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Constrained model predictive control for T–S fuzzy system with randomly occurring actuator saturation and packet losses via PDC and non-PDC strategies

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Abstract

This paper studies model predictive control for a Takagi–Sugeno (T–S) fuzzy system with randomly occurring actuator saturation and packet losses. The nonlinearity of the actuator saturation is transformed into a set of convex hulls, while the packet losses are assumed to obey the rules of Bernoulli distribution. Both parallel-distributed-compensation (PDC) and non-parallel-distributed-compensation (non-PDC) strategies are adopted to design the controller for the system. In addition, sufficient conditions of the stability for the closed-loop system are given in terms of linear matrix inequalities. It is shown that the non-PDC strategy behaves less conservatively than the PDC strategy in controlling the considered T–S fuzzy system, when the input and output constraints are explicitly considered. Two simulation examples are provided to illustrate the effectiveness of the proposed design techniques.

Keywords

Actuator saturation, packet losses, model predictive control, fuzzy system

Introduction

Over the past few decades, model predictive control, which features in solutions to constrained control problems, has been intensively studied. As one of the most popular advanced control theories, model predictive control predicts future system behaviour and selects optimal control sequences based on an explicit model and an objective function (Cole et al., 2014; Corbin et al., 2013; Wu, 2015). So far, model predictive control technology has been utilized in many application areas, such as processing industries, where plants are sufficiently slow to implement operations (Chai et al., 2013; Forbes et al., 2015; O'Brien et al., 2011). Moreover, to eliminate geographical restrictions and meet the increasingly demanding requirements in complex control industries, model predictive control technology can now be commonly found in network control systems.

Network control systems have attracted great attention in both practical applications and theoretical research in the past decades, since they have huge advantages over traditional point-to-point control systems, such as simplification of maintenance and installation or reduction in the cost of the cables and power (Heemels et al., 2010; Wang and Lemmon MD, 2011; Yue et al., 2013).

However, there still exist new challenges, owing to unreliable networked transmissions (Chen et al., 2010; Donkers et al., 2011; Heemels et al., 2013). New approaches for network control systems should be proposed to eliminate poor performance caused by the undesired networked environment, for instance, packet losses (Ding, 2011; Xu et al., 2012; Zhang and Tian, 2010), time delay (Garcia and Antsaklis, 2013; Song et al., 2011) and quantized signals (Coutinho et al., 2010; Liu et al., 2015; Yang et al., 2011). Packet losses, as the main issue to be discussed in this paper, have turned out to be a hot topic, with unsolved challenges in the network control system. Xiong and Lam (2007) addressed the stabilization problem of network control systems with bounded packet losses and Markov packet losses, and established stability conditions for both types of packet loss. By modelling the packet losses as a Bernoulli process, Sahebsara et al. (2007) presented a control strategy for a discrete-time network control system, considering the problem of optimal filtering for discrete-time systems with random sensor delay, multiple packet losses and uncertain observation. Zhang et al.

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(2012) considered the estimation problem over networks with packet losses. The estimator was presented by solving a deterministic Riccati equation, which shows that it has a smaller error covariance and wider applications.

Some interesting results were presented in the aforementioned documents. However, all of the results were obtained under the assumption that the controlled plants were linear systems. However, most systems are complex and nonlinear in actual industrial production; hence, more attention should be paid to the design of nonlinear networked controllers and it is urgently necessary to study the relevant results further. In the field of nonlinear control systems, fuzzy control has aroused great attention, especially for the T–S fuzzy model, which is an effective solution to narrow the gap between the fruitful linear control systems and complex nonlinear fuzzy control ones (Bouyahya et al., 2013; Lu and Shih, 2010; Ren et al., 2014).

Saturation is one of the most common factors to deteriorate system performance in engineering (Mahjoub et al., 2014;Wen et al., 2011; Zhao et al., 2016, Song et al., 2014); owing to the limitation of the communication bandwidth, large information-carrying packets might be randomly saturated or lost while being transmitted, with only a small number of packets being transmitted successfully to the actuator. The problem of saturation and packet loss at the actuator node are highly likely to occur. Therefore, it is essential to take both issues into account. Most recently, we are delighted to see that there are nice works addressing packet loss and the input saturation problem of robust stabilization for T–S fuzzy discrete systems (Kaleybar and Esfanjani, 2014; Zhao and Li, 2015; Zhou and Zhang, 2012). Kaleybar and Esfanjani (2014) investigated a linear network control system subject to input saturation and packet losses. Two methods have been presented to synthesize a stabilizing controller for the considered control system. The output feedback control problem was studied for networked discrete-time systems with actuator saturation and packet losses by Zhou and Zhang (2012), who proposed an output feedback controller that is dependent on both saturation and packet loss. Zhao and Li (2015) studied the problem of robust stabilization of T–S fuzzy discrete systems with actuator saturation. By using a parameter-dependent Lyapunov function, both parallel-distributed-compensation (PDC) and non-PDC strategies were designed. All these works gave us good ideas.

However, unlike these works, we would like to emphasize that the contribution of this paper is not just a simple comparison between the PDC and the non-PDC strategy. In fact, compared with the existing literature, the main contribution of this paper is that we consider the synthesis approach of model predictive control for network control systems with randomly occurring actuator saturation and packet losses. For model predictive control, the synthesis approach means that the closed-loop system is stable once the optimization problem is feasible at the initial time. Although there are many nice papers considering the design of model predictive control for network control systems, only a limited number of works have been found on the synthesis approach of model predictive control. The main difficulty in extending the synthesis approach of model predictive control to the networked environment is guaranteeing closed-loop stability. For non-networked model predictive control, closed-loop stability can be guaranteed by imposing appropriate constraints on the optimization problem. However, when communication networks are taken into consideration, these constraints do not maintain the desired closed-loop stability. Hence, the feature of this paper is that we provide a new solution to the synthesis approach of model predictive control for network control systems, which has not, to our knowledge, yet appeared in the literature.

Notation

I is the identity matrix, with appropriate dimensions. For any vector x and matrix W, $\| x \|_W^2 = x^{\mathrm{T}} W x$. $x(k+i|k)$ is the value of vector x at a future time $k+i$, predicted at time k. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x. The symbol $(*)$ induces a symmetrical structure in the linear matrix inequalities. $\lambda_{\min(\cdot)}$ denotes the minimal eigenvalue of the matrix '$\cdot$'.

Problem statement

Fuzzy model

The structure of a fuzzy network control system with randomly occurring actuator saturation and packet losses is presented in Figure 1.

The fuzzy system is supposed to be represented by the following discrete-time nonlinear model, in which the jth rule is provided as follows.

Plant rule j. If $\theta_1(k)$ is M_{j1}, $\theta_2(k)$ is M_{j2} and $\cdots$ and $\theta_q(k)$ is M_{jq} then

$$x_j(k+1) = A_j x(k) + B_j u(k) \tag{1}$$

$$y_j(k) = C_j x(k) \tag{2}$$

where $\theta_1(k), \theta_2(k), \ldots, \theta_q(k)$ are the premise variables, M_{jl} are fuzzy sets, $j = 1, 2, \ldots, r; l = 1, 2, \ldots, q$; $x(k) \in R^n$ is the state variable, $u(k) \in R^q$ is the control input, $y(k) \in R^p$ is the output of the system, $A_j, B_j, C_j, j \in [1, r]$ are constant matrices and r is the number of if-then rules.

By utilizing fuzzy blending, the model of the fuzzy system can be expressed as

$$x(k+1) = A_z x(k) + B_z u(k) \tag{3}$$

$$y(k) = C_z x(k) \tag{4}$$

where

$$A_z = \sum_{j=1}^{r} f_j(\theta(k)) A_j$$

$$B_z = \sum_{j=1}^{r} f_j(\theta(k)) B_j$$

$$C_z = \sum_{j=1}^{r} f_j(\theta(k)) C_j$$

$$f_j(\theta(k)) = \omega_j(\theta(k)) / \sum_{j=1}^{r} \omega_j(\theta(k))$$

$$\omega_j(\theta(k)) = \prod_{l=1}^{q} M_{jl}(\theta_l(k))$$

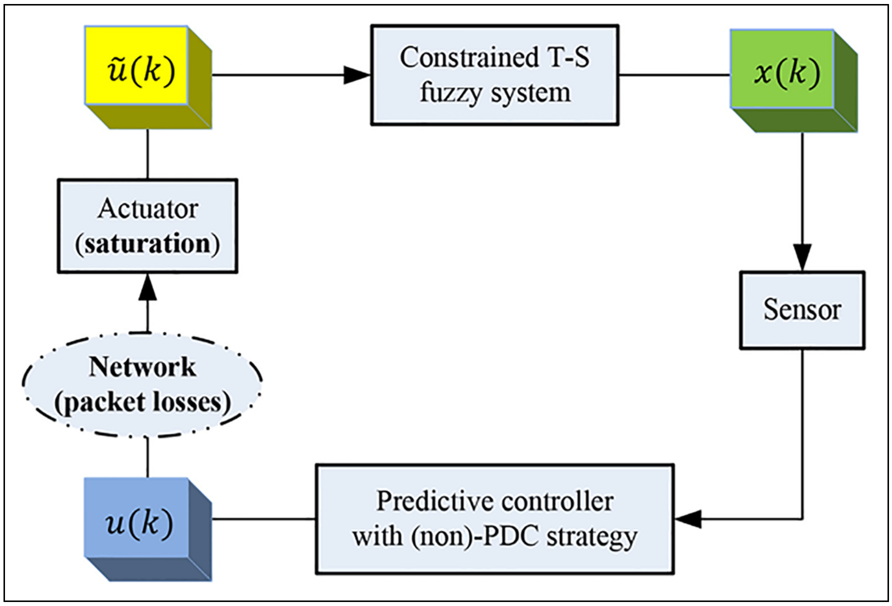

Figure 1. Structure of the fuzzy network control system.
PDC: parallel distributed compensation.

where $M_{jl}(\theta_l(k))$ is the membership degree of $\theta_l(k)$ in M_{jl}. It should be noted that $\omega_j(\theta(k)) > 0$ and $\sum_{j=1}^{r} f_j(\theta(k)) = 1$. For the sake of convenience, $f_j(\theta(k))$ is written as f_j.

Actuator saturation and packet losses. Next, consider the fuzzy system with saturated input

$$x(k+1) = A_z x(k) + B_z \mathrm{sat}\{u(k)\} \tag{5}$$

where $\mathrm{sat}\{u(k)\}$ is a standard saturation function of appropriate dimensions, defined as

$$\begin{aligned}\mathrm{sat}\{u(k)\} &= \mathrm{sgn}(u(k)) \min\{1, \| u(k) \|\} \\ &= [\mathrm{sat}\{u_1(k)\}, \mathrm{sat}\{u_2(k)\}, \cdots, \mathrm{sat}\{u_q(k)\}]^{\mathrm{T}}\end{aligned}$$

Before proceeding, the following definitions and lemmas are provided to deal with the main issues of the random actuator saturation and Bernoulli packet losses in later sections.

Definition 1. An ellipsoid set: $\Omega(P, \gamma) = \{x(k) \in R^n : x(k)^{\mathrm{T}} P x(k) \leq \gamma\}$, where $P \in R^{n \times n} > 0$ is a positive symmetric matrix and γ is a positive scalar. If there exists inequality $\Delta V(x(k)) < 0$, $V(x(k)) = x(k)^{\mathrm{T}} P x(k)$, for all $x(k) \in \Omega(P, \gamma)$, then the ellipsoid $\Omega(P, \gamma)$ is said to be a contractive invariant.

Definition 2. A set: $\mathbb{L}(H) = \{x(k) \in R^n : |h_t x(k)| \leq 1\}$, where h_t is the tth row of the matrix H, $t = 1, 2, \ldots, q$.

Lemma 1 (Zhao and Li, 2015). *For the positive weight matrix $P \in R^{n \times n} > 0$, the positive scalar $\gamma > 0$, an ellipsoid set $\Omega(P, \gamma) = \{x(k) \in R^n : x(k)^{\mathrm{T}} P x(k) \leq \gamma\}$ is inside $\mathbb{L}(H)$ if and only if $h_t P h_t^{\mathrm{T}} \leq \gamma$.*

Lemma 2 (Zhao and Li, 2015). *Let $F \in R^{q \times n}$ and $H \in R^{q \times n}$ be given. If $|h_t x| \leq 1$, for all $t = 1, 2, \ldots, q$, then $\mathrm{sat}\{u(k)\}$ can be expressed as*

$$\mathrm{sat}\{u(k)\} = \mathrm{sat}\{Fx(k)\} = \sum_{s=1}^{2^m} \eta_s(k)(E_s F + E_s^- H) x(k) \tag{6}$$

$$\sum_{s=1}^{2^m} \eta_s(k) = 1, \qquad 0 \leq \eta_s(k) \leq 1, \qquad s = 1, 2, \ldots, 2^m \tag{7}$$

where F is the feedback gain to be designed. Let $\mathbb{M}$ be the $m \times m$ diagonal matrix whose diagonal elements are either 1 or 0. Suppose that each element of $\mathbb{M}$ is labelled $E_s, s = 1, 2, \ldots, 2^m$. Denote $E_s^- = I - E_s$; obviously E_s^- is also an element of $\mathbb{M}$ if $E_s \in \mathbb{M}$. Therefore, the saturated feedback $\mathrm{sat}\{Fx(k)\}$ can be transformed into a set of linear ones.

Owing to the limitation of the communication bandwidth, the large information-carrying packets might be randomly saturated or lost while being transmitted; only a small number of packets can be successfully transmitted to the actuator. The problem of actuator saturation and packet losses is highly likely to appear. Therefore, it is essential to take both issues into account. By considering Bernoulli packet losses, the control input $\tilde{u}(k)$ can be described as

$$\begin{aligned}\tilde{u}(k) &= \alpha(k)\mathrm{sat}\{u(k)\} + (1 - \alpha(k))\beta(k)u(k) \\ u(k) &= Fx(k)\end{aligned} \tag{8}$$

To analyse the control system conveniently, we introduce the random variables $\alpha(k)$ and $\beta(k)$; thus, the different cases of whether there are packet losses or actuator saturation at every sampling time can be presented in one equation, where $\alpha(k), \beta(k)$ are Bernoulli white sequences taking values of 0 or 1 and the probabilities are defined as

$$\mathrm{prob}\{\alpha(k) = 1\} = \mu, \qquad \mathrm{prob}\{\alpha(k) = 0\} = 1 - \mu \tag{9}$$

$$\text{prob}\{\beta(k)=1\}=\nu, \qquad \text{prob}\{\beta(k)=0\}=1-\nu \tag{10}$$

where $\mu, \nu \in [0,1]$ are known constants and the stochastic variables $\alpha(k), \beta(k)$ are both mutually independent.

Remark 1. It is shown that there are three possible cases in equation (8):

1. If $\alpha(k)=1$, then $\widetilde{u}(k)=\text{sat}\{u(k)\}$. This means that the actuator is subjected to saturation.
2. If $\alpha(k)=0$, $\beta(k)=1$, then $\widetilde{u}(k)=u(k)$. It can be seen that the actuator works normally.
3. If $\alpha(k)=0$, $\beta(k)=0$, then $\widetilde{u}(k)=0$. This indicates that the data packet transmitted to actuator is lost.

Finally, considering both the random actuator saturation and Bernoulli packet losses, the system model can be presented as

$$\begin{aligned} x(k+1) &= A_z x(k) + B_z \widetilde{u}(k) \\ &= A_z x(k) + B_z[\alpha(k)\text{sat}\{u(k)\} + (1-\alpha(k))\beta(k)u(k)] \end{aligned} \tag{11}$$

$$y(k) = C_z x(k), \qquad k > 0 \tag{12}$$

Remark 2. The aim of this paper is to find a proper state-feedback law (equation (8)) for the fuzzy system with random actuator saturation and Bernoulli packet losses.

Model predictive control optimization

Let us consider the formulated model predictive control problem

$$\begin{gathered} \min_{F(k|k)} \max_{\Omega} J^{\infty}(k) \\ J^{\infty}(k) = \mathbb{E}\left\{\sum_{i=0}^{\infty}[X(k+i)+U(k+i)]\right\} \\ X(k+i) = x(k+i|k)^{\mathrm{T}} Q_0 x(k+i|k) \\ U(k+i) = \widetilde{u}(k+i|k)^{\mathrm{T}} R_0 \widetilde{u}(k+i|k) \end{gathered} \tag{13}$$

where $J^{\infty}(k)$ is the worst-case quadratic objective function, Q_0 and R_0 are two known weighing matrices, $A_j, B_j, C_j, \in \Omega, j=1,2,\ldots,r$.

The input and output constraints are

$$|\widetilde{u}_c(k+i|k)| \leqslant \overline{u}_{c,\max}, i \geqslant 0, c=1,2,\ldots,q \tag{14}$$

$$|y_c(k+i|k)| \leqslant \overline{y}_{c,\max}, i \geqslant 1, c=1,2,\ldots,p \tag{15}$$

where $u(k+i|k), y(k+i|k)$ are the state and output, respectively, of the system at time $k+i$, and are predicted based on the measurements at time k; $\widetilde{u}(k+i|k)$ is the control move with random actuator saturation and Bernoulli packet losses.

By taking equations (11) and (12) as the predictive model, we can obtain the predictive states at the future sampling time as

$$\begin{aligned} x(k+1+i|k) &= A_z x(k+i|k) + B_z \widetilde{u}(k+i|k) \\ &= \sum_{s=1}^{2^m} \eta_s(k)\{[A_z + \alpha(k+i|k)B_z[E_s F + E_s^- H] \\ &\quad + (1-\alpha(k+i|k))\beta(k+i|k)B_z F]\}x(k+i|k) \end{aligned} \tag{16}$$

$$y(k+i|k) = C_z x(k+i|k), \quad i \geqslant 0 \tag{17}$$

To derive an upper bound of $J^{\infty}(k)$ and solve equations (13), (14) and (15), let us define the Lyapunov function as

$$\begin{gathered} V(x(k+i|k)) = x(k+i|k)^{\mathrm{T}} S_{zz}^{-1} x(k+i|k), \qquad i \geqslant 0 \\ S_{zz}^{-1} = \sum_{f=1}^{r}\sum_{g=1}^{r} f_j f_g S_{fg}^{-1}, \quad S_{fg}^{-1} \in R^{n\times n}, \quad S_{fg}^{-1} > 0 \end{gathered} \tag{18}$$

Furthermore, $V(x(k+i|k))$ should satisfy the following stability condition at each time k

$$\begin{aligned} \mathbb{E}\{\Delta V(x(k+i|k)\} \leqslant &-\mathbb{E}\{x(k+i|k)^{\mathrm{T}} Q_0 x(k+i|k) \\ &+ \widetilde{u}(k+i|k)^{\mathrm{T}} R_0 \widetilde{u}(k+i|k)\} \end{aligned} \tag{19}$$

where

$$\begin{aligned} \Delta V &= V(x(k+i+1|k)) - V(x(k+i|k)) \\ &= \| x(k+i+1|k) \|^2_{S_{zz}^{-1}} - \| x(k+i|k) \|^2_{S_{zz}^{-1}} \\ &= \sum_{s=1}^{2^m} \eta_s(k)[\| \Phi_1 x(k+i|k) \|^2_{S_{zz}^{-1}} - \| x(k+i|k) \|^2_{S_{zz}^{-1}}] \\ \Phi_1 &= A_z + \alpha(k)B_z[E_s F + E_s^- H] + (1-\alpha(k))\beta(k)B_z F \end{aligned} \tag{20}$$

Summing equation (19) from $i=0$ to ∞ and requiring $\lim_{i\to\infty} V(x(k+i|k))=0$, it follows that $J^{\infty}(k) \leqslant V(x(k|k)) = x(k)^{\mathrm{T}} S_{zz}^{-1} x(k)$.

Let us define $V(x(k|k)) \leqslant \gamma$; where γ is a positive scalar, we have

$$J^{\infty}(k) \leqslant V(x(k|k)) \leqslant \gamma \tag{21}$$

Thus the min-max optimization problem for the proposed model predictive control can be developed as equation (21), which gives an upper bound on $J^{\infty}(k)$.

Main results

Non-parallel distributed compensation

In this section, the non-PDC strategy will be utilized for the fuzzy model predictive control problem. For the considered fuzzy system (equation (16)), the non-PDC state-feedback fuzzy control strategy is given as

$$\begin{aligned} u(k) &= -\left(\sum_{j=1}^{r} f_j(\theta(k))Y_j\right)\left(\sum_{j=1}^{r} f_j(\theta(k))G_j\right)^{-1} x(k) \\ &= -Y_z G_z^{-1} x(k) \end{aligned} \tag{22}$$

By defining

$$Y_z = \sum_{j=1}^{r} f_j(\theta(k))Y_j$$
$$G_z = \sum_{j=1}^{r} f_j(\theta(k))G_j \tag{23}$$

and considering equation (8), we can obtain the closed-loop system

$$\begin{aligned} x(k+i+1|k) &= A_z x(k+i|k) + B_z \widetilde{u}(k+i|k) \\ &= \sum_{s=1}^{2^m} \eta_s(k)\{A_z + \alpha(k)B_z[-E_s Y_z G_z^{-1} + E_s^- H] \\ &- (1-\alpha(k))\beta(k)B_z Y_z G_z^{-1}\}x(k+i|k), \qquad i \geqslant 0 \end{aligned} \tag{24}$$

Theorem 1. *Considering the closed-loop fuzzy system (equation (24)) at each time k, assume $x(k|k)$ is the measured state $x(k)$, if there exist symmetric positive matrices S_{jg}, S_{cd}, G_j, positive matrices Y_j, D_{tj} satisfying the following linear matrix inequality optimization problem*

$$\min_{S_{jg}, S_{cd}, G_j, Y_j, D_{tj}} \gamma \tag{25}$$

$$\begin{bmatrix} G_j^{\mathrm{T}} + G_j - \widehat{S}_{jg} & * \\ D_{tj} & I \end{bmatrix} \geqslant 0, \quad \text{with} \quad j, g = 1, 2, \ldots, r, \quad t = 1, 2, \ldots, q \tag{26}$$

$$\begin{bmatrix} \gamma & * \\ x(k) & S_{jg} \end{bmatrix} \geqslant 0, \quad \text{with} \quad f, g = 1, 2, \ldots, r \tag{27}$$

$$\begin{bmatrix} G_j^{\mathrm{T}} + G_j - S_{jg} & * & * & * \\ \Psi_1 G_j & S_{cd} & * & * \\ \Psi_2 G_j R_0 & 0 & R_0 & * \\ G_j Q_0 & 0 & 0 & Q_0 \end{bmatrix} \geqslant 0 \quad \text{with} \quad j, g, c, d = 1, 2, \ldots, r, s = 1, 2, \ldots, 2^m \tag{28}$$

where

$$\Psi_1 = A_j + B_j\{\mu[-E_s Y_j G_j^{-1} + E_s^- H] - (1-\mu)\nu Y_j G_j^{-1}\}$$
$$\Psi_2 = \mu[-E_s Y_j G_j^{-1} + E_s^- H] - (1-\mu)\nu Y_j G_j^{-1}$$
$$S_{zz} = \sum_{j=1}^{r}\sum_{g=1}^{r} f_j f_g S_{jg} = \sum_{c=1}^{r}\sum_{d=1}^{r} f_c f_d S_{cd}$$
$$D_{tj} = h_t G_j, \qquad \widehat{S}_{jg} = (1/\gamma)S_{jg}$$

then the Lyapunov function can be taken as $V(x(k|k)) = x(k)^{\mathrm{T}} S_{zz}^{-1} x(k)$, $u(k|k) = Fx(k|k) = -Y_z G_z{}^{-1} x(k|k)$, such that the upper bound γ of the closed-loop fuzzy system is minimized with performance objective function at each sampling time k.

Proof. As a prerequisite for utilizing Lemma 2 in equation (24), the constraint $|h_t x(k)| \leqslant 1$ must be satisfied. Since $|h_t x(k)| \leqslant 1$, there is $|h_t x(k)|^2 \leqslant 1$, based on equations (18) and (21); then we can get $x(k)^{\mathrm{T}} S_{zz}^{-1} x(k) \leqslant \gamma$. It is supposed that $(h_t x(k))^{\mathrm{T}} h_t x(k) \leqslant x(k)^{\mathrm{T}} \widehat{S}_{zz}^{-1} x(k) \leqslant 1$, $\widehat{S}_{zz}^{-1} = (1/\gamma) S_{zz}^{-1}$, which can be transformed into

$$\widehat{S}_{zz}^{-1} - h_t^{\mathrm{T}} h_t \geqslant 0, \qquad t = 1, 2, \ldots, q \tag{29}$$

Pre- and post-multiply G_z^{T} and G_z on equation (29), since

$$G_z = \sum_{j=1}^{r} f_j G_j, \qquad S_{zz}^{-1} = \sum_{j=1}^{r}\sum_{g=1}^{r} f_j f_g S_{jg}$$

Then we can get

$$\begin{aligned} &G_z^{\mathrm{T}} \widehat{S}_{zz}^{-1} G_z - (h_t G_z)^{\mathrm{T}} h_t G_z \geqslant 0 \\ &\sum_{j=1}^{r}\sum_{g=1}^{r} f_j f_g \{G_j^{\mathrm{T}} \widehat{S}_{jg}^{-1} G_j - (h_t G_j)^{\mathrm{T}} h_t G_j\} \geqslant 0 \end{aligned} \tag{30}$$

It should be noticed that

$$G_j^{\mathrm{T}} S_{jg}^{-1} G_j \geqslant G_j^{\mathrm{T}} + G_j - S_{jg} \tag{31}$$

then based on the Schur's complement, equation (26) can be obtained.

Based on Schur's complement, equation (21), which is equivalent to $\sum_{j=1}^{r}\sum_{g=1}^{r} f_j f_g \times \{x(k)^{\mathrm{T}} S_{jg}^{-1} x(k)\} \leqslant \gamma$ can be guaranteed by

$$\begin{bmatrix} \gamma & * \\ x(k) & S_{jg} \end{bmatrix} \geqslant 0, \qquad j, g = 1, 2, \ldots, r$$

The stability of the presented system is guaranteed by equation (19); by computing the mathematical expectation of both sides, it can be rewritten as

$$\begin{aligned} &\sum_{s=1}^{2^m} \eta_s(k)\Big\{\| \big[A_z + B_z\{\mu[-E_s Y_z G_z^{-1} + E_s^- H] - (1-\mu)\nu Y_z G_z^{-1}\} \|_{S_{zz}^{-1}}^2 \Big\} \\ &- S_{zz}^{-1} \leqslant -Q_0 - \sum_{s=1}^{2^m} \eta_s(k) \\ &\Big\{\| [\mu[-E_s Y_z G_z^{-1} + E_s^- H] - (1-\mu)\nu Y_z G_z^{-1} \|_{R_0}^2 \Big\} \end{aligned} \tag{32}$$

After pre- and post-multiplying by G_z^{T} and G_z, equation (32) becomes

$$\begin{aligned} &\sum_{s=1}^{2^m} \eta_s(k)\{\| [A_z G_z + B_z\{\mu[-E_s Y_z + E_s^- H G_z] - (1-\mu)\nu Y_z\} \|_{S_{zz}^{-1}}^2\} \\ &- G_z^{\mathrm{T}} S_{zz}^{-1} G_z \leqslant -G_z^{\mathrm{T}} Q_0 G_z - \sum_{s=1}^{2^m} \eta_s(k)\{\| [\mu[-E_s Y_z + E_s^- H G_z] \\ &- (1-\mu)\nu Y_z \|_{R_0}^2\} \end{aligned} \tag{33}$$

therefore equation (33) can be unfolded as

$$\begin{aligned} &\sum_{s=1}^{2^m}\sum_{j=1}^{r}\sum_{g=1}^{r}\sum_{c=1}^{r}\sum_{d=1}^{r} \eta_s(k) f_j f_g f_c f_d \\ &\times \{\| G_j \|_{S_{jg}^{-1}}^2 - \| \Psi_1 G_j \|_{S_{cd}^{-1}}^2 - \| \Psi_2 G_j \|_{R_0}^2 - \| G_j \|_{Q_0}^2\} \geqslant 0 \end{aligned} \tag{34}$$

Thus, based on equation (31) and the Schur's complement, equation (34) can be transformed into the following linear matrix inequalities

$$\begin{bmatrix} G_j^{\mathrm{T}} + G_j - S_{jg} & * & * & * \\ \Psi_1 G_j & S_{cd} & * & * \\ \Psi_2 G_j R_0 & 0 & R_0 & * \\ G_j Q_0 & 0 & 0 & Q_0 \end{bmatrix} \geqslant 0,$$

$$j, g, c, d = 1, 2, \ldots, r; s = 1, 2, \ldots, 2^m \tag{35}$$

That completes the proof.

Constraints handling

Theorem 2. *Suppose there exist symmetric positive matrix $\widehat{S}_{jg}$ and positive matrices Y_j, G_j, H satisfying the following optimization problem*

$$\begin{bmatrix} G_j^{\mathrm{T}} + G_j - \widehat{S}_{jg} & * \\ \Psi_2 G_j & Z \end{bmatrix} > 0 \tag{36}$$

with $Z_{cc} < \overline{u}_{c.\max}^2, j, g = 1, 2, \ldots, r; c = 1, 2, \ldots, q;$

$$\begin{bmatrix} G_j^{\mathrm{T}} + G_j - \widehat{S}_{jg} & * \\ C_j \Psi_1 G_j & M \end{bmatrix} > 0 \tag{37}$$

with $M_{cc} < \overline{y}_{c.\max}^2$, $j, g = 1, 2, \ldots, r; c = 1, 2, \ldots, p$, such that the constraints of $\widetilde{u}(k+i|k)$ and $y(k+i|k)$ are satisfied.

Proof. The constrained input $\widetilde{u}(k)$ satisfies $\mathbb{E}\{\widetilde{u}(k+i|k)\} \leqslant \overline{u}_{c.\max}$; based on equation (24), the state constraint can be rewritten as

$$\mathbb{E}\left\{\sum_{s=1}^{2^m} \eta_s(k)\{\alpha(k)[-E_s Y_z G_z^{-1} + E_s^- H] - (1-\alpha(k))\beta(k) Y_z G_z^{-1}\} x(k+i|k)\right\} \leqslant \overline{u}_{c.\max} \tag{38}$$

Based on equations (9) and (10), obviously, equation (38) can be rewritten as

$$\max_{i \geqslant 0} \mathbb{E}\{|\widetilde{u}_c(k+i|k)|^2\} \tag{39}$$

$$= \sum_{s=1}^{2^m} \eta_s(k) \Big\{ \big[\mu(-E_s Y_z G_z^{-1} + E_s^- H) - (1-\mu)\nu Y_z G_z^{-1}\big] \widehat{S}_{zz}^{1/2} \Big[\big[(\mu(-E_s Y_z G_z^{-1} + E_s^- H) - (1-\mu \times \nu Y_z G_z^{-1}))\widehat{S}_{zz}^{1/2}\big]^T\Big]_{cc} \Big\} x(k+i|k)^T \widehat{S}_{zz}^{-1} x(k+i|k) \tag{40}$$

Since $x(k+i|k)^{\mathrm{T}} \widehat{S}_{zz}^{-1} x(k+i|k) \leqslant 1$, equation (39) can be rewritten as

$$\mathbb{E}\left\{\max_{i \geqslant 0} |\widetilde{u}_c(k+i|k)|^2\right\} \leqslant \sum_{s=1}^{2^m} \eta_s(k) [\Psi_2 \widehat{S}_{zz}^{1/2} (\Psi_2 \widehat{S}_{zz}^{1/2})^{\mathrm{T}}]_{cc} \tag{41}$$

After pre- and post-multiplying by G_z and G_z^{T}, and using the fact that $G_z^{\mathrm{T}} + G_z - \widehat{S}_{zz} < G_z^{\mathrm{T}} \widehat{S}_{zz}^{-1} G_z$, equation (41) can be rewritten as

$$\sum_{s=1}^{2^m} \sum_{j=1}^{r} \sum_{g=1}^{r} \eta_s(k) f_j f_g \left\{ \Psi_2 G_j [G_j^{\mathrm{T}} + G_j - \widehat{S}_{jg}]^{-1} (\Psi_2 G_j)^{\mathrm{T}} \right\} \leqslant Z \tag{42}$$

Thus, the constraint of the input holds if equation (36) is satisfied.

With the non-PDC strategy obtained at sampling time k, we have

$$\begin{aligned} &\max_{i \geqslant 0} \mathbb{E}\{|y_c(k+i+1|k)|^2\} \\ &= \max_{i \geqslant 0} |C_z x(k+i+1|k)|^2 \\ &= \sum_{s=1}^{2^m} \eta_s(k) \{|C_z[A_z + \mu B_z[-E_s Y_z G_z^{-1} + E_s^- H] \\ &\quad - (1-\mu)\nu B_z Y_z G_z^{-1}] x(k+i|k)|^2\}_{cc} \\ &= \sum_{s=1}^{2^m} \eta_s(k) \{C_z[A_z + \mu B_z[-E_s Y_z G_z^{-1} + E_s^- H] \\ &\quad - (1-\mu)\nu B_z Y_z G_z^{-1}] \widehat{S}_{zz} \{C_z[A_z + \mu B_z \\ &\quad \times [-E_s Y_z G_z^{-1} + E_s^- H] - (1-\mu)\nu B_z Y_z G_z^{-1}]\}^{\mathrm{T}}\}_{cc} \\ &\quad x(k+i|k)^{\mathrm{T}} \widehat{S}_{zz}^{-1} x(k+i|k) \end{aligned} \tag{43}$$

Similarly, one can obtain

$$\mathbb{E}\left\{\max_{i \geqslant 0} |y_c(k+i|k)|^2\right\} \leqslant \sum_{s=1}^{2^m} \eta_s(k) \left\{ C_z \Psi_1 \widehat{S}_{zz} (C_z \Psi_1)^{\mathrm{T}} \right\}_{cc} \tag{44}$$

Using Schur's complement and the fact that $G_z^{\mathrm{T}} + G_z - \widehat{S}_{zz} < G_z^{\mathrm{T}} \widehat{S}_{zz}^{-1} G_z$, equation (37) can be obtained.

Remark 3. The complexity of solving the linear matrix inequality optimization problem is in polynomial time, which is proportional to $K^3 L$, where K is the total number of scalar variables and L is the total row size of the linear matrix inequality system. For Theorems 1 and 2

$$K = \tfrac{3n(n+1)}{2} + 2qn + \tfrac{q(1+q)}{2} + \tfrac{p(1+p)}{2}$$
$$L = 2^m r^4 (3n+q) + 2r^2 q(n+q) + r^2(n+1) + r^2 p(n+p)$$

Hence, by increasing r, m, n, p, q, the computational burden is increased with a power law. For $n = q = p = r = m$, keeping the most influential parameters in K and L, we can say that the computational complexity of solving Theorems 1 and 2 is proportional to $2^{n+8} n^{11}$.

Feasibility

Lemma 3. (Lu et al., 2015). *The closed-loop system is stochastically stable, if there exists a finite matrix $Y > 0$, such that*

$$\mathbb{E}\{\sum_{k=0}^{\infty}|x(k)|^2|x(0)\}<x(0)^{\mathrm{T}}Yx(0)$$

Theorem 3. *At time $t = k$, assume that the optimization (equation (25)) is feasible, and that the optimization is also feasible at time $k+1$. Moreover, the feasible receding-horizon state-feedback control obtained by optimization at each sampling time, $u(k|k) = Fx(k) = -Y_zG_z^{-1}x(k)$ asymptotically stabilizes the resulting closed-loop fuzzy system (equation (24)).*

Proof.

Recursive feasibility. It can be known that the constrained model predictive control optimization problem is feasible at the sampling time k. Assume that the optimal solutions at k are γ, S_{jg}, G_j, Y_j and H. When $t = k+1$, it should be proved that γ, S_{jg}, G_j, Y_j and H satisfy

$$\begin{bmatrix} G_{j+}^{\mathrm{T}} + G_{j+} - \widehat{S}_{jg+} & * \\ D_{tj+} & I \end{bmatrix} > 0, \quad j,g = 1,2,\ldots,r, t = 1,2,\ldots,q \tag{45}$$

$$\begin{bmatrix} \gamma & * \\ x(k+1|k+1) & S_{jg+} \end{bmatrix} > 0, \qquad j,g = 1,2,\ldots,r \tag{46}$$

$$\begin{bmatrix} G_{j+}^{T} + G_{j+} - S_{jg+} & * & * & * \\ \Psi_{1+}G_{j+} & S_{cd+} & * & * \\ \Psi_{2+}G_{j+}R_0 & 0 & R_0 & * \\ G_{j+}Q_0 & 0 & 0 & Q_0 \end{bmatrix} > 0, \quad j,g,c,d = 1,2,\ldots,r, s = 1,2,\ldots,2^m \tag{47}$$

where ' + ' is the next sampling time, which means $f_j(\theta(k))$ is replaced by $f_j(\theta(k+1))$, and

$$\begin{aligned}\Psi_{1+} &= A_{j+} + \mu B_{j+}[-E_{s+}Y_{j+}G_{j+}^{-1} + E_{s+}^{-}H+] \\ &\quad -\nu(1-\mu)B_{j+}Y_{j+}G_{j+}^{-1} \\ \Psi_{2+} &= \mu[-E_{s+}Y_{j+}G_{j+}^{-1} + E_{s+}^{-}H+] - (1-\mu)\nu Y_{j+}G_{j+}^{-1}\end{aligned}$$

According to Theorem 1, an upper bound of γ is minimized at sampling time k with optimal solutions. Based on equation (28), the following can be obtained

$$\begin{aligned}&-G_j - G_j^{\mathrm{T}} + S_{jg} + (\Psi_1 G_j)^{\mathrm{T}}S_{cd}^{-1}G_j\Psi_1 \\ &+ G_j^{\mathrm{T}}Q_0G_j + (\Psi_2 G_j)^{\mathrm{T}}R_0G_j\Psi_2 < 0\end{aligned} \tag{48}$$

After pre- and post-multiplying this inequality by $G_j^{-1}, x(k|k)$ and $G_j^{-\mathrm{T}}, x(k|k)^{\mathrm{T}}$ on both sides and utilizing the fact $G_j + G_j^{\mathrm{T}} - S_{jg} \leqslant G_j^{\mathrm{T}}S_{jg}^{-1}G_j$, one can obtain

$$\begin{aligned}&(\Psi_1 x(k|k))^{\mathrm{T}}S_{cd}^{-1}\Psi_1 x(k|k) - x(k|k)^{\mathrm{T}}S_{jg}^{-1}x(k|k) < \\ &-x(k|k)^{\mathrm{T}}Q_0x(k|k) - (\Psi_2 x(k|k))^{\mathrm{T}}R_0\Psi_2 x(k|k)\end{aligned} \tag{49}$$

It should be noted that

$$x(k+1|k+1) = \sum_{s=1}^{2^m}\sum_{j=1}^{r}\eta_s(k)f_j\{\Phi_1 x(k|k)\}$$

$$S_{zz}^{-1} = \sum_{c=1}^{r}\sum_{d=1}^{r}f_cf_dS_{cd}^{-1} = \sum_{j=1}^{r}\sum_{g=1}^{r}f_jf_gS_{jg}^{-1}$$

where $x(k+1|k+1)$ means the state measured at $k+1$. For $x(k|k) \neq 0$, equation (49) can be rewritten as

$$x(k+1|k+1)^{\mathrm{T}}S_{zz}^{-1}x(k+1|k+1) < x(k|k)^{\mathrm{T}}S_{zz}^{-1}x(k|k) \tag{50}$$

Certainly, $\gamma, S_{jg}, S_{cd}, G_j, D_{tj}$ and H at $t = k+1$ satisfies equation (46). Moreover, by replacing $f_j(\theta(k))$ with $f_j(\theta(k+1))$, it can be proved that $\gamma, S_{jg}, S_{cd}, G_j, D_{tj}$ satisfies equations (45) and (47).

Closed-loop stability

We assume that

$$\| x(k+i|k) \|_{Q_0}^2 + \| \widetilde{u}(k+i|k) \|_{R_0}^2 = \| x(k+i|k) \|_{\Theta}^2 \tag{51}$$

where $\Theta = Q_0 + \Phi_2^{\mathrm{T}}R_0\Phi_2$.

As mentioned in equations (19) and (51), we obtain

$$V(x(k+i+1|k))) - V(x(k+i|k)) \leqslant -\| x(k+i|k) \|_{\Theta}^2 \tag{52}$$

Computing the mathematical expectation of both sides of equation (52), and for any $w > 1$, summing up the inequality on both sides from $k = 0$ to $k = w$, equation (52) can be rewritten as

$$\mathbb{E}\{V(x(w+i+1|w))) - V(x(0))\} \leqslant -\lambda_{\min}(\Theta)\,\mathbb{E}\left\{\sum_{k=0}^{w}x(k+i|k)^{\mathrm{T}}x(k+i|k)\right\} \tag{53}$$

$$\mathbb{E}\left\{\sum_{k=0}^{w}x(k+i|k)^{\mathrm{T}}x(k+i|k)\right\} \leqslant \lambda_{\min}^{-1}(\Theta)(V(x(0)) - \mathbb{E}\{V(x(w+i+1|k))\}) \tag{54}$$

considering $\lim_{w\to\infty}\mathbb{E}\{V(x(w+i+1|k))\} \geqslant 0$, so equation (54) is equal to

$$\begin{aligned}\mathbb{E}\left\{\sum_{k=0}^{\infty}x(k+i|k)^{\mathrm{T}}x(k+i|k)\right\} &\leqslant \lambda_{\min}^{-1}(\Theta)V(x(0)) \\ &= x(0)^{\mathrm{T}}\lambda_{\min}^{-1}(\Theta)S_{zz}^{-1}x(0) \\ &= x(0)^{\mathrm{T}}Yx(0)\end{aligned} \tag{55}$$

where $Y = \lambda_{\min}^{-1}(\Theta)S_{zz}^{-1}, Y > 0$. Therefore, the feasible receding-horizon state-feedback control law, which is obtained in Theorem 1, asymptotically stabilizes the closed-loop fuzzy system (equation 24).

That completes the proof.

Parallel distributed compensation

In this section, the PDC strategy for a fuzzy system with random actuator saturation and packet losses is adopted:

Plant rule j: IF $\theta_1(k)$ is M_{j1}, $\theta_2(k)$ is M_{j2} and $\cdots$ and $\theta_q(k)$ is M_{jq} THEN

$$u_j(k|k) = -K_j x(k|k), \qquad j = 1, 2, \ldots, r \tag{56}$$

Therefore, $u(k|k) = -K_z x(k|k)$, where $K_z = \sum_{j=1}^{r} f_j(\theta(k))K_j$, and the fuzzy control input with actuator saturation and packet losses is presented as

$$\begin{aligned}\widetilde{u}(k|k) &= \alpha(k)\mathrm{sat}\{u(k)\} - (1-\alpha(k))\beta(k)K_z x(k) \\ &= \sum_{s=1}^{2^m} \eta_s(k)\{\alpha(k)(-E_s K_z + E_s^- H) - (1-\alpha(k))\beta(k)K_z\}x(k|k)\end{aligned} \tag{57}$$

Thus, the closed-loop fuzzy system under the PDC strategy can be obtained as

$$\begin{aligned} x(k+1) = &\sum_{s=1}^{2^m} \eta_s(k)\{A_z + \alpha(k)B_z[-E_s K_z + E_s^- H] \\ &- (1-\alpha(k))\beta(k)B_z K_z\}x(k) \\ y(k) = &\, C_z x(k)\end{aligned} \tag{58}$$

In the following, the infinite-horizon model predictive control problem for fuzzy system with PDC strategy will be presented.

Corollary 1. *Let us consider the closed-loop fuzzy system (equation (58)) at time k, and let $x(k|k)$ be the measured state $x(k)$. If there exist symmetric matrices Q_{jg}, Q_{cd} and positive matrices $Y_j, G, \widetilde{D}_{tj}$ satisfying the optimization problem*

$$\min_{Q_{jg}, Q_{cd}, G, K_j, \widetilde{D}_{tj}} \gamma \tag{59}$$

$$\begin{bmatrix} G^{\mathrm{T}} + G - \widehat{Q}_{jg} & * \\ \widetilde{D}_t & I \end{bmatrix} > 0, \quad j, g = 1, 2, \cdots, r.t = 1, 2, \ldots, q \tag{60}$$

$$\begin{bmatrix} \gamma & * \\ x(k) & Q_{jg} \end{bmatrix} > 0, \quad j, g = 1, 2, \ldots, r \tag{61}$$

$$\begin{bmatrix} G^{\mathrm{T}} + G - Q_{jg} & * & * & * \\ \Psi_{1-PDC}G & Q_{cd} & * & * \\ \Psi_{2-PDC}GR_0 & 0 & R_0 & * \\ GQ_0 & 0 & 0 & Q_0 \end{bmatrix} > 0, \quad j, g, c, d = 1, 2, \cdots, r, s = 1, 2, \ldots, 2^m \tag{62}$$

where

$$\begin{aligned} \Psi_{1-\mathrm{PDC}} &= A_j + \mu B_j[-E_s Y_j G^{-1} + E_s^- H] - (1-\mu)\nu\, B_j Y_j G^{-1} \\ \Psi_{2-\mathrm{PDC}} &= \mu[-E_s Y_j G^{-1} + E_s^- H] - (1-\mu)\nu Y_j G^{-1} \\ Q_{zz} &= \sum_{j=1}^{r}\sum_{g=1}^{r} f_j f_g Q_{jg} = \sum_{c=1}^{r}\sum_{d=1}^{r} f_c f_d Q_{cd} \\ \widetilde{D}_{tj} &= h_t g_j \\ \widehat{Q}_{jg} &= (1/\gamma)Q_{jg} \end{aligned} \tag{63}$$

then the Lyapunov function can be taken as $V(x(k|k)) = x(k|k)^{\mathrm{T}} Q_{zz}^{-1} x(k|k)$, $Q_{zz} \in R^{n \times n}, Q_{zz} > 0$, the state-feedback control gain matrix in the PDC strategy $u(k+i|k) = -K_z x(k+i|k) = -Y_z G^{-1} x(k+i|k), i \geqslant 0$, such that the resulted closed-loop fuzzy system (equation (58)) is minimized at each sampling time k on the performance objective function (equation (13)).

Corollary 2. *Suppose there exist a symmetric matrix $\widehat{Q}_{jg}$ and positive matrices Y_j, G and H satisfying the following optimization problem*

$$\begin{bmatrix} G^{\mathrm{T}} + G - \widehat{Q}_{jg} & * \\ \Psi_{2-PDC}G & \widehat{Z} \end{bmatrix} > 0, \qquad j, g = 1, 2, \ldots, r, s = 1, 2, \ldots, 2^m \tag{64}$$

$$\begin{bmatrix} G^{\mathrm{T}} + G - \widehat{Q}_{jg} & * \\ C_j G \Psi_{1-PDC} & \widehat{M} \end{bmatrix} > 0, \qquad j, g = 1, 2, \ldots, r, s = 1, 2, \ldots, 2^m \tag{65}$$

with $\widehat{Z}_{cc} < \bar{u}_{c.\max}^2, c = 1, 2, \ldots, q$; $\widehat{M}_{cc} < \bar{y}_{c.\max}^2, c = 1, 2, \ldots, p$, the constraints of $\widetilde{u}(k+i|k)$ and $y(k+i|k)$ are satisfied.

Corollary 3. *The feasible receding-horizon fuzzy PDC strategy, which is obtained by optimization at each sampling time in Corollary 2, i.e., $(\widetilde{u}(0|0), \widetilde{u}(1|1), \ldots, \widetilde{u}(k|k), k \to \infty)$ asymptotically stabilizes the resulted closed-loop fuzzy system (equation (58)).*

Illustrative examples

In this section, we present two simulations to illustrate the method developed in this paper.

Example 1

The considered fuzzy system with actuator saturation and packet losses is described as follows.

Rule 1. *If $x_1(k)$ is M_1, then*

$$x(k+1) = A_1 x(k) + B_1 u(k), \qquad y(k) = C_1 x(k)$$

Rule 2. *If $x_1(k)$ is M_2, then*

$$x(k+1) = A_2 x(k) + B_2 u(k), \qquad y(k) = C_2 x(k)$$

The membership functions have the forms

$$M_1(x_1) = \frac{1}{1 + \exp(-2x_1)}, \qquad M_2(x_1) = 1 - M_1(x_1)$$

where the system matrices are presented by the following vertices

$$A_1 = \begin{bmatrix} -0.5 & 0.7 \\ -0.1 & 0.7 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 3 \\ 1.7 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -0.2 & 0.5 \\ -0.1 & -0.9 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$
$$C_1 = [\,1 \quad 0.3\,], \qquad C_2 = [\,1.8 \quad 0.2\,]$$

Table 1. Comparison of different strategies.

	Non-PDC of this paper	PDC of this paper	Non-PDC of Zhao and Li (2015)
Total time	11.863 s	11.516 s	11.929 s
$J^{\infty}(k)$	0.5812	0.7048	1.2402

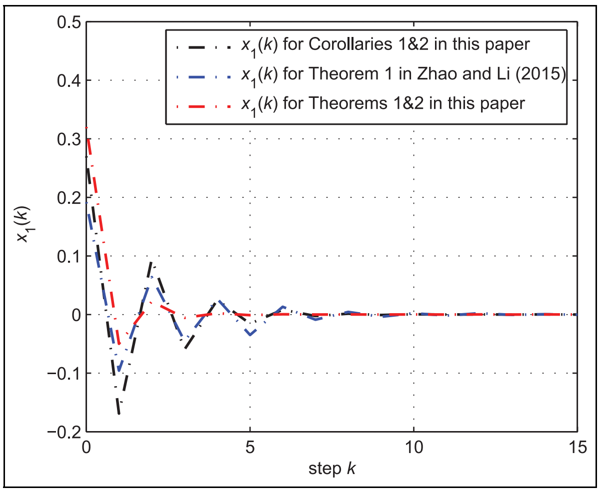

Figure 2. State x_1 for the system.

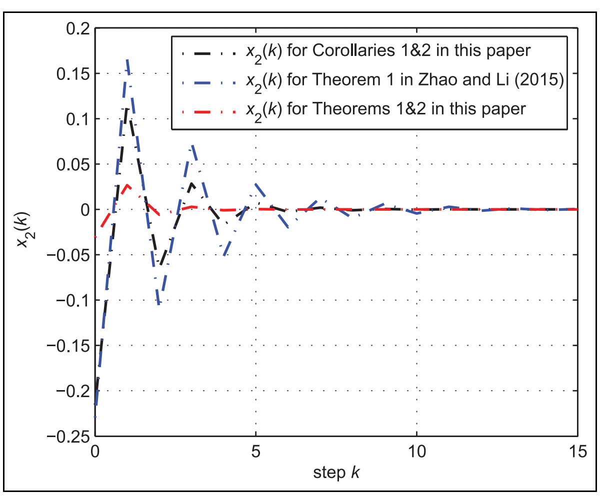

Figure 3. State x_2 for the system.

A comparison of the different strategies is presented in Table 1.

We have given a comparison between the non-PDC strategy (Theorems 1 and 2) and PDC strategy (Corollaries 1 and 2) proposed in this paper and the non-PDC strategy (Theorem 1) proposed in Zhao and Li (2015). For the non-PDC strategy, the states of the control system are given in Figures 2 and 3 and the evolution of γ is given in Figure 4. We can see that the stability of the closed-loop system can be guaranteed by using these three strategies; however, the proposed non-PDC strategy shows the fastest convergence speed, and the performance with the proposed PDC strategy

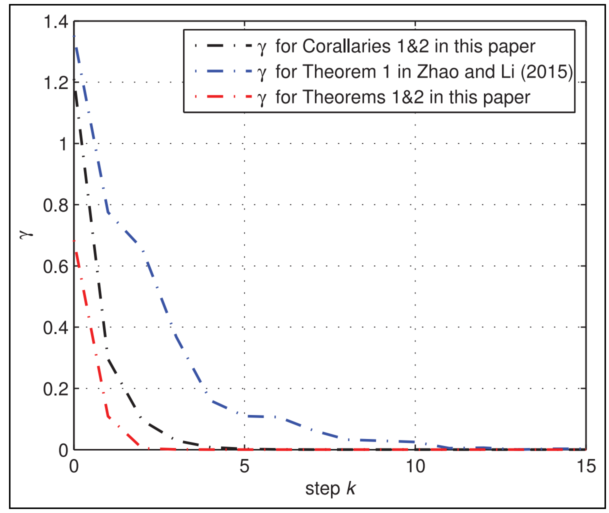

Figure 4. Evolution of γ for the system.

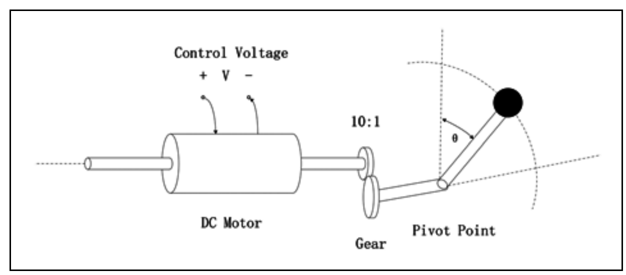

Figure 5. Inverted pendulum controlled by a direct-current (DC) motor.

behaves better than the non-PDC one given in Zhao and Li (2015).

Example 2

Consider an inverted pendulum controlled by a direct-current motor, as shown in Figure 5. Its discrete-time model is presented as

$$x(k+1) = \sum_{i}^{2} h_i(k)(A_i x(k) + B_i u(k))$$

where

$$A_1 = \begin{bmatrix} 1.002 & 0.02 & 0.02 \\ 0.196 & 1.0001 & 0.0181 \\ -0.0184 & -0.1813 & 0.8170 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0.02 & 0.0002 \\ 0 & 0.9981 & 0.0181 \\ 0 & -0.1811 & 0.8170 \end{bmatrix},$$
$$B_1 = B_2 = [0 \quad 0.0019 \quad 0.1811]^{\mathrm{T}}$$

To demonstrate the effectiveness of the obtained results, we assume the membership function at the plant and controller side to be

$$h_1(x_1) = 1 - 0.25x_1^2, \qquad h_2(x_1) = 1 - h_1(x_1)$$

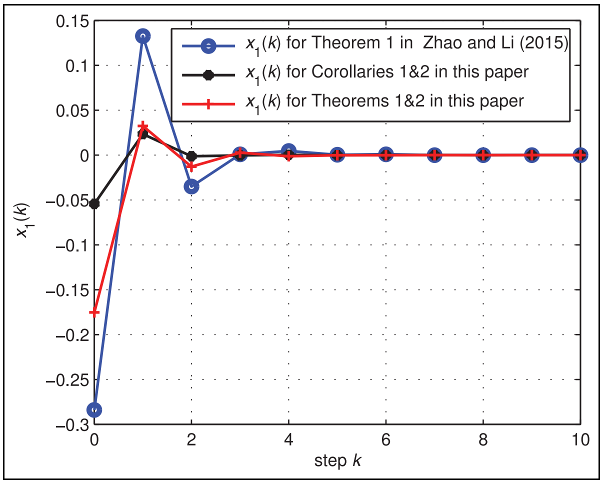

Figure 6. State $x_1(k)$ for the system.

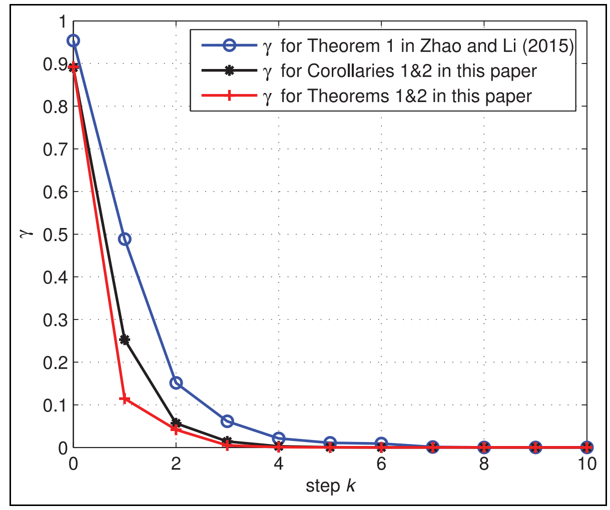

Figure 9. Evolution of γ for the system.

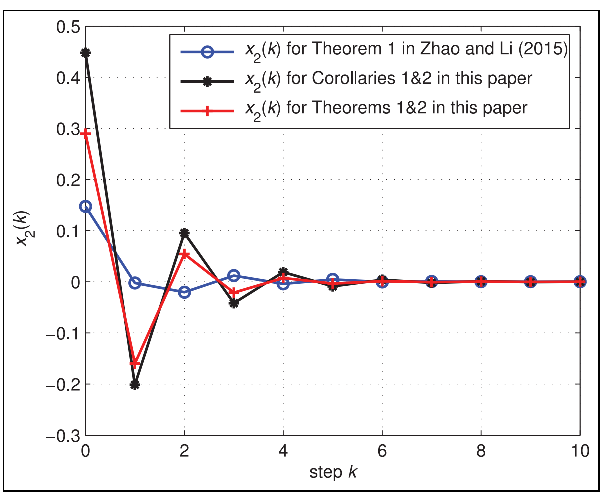

Figure 7. State $x_2(k)$ for the system.

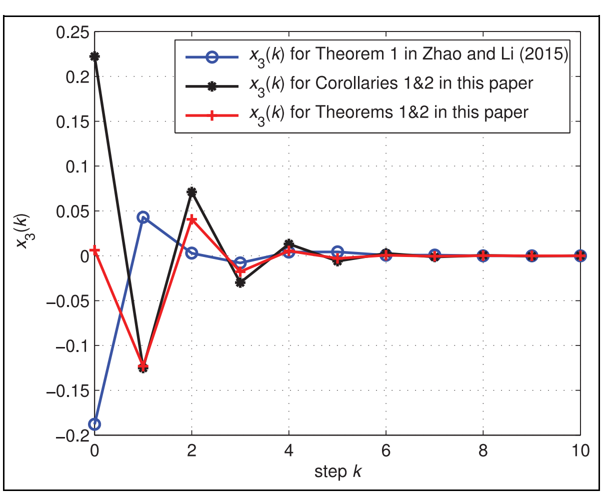

Figure 8. State $x_3(k)$ for the system.

Table 2. Comparison of different strategies.

	Non-PDC of this paper	PDC of this paper	Non-PDC of Zhao and Li (2015)
Total time	18.216 s	18.396 s	12.828 s
$J^{\infty}(k)$	0.7963	0.8004	0.9204

Different states for the fuzzy control system are given in Figures 6, 7, 8 and 9, from which it can be seen that different strategies give different performances. The proposed non-PDC strategy is less conservative than the PDC strategy, and the proposed PDC strategy behaves better than the non-PDC one given in Zhao and Li (2015).

A comparison of the different strategies is presented in Table 2.

Conclusion

This paper presents a study of model predictive control for a fuzzy system with randomly occurring actuator saturation and packet losses. Two sets of Bernoulli sequences are introduced to express randomly occurring actuator saturation and packet losses. An online model predictive controller is obtained by solving an infinite-horizon optimization problem online, which explicitly considers the actuator saturation and packet losses. Based on the solution of the model predictive control optimization problem, we provided state-feedback controllers using both PDC and non-PDC strategies; our results show that the non-PDC strategy leads to less conservatism than applying does the PDC strategy. Moreover, the proposed controllers meet the restriction of the input and output. In addition, results show that there is a faster convergence when using the non-PDC strategy than when using the PDC strategy.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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