# Taller 1 (Pregunta teórica)

5. (**Theoretical**) Show that the $D^4 f$ operator is given by:

$$D^4 f(x_j) \cong \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4} \tag{3.33}$$

For this operator, what is the order ($\mathcal{O}(h^k)$) of the approximation?

Tenemos la definición del libro

$$f(x_j) = \frac{f(x_{j+1}) - f(x_{j-1})}{2h} \underbrace{- \frac{h^2}{3} f'''(x)}_{O(h^2)}$$

$$f(x_j) \doteq \frac{f(x_{j+1}) - f(x_{j-1})}{2h}$$

El orden de nuestra aproximación es $h^2$

Aplicando el concepto de derivada central

$$f''(x_j) \doteq \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2}$$

encontramos $f^4(x)$ usando la anterior definición de derivada central

$$f''(f''(x_j)) = \frac{\frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j)}{h^2} - 2\frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} + \frac{f(x_j) - 2f(x_{j-1}) + f(x_{j-2})}{h^2}}{h^2}$$

$$f^4(x_j) = \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

La derivada tiene orden $O(h^2)$