Computation of the field radited by a metasurface antenna

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For this assignment, the first step was to obtain the equivalent surface currents from the coefficients of the Fourier-Bessel functions. It was also asked to obtain the radiation pattern of this antenna's metasurface using 2 methods : the first one being using the Fourier transforms(FT) of the Fourier-Bessel Basis Function (FBBF) and the second one by realizing the FFT of the current obtained using the surfaces equivalent currents. In this report, the different steps taken to answer this questions as well as the results obtained will be detailed for the reader.

1 Convention used

In the following report, it was decided the same angle conventions as the ones used in the course.

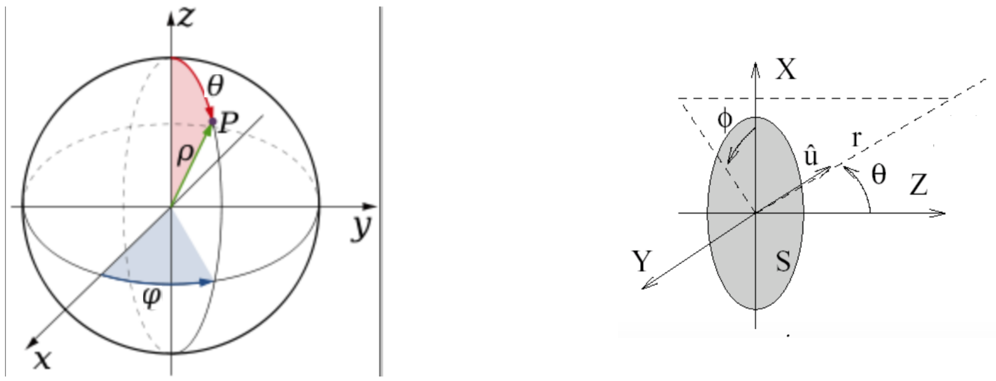

Figure 1 – Used Conventions

2 Calculation of equivalent surface currents

At first, using a Fourier-Bessel decomposition, we calculated the 2 components of the tangential field (Ex and Ey) in polar coordinates. These are given by the following equation :

$$\overrightarrow{E}(r,\theta) = \sum_{n=-8}^{n=8} \sum_{m=1}^{m=46} C_{m,n} \cdot J_n(\lambda_n^m \frac{r}{R}) \cdot e^{-jn\theta} \tag{1}$$

where $C_{m,n}$ represents the coefficients of the Fourier-Bessel functions and λ_n^m represents the zeros of these functions (calculated using the function provided : besselzero). The $C_{m,n}$ coefficiants were then provided for both the x and y field.

Given these fields the surface current distribution can be find by applying the equivalence principle on the aperture. This is done with the following well-known relation :

$$\begin{cases} \overrightarrow{K}_m = \overrightarrow{E} \times \hat{n} \\ \overrightarrow{K} = \hat{n} \times \overrightarrow{H} \end{cases}$$

Using the image method applied on planar apertures it is the assumed that $\overrightarrow{K}_m + \overrightarrow{K} = 2 \cdot \overrightarrow{K}_m$, the expression of the 2 components of these surface currents is given by :

$$\begin{cases} \overrightarrow{K}_x = 2 \cdot \overrightarrow{E}_y \\ \overrightarrow{K}_y = -2 \cdot \overrightarrow{E}_x \end{cases}$$

Figure 2 and 3 display representations of the distribution of the 2 components of the surface current on the circular aperture using Matlab.

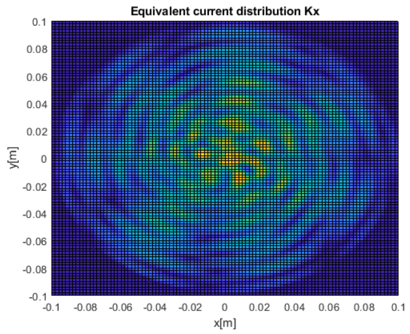

FIGURE 2 – Equivalent current distribution kx

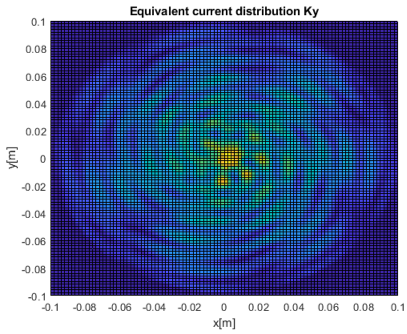

FIGURE 3 – Equivalent current distribution ky

3 Fourier transform (FT) of F-B basis functions

The Fourier transforms of Fourier-Bessel function are known to be given by :

$$\tilde{R}_m^n(k_\rho, \phi) = -2\pi j^n e^{-jn\phi} \left(\frac{a^2 \lambda_n^m J_{n-1}(\lambda_n^m) J_n(k_\rho a)}{(\lambda_n^m)^2 - (k_\rho a)^2} \right) \qquad k_\rho = k\sin(\theta) \tag{2}$$

With "a" being the radius of the apperture.

The Fourier Transform of the currents can then be obtained by the Fourier Transform of the fields, using FT of FBBF we obtain its analytical expression :

$$f_t(\theta, \phi) = \sum_{n=-8}^{n=8} \sum_{m=1}^{m=46} C_{m,n} \cdot \tilde{R}_m^n \tag{3}$$

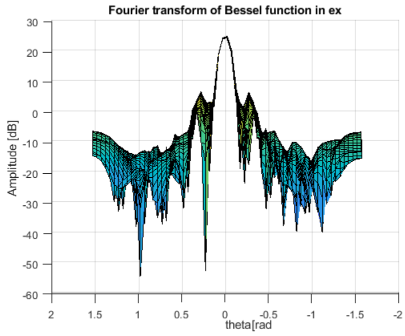

FIGURE 4 – FT of Fourier-Bessel function in direction x

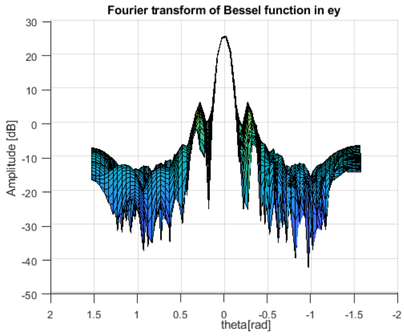

FIGURE 5 – FT of Fourier-Bessel function in direction y

4 Computation of the radiation pattern using Fourier-Bessel method

From the expression of the Fourier Transform of the field $\overrightarrow{f_t}$ the radiation pattern can be derived with twice the magnetic current :

$$\overrightarrow{F}(\theta,\phi) = \frac{-j}{\lambda}[\hat{u} \times (\hat{n} \times \overrightarrow{f_t})] \tag{4}$$

This equation is given in cartesian coordinates and can be found in figure 6 and 7 . In order to draw the $\overrightarrow{e_\theta}$ and $\overrightarrow{e_\theta}$ components of the pattern we have to express it in spherical coordinates such that $(\overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z}) \rightarrow (\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_\theta})$. These can be found in figure 8 and 9

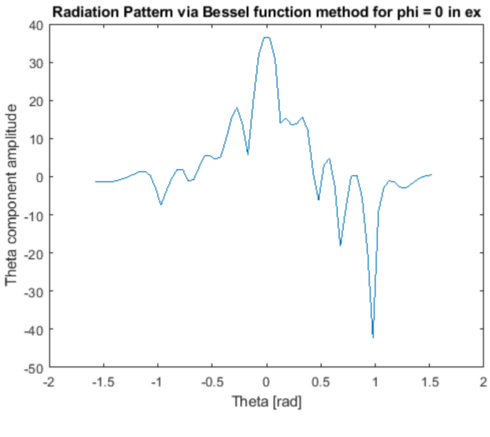

FIGURE 6 – Component E_x of the radiation pattern

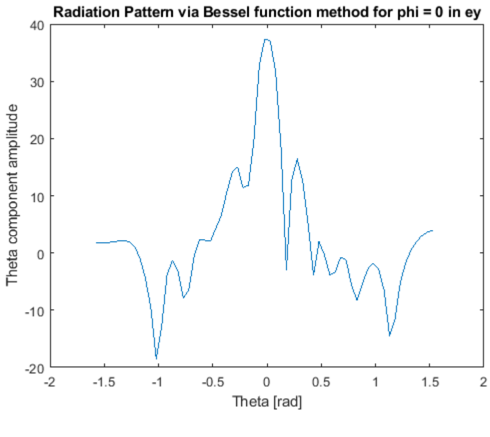

FIGURE 7 – Component E_y of the radiation pattern

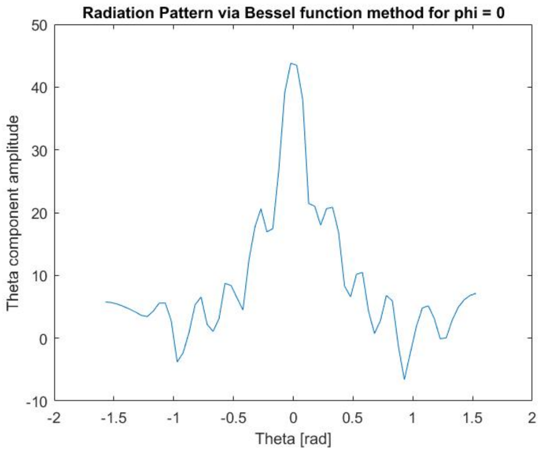

FIGURE 8 – Component E_θ of the radiation pattern

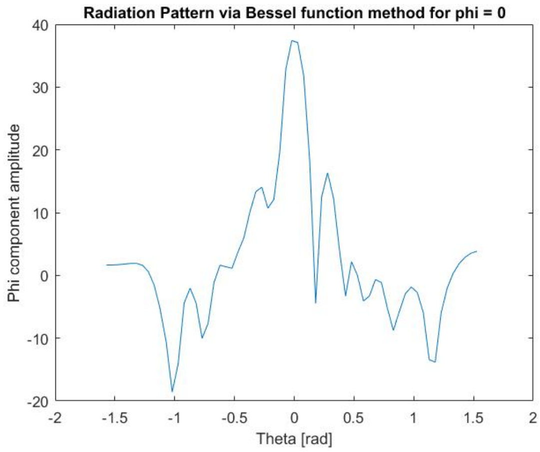

FIGURE 9 – Component E_ϕ of the radiation pattern

5 Computation of the radiation pattern using FFT method

This time the equivalent surface current was used through FFt to evaluate the radiation pattern. The FFT should be evaluated only in certain direction. It is in fact a 2D-FFT, it is performed using fft2 function in Matlab. The 2D-fft is equivalent at making the DFT but do it using a particular algorithm. It gives us as much output values as its inputs. Considering the aperture, it was discretised in cartesian domain. A grid was considered and meshed [-a,a] in x and y direction.

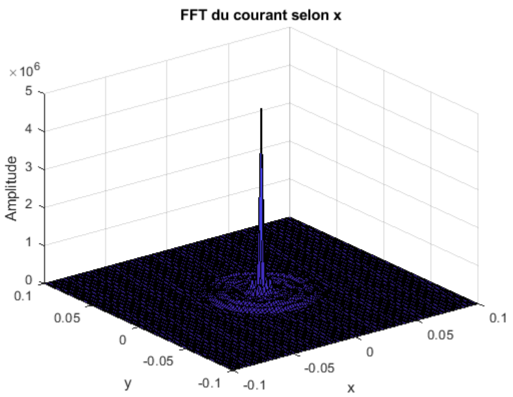

FIGURE 10 – FFT du courant selon x

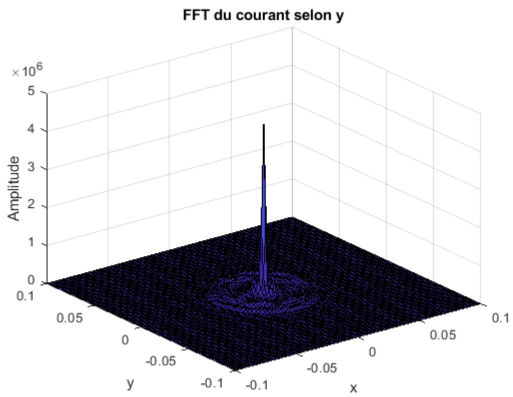

FIGURE 11 – FFT du courant selon y

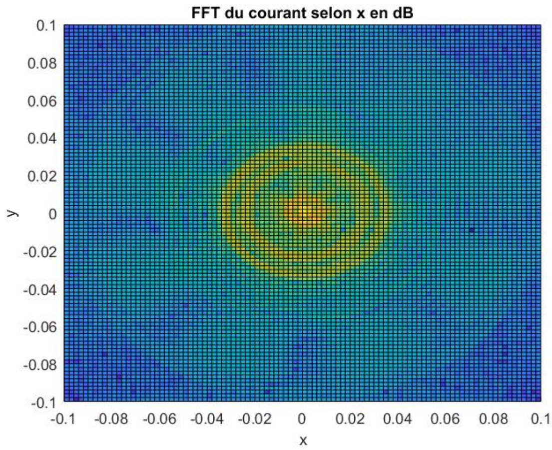

FIGURE 12 – FFT du courant selon x [dB]

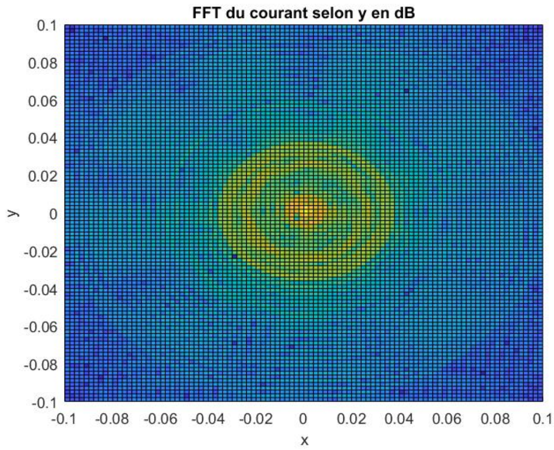

FIGURE 13 – FFT du courant selon y [dB]

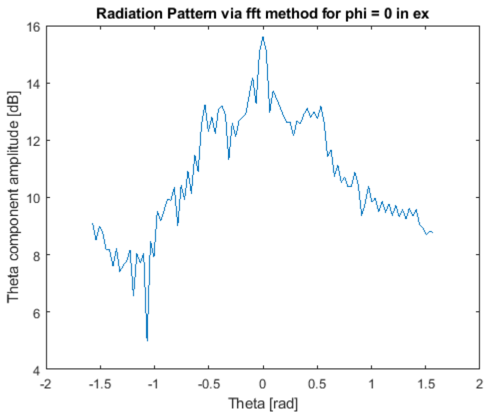

FIGURE 14 – Component x of the radiation Pattern

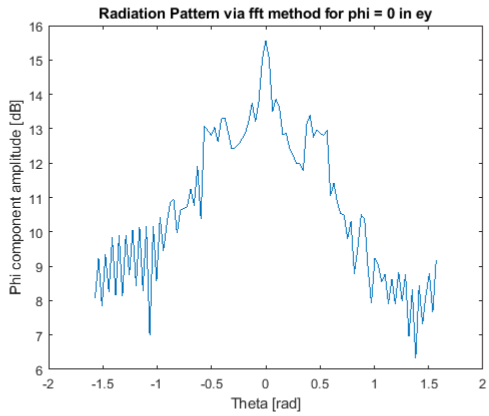

FIGURE 15 – Component y of the radiation Pattern

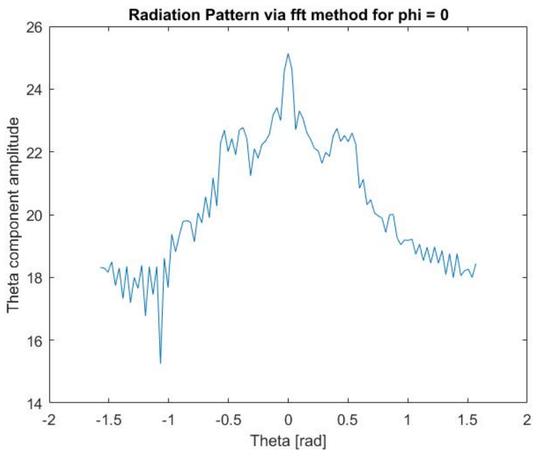

FIGURE 16 – Component theta of the radiation Pattern

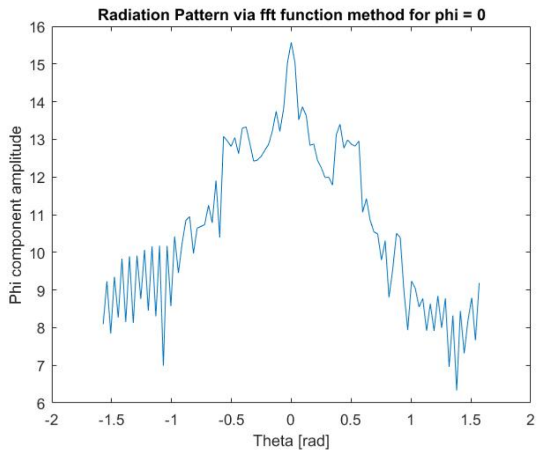

FIGURE 17 – Component phi of the radiation Pattern

6 Computation time

The computation time of the analytical part of the code is 28.327318 seconds and the fft part is 0.2472 seconds.

```
>> FourierBesselWain
Elapsed time is 28.327318 seconds.
Elapsed time is 0.247200 seconds.
```

FIGURE 18 – Computation time

The analytic part takes a lot longer because we had to implement it with 5 loops which is very expensive in computing time. On the other hand, the fft (fast fourier transform) method is a fast function by its name but also optimized by Matlab.