Punto 1

1. (**Theoretical**) Hacer pasos intermedios para regla de trapecio simple, Ecuación (3.94).

$$f(x) \approx p_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b), \qquad \forall x \in [a,b]. \tag{3.93}$$

$$I = \int_a^b f(x)dx \cong \int_a^b p_1(x)dx = \frac{b-a}{2}(f(a)+f(b)) \tag{3.94}$$

Integrando $p_1(x)$

$$\int_a^b p_1(x) = \int_a^b \frac{x-b}{a-b} f(a) + \int_a^b \frac{x-a}{b-a} f(b)$$

$$= \frac{f(a)}{a-b} \int_a^b x-b + \frac{f(b)}{b-a} \int_a^b x-a$$

$$= \frac{f(a)}{a-b} \left(\frac{b^2}{2} - b^2 - \left(\frac{a^2}{2} - ba \right) \right) + \frac{f(b)}{b-a} \left(\frac{b^2}{2} - ab - \left(\frac{a^2}{2} - a^2 \right) \right)$$

$$= \frac{f(a)}{a-b} \left(\frac{-b^2 - a^2 + 2ba}{2} \right) + \frac{f(b)}{b-a} \left(\frac{b^2 - 2ab + a^2}{2} \right)$$

$$= \frac{f(a)}{a-b} \left(-\frac{(b^2 + a^2 - 2ab)}{2} \right) + \frac{f(b)}{b-a} \left(\frac{(b-a)^2}{2} \right)$$

$$= \frac{f(a)}{a-b} \left(\frac{-(a-b)^2}{2} \right) + \frac{f(b)}{b-a} \left(\frac{(b-a)^2}{2} \right)$$

$$= \frac{f(a) \cdot (-(a-b))}{2} + \frac{f(b)(b-a)}{2}$$

$$= \frac{f(a)(b-a)}{2} + \frac{f(b)(b-a)}{2}$$

$$= \left(f(a) + f(b) \right) \frac{b-a}{2}$$