICAL PROPERTIES OF MATERIALS AND APPLICATIONS**

1. Explain the failures of Classical free electron theory and Enumerate the assumptions of quantum free electron theory.
2. Explain the temperature dependence of resistivity of a metal and Matthiessen's rule.
3. Define fermi energy and fermi factor. Discuss the variation of fermi energy with temperature and energy.
4. Explain the phenomenon of superconductivity and discuss qualitatively the BCS theory of superconductivity.
5. Explain the temperature dependence of resistivity of a superconductor.
6. Describe Meissner's effect and the variation of critical field with temperature.
7. Classify superconductors into Soft and Hard superconductors using M-H graphs.

**OR**

**Explain the Type-I and Type-II superconductors using M-H graphs.**

8. What is Quantum Tunneling? Explain DC and AC/RF Josephson effects.
9. What are SQUIDS? Explain DC and AC/RF SQUIDS with the help of neat diagram.
10. Mention the applications of superconductivity in quantum computing.