

1. Encontrar el n-ésimo término de la ecuación

$$a) \quad x_{n+1} = 4x_n - x_n^2$$

$$x_0 = 4 \sin^2 \theta$$

$$x_1 = 4(4 \sin^2 \theta) - (4 \sin^2 \theta)^2$$

$$= 16 \sin^2 \theta - 16 \sin^4 \theta$$

$$x_1 = 16 \sin^2 \theta (1 - \sin^2 \theta)$$

$$= 16 \sin^2 \theta \cos^2 \theta$$

$$= 4 \sin^2(2\theta)^2$$

$$\text{Usando } 2 \sin(\theta) \cos(\theta) = \sin(2\theta)$$

$$x_2 = 4(4 \sin^2(2\theta)) - (4 \sin^2(2\theta))^2$$

$$= 4 \sin^2(4\theta)$$

$$x_n = 4 \sin^2(2^n \theta)^2$$

$$x_{n+1} = 4(4 \sin^2(2^n \theta)) - (4 \sin^2(2^n \theta))^2$$

$$= 16 \sin^2(2^n \theta) (1 - \sin^2(2^n \theta))^2$$

$$= 16 \sin^2(2^n \theta) \cos^2(2^n \theta)$$

$$= 4 \sin^2(2^{n+1} \theta)$$

2. De la misma forma tenemos

$$x_0 = \sin^2 x$$

$$x_{n+1} = \sin^2(2^{n+1} x)$$

$$x_{n+1} = 4x_n - 4x_n^2$$

$$x_1 = 4 \sin^2 x - 4(\sin^2 x)^2$$

$$= 4 \sin^2 x (1 - \sin^2 x)$$

$$= 4 \sin^2 x \cos^2 x$$

$$= \sin^2(2x)$$

$$x_n = \sin^2(2^n x)$$

$$x_{n+1} = \sin^2(2^{n+1} x)$$