$$P_2(x) = dx^2 \quad ex + f$$

$$P_2(a) = f + e(a) + d(a)^2 \quad (*)$$

$$P_2(x_m) = f + e(x_m) + d\,x_m^2 \quad (**)$$

$$P_2(b) = f + eb + db^2 \quad (***)$$

Queremos entonces $\int_a^b P_2(x)\,dx$

$$= \int_a^b f + ex + dx^2\,dx$$

$$= fx + \frac{ex^2}{2} + \frac{dx^3}{3}\Big]_a^b$$

$$= f(b-a) + \frac{e}{2}(b-a)^2 + \frac{f}{3}(b-a)^3$$

Ahora despejamos los coeficientes (Wolfram) con las ecuaciones (*), (**), (***)

$$f = \frac{a^2 P_2(b) + ab\, P_2(b) - 4ab\, P_2(x_m) + ab\, P_2(a) + b^2 p_2}{a^2 - 2ab + b^2}$$

$$e = \frac{a P_2(a) - 4a\, p_2(x_m) + 3a\, P_2(b) + 3b\, P_2(a) - 4b\, P_2(x_m) + b\, p_2(b)}{a^2 - 2ab + b^2}$$

$$d = \frac{2 P_2(a) - 4 P_2(x_m) + 2 P_2(b)}{a^2 - 2ab + b^2}$$

Simplificando (Wolfram Alpha): Reemplazamos

$$\int_a^b P_2(x)\,dx = \frac{b-a}{6}\left[P_2(a) + 4P_2(x_m) + P_2(b)\right]$$

Si tomamos $h = cte = \frac{b-a}{2}$ obtenemos:

$$\frac{h}{3}\left[P_2(a) + 4P_2(x_m) + P_2(b)\right]$$