LHOEST Simon A4 ELS-S

Homework of Dynamic Stability
Airplane E (case 1- power approach)

Airplane E is a supersonic fighter bomber airplane. The objective of this homework is to study the stability modes of an aircraft which we knew his geometric, inertial, and aerodynamic data. The aircraft is of class IV. We will study the aircraft in flight phase category A, as it is the one that includes air-to-air combat, ground attack and inflight refueling.

I- Longitudinal Dynamic Stability of Airplane

1. The aircraft matrix A and control matrix B of aircraft in longitudinal motion

We build the aircraft matrix A and control matrix B of our aircraft in longitudinal motion using Matlab and the formulas bellow :

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta_0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 \\ M_u + Z_u M_{\dot{w}} & M_w + Z_w M_{\dot{w}} & M_q + U_0 M_{\dot{w}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} + Z_{\delta_e} M_{\dot{w}} & M_{\delta_T} + Z_{\delta_T} M_{\dot{w}} \\ 0 & 0 \end{bmatrix}$$

Knowing the equation of each parameter :

Equation	*X*	*Z*	*M*
u	$X_u = -\frac{\bar{q}S}{mU_0}(2C_{D_0} + C_{D_u})$	$Z_u = -\frac{\bar{q}S}{mU_0}(2C_{L_0} + C_{L_u})$	$M_u = \frac{\bar{q}S\bar{c}}{I_{yy}U_0}C_{m_u}$
w	$X_w = \frac{\bar{q}S}{mU_0}\left(C_{L_0}\left(1 - \frac{2}{\pi e A}C_{L_\alpha}\right)\right)$	$Z_w = -\frac{\bar{q}S}{mU_0}(C_{D_0} + C_{L_\alpha})$	$M_w = \frac{\bar{q}S\bar{c}}{I_{yy}U_0}C_{m_\alpha}$
$\dot{w}$	*0*	$Z_{\dot{w}} = \frac{\bar{q}S\bar{c}}{2mU_0^2}[C_{D_0} + C_{L_\alpha}]$	$M_{\dot{w}} = \frac{\bar{q}S\bar{c}^2}{2I_{yy}U_0^2}C_{m_{\dot{\alpha}}}$
q	*0*	$Z_q = \frac{\bar{q}S\bar{c}}{2mU_0}C_{L_q}$	$M_q = \frac{\bar{q}S\bar{c}^2}{2I_{yy}U_0}C_{m_q}$

$$X_{\delta_e} = \frac{\frac{1}{2}\rho U_0^2 S}{mU_0}C_{D_{\delta_e}} \qquad X_{\delta_T} = \frac{\frac{1}{2}\rho U_0^2 S}{mU_0}C_{D_{\delta_T}}$$

$$Z_{\delta_e} = \frac{\frac{1}{2}\rho U_0^2 S}{mU_0}C_{L_{\delta_e}} \qquad Z_{\delta_T} = \frac{\frac{1}{2}\rho U_0^2 S}{mU_0}C_{L_{\delta_T}}$$

$$M_{\delta_e} = \frac{\frac{1}{2}\rho U_0^2 S\bar{c}}{I_{yy}U_0}C_{m_{\delta_e}} \qquad M_{\delta_T} = \frac{\frac{1}{2}\rho U_0^2 S\bar{c}}{I_{yy}U_0}C_{m_{\delta_T}}$$

We do the calculations in Matlab and find :

$$A=\begin{bmatrix} -0.0562 & +0.0311 & 0 & -9.6064 \\ -0.2810 & -0.4215 & +70.100 & -1.9883 \\ +0.0006 & -0.0054 & -0.4663 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \; and \; B = \begin{bmatrix} -0.0197 & 0 \\ +0.0337 & 0 \\ -0.0209 & 0 \\ 0 & 0 \end{bmatrix}$$

2. The characteristic equation

To find the characteristic equation of the aircraft longitudinal matrix A, we simply use the function poly() of Matlab. It gives us the coefficient of each lambda : [1.0000 0.9440 0.6398 0.0311 0.0166]

$$Characteristic\ equation = \lambda^4 + 0.944 * \lambda^3 + 0.6398 * \lambda^2 + 0.0311 * \lambda^1 + 0.0166$$

3. The eigenvalues of the system

In Matlab, we can find the eigenvalues of the system by using the function eig(). By making the input a list of 2 variables, it will give us the eigenvectors and the eigenvalues, both in a [4,4] matrix.

$$\lambda_1 = -0.4675 + 0.6208\mathrm{i};\ \lambda_2 = -0.4675 - 0.6208\mathrm{i};$$

$$\lambda_3 = -0.0045 + 0.1656\mathrm{i};\ \lambda_4 = -0.0045 - 0.1656\mathrm{i}$$

4. Different modes of longitudinal stability

a. Short period mode

To find the short period mode natural frequency and damping factor we use the eigenvalues :

$$\lambda^2 + 2\xi_{sp}\omega_{n\,sp}\lambda + \omega_{n\,sp}^2 = (\lambda - \lambda_1) * (\lambda - \lambda_2)$$

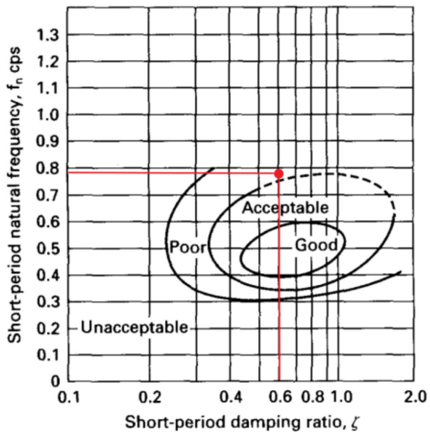
Using Matlab, we can first find $\omega_{n\,sp}^2$ by calculating the equation at $\lambda = 0$ and taking the square root of the result.

$$\omega_{n\,sp} = 0.77712\ rad/s$$

Then, we take the same equation, differentiate it, and find $2\xi_{sp}\omega_{n\,sp}$ by taking $\lambda = 0$ again. Finally isolate ξ_{sp}.

$$\xi_{sp} = 0.60161$$

The damping factor in short period mode being $\xi_{sp} = 0.6016$, with an aircraft in category A, it fits in level 1. We can evaluate the flying qualities by plotting the short period natural frequency against the short period damping ratio. We find that the flying qualities are poor, near acceptable. It is fine for a fighter airplane as we aim to have an agile aircraft rather than a stable one, and the trained pilot will keep it in air while being able to fight freely.



b. Phugoid mode

Same method using the equation :

$$\lambda^2 + 2\xi_{ph}\omega_{n\,ph}\lambda + \omega_{n\,ph}^2 = (\lambda - \lambda_3) * (\lambda - \lambda_4)$$

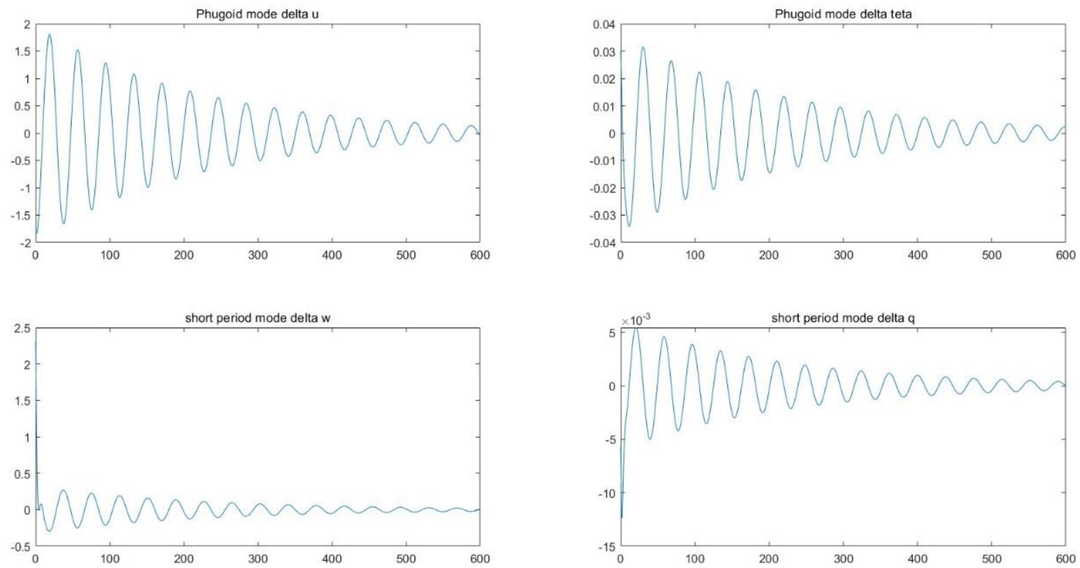
We find :

$$\omega_{n\,ph} = 0.16569\, rad/s\ \xi_{ph} = 0.02716$$

With a damping factor in phugoid mode $\xi_{ph} = 0.02716 > 0$, our aircraft is in level 2 flying qualities, meaning the flying qualities are adequate to the mission but with some increase in pilot workload.

5. Curves of longitudinal motion

We plot the axial velocity, the angle of attack, the pitch rate, and the pitch angle in function of time.



6. Transfer functions of each variable

The function mineral($(s * I_{4,4} - A)\backslash B$) helps us find the transfer function of each variable. And combining it with zpk() we can easily identify each parameter.

$$\frac{u(s)}{\delta_e(s)} = \frac{-0.01967 * (s - 2.518) * (s + 2.666) * (s + 0.6698)}{(s^2 + 0.008999s + 0.02745) * (s^2 + 0.935s + 0.6039)}$$

$$\frac{w(s)}{\delta_e(s)} = \frac{0.03372 * (s - 42.86) * (s^2 + 0.02731s + 0.03733)}{(s^2 + 0.008999s + 0.02745) * (s^2 + 0.935s + 0.6039)}$$

$$\frac{q(s)}{\delta_e(s)} = \frac{-0.020934s * (s + 0.3917) * (s + 0.09538)}{(s^2 + 0.008999s + 0.02745) * (s^2 + 0.935s + 0.6039)}$$

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-0.020934 * (s + 0.3917) * (s + 0.09538)}{(s^2 + 0.008999s + 0.02745) * (s^2 + 0.935s + 0.6039)}$$

II- Lateral Dynamic Stability of Airplane

1. The aircraft matrix A and control matrix B of aircraft in lateral motion

We build the aircraft matrix A and control matrix B of our aircraft in lateral motion using Matlab and the formulas bellow :

$$A_{Lateral} = \begin{bmatrix} Y_v & Y_p & -(U_0 - Y_r) & g\cos\theta_0 \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_{Lateral} = \begin{bmatrix} Y_{\delta_r} & Y_{\delta_a} \\ L_{\delta_r} & L_{\delta_a} \\ N_{\delta_r} & N_{\delta_a} \\ 0 & 0 \end{bmatrix}$$

Knowing the equation of each parameter :

Equation	*Y*	*L*	*N*
v	$Y_v = \frac{\bar{q}\,S}{mU_0} C_{Y_\beta}$	$L_v = \frac{\bar{q}\,S\,b}{I_{xx}U_0} C_{l_\beta}$	$N_v = \frac{\bar{q}\,S\,b}{I_{zz}U_0} C_{n_\beta}$
p	$Y_p = \frac{\bar{q}\,Sb}{2mU_0} C_{Y_p}$	$L_p = \frac{\bar{q}\,S\,b^2}{2I_{xx}U_0} C_{l_p}$	$N_p = \frac{\bar{q}\,S\,b^2}{2I_{zz}U_0} C_{n_p}$
r	$Y_r = \frac{\bar{q}\,Sb}{2mU_0} C_{Y_r}$	$L_r = \frac{\bar{q}\,S\,b^2}{2I_{xx}U_0} C_{l_r}$	$N_r = \frac{\bar{q}\,S\,b^2}{2I_{zz}U_0} C_{n_r}$

Rudder Derivatives

$$Y_{\delta_r} = \frac{\bar{q}S}{mU_0} C_{y_{\delta_r}}$$

$$L_{\delta_r} = \frac{\bar{q}Sb}{I_{xx}U_0} C_{l_{\delta_r}}$$

$$N_{\delta_r} = \frac{\bar{q}Sb}{I_{zz}U_0} C_{n_{\delta_r}}$$

Aileron Derivatives

$$Y_{\delta_a} = \frac{\bar{q}S}{mU_0} C_{y_{\delta_a}}$$

$$L_{\delta_a} = \frac{\bar{q}Sb}{I_{xx}U_0} C_{l_{\delta_a}}$$

$$N_{\delta_a} = \frac{\bar{q}Sb}{I_{zz}U_0} C_{n_{\delta_a}}$$

We do the calculations in Matlab and find :

$$A=\begin{bmatrix} -0.0920 & 0 & -70.100 & +9.6064 \\ -0.1212 & -1.2464 & +0.9394 & 0 \\ +0.0274 & +0.0106 & -0.2599 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \; and \; B = \begin{bmatrix} +0.0174 & -0.0050 \\ +0.0007 & +0.0443 \\ -0.0099 & -0.0006 \\ 0 & 0 \end{bmatrix}$$

2. The characteristic equation

To find the characteristic equation of the aircraft longitudinal matrix A, we simply use the function poly() of Matlab. It gives us the coefficient of each lambda : [1.0000 1.5984 2.3733 3.4971 0.0553]

$$Characteristic\ equation = \lambda^4 + 1.5984 * \lambda^3 + 2.3733 * \lambda^2 + 3.4971 * \lambda^1 + 0.0553$$

3. The eigenvalues of the system

In Matlab, we can find the eigenvalues of the system by using the function eig(). By making the input a list of 2 variables, it will give us the eigenvectors and the eigenvalues, both in a [4,4] matrix.

$$\lambda_1 = -0.0273 + 1.5046\mathrm{i};\ \lambda_2 = -0.0273 - 1.5046\mathrm{i};$$

$$\lambda_3 = -1.5277 + 0\mathrm{i};\ \lambda_4 = -0.0160 - 0\mathrm{i}$$

4. Different modes of lateral stability

a. Spiral mode

The root of spiral mode is $\lambda_4 = -0.0160 = \lambda_{spiral}$.

The time to double is $\frac{\ln(2)}{|\lambda_{spiral}|} = 43.32 sec$.

In spiral mode, the minimum time to double amplitude flying quality requirement is of level 1 as it is greater than 12 seconds.

b. Rolling mode

The root of rolling mode is $\lambda_3 = -1.5277 = \lambda_{roll}$.

The roll time constant is $\frac{1}{|\lambda_{roll}|} = 0.65\ sec$.

In rolling mode, the maximum roll time constant flying quality requirement is of level 1 as it is lower than 1 second.

c. Dutch roll mode

We can find by approximation the Dutch roll mode natural frequency and damping factor :

$$\omega_{n\,dr} = \sqrt{\lambda_1 * \lambda_2} = 1.5048\, rad/s$$

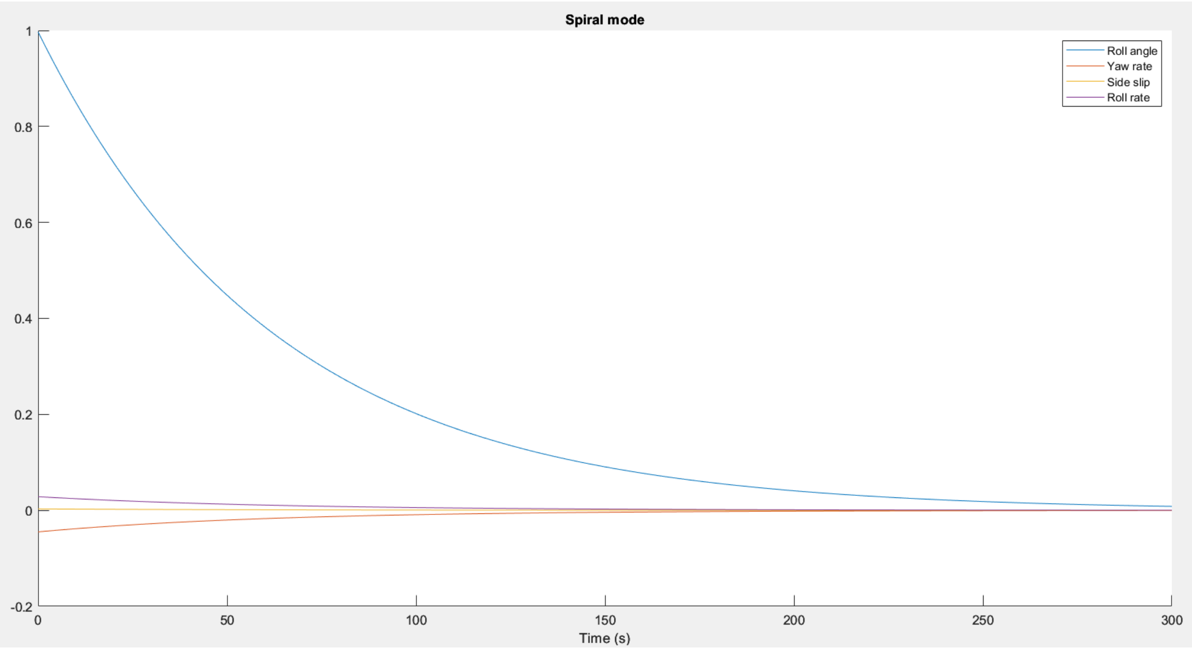
$$\xi_{dr} = \frac{|\lambda_1 + \lambda_2|}{2 * \omega_{n\,dr}} = 0.0182$$

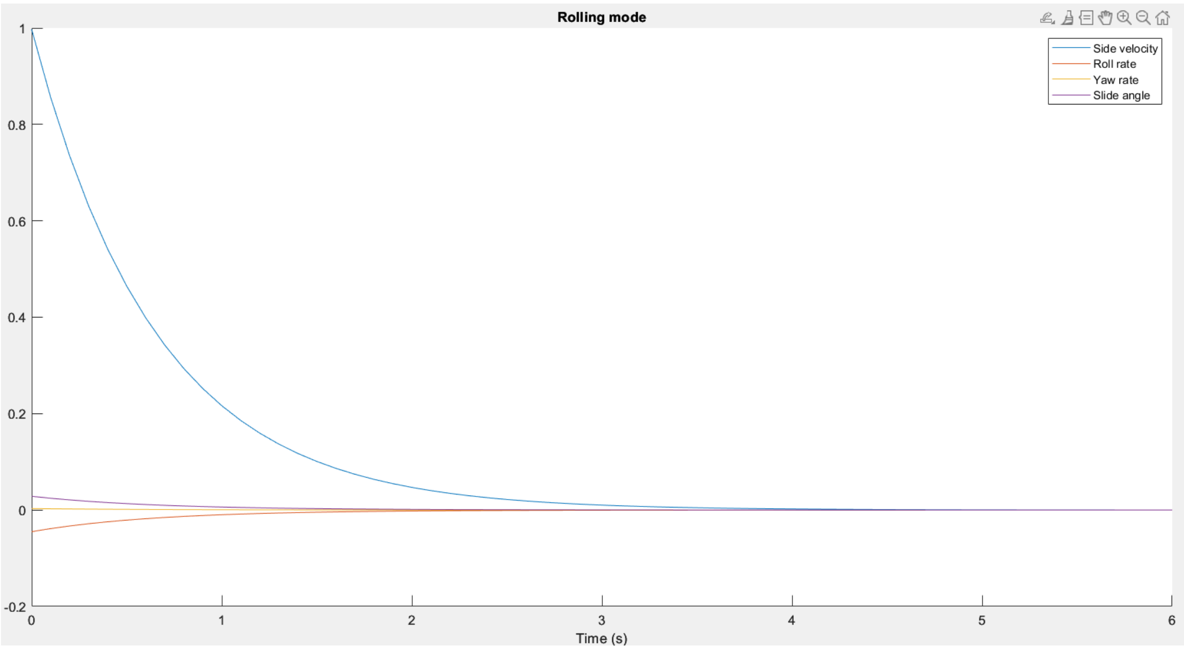
And the product $\omega_{n\,dr} * \xi_{dr} = 0.0274\, rad/s$.

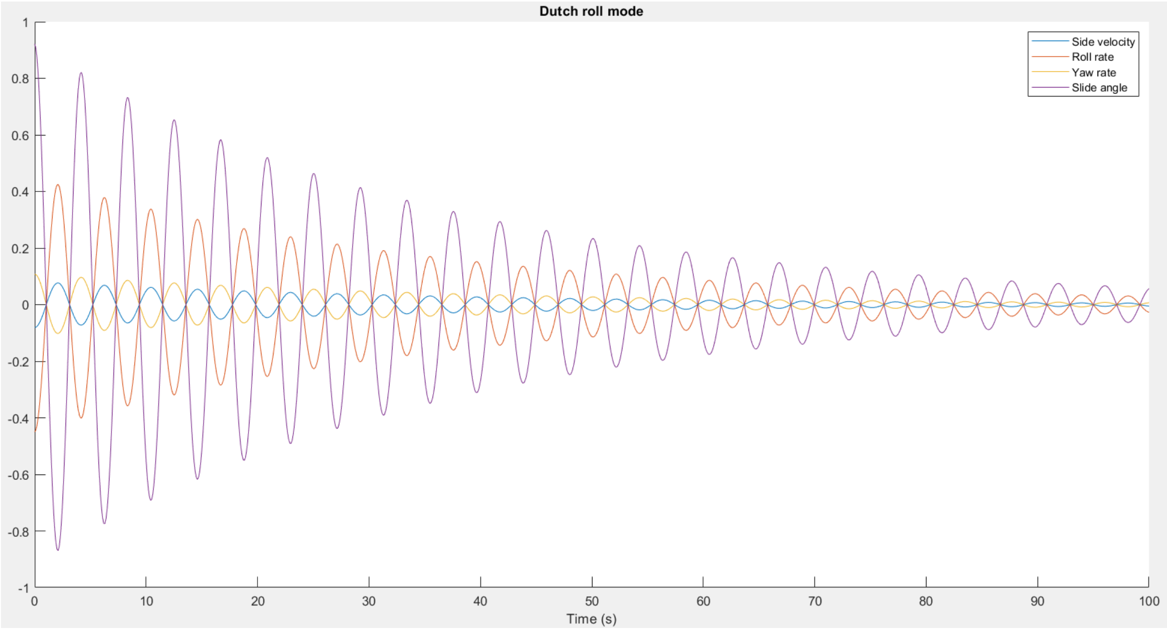
Considering all these parameters, the Dutch roll flying quality requirement is of level 1.

5. Curves of lateral motion

We plot the curves of lateral motion :







6. Transfer Functions of Each variable

In the same manner as with the longitudinal parameters, we find the transfer equations :

$$\frac{v(s)}{\delta_a(s)} = \frac{0.017422 * (s + 40.13) * (s + 1.351) * (s - 0.09283)}{(s + 1.528) * (s + 0.01599) * (s^2 + 0.05467s + 0.2.264)}$$

$$\frac{p(s)}{\delta_a(s)} = \frac{0.00069903 * s * (s - 21.55) * (s + 5.562)}{(s + 1.528) * (s + 0.01599) * (s^2 + 0.05467s + 0.2.264)}$$

$$\frac{r(s)}{\delta_a(s)} = \frac{-0.0099128 * (s + 1.667) * (s^2 - 0.3779s + 0.6869)}{(s + 1.528) * (s + 0.01599) * (s^2 + 0.05467s + 0.2.264)}$$

$$\frac{\varphi(s)}{\delta_a(s)} = \frac{0.00069903 * (s - 21.55) * (s + 5.562)}{(s + 1.528) * (s + 0.01599) * (s^2 + 0.05467s + 0.2.264)}$$