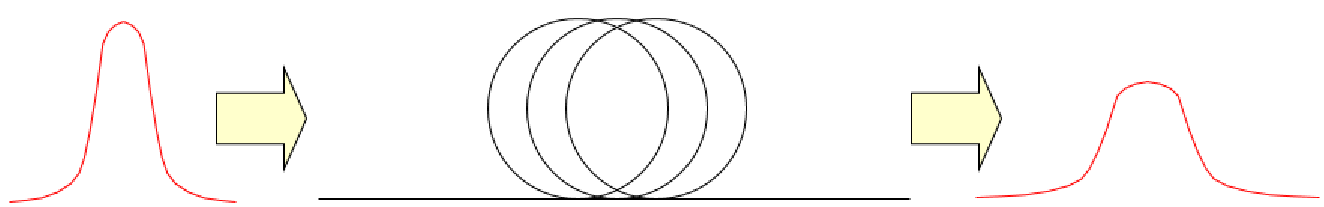
Hiroyuki Tsuda's Laboratory - 2019

Report of exercise 3 - Pulse propagation in optical fiber



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1 Introduction

This report investigates the propagation of pulses in single-mode optical fibers. Beside losses, the notable effect on the propagation of pulses is their dispersion. More precisely, several kinds of dispersion are observed in optical fibers.

In multi-mode fibers an effect called "Mode dispersion" arises. While solving Maxwell's equations, the solution to the propagation of the electric field (energy) is not unique and different spatial distribution of the fields can exits simultaneously. Therefore, for one fiber, we have different field components propagating at the same speed but taking different paths. The larger the path, the greater the delay between input and output and the smaller the path (closer from the center of the fiber) the faster the mode travels through the fiber. As single mode fibers allow only the propagation of the first mode this effect does not emerge.

Even if the mode dispersion does not occur in single mode fibers, another dispersive phenomenon is present, it is called "chromatic dispersion". This last one can be divided in three main categories: waveguide dispersion, material dispersion and polarisation dispersion. In this framework, polarisation dispersion is omitted. Waveguide dispersion is due to the dependence of the effective refractive index $n_{eff} = \frac{\beta}{k_0}$ on the normalized frequency $V = \frac{2\pi a}{\lambda_0}\sqrt{{n_1}^2 - n_2^2}$. The higher the value of the the normalized frequency, the more the power is contained inside the core, the lower the normalized frequency, the more the power is contained in the cladding. This means that there is typically a change in the distribution of the power between the cladding and the core. Therefore, the power contained in one or the other being different with the frequency V, the shape of the power profile will also be different.
Chromatic dispersion itself just depends on the frequency dependence of the refractive index. Those different refractive index for different frequencies are such that different fequency component will travel at different speed and reach the receiver at different time instant. Resulting in a broadening of the received pulse.

2 Propagation equations and influence of the dispersion

Maxwell's equations give the result that frequencies components of a particular signal in single-mode fiber propagate such that

$$\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \bar{\mathbf{v}} F(x, y) \tilde{B}(0, \omega) \exp\{i\beta z\}. \tag{1}$$

For this equation, the $\bar{\mathbf{v}}$ is the polarization unit vector, $\tilde{B}(0, \omega)$ is the initial amplitude's transform and β the propagation constant. Of course, we are rather interested in finding the pulse propagation equation in the time domain. To do so, we will just do the inverse Fourier transform of the signal's spectrum. This is given as follow

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{B}(z, \omega) \mathrm{e}^{-i\omega t} \, d\omega. \tag{2}$$

As precised in the introduction, the pulse broadening is due to the frequency dependence of $\beta = \beta(\omega)$. We therefore expand β using Taylor series and obtain

$$\beta(\omega) \approx \beta_0 + \beta_1(\Delta\omega) + \frac{\beta_2}{2}(\Delta\omega)^2 + \frac{\beta_3}{6}(\Delta\omega)^3. \tag{3}$$

With an additional slowly amplitude of the envelope

$$B(z, t) = A(z, t) \exp\{i[\beta_0 z - \omega_0 t]\}. \tag{4}$$

The final relation is therefore obtained as

$$A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d(\Delta\omega) \tilde{A}(0, \Delta\omega) \mathrm{e}^{[i\beta_1 z(\Delta\omega) + i\frac{\beta_2}{2} z(\Delta\omega)^2 + i\frac{\beta_3}{6} z(\Delta\omega)^3 - i(\Delta\omega)t]}. \tag{5}$$

3 Numerical method discussion

From a previous section, it is obvious that numerically, at a particular distance z = L representing the length of the fiber, it it necessary to resolve equation 5. We therefore need the Fourier transform of the input pulse A(z,t) given by $\tilde{A}(0, \Delta\omega)$. Two method could therefore be used :

1. Compute analytically the Fourier Transform of the input pulse and then integrate using Matlab's integrals

2. Use fft to compute the Fourier Transform of the input pulse and then integrate by multiplying and summing

3.1 Gaussian Pulses

Both method 1. and 2. can be used as the Fourier transform of the Gaussian pulse is defined and can be computed analytically.

$$\exp\left(\frac{-t^2}{2T_0^2}\right) \xrightarrow{\mathcal{F}} \sqrt{2\pi T_0^2}\exp\big(-\pi^2 2T_0^2(\Delta\omega)^2\big). \tag{6}$$

3.2 Super Gaussian Pulses

Only method 2. can be used in this case as the Fourier Transform from $-\infty$ to ∞ is not known.

$$\exp\left(-0.5\left(\frac{t}{T_0}\right)^6\right) \xrightarrow{\mathcal{F}} \quad ? \tag{7}$$

4 Simulation results

4.1 Gaussian Pulse

4.1.1 Second Order Dispersion

The effect of second order dispersion is displayed on figure 1, 2 and 3. From the left-upper corner, displaying the input signal to the fiber, it is observed in both the analytical method and the numerical one a broadening of the pulse at the output. Also, the maximum amplitude is lower compare to the input pulse. Both analytical formula and numerical method give exactly the same result: the resulting beams on the right of figures 1, 2 and 3 are the same.

Another effect which can be denoted is that the dispersion becomes more important as the width of the peak is smaller. This happens because, the narrower the Gaussian pulse is spatially, the wider it will be frequencially. Therefore, the beam having a broader range of frequency components, those components, travelling at different speeds will disperse more importantly. This phenomenon is displayed by observing how the output beams behave while the width of the initial (input) signal is decreasing. This is displayed below by looking sequentially at the right graphs of figure 1 then 2 and finally 3.

It is important to note that the right-upper graph of figure 3 is not accurate due to the MATLAB integration method used and the precision it can perform.

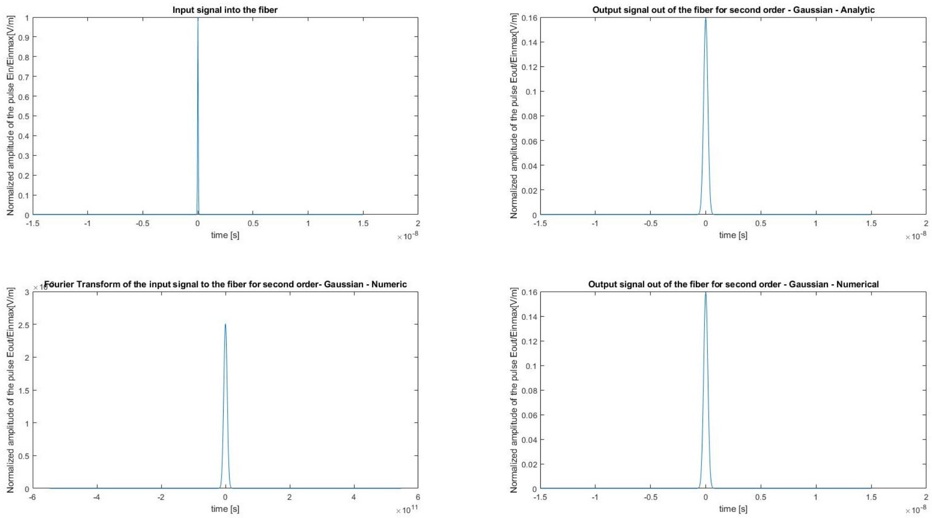

Figure 1: Effect of second order dispersion on a pulse of 50 ps width, both using analytical formula and numerical method technique

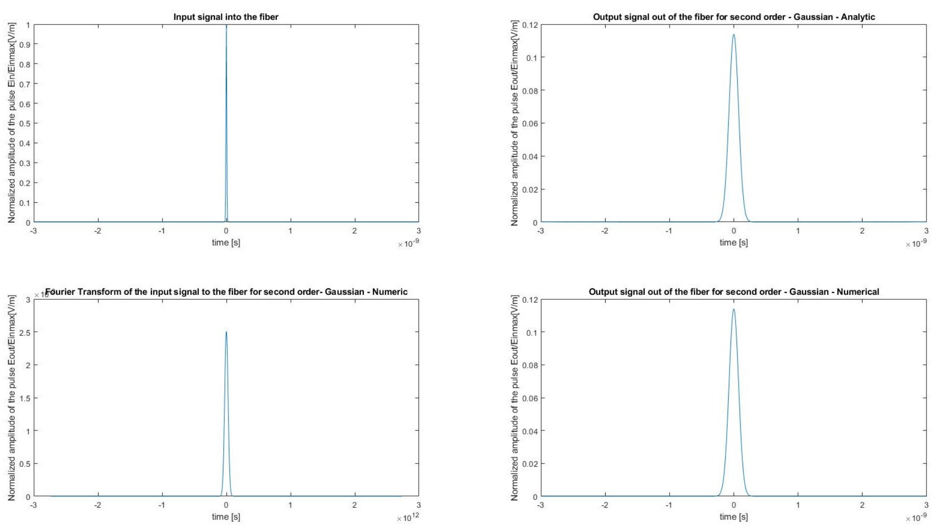

Figure 2: Effect of second order dispersion on a pulse of 10 ps width, both using analytical formula and numerical method technique

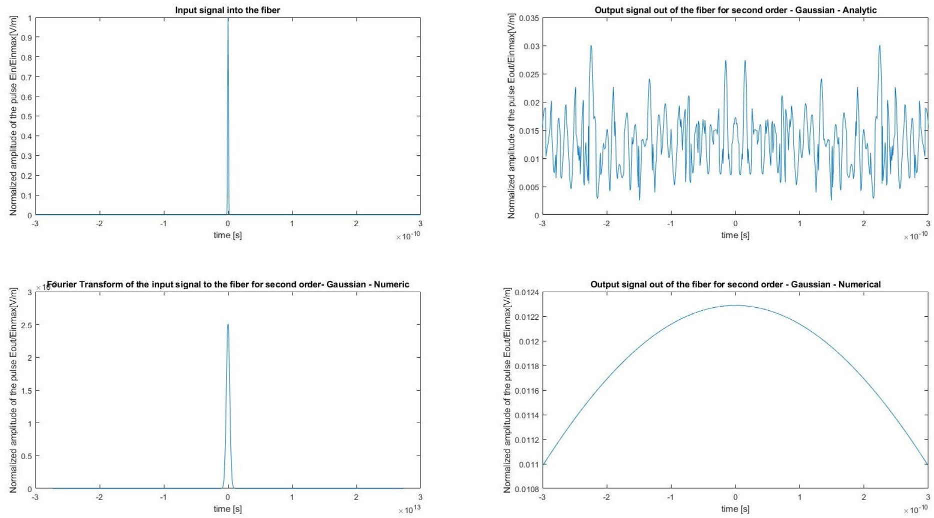

Figure 3: Effect of second order dispersion on a pulse of 1 ps width, both using analytical formula and numerical method technique

4.1.2 Third Order Dispersion

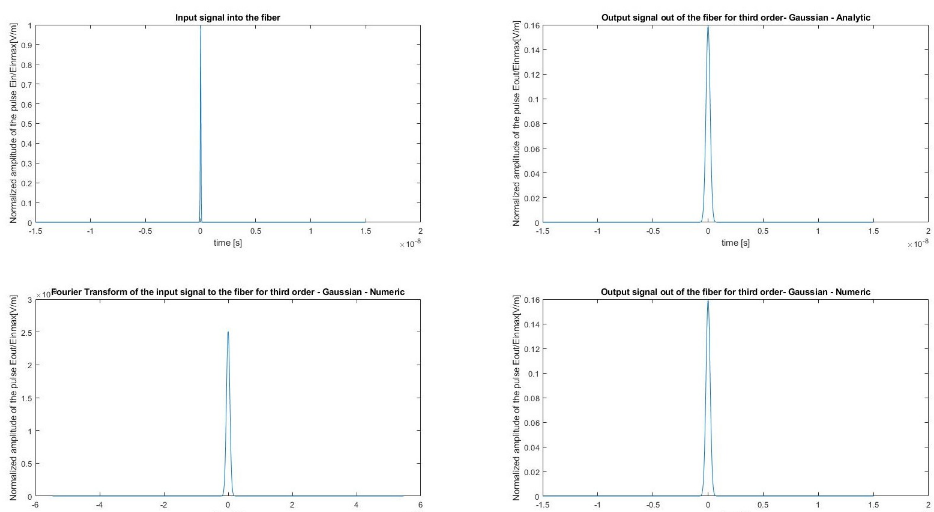

Figure 4: Effect of third order dispersion on a pulse of 50 ps width, both using analytical formula and numerical method technique

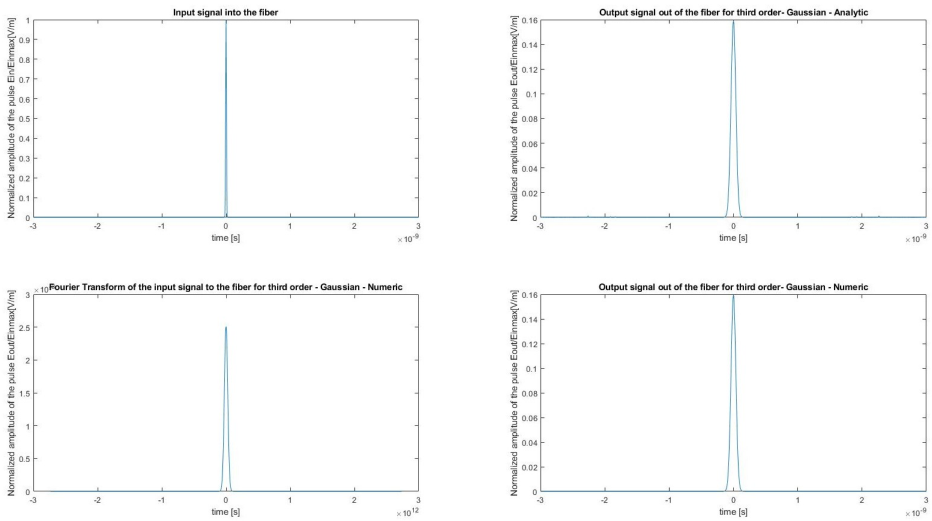

Figure 5: Effect of third order dispersion on a pulse of 10 ps width, both using analytical formula and numerical method technique

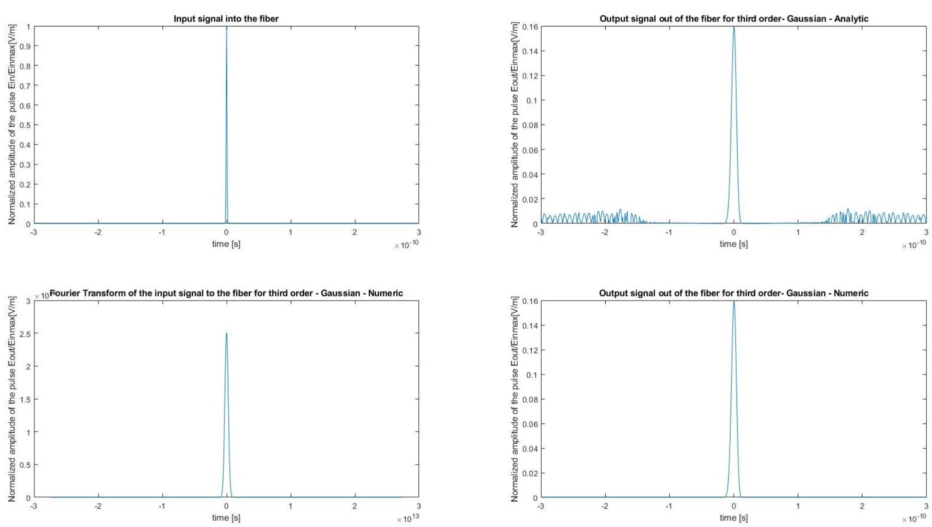

Figure 6: Effect of third order dispersion on a pulse of 1 ps width, both using analytical formula and numerical method technique

4.2 Super-Gaussian Pulse

In the Super-Gaussian case, second order dispersion shows the same behavior as the Gaussian shape. This means a broadening of the output pulse comparing to the input pulse also, similarly, the broadening becomes more important as the width of the input pulse decreases. An important difference is the shape of the output pulse, this one is not anymore Gaussian but it represents a pulse with secondary lobes. This shape is a direct consequence of the frequency components of the Super-Gaussian pulse, this one being a peak followed by secondary lobes. The dispersion is such that those frequency lobes will travel at different speeds and the energy therefore disperse in time domain. Different peaks therefore arrive at different time instants.

For the third order dispersion it is important to note that at first, its effect is smaller than second order dispersion in this case. Also it appears significantly when the width of the peaks get really small (around 1 ps). This is because the third order effects on the signal depend on how big is the dispersion length $L_D = \frac{T_0^2}{\beta_2}$ compare to the length of the fiber L. The lower the dispersion length, the more it will be dispersed. This phenomena can be observed on the right lower corner of figure 7, 8 and 9. The effect on the signal is an odd effect of distortion as seen on the right lower corner of figure 9. This distortion extends the signal in the positive domain.

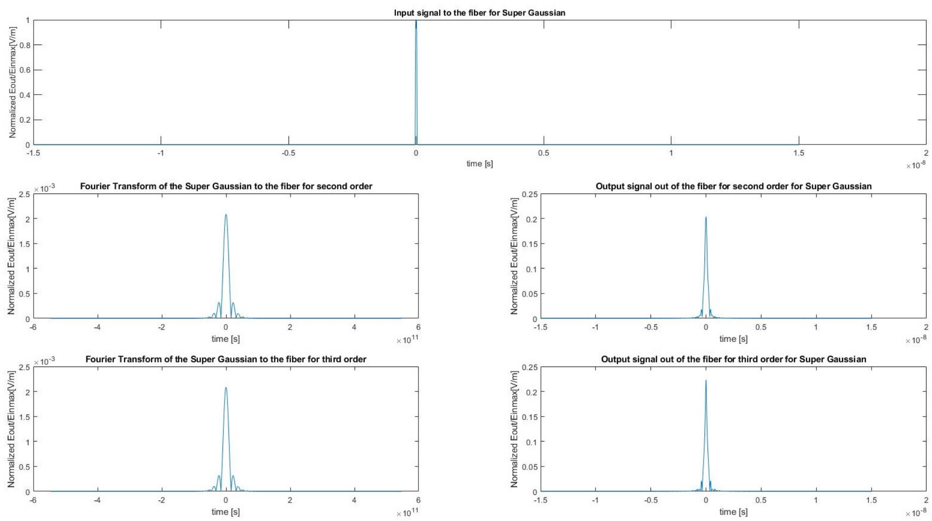

Figure 7: Effect of second and third order dispersion on a pulse of 50 ps width, both using analytical formula and numerical method technique

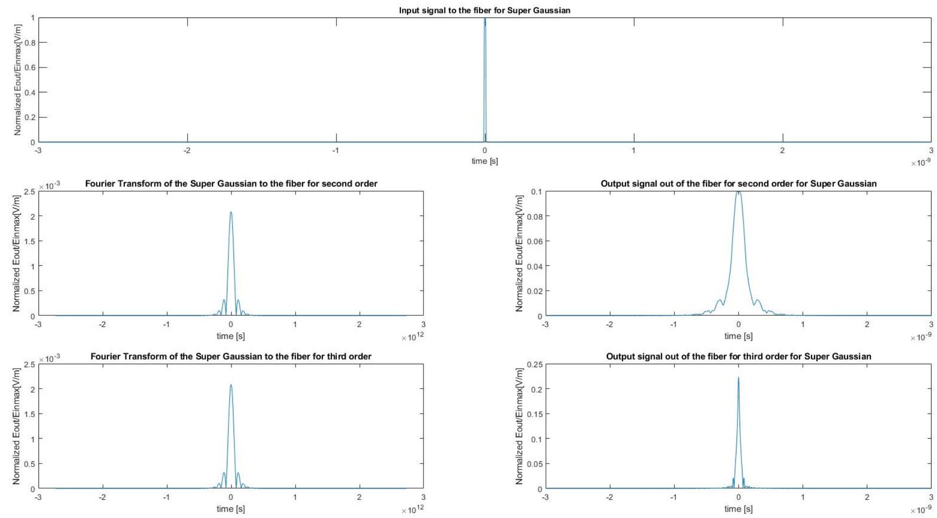

Figure 8: Effect of second and third order dispersion on a pulse of 10 ps width, both using analytical formula and numerical method technique

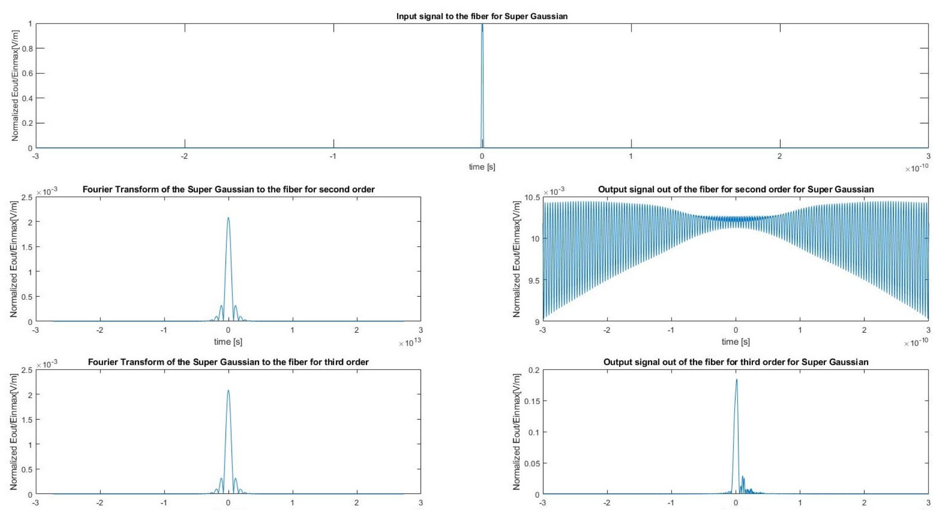

Figure 9: Effect of second and third order dispersion on a pulse of 1 ps width, both using analytical formula and numerical method technique

4.3 Pseudo-Random bit sequence

Using the same approach as in the previous cases a new simulation was developed, this time dealing with a pseudo-random bit sequence of 15 values. The goal was to simulate the effects of the fiber on the sequence and compare the output pulses using an eye pattern, i.e. superposing the different output pulses reaching the output end of the fiber at different time instants. This approach helps to characterized the behavior of the fiber through time, not anymore for one isolated pulse. Therefore, the quantification of a system using random, varying values improve our knowledge on how this system responds to partly random, varying signal as it is in real operation conditions. Additionally, the pseudo-random sequences are random sequences which have been recorded deterministically. In other word, even if those pulses are the result of a random process while recording those, those can be reused several time. This recording and reuse of such values make them loose their pure randomness, those then become pseudo-random. Such signals are very useful practically for various reasons. Usually it's quite of a harsh to obtain pure random values, having pseudo-random sequence help to overcome this problem.

4.3.1 Simulations method

Generations of pseudo-random bit sequence are performed, those are sent into the fiber under the form of Gaussian pulses.

It has to be noted that :

- on all the pictures the x-axis represent the time coordinate (in seconds) of the pseudo-random bit relative to a particular time referential, i.e. index 0.
- the pulse (bits) initial amplitude is always 1
- the final pulses, out of the fiber, are normalized to the input pulses.

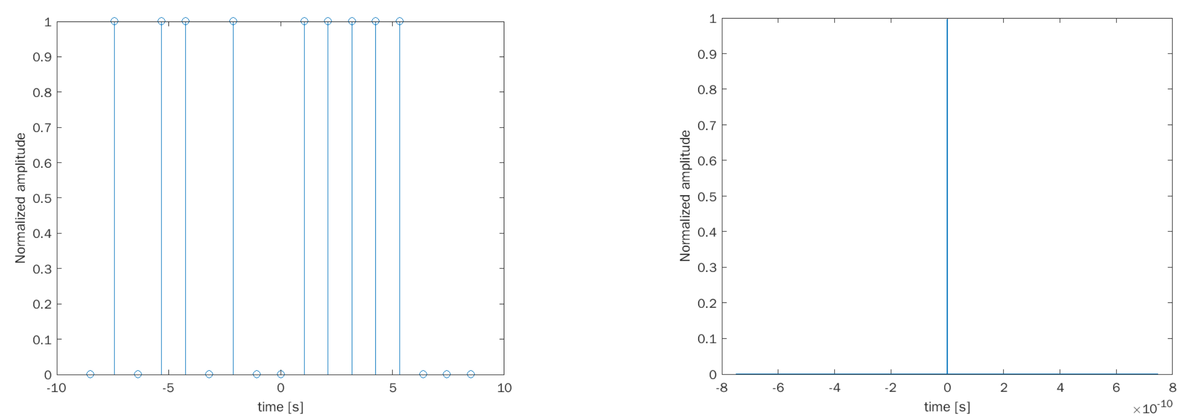

Figure 10: On the left, the pseudo random bit sequence of length n = 15. On the right, the Gaussian signal under which we send the pulses.

The signal sent at into the fiber is therefore the following.

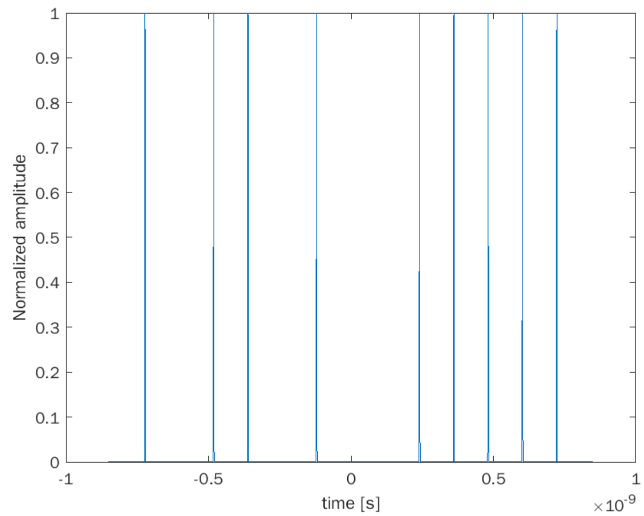

Figure 11: Random bit sequence of Gaussian pulses sent into the fiber

Following the same procedure as in the part of the report, the pulses, crossing the fiber, undergo the effect of dispersion. Second and third order dispersion were simulated and are represented below.

4.3.2 Pulse width of 50ps

Second order dispersion :

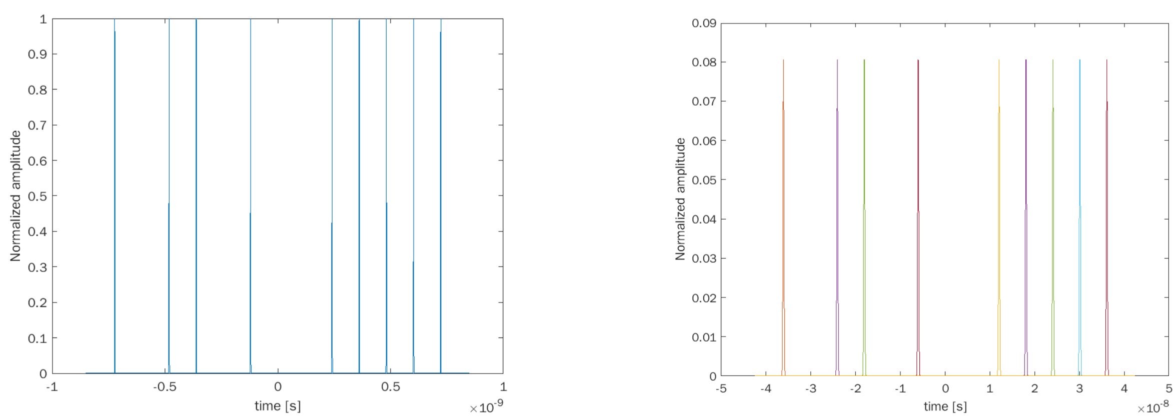

Figure 12: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit second order dispersion.

Third order dispersion :

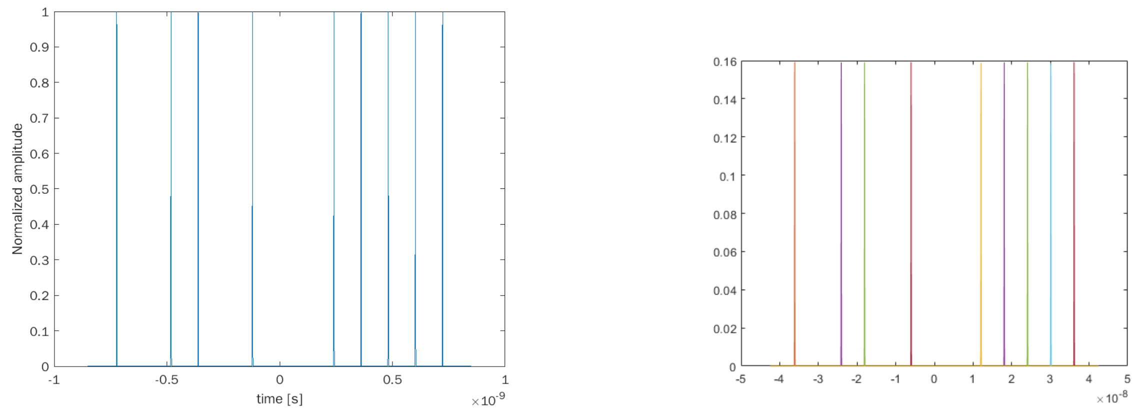

Figure 13: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit third order dispersion.

An important observation is that is the case of a pulse width of 50ps, the dispersion for both second and third order is not really significant as there is not ISI(Intersymbol Interference), the system is therefore still functional.

4.3.3 Pulse width of 10ps

Second order dispersion :

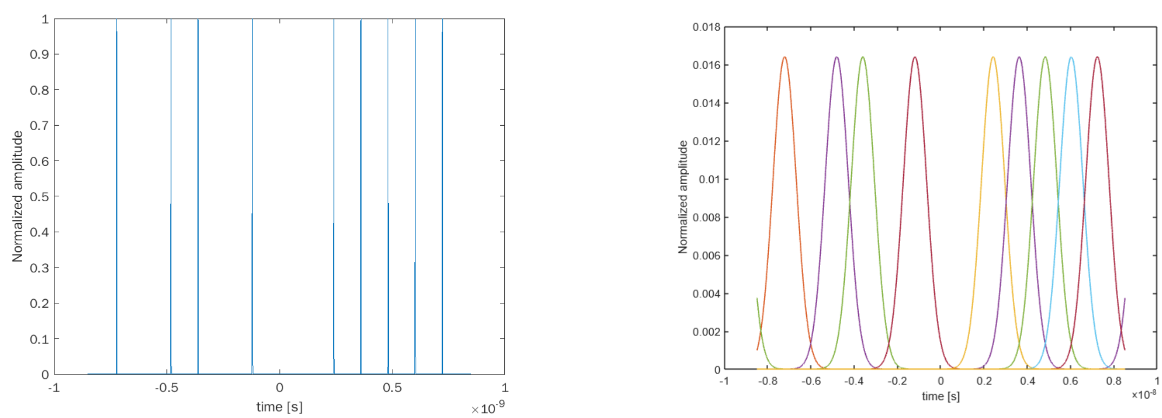

Figure 14: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit third order dispersion.

Third order dispersion :

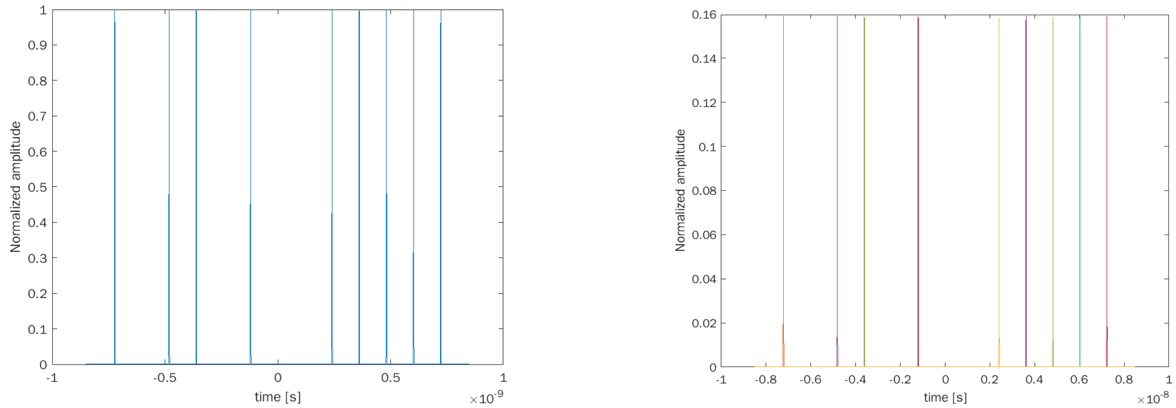

Figure 15: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit third order dispersion.

Reaching a pulse width of 10 ps the system's functionality strongly decreases. The 3rd order dispersion is not really significant but the second order dispersion greatly broaden the pulses, resulting in a big overlap between adjacent pulses. Making it harder to detect individually, the phenomenon is called ISI.

4.3.4 Pulse width of 1ps

Second order dispersion :

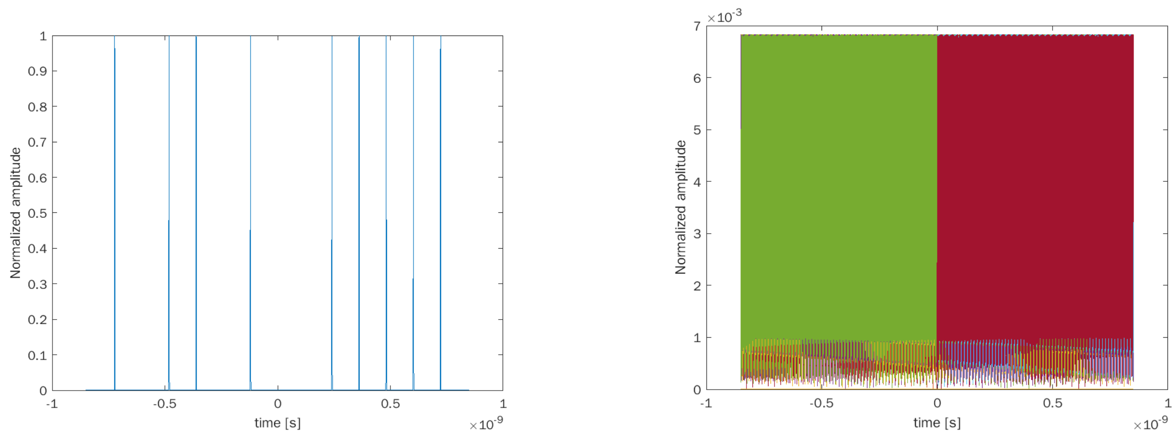

Figure 16: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit second order dispersion.

Third order dispersion :

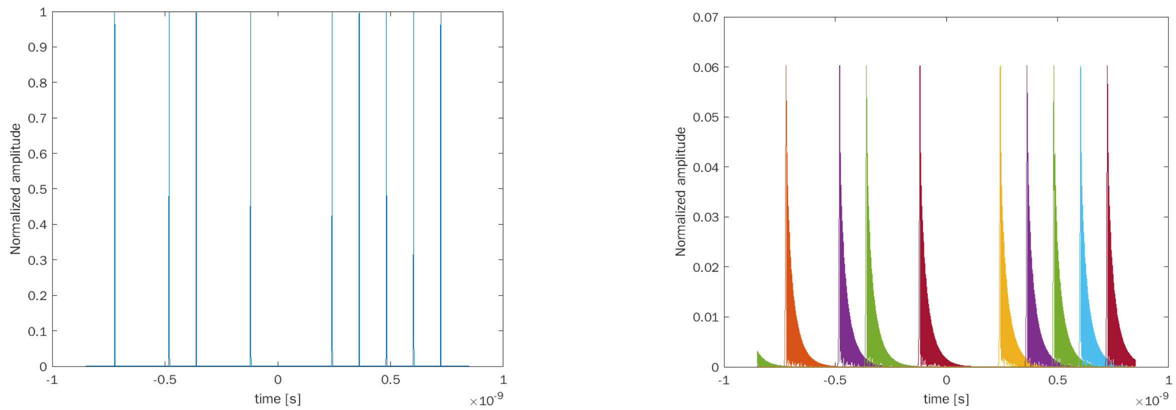

Figure 17: On the left, the pseudo random bit sequence of Gaussian pulses entering the fiber. On the right, the Gaussian signals at the output of the fiber after they exhibit third order dispersion.

Finally, reaching a pulse width of 1ps the system's functionality plummet, the second order dispersion makes it impossible to detect different pulses, not only adjacent pulses overlap each other but all the pulses at the same time. The energy, representing the message, the useful information is not anymore located around a particular time index but is spread all over the time domain. This last observation means that no information can be detected at the output end of the fiber. This is observed on the right of figure 16. In this case the profile of third order dispersion can be very well seen on the right of figure with the exponential decrease of figure 17.

5 Conclusion

Throughout this report the effect of optical fiber on different pulses shapes and numbers were performed analytically and by simulations. Two main phenomena were studied : the second and third order dispersion. As predicted and observed, the first one results in a broaden of the pulse at the output, this effect increasing as the width of the pulse decreases, this being caused by the increasing number of frequencies in the spectrum, each one travelling at a different speed and then arriving at different time instants at the output end. The second one, third order dispersion, is an effect appearing when the width of the pulse is very small, around 1ps, this effect results in a decreasing exponential spread of the pulse. Finally, the last part was dedicated to study the effect of several, independent pulses sent into the fiber, this in order to quantify the functionality of the fiber under near real conditions. The major effect that came out is that the quality of the system under near real condition drops rapidly as the pulses' width decreases, due to overlapping of the pulses or, in order word, to the spreading of the energy through time, making the deduction of independent pulses harder or impossible. This last observation making the system non functional for pulses below 10ps. A track of improvement, to increase the range of width of the pulses under which the system is functional, would be to use some recovery methods that detect were the maximum of the energy (peak detection) would be located.