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Dynamical simulation of the asteroid population

Ci421g Exam

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Context

A co-orbital configuration is a configuration where two astronomical objects orbit at the same distance from the Sun, we call this a 1/1 mean motion resonance. Asteroid 2007VW266 is among the rare objects with a heliocentric retrograde orbit, and its semimajor axis is within a Hill sphere radius of that of Jupiter, the region in which gravitational attraction of Jupiter and the Sun on asteroid 2007VW266 are at equilibrium. It was recently discovered that the object is in 13/14 retrograde mean motion resonance.

Aims

In this project we will study the trajectory of the asteroid 2007VW266 and the influence of Jupiter on its path. To do so, we will compute numerical integrations on python, using Runge-Kutta method on order four. We will first code the method and check its induced error. Then we will implement the trajectory of Jupiter, first on a circular orbit, then on a realistic orbit and study the influence of the planet on an arbitrary asteroid. Finally, we will use the data of the 2007VW266 asteroid and compute its trajectory. Adding clones of the asteroid, we will study the impact of observations error to the trajectory.

Methods

To simplify our calculations and have a cleaner code, we will use the following units : distance will be in astronomical units, mass in solar mass, and time in day. Following this, we have a gravitational constant :

$$G = 2.95824 * 10^{-4}\ au^3 Mo^{-1} day^{-2}$$

In order to compute our asteroid trajectory, we need to find the equation of motion of it. Using heliocentric position and Newton's second law and considering only the perturbation of the Sun as well as Jupiter, we find the equation of motion :

$$\ddot{x} = -\frac{GM_o}{r^3}x - GM_J\left(\frac{x - x_J}{\Delta^3} + \frac{x_J}{{r_J}^3}\right)$$
$$\ddot{y} = -\frac{GM_o}{r^3}y - GM_J\left(\frac{y - y_J}{\Delta^3} + \frac{y_J}{{r_J}^3}\right)$$
$$\ddot{z} = -\frac{GM_o}{r^3}z - GM_J\left(\frac{z - z_J}{\Delta^3} + \frac{z_J}{{r_J}^3}\right)$$

The mass of Jupiter is in our system : $M_j = \frac{1}{1047.348625}\ Mo$.

The distance of the asteroid with the center of our frame $r = \sqrt{x^2 + y^2 + z^2}$.

The distance asteroid-Jupiter $\Delta = \sqrt{(x - x_J)^2 + (y - y_J)^2 + (z - z_J)^2}$.

The distance of Jupiter with the center of our frame $r_J = \sqrt{{x_J}^2 + {y_J}^2 + {z_J}^2}$.

Where (x,y,z) are the coordinates of the asteroid and (x_J,y_J,z_J) the coordinates of Jupiter.

To find the coordinates of Jupiter assuming it follows a circular and planar orbit we take its semi major axis and do simple trigonometry :

$$a_J = 5.2\ au$$

$$x_J(t) = 5.2 * \cos\left(\frac{2\pi t}{T}\right)\ ;\ y_J(t) = 5.2 * \sin\left(\frac{2\pi t}{T}\right)\ ; z = 0$$

Using Kepler third law we find the orbit period :

$$\frac{a^3}{T^2} = \frac{GMo}{4\pi^2}$$
$$T = \frac{a^{3/2}}{\sqrt{G}} * 2\pi = 4331.8\ days$$

To get the position of Jupiter on an elliptic orbit, we take its orbital elements :

$$a = 5.202\ au$$
$$e = 0.048908$$
$$i = 1.3038\ deg$$
$$\Omega = 100.51\ \mathrm{deg}$$
$$\omega = 273.88\ deg$$
$$Mo = 80.04\ deg$$
$$epoch = 2456600.5\ s$$

Using the latter elements, we can find the position :

$$n = \frac{k}{\sqrt{a^3}} \text{ } with \text{ } k = \sqrt{G}$$
$$M = Mo + n * (t - epoch)$$

To find the eccentric anomaly, we do the iterative process :

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \text{ } with \text{ } f(E) = E - esinE - M$$

We start with Eo=Mo, and stop when x_{k+1} is inferior to 10^{-6}.

$$X = a(cosE - e) \text{ } ; Y = a\sqrt{1 - e^2} * sinE$$
$$\dot{X} = -\frac{na^2}{r} sinE \text{ } ; \dot{Y} = \frac{na^2}{r}\sqrt{1 - e^2} * cosE$$

With $r = a(1 - ecosE)$, then :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\omega)\begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

With :

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{pmatrix}; \text{ } R_3(\theta) = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To predict the trajectories of our asteroid, we use Runge-Kutta on order 4 :

$$k1 = f(u_i, t_i) \text{ } ; k2 = f\left(u_i + \frac{h}{2}k1, t_i + \frac{h}{2}\right)$$
$$k3 = f\left(u_i + \frac{h}{2}k2, t_i + \frac{h}{2}\right) \text{ } ; k4 = f(u_i + hk3, t_i + h)$$
$$y_{i+1} = y_i + \frac{h}{6}(k1 + 2k2 + 2k3 + k4)$$

In our case, we consider $\ddot{x} = x_{i+2}, \dot{x} = x_{i+1}$ and $x = x_i$ and same for y and z. So we initiate our values as :

$$u_0 = \left\{\begin{matrix} x = x_0 \\ y = y_0 \\ z = z_0 \\ \dot{x} = \dot{x}_0 \\ \dot{y} = \dot{y}_0 \\ \dot{z} = \dot{z}_0 \end{matrix}\right\}$$

Entering our values in our Runge-Kutta function, $(\ddot{x}, \ddot{y}, \ddot{z})$ will be calculated and the next u will be defined as :

$$u_i = \left\{\begin{matrix} x = \dot{x}_{i-1} \\ y = \dot{y}_{i-1} \\ z = \dot{z}_{i-1} \\ \dot{x} = \ddot{x}_i \\ \dot{y} = \ddot{y}_i \\ \dot{z} = \ddot{z}_i \end{matrix}\right\}$$

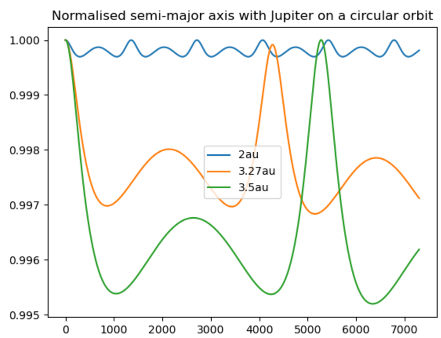
Results

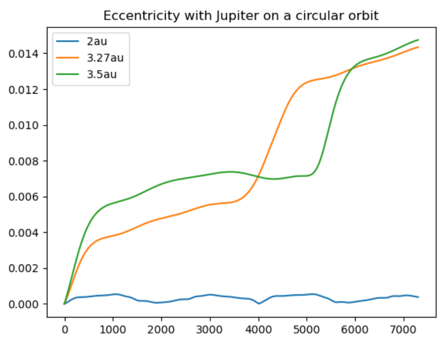
First, we take a look at our Runge-Kutta function and look for the optimal step for which we get a good ratio of precision compared to processing time. For this, we run our program on 100 years of simulation back and forth, and see the error produced on compared to our initial value :

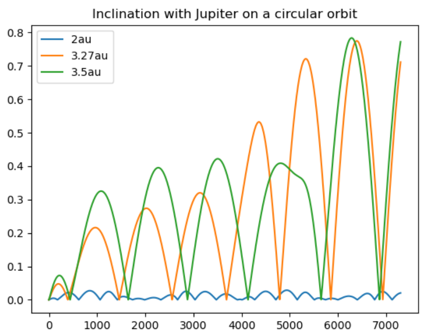
Initial a\time step	0.1 day	0.5 day	1 day	10 days
2 au	2.87e-11, 1 min	1.05e-09, 10 sec	3.42e-08, 5 sec	3.42e-04, 1 sec
3.27 au	3.99e-11, 1 min	1.01e-11, 10 sec	3.79e-10, 5 sec	3.75e-05, 1 sec
3.5 au	3.98e-11, 1 min	1.46e-11, 10 sec	1.71e-10, 5sec	1.72e-05, 1sec

We choose to use a step of 1 day, as it provides sufficient precision for a comfortable processing time.

We proceed to show the influence of Jupiter perturbation on an arbitrary asteroid when the planet is on a circular orbit over 20 years. We place our object at 3 different semi-major axis : 2au, 3.27 au and 3.5 au. We study the variation of semi-major axis, the eccentricity, and the inclination :

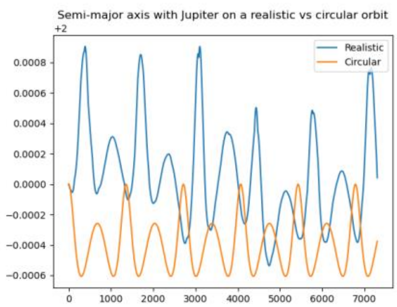






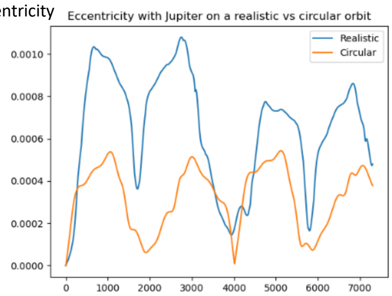
We do the same but with Jupiter on a realistic orbit. We compare both situation, first with the asteroid at 2au :

Semi-major axis (au)

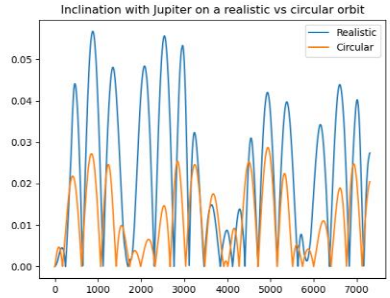


Time (day)

Eccentricity



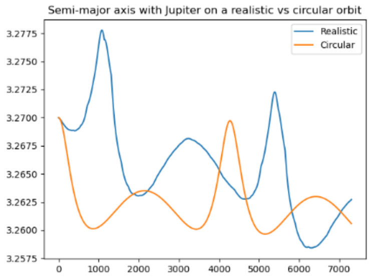
Time (day)

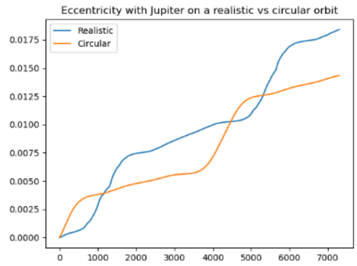


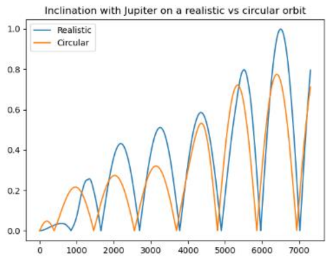
Inclination (deg)

Time (day)

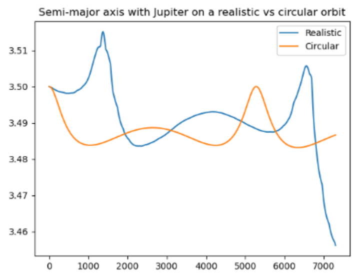
With 3.27 au :

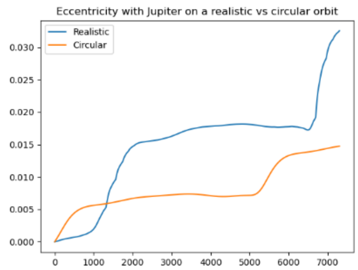


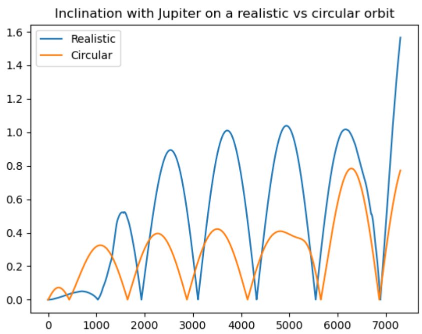




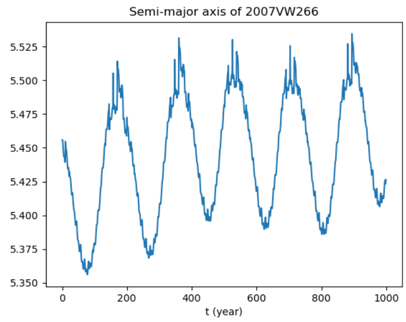
And 3.5 au :

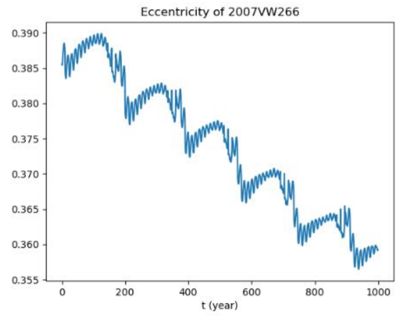


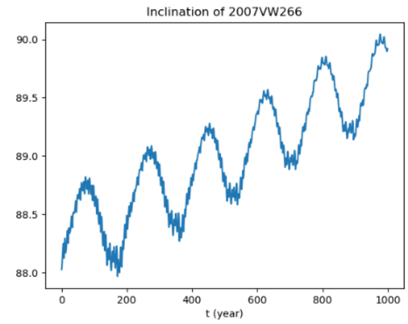




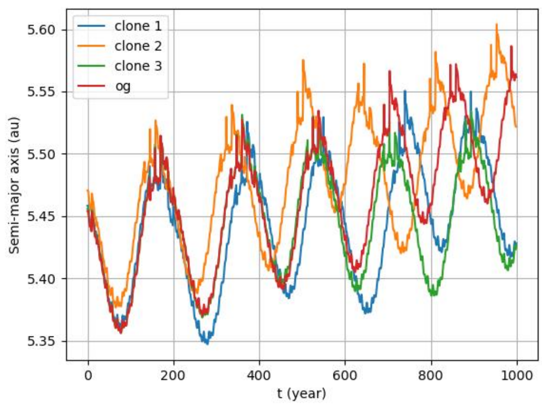
Finally, we do the simulation with the asteroid 2007VW266. We choose a step of 2 days as we are going to study it over 1000 years, and a step of 1 makes the processing time too long.

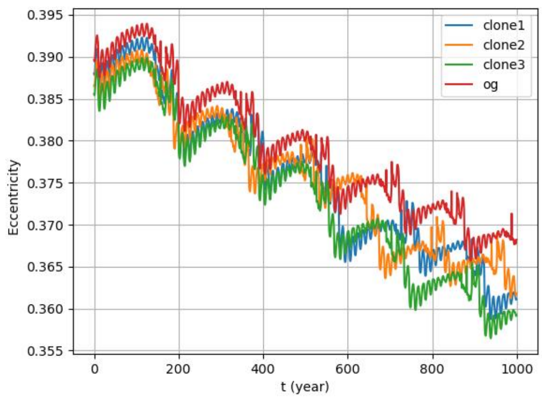






To study the impact of measurement approximation, we implement 3 clones of 2007VW266, taking random initial values included in the range of approximation given. We use a uniform law of probability. "Og" represents the original values of the asteroid.





Discussion

For our arbitrary asteroid, we've seen great results when varying the initial semi-major axis. This is due to the resonance with Jupiter. At 3.27 au, the period of our asteroid is half the period of Jupiter, which makes the perturbations greater as they are less diluted on the whole orbit but concentrated at one point. At 3.5 au, we also observe great perturbations. This is due to the distance of the asteroid increasing with the sun and decreasing with Jupiter, so the ratio of influence grows in favor of Jupiter's perturbations.

Changing Jupiter orbit from circular to elliptic also has an impact on the asteroid trajectory. It increases the perturbations. The elliptic perturbations have a less periodic feeling compared to the circular ones. This is due to the distance with Jupiter varying differently.

Lastly, implanting the 2007VW266 asteroid, we see the effect of the perturbations on a span of 1000 years. We can note that it follows a trend, either upward or downwards for each parameter. The semi-major axis and the inclination increases whereas the eccentricity decreases. We tend to a perpendicular to the solar plane circular orbit. In

Adding the clones to the computation, we see how a small difference in orbit parameters have an impact on the trajectory over the years. For example, clone 3 (green) seem to have the same semi-major axis as the Og (red) when it suddenly drops around year 600. We see when reaching year 1000, all the clones have a significantly different orbit.