Ejercicio 1 Probabilidad continua

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a) Tenemos que verificar $\int_0^1 \int_0^1 f(x,y)\,dx\,dy = 1$

$$\int_0^1 \int_0^1 \frac{2}{3}(x+2y)\,dx\,dy$$

$$= \frac{2}{3}\int_0^1 \int_0^1 x + 2y\,dx\,dy$$

$$= \frac{2}{3}\int_0^1 \frac{x^2}{2} + 2yx \Big|_0^1 dy$$

$$= \frac{2}{3}\int_0^1 \frac{1}{2} + 2y\,dy$$

$$= \frac{2}{3}\left[\frac{1}{2}y + y^2 \Big|_0^1\right]$$

$$\boxed{= \frac{2}{6} + \frac{2}{3} = \frac{2}{6} + \frac{4}{6} = 1}$$

Además verificamos que para cualquier $x, y \in [0,1]$

$f(x,y) > 0$

$f(0,0) = 0$

$f(1,1) = 2$

$$g(x) = \int_0^1 \frac{2}{3}(x+2y)\,dy$$

$$= \frac{2}{3}\int_0^1 x+2y \quad dy$$

$$= \frac{2}{3}\left(xy + y^2\right)\Big|_0^1$$

$$= \frac{2}{3}(x+1)$$

$$= \frac{2}{3}x + \frac{2}{3}$$

$$h(y) = \frac{2}{3}\int_0^1 (x+2y)\,dx$$

$$= \frac{2}{3}\left(\frac{x^2}{2} + 2xy\right)\Big|_0^1$$

$$= \frac{2}{3}\left(\frac{1}{2} + 2y\right)$$

$$= \frac{2}{6} + \frac{4}{3} y$$

$$E(x) = \int_0^1 g(x) \cdot x \quad dx$$

$$= \int_0^1 \frac{2}{3} x^2 + \frac{2}{3} x \quad dx$$

$$= \frac{2}{9} x^3 + \frac{1}{3} x^2 \Big|_0^1$$

$$= \frac{2}{9} + \frac{1}{3} = \frac{5}{9} = \frac{10}{18}$$

$$E(y) = \int_0^1 h(y) \cdot y \quad dy$$

$$= \int_0^1 \frac{2}{6} y + \frac{4}{3} y^2 \quad dy$$

$$= \frac{1}{6} y^2 + \frac{4}{9} y^3 \Big|_0^1$$

$$= \frac{1}{6} + \frac{4}{9} = \frac{9}{54} + \frac{24}{54} = \frac{33}{54} = \frac{11}{18}$$

$$E(x,y) = \int_0^1\int_0^1 \frac{2}{3}(x+2y)\,xy \cdot dx\,dy$$

$$= \frac{2}{3}\int_0^1\int_0^1 x^2y + 2y^2x \quad dy\,dx$$

$$= \frac{2}{3}\int_0^1 \left.\frac{x^2y^2}{2} + \frac{2y^3}{3}x\right|_0^1 dx$$

$$= \frac{2}{3}\int_0^1 \frac{x^2}{2} + \frac{2}{3}x \quad dx$$

$$= \frac{2}{3}\left(\left.\frac{x^3}{6} + \frac{1}{3}x^2\right|_0^1\right)$$

$$= \frac{2}{3}\left(\frac{1}{6} + \frac{1}{3}\right)$$

$$= \frac{2}{6} = \frac{1}{3}$$

Covarianza $E(xy) - E(x)E(y) =$

$$\frac{1}{3} - \frac{11}{18}\,\frac{10}{18} = -0{,}00617$$

f) $$\int_0^1\int_0^1 \left(x - \frac{10}{18}\right)\left(y - \frac{11}{18}\right)\frac{2}{3}(x+2y)\,dy$$

$\vdots$

$$\int_0^1 -\frac{2}{27}x^2 + \frac{15}{243}x - \frac{5}{243}\,dx$$

$$-\frac{1}{27}x^3 + \frac{15}{243\cdot 2}x^2 - \frac{5}{243}x\Big|_0^1$$

$$= \frac{1}{27} + \frac{15}{243\cdot 2} - \frac{5}{243} = -\frac{1}{162} = -1{,}00617$$

g) La covarianza es diferente de cero

Entonces las variables son dependientes