HW #2

1.

a. False: Na+ conductance increases the most first during the action potential and is responsible for the larger current
b. False: Na+ current depolarizes/increases membrane potential (over 0mv) and K+ current acts to repolarize the cell (decrease the cell potential under 0mv)
c. True
d. False: the EEG does not have enough resolution to measure individual action potential but rather measures general neural activity like LFP's
e. False: in Poisson process all events are independent and will have the same chance of firing at any given time regardless of when the last firing occurred.
f. False: it means the variance Is greater than the mean if it is greater than 1
g. True : <If the time window was infinity, it would approach 1 if it was a truly Poisson process>
h. False: in the exponential interspike distribution will have a skewed distribution close to zero but refectory period limits a minimum time in between spikes
i. True
j. True
k. True
l. True
m. False: It's a type of low pass filtering
n. False: Static tuning curves don't well represent activity in motor systems because these are time varying signals
o. False: very unlikely not impossible

2.

a) If all variables except θ are constant, then the only thing that changes is what is inside the cosine function. The cosine of a function is at a maximum (of 1) when its inside is equal to zero, and for that to happen θ = θo.
b) “You’ve made a mistake.”: With those numbers, the equation will be f(θ) = -11 + 8*cos(θ - 125) and for all θ it will predict negative frequency firing rates which is not possible

C, D, and E see notes Below:

$\boxed{2c}$ $\cos(\theta - \theta_0) = \mathrm{Re}\left(e^{i(\theta - \theta_0)}\right)$

$= \mathrm{Re}\left(e^{i\theta}\, e^{-i\theta_0}\right)$

euler euler $(-\cos(-\theta_0) - i\sin(-\theta_0))$

$= \mathrm{Re}\left((\cos\theta + i\sin\theta)(\cos(-\theta_0) + i\sin(-\theta_0))\right)$

$\cos(\theta - \theta_0) = \boxed{\cos(\theta)\cos(\theta_0) + \sin(\theta)\sin(\theta_0)}$

$\boxed{2d}$ $f(\theta) = c_0 + c_1 \cos(\theta - \theta_0)$

$f(\theta) = c_0 + c_1(\cos(\theta)\cos(\theta_0)) + c_1(\sin\theta \sin(\theta_0))$

$f(\theta) = k_0 + k_1 \sin(\theta) + k_2 \cos(\theta)$

$\boxed{k_0 = c_0}$, $\boxed{k_1 = c_1 \cdot \sin(\theta_0)}$, $\boxed{k_2 = c_1\cos(\theta_0)}$

$\boxed{2e}$ $y(120) = k_0 + k_1 \sin(120) + k_2 \cos(120) = k_0 + .86 k_1 - \frac{1}{2}(k_2)$

$y(240) = k_0 + k_1 \sin(240) + k_2 \sin(240) = k_0 - .86 k_1 - \frac{1}{2}(k_2)$

$y(0) = k_0 + k_1 \sin(0) + k_2 \cos(0) = k_0 + 0 + k_2$

$y(0) + y(120) + y(240) = 3k_0$ $\boxed{k_0 = \frac{y(0) + y(120) + y(240)}{3}}$

$k_2 = y_0 - k_0 =$ $\boxed{k_2 = -\left(\frac{y(0) + y(120) + y(240)}{3}\right) + y_0}$

2E continued below, 2G

2e) continued ...

$$y_{240} - y_{120} = -\frac{\sqrt{3}}{2}k_1 - \frac{\sqrt{3}}{2}k_1 = -\sqrt{3}k_1 = y240 - y120$$

$$\boxed{k_1 = \frac{-y(240)}{\sqrt{3}} + \frac{y(120)}{\sqrt{3}}}$$

$$k_1 = \frac{1}{\sqrt{3}}y(120) - \frac{1}{\sqrt{3}}y(240)$$

$$k_0 = \frac{1}{3}y(0) + \frac{1}{3}y(120) + \frac{1}{3}y(240)$$

$$k_2 = \frac{2}{3}y(0) - \frac{1}{3}y(120) - \frac{1}{3}y240$$

2F.)

```
def ptc(y0 , y1 , y2):
    #PTC calculates the tuning curve given average firing rates for certain directions.

    # ================================================================= #
    # YOUR CODE HERE:
    #  The function takes three inputs corresponding to the average
    #  firing rate of a neuron during a reach to 0 degrees (y0), 120
    #  degrees (y1) and 240 degrees (y2). The outputs, c0, c1, and
    #  theta0 are the parameters of the tuning curve.
    # ================================================================= #
    k0 = (y0 + y1 + y2)/3
    k2 = -y1/3 + (2/3)*y0 -y2/3
    k1 = y1/1.73 - y2/1.73

    c0 = k0
    theta0 = np.arctan(k1/k2 )
    c1 = k1/np.sin(theta0 )
    theta0 = theta0 * 180/np.pi

    # ================================================================= #
    # END YOUR CODE HERE
```

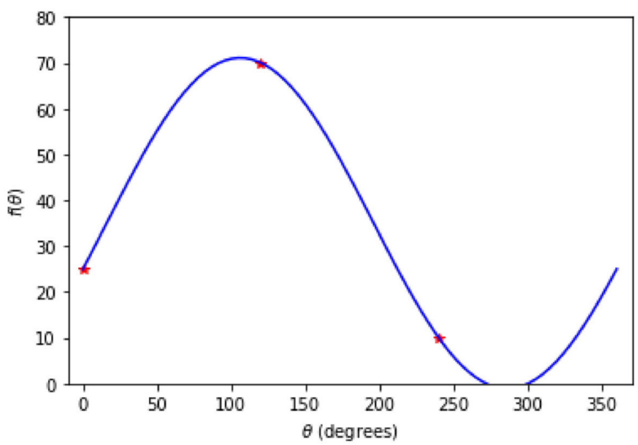
Plot the figure

The following cells execute your PTC function, printing out the values and plotting the tuning curve.

```
c0,c1,theta0=ptc(25,70,10)
print('c0 = ', c0)
print('c1 = ', c1)
print('theta0 = ', theta0)
```

```
c0 =  35.0
c1 =  -36.094968309699695
theta0 =  -73.91596521992966
```

```
theta = np.linspace(0, 2*np.pi, num=80)
plt.plot([0, 120, 240 ],[25, 70, 10],'r*',10)
plt.plot(theta * 180 / np.pi,c0 + c1 *np.cos(theta - theta0 * np.pi/180),'b',2)
plt.xlim ([-10 ,370])
plt.ylim ([0,80])
plt.xlabel(r'$\theta$ (degrees)');
plt.ylabel(r'$f(\theta)$');
```



2G

2G) $f_\theta = k_0 + k_1 \sin(\theta) + k_2 \cos(\theta)$

$$
B = \begin{bmatrix} 25 \\ 40 \\ 70 \\ 30 \\ 10 \\ 15 \end{bmatrix}
\begin{matrix} y_0 \\ y_{60} \\ y_{120} \\ y_{180} \\ y_{240} \\ y_{300} \end{matrix}
=
\begin{matrix}
k_0 + 0\,k_1 + 1\,k_2 \\
k_0 + .866\,k_1 + .5\,k_2 \\
k_0 + .866\,k_1 + -.5\,k_2 \\
k_0 + 0\,k_1 + -1\,k_2 \\
k_0 + -.866\,k_1 + -.5\,k_2 \\
k_0 + -.866\,k_1 + .5\,k_2
\end{matrix}
$$

$$
\underset{Y}{\begin{bmatrix} 25 \\ 40 \\ 70 \\ 30 \\ 10 \\ 15 \end{bmatrix}}
=
\underset{F}{\begin{bmatrix} 1 & 0 & 1 \\ 1 & .866 & .5 \\ 1 & .866 & -.5 \\ 1 & 0 & -1 \\ 1 & -.866 & -.5 \\ 1 & -.866 & .5 \end{bmatrix}}
\underset{W}{\begin{bmatrix} k_0 \\ k_1 \\ k_2 \end{bmatrix}}
$$

$$W^* = (F^T F)^{-1} F^T Y$$

From Computer

$k_0 = 31.66 \quad k_1 = 24.5 \quad k_2 = -5.83$

$\boxed{C_0 = k_0 = 31.66}$ $\quad \theta_0 = \tan^{-1}\left(\frac{k_1}{k_2}\right) = \boxed{-76.6 = \theta_0}$

$C_1 = \frac{k_1}{\sin(\theta_0)} = \boxed{-25.18}$

…Computer Matrix Calculations used above to solve for K values

A

Dimensions: 6 by 3

1	0	1
1	.866	.5
1	.866	-.5
1	0	-1
1	-.866	-.5
1	-.866	.5

B

Dimensions: 6 by 1

25
40
70
30
10
15

Calculate least squares solution

Least Squares Solution: X =

31.666666666666664
24.53810623556582
-5.833333333333333

3).

a). **No it does not**. The exponential distribution model assumes there will be non-zero probability amounts of firing directly after time 0 for ISI activity, however with refractory periods we know that there will be no firing for the first ~0-4ms (refectory period).

b).

Given λ = 50 spikes/sec

P(T<.001) = $(1-e^{-\lambda t})$ = .0487

Answer: around 5% would violate the 1ms refractory period

4.

a) Mean = 1/ λ
b) P(T > 1/ λ) = e^-1 = 36.7%

4C) $E\left[T \mid T > \frac{1}{\lambda}\right] \rightarrow E[T \mid A]$

$$f_T(t \mid A) = \frac{\lambda e^{-\lambda t}}{P(A)} = \begin{cases} \dfrac{\lambda e^{-\lambda t}}{e^{-\lambda \frac{1}{\lambda}}}, & t > \frac{1}{\lambda} \\ 0, & t < \frac{1}{\lambda} \end{cases}$$

$$E[T \mid A] = \int_{\frac{1}{\lambda}}^{\infty} (t)\frac{\lambda e^{-\lambda t}}{e^{-1}}\,dt = e\lambda \int_{\frac{1}{\lambda}}^{\infty} t e^{-\lambda t}\,dt = e\lambda \frac{(2e^{-1})}{\lambda^2} = \boxed{\frac{2}{\lambda}}$$

4D) $E\left[T \mid T < \frac{1}{\lambda}\right]$

$$E[T \mid A] = \int t\, f_T(t)\,dt \qquad f_{t \mid A} = \begin{cases} 0 & t > \frac{1}{\lambda} \\ \dfrac{\lambda e^{-\lambda t}}{(1 - e^{-\lambda \frac{1}{\lambda}})} & 0 < t < \frac{1}{\lambda} \end{cases}$$

$$\int_0^{\frac{1}{\lambda}} t \cdot \frac{\lambda e^{-\lambda t}}{(1 - e^{-1})}\,dt = \left(\frac{\lambda}{1 - e^{-1}}\right) \int_0^{1/\lambda} e^{-\lambda t}\, t\,dt = \boxed{\frac{-e+2}{(e-1)\lambda}}$$

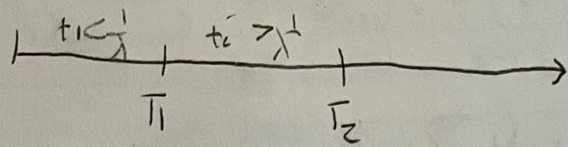
4E) $\underline{t < \frac{1}{\lambda}}$ $\underline{t < \frac{1}{\lambda}}$ ... $\underset{k-1}{\underline{t < \frac{1}{\lambda}}}$ $\underset{k}{\underline{t > \frac{1}{\lambda}}}$

$$P(N = k) = (1 - p)^k (p)$$

$$E[N] = \frac{1}{P(T > e^{-\lambda t})} = \boxed{\frac{1}{e^{-1}}}$$

neuron 1 neuron 2

4F)



$T = t_1 + t_2$

Let X = # spikes before ISI $> 1/\lambda$

$$T = \sum_{i=1}^{X} t_i \qquad f_{T|X}(t|x) \sim \text{gamma}$$

$$f_{T|X}(t|x) = f_{T_X}(t) = \frac{\lambda e^{-\lambda t} (\lambda t)^{(x-1)}}{(x-1)}$$

$$E[T|X=x] = \int_0^\infty t \cdot f_{T|X}(t)\,dt = \frac{x}{\lambda}$$

$$E[T] = E[E[T|X=k]]$$

$$E\left[\frac{X}{\lambda}\right] = e \Rightarrow E[T] = \boxed{\frac{e}{\lambda} = E[T]}$$

5)

5a) neuron 1 · neuron 2

$P(T > 60ms) \cdot P(T > 60ms)$

$\lambda_1 = \frac{1}{20ms} = 50 \qquad \lambda_2 = \frac{1}{30ms} = 33.3$

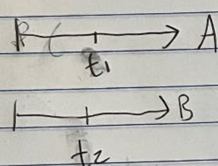
$= (e^{-\lambda 60ms}) \cdot (e^{-\lambda 60ms}) = e^{-3} \cdot e^{-2} = \boxed{.67\%}$

5b) $P(T_1 > s+t \mid T_1 > s) \cdot P(T_2 > s+t \mid T_2 > s)$ Since $t_1 \perp\!\!\!\perp t_2$

$= P(T_1 > t) \cdot P(T_2 > t)$

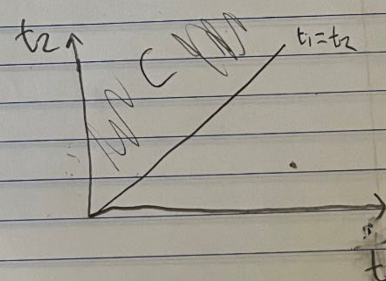
$= e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} = \boxed{e^{-83.3t}}$

5c)



Let t_1 denote spike time at a

Let t_2 denote spike time arrival at b



Compute $P(t_2 > t_1)$

$$P(t_2 > t_1) = \iint_C f_{t_1, t_2}(t_1, t_2)$$

$$\iint f_1(t_1) f_2(t_2) = \iint \lambda_1 e^{-\lambda_1 t} \cdot \lambda_2 e^{-\lambda_2 t_2} \, dt_1 dt_2$$

$$= \lambda_1 \cdot \lambda_2 \int_0^{\infty} \int_0^{t_2} e^{-\lambda_1 t_1} e^{-\lambda_2 t_2} \, dt_1 dt_2$$

$$= \lambda_1 \lambda_2 \int_0^{\infty} e^{-\lambda_2 t_2} \left(\frac{1}{\lambda_1} (1 - e^{-\lambda_1 t_2}) \right) dt_2 = \lambda_2 \left[-\frac{1}{\lambda_2} e^{-\lambda_2 t_2} + \frac{1}{\lambda_1 + \lambda_2} e^{-(\lambda_1 + \lambda_2) t_2} \right.$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{50}{50 + 33.33} = \boxed{.6}$$