GLOBAL STABILITY FOR AN MSEIR MODEL

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Abstract: In this paper, we consider an MSEIR model of a disease spreading though a population. This model accounts for passive immunity given to a certain fraction of the population, which makes them immediately immune to a disease, but only for a temporary amount of time. Later on, a Lyapunov function is created for an MSEIR disease model to resolve the global dynamics for $\mathcal{R}_0 > 1$. We show the endemic equilibrium is globally stable for $\mathcal{R}_0 > 1$.

1. Introduction

In this paper, we consider an MSEIR model of a disease spreading though a population with mass action incidence $\beta S(t)I(t)$. The M class is the number of individuals in the population who have temporary passive immunity to a disease. The SEIR remains as a general SEIR model, which will be discussed thoroughly later in the paper.

There are two types of immunity: active and passive. Active immunity is when the body naturally creates the antibodies needed to make the body immune. Active immunity is long-lasting (sometimes life-long), but it does not have an immediate effect. Passive immunity, on the other hand, is provided when a person is given antibodies to a disease (such as a mother passing it down to her child) which has an immediate effect. However, this effect is only temporary [1]. We will separate the passively immune humans from the humans who gain immunity after acquiring the disease.

In this paper, we show that if the basic reproduction number $\mathcal{R}_0 < 1$, then the disease-free equlibrium is stable. If $\mathcal{R}_0 > 1$ then the disease-free equilibrium is unstable and there is an endemic equilibrium which is locally stable. Furthermore, we can use a Lyapunov functional to show that this endemic equilibrium is globally asymptotically stable.

2. Model

A population is divided into five classes - **M** (Passively immune), **S** (Susceptible), **E** (Exposed), **I** (Infectious), and **R** (Recovered), which represent the total population of each class. Individuals are either born into the M or the S class. We will denote p to be the fraction of individuals who do not have passive immunity. We will take Λ to be the total birth rate into the population, so $(1-p)\Lambda$ is the birth rate into the M class, and $p\Lambda$ is the birth into the S class. There is a natural death rate d associated with each class. Passively immune individuals will leave class M and join S at some rate δM. Susceptible individuals are infected at a constant rate β. After being infected, susceptibles move into the E class. After some time, they will move from E to I at a rate ϵE, and after some time, an individual will recover and move from I to R at some rate γI. We can create a transfer diagram as follows:

$$\begin{tikzcd}
{} \arrow[d, "(1-p)\Lambda"'] & {} \arrow[d, "p\Lambda"'] & & & \\
M \arrow[r, "\delta M"] \arrow[d, "dM"'] & S \arrow[r, "\beta SI"] \arrow[d, "dS"'] & E \arrow[r, "\epsilon E"] \arrow[d, "dE"'] & I \arrow[r, "\gamma I"] \arrow[d, "dI"'] & R \arrow[d, "dR"'] \\
{} & {} & {} & {} & {}
\end{tikzcd}$$

Note: all parameter values are positive.

The differential equations associated with the above transfer diagrams are:

(1) $\frac{dM}{dt} = (1-p)\Lambda - M(d+\delta)$
(2) $\frac{dS}{dt} = p\Lambda + \delta M - dS - \beta SI$
(3) $\frac{dE}{dt} = \beta SI - E(d+\epsilon)$
(4) $\frac{dI}{dt} = \epsilon E - I(d+\gamma)$
(5) $\frac{dR}{dt} = \gamma I - dR$

Since R does not appear in any of the other equations, we can say it decouples from the system, and we only need to look at the first four equations.

3. Equilibria

There will be two equlibria that arise in this system. One which will be the disease-free equilibrium (DFE), and an endemic equlibrium point (EEP). Studying the stability of these equilibria will help determine how the system will behave. To determine stability, we must first determine the coordinates of each equilibrium. To find the coordinates of the DFE, we set all equations equal to 0, and solve for the coordinates. Since we are at a DFE, then I=0. We proceed with:

(1) $\frac{dM}{dt} = 0 = (1-p)\Lambda - M(d+\delta)$
(2) $\frac{dS}{dt} = 0 = p\Lambda + \delta M - dS - \beta S \cdot \mathbf{0}$
(3) $\frac{dE}{dt} = 0 = \beta S \cdot \mathbf{0} - E(d+\epsilon)$
(4) $\frac{dI}{dt} = 0 = \epsilon E - \mathbf{0}(d+\gamma)$

We can easily see that equation (4) implies that E must be 0 since all parameters are positive. We are just left with the first two equations:

(1) $0 = (1-p)\Lambda - M(d+\delta)$
(2) $0 = p\Lambda + \delta M - dS$

To solve this two equation, two unknown system, we can solve for M in (1) and substitute into (2):

(1) $\implies M = \frac{(1-p)\Lambda}{d+\delta}$

Substitute into (2):

(2) $\implies S = \frac{p\Lambda + \delta\frac{(1-p)\Lambda}{d+\delta}}{d} = \frac{\Lambda(dp+\delta)}{d(d+\delta)}$.

Therefore, our DFE point is $(M, S, E, I) = \left(\frac{(1-p)\Lambda}{d+\delta}, \frac{\Lambda(dp+\delta)}{d(d+\delta)}, 0, 0\right)$

Similarily, we can solve the EEP by setting all equations equal to 0 and solving, however, $I \neq 0$. Using Maple to solve this system will yield the EEP:

$$M^* = \frac{\lambda(1-p)}{d+\delta}$$

$$S^* = \frac{(d+\epsilon)(d+\gamma)}{\epsilon \mathrm{B}}$$

$$E^* = \frac{1}{\epsilon(d+\epsilon)(d+\delta)\mathrm{B}}\left(-d^4 + (-\delta-\gamma-\epsilon)d^3 + ((-\delta-\gamma)\epsilon - \delta\gamma)d^2 + \epsilon(\mathrm{B}\lambda p - \delta\gamma)d + \delta\lambda\mathrm{B}\epsilon\right).$$

$$\mathrm{I}^* = \frac{1}{\mathrm{B}(d+\epsilon)(d+\gamma)(d+\delta)}\left(-d^4 + (-\delta-\gamma-\epsilon)d^3 + ((-\delta-\gamma)\epsilon - \delta\gamma)d^2 + \epsilon(\mathrm{B}\lambda p - \delta\gamma)d + \delta\lambda\mathrm{B}\epsilon\right).$$

Finding the stability of these equilibria is essential to understanding how the system behaves. One approach to find the stability of the DFE is to calculate the Next Generation Matrix. However, the downfall of doing this calculation is that the Next Generation Matrix only determines locally stability around the DFE. This method fails if initial values are chosen far away from the DFE. Another method uses a Lyapunov function which will determine global stability of the EEP if it exists.

4. Lyapunov Functional

Suppose $\mathcal{R}_0 > 1$. Let $g(x) = x - 1 - ln(x)$ be our test function used, and (M^*, S^*, E^*, I^*) is the EEP calculated in section 3. We define V as:

$$\begin{aligned}
V(t) &= M^* g(\frac{M}{M^*}) + A_1 S^* g(\frac{S}{S^*}) + A_2 E^* g(\frac{E}{E^*}) + A_3 I^* g(\frac{I}{I^*}) \\
&= M^*(\frac{M}{M^*} - 1 - ln(\frac{M}{M^*})) + A_1 S^*(\frac{S}{S^*} - 1 - ln(\frac{S}{S^*})) \\
&\qquad + A_2 E^*(\frac{E}{E^*} - 1 - ln(\frac{E}{E^*})) + A_3 I^*(\frac{I}{I^*} - 1 - ln(\frac{I}{I^*}))
\end{aligned} \tag{4.1}$$

Then

$$\begin{aligned}
V' &= M^*(\frac{1}{M^*} - \frac{M^*}{M}\frac{1}{M^*})\frac{dM}{dt} + A_1 S^*(\frac{1}{S^*} - \frac{S^*}{S}\frac{1}{S^*})\frac{dS}{dt} \\
&\qquad + A_2 E^*(\frac{1}{E^*} - \frac{E^*}{E}\frac{1}{E^*})\frac{dE}{dt} + A_3 I^*(\frac{1}{I^*} - \frac{I^*}{I}\frac{1}{I^*})\frac{dI}{dt} \\
&= \boxed{(1 - \frac{M^*}{M})((1-p)\Lambda - (\delta+d)M)} + \boxed{A_1(1 - \frac{S^*}{S})(p\Lambda + \delta M - \beta SI - dS)} \\
&\qquad + \boxed{A_2(1 - \frac{E^*}{E})(\beta SI - (\epsilon+d)E)} + \boxed{A_3(1 - \frac{I^*}{I})(\epsilon E - (\gamma+d)I)}
\end{aligned} \tag{4.2}$$

At the EEP, $0 = \frac{dM}{dt} = (1-p)\Lambda - \delta M^* - dM^* \implies \ldots \implies p\Lambda = \Lambda - \delta M^* - dM^*$
At the EEP, $0 = \frac{dS}{dt} = p\Lambda + \delta M^* - dS^* - \beta S^* I^*$ Substituting in the expression for $p\Lambda$ above, we eventually get $\Lambda = dM^* + dS^* + \beta S^* I^*$. These expressions for $p\Lambda and \Lambda$ will be used in the following calculations.
Looking at the first boxed term of V' (4.2) and making these substitutions:

$$
\begin{aligned}
&= (1 - \frac{M^*}{M})(\Lambda - p\Lambda - \delta M - dM) \\
&= (1 - \frac{M^*}{M})(\Lambda - (\Lambda - dM^* - \delta M^*) - \delta M - dM) \\
&= (1 - \frac{M^*}{M})(\delta M^* + \delta M + dM^* + dM) \\
&= (1 - \frac{M^*}{M})(\delta M^*(1 - \frac{M}{M^*}) + dM^*(1 - \frac{M}{M^*}) \\
&= \delta M^*(1 - \frac{M^*}{M})(1 - \frac{M}{M^*}) + dM^*(1 - \frac{M^*}{M})(1 - \frac{M}{M^*}) \\
&\boxed{= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M})}
\end{aligned} \tag{4.3}
$$

We will leave this as is and move on to looking at the second boxed term of V' (4.2) and making the substitutions given from the first term:

$$
\begin{aligned}
&= A_1(1 - \frac{S^*}{S})(p\Lambda + \delta M - \beta SI - dS)) \\
&= A_1(1 - \frac{S^*}{S})(\Lambda - dM^* - \delta M^* + \delta M - \beta SI - dS) \\
&= A_1(1 - \frac{S^*}{S})(\beta S^* I^* + \delta M^* + dM^* - dM^* - \delta M^* + \delta M - \beta SI - dS) \\
&= A_1(1 - \frac{S^*}{S})(\beta S^* I^* - \beta SI + dS^* - dS - \delta M^* + \delta M) \\
&= A_1(1 - \frac{S^*}{S})\left(\beta S^* I^*(1 - \frac{SI}{S^* I^*}) + dS^*(1 - \frac{S}{S^*}) + \delta M^*(-1 + \frac{M}{M^*})\right) \\
&\boxed{= A_1 \beta S^* I^*(1 - \frac{SI}{S^* I^*} - \frac{S^*}{S} + \frac{I}{I^*})} \\
&\boxed{\qquad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S})} \\
&\boxed{\qquad + A_1 \delta M^*(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^* S})}
\end{aligned} \tag{4.4}
$$

Looking at the third boxed term of V' (4.2):
Note: Let $\sigma = (\epsilon + d)$. For $E' = 0$, then at the EEP, $\sigma = \frac{\beta S^* I^*}{E^*}$

$$
\begin{aligned}
&= A_2(1-\frac{E^*}{E})(\beta SI - \sigma E)\\
&= A_2(1-\frac{E^*}{E})(\beta SI - \frac{\beta S^* I^*}{E^*}E)\\
&= A_2(1-\frac{E^*}{E})\big(\beta S^* I^*(\frac{SI}{S^*I^*} - \frac{E}{E^*})\big)\\
&\boxed{= A_2\beta S^* I^*(\frac{SI}{S^*I^*} - \frac{E}{E^*} - \frac{SE^*I}{S^*EI^*} + 1)}
\end{aligned}
\tag{4.5}
$$

Looking at the fourth boxed term of V' (4.2):
Note: Let $\omega = (\gamma + d) \implies$ at the EEP, $\omega = \frac{\epsilon E^*}{I^*} \implies \epsilon = \frac{\omega I^*}{E^*}$

$$
\begin{aligned}
&= A_3(1-\frac{I^*}{I})(\epsilon E - (\gamma + d)I)\\
&= A_3(1-\frac{I^*}{I})(\epsilon E - \omega I)\\
&= A_3(1-\frac{I^*}{I})(\epsilon E - \epsilon E^* \frac{I}{I^*})\\
&= A_3\epsilon E^*(1-\frac{I^*}{I})(\frac{E}{E^*} - \frac{I}{I^*})\\
&\boxed{= A_3\epsilon E^*(\frac{E}{E^*} - \frac{I}{I^*} - \frac{E}{E^*}\frac{I^*}{I} + 1)}
\end{aligned}
\tag{4.6}
$$

Combining our boxed equations from (4.3) to (4.6) back together to do more factoring:

$$
\begin{aligned}
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M})\\
&\qquad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M})\\
&\qquad + A_1\beta S^* I^*(1 - \frac{SI}{S^*I^*} - \frac{S^*}{S} + \frac{I}{I^*})\\
&\qquad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S})\\
&\qquad + A_1\delta M^*(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S})\\
&\qquad + A_2\beta S^* I^*(\frac{SI}{S^*I^*} - \frac{E}{E^*} - \frac{SE^*I}{S^*EI^*} + 1)\\
&\qquad + A_3\epsilon E^*(\frac{E}{E^*} - \frac{I}{I^*} - \frac{E}{E^*}\frac{I^*}{I} + 1)
\end{aligned}
\tag{4.7}
$$

Grouping like terms:

$$\begin{aligned}
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M} + A_1(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S})) \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^*(A_1(1 - \frac{SI}{S^*I^*} - \frac{S^*}{S} + \frac{I}{I^*}) + A_2(\frac{SI}{S^*I^*} - \frac{E}{E^*} - \frac{SE^*I}{S^*EI^*} + 1)) \\
&\quad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S}) \\
&\quad + A_3 \epsilon E^*(\frac{E}{E^*} - \frac{I}{I^*} - \frac{E}{E^*}\frac{I^*}{I} + 1)
\end{aligned} \tag{4.8}$$

Taking $A_1 = A_2$ and $A_3 = A_1 \frac{\beta S^* I^*}{\epsilon E^*}$, we get:

$$\begin{aligned}
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M} + A_1(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S})) \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^* A_1(1 - \frac{SI}{S^*I^*} - \frac{S^*}{S} + \frac{I}{I^*} + \frac{SI}{S^*I^*} - \frac{E}{E^*} - \frac{SE^*I}{S^*EI^*} + 1) \\
&\quad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S}) \\
&\quad + A_1 \beta S^* I^*(\frac{E}{E^*} - \frac{I}{I^*} - \frac{E}{E^*}\frac{I^*}{I} + 1) \\
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M} + A_1(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S})) \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^* A_1(2 - \frac{S^*}{S} + \frac{I}{I^*} - \frac{E}{E^*} - \frac{SE^*I}{S^*EI^*} + \frac{E}{E^*} - \frac{I}{I^*} - \frac{E}{E^*}\frac{I^*}{I} + 1) \\
&\quad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S}) \\
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M} + A_1(-1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S})) \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^* A_1(3 - \frac{S^*}{S} - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I}) \\
&\quad + A_1 dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S})
\end{aligned} \tag{4.9}$$

Sub in $A_1 = 1$:

$$\begin{aligned}
&= \delta M^*(2 - \frac{M}{M^*} - \frac{M^*}{M} - 1 + \frac{M}{M^*} + \frac{S^*}{S} - \frac{MS^*}{M^*S}) \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^*(3 - \frac{S^*}{S} - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I}) \\
&\quad + dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S}) \\
&= \boxed{\delta M^*(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S})} \\
&\quad + dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^*(3 - \frac{S^*}{S} - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I}) \\
&\quad + dS^*(2 - \frac{S}{S^*} - \frac{S^*}{S})
\end{aligned} \tag{4.10}$$

From $0 = S' = p\Lambda + \delta M^* - \beta S^* I^* - dS^*$
$\delta M^* = \beta S^* I^* + dS^* - p\Lambda$
$\delta M^* \le \beta S^* I^* + dS^*$
Then, looking at the boxed term in (4.10):

$$\begin{aligned}
\delta M^*(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S}) \le \beta S^* I^* &\left(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S}\right) \\
+ dS^* &\left(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S}\right)
\end{aligned} \tag{4.11}$$

Substituting (4.11) into (4.10) and regrouping, we get:

$$\begin{aligned}
V' &\le dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^* \left(3 - \frac{S^*}{S} - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I} + \left(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S}\right)\right) \\
&\quad + dS^* \left(2 - \frac{S}{S^*} - \frac{S^*}{S} + \left(1 - \frac{M^*}{M} + \frac{S^*}{S} - \frac{MS^*}{M^*S}\right)\right) \\
V' &\le dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
&\quad + \beta S^* I^*(4 - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I} - \frac{M^*}{M} - \frac{MS^*}{M^*S}) \\
&\quad + dS^*(3 - \frac{S}{S^*} - \frac{M^*}{M} - \frac{MS^*}{M^*S})
\end{aligned} \tag{4.12}$$

Reviewing what we have:

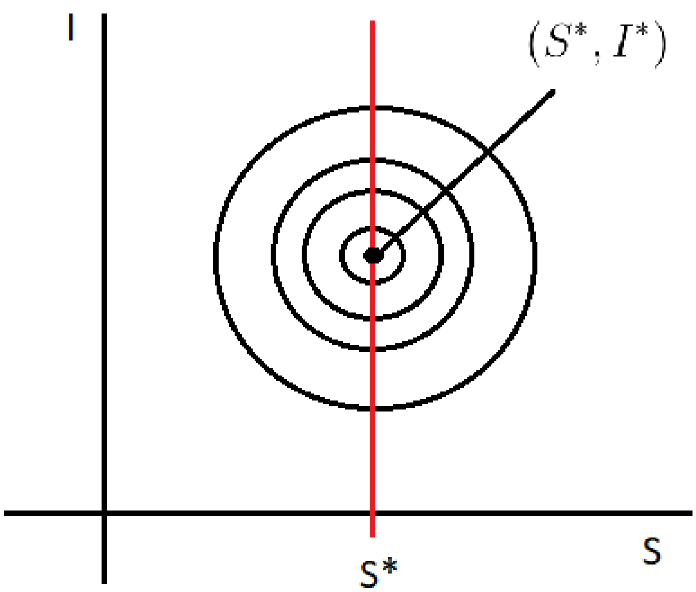
$$
\begin{aligned}
V' \leq dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M}) \\
+ \beta S^* I^*(4 - \frac{SE^*I}{S^*EI^*} - \frac{E}{E^*}\frac{I^*}{I} - \frac{M^*}{M} - \frac{MS^*}{M^*S}) \\
+ dS^*(3 - \frac{S}{S^*} - \frac{M^*}{M} - \frac{MS^*}{M^*S})
\end{aligned} \tag{4.13}
$$

Theorem 4.1. *Arithmetic Mean - Geometric Mean Inequality (AMGM)*
If $x_1, x_2, ..., x_n > 0$ *and* $x_1 \cdot x_2 \cdot ... \cdot x_n = 1$ *then* $x_1 + x_2 + ... + x_n \geq n$
$\implies n - (x_1 + x_2 + ... + x_n) \leq 0$

Looking at (4.13), we can confirm that the three terms are all less than or equal to 0 from the AMGM.
For example, look at the term $dM^*(2 - \frac{M}{M^*} - \frac{M^*}{M})$. All terms are positive.
In the brackets, we have $2 - \frac{M}{M^*} - \frac{M^*}{M}$
So, from the AMGM, n = 2. It is easy to see that $\frac{M}{M^*} \cdot \frac{M^*}{M} = 1$
So, by the last statement of the AMGM, $2 - \frac{M}{M^*} - \frac{M^*}{M} \leq 0$.
A similar method can be done for the other two terms in (4.13).
$\therefore V' \leq 0$

5. LaSalle's Invariance Principle

To interpret this result, we will use LaSalle's Invariance Principle with a more intuitive definition in lower dimension.



The above graph shows level curves in a 2D system of S and I, with a red line denoting $S = S^*$. This red line is also when $V' = 0$. With the V and g chosen at the start of section 4, as well as the calculation shown at the end of section 4, we can see that $V' \leq 0$ for S,I > 0, but $V' = 0$ if $S = S^*$. We need to confirm that $V' \neq 0$ unless we are at the equilibrium, otherwise, we do not know the EEP is globally stable.

Theorem 5.1. *Let* $M_1 = \{x : V'(x) = 0\} = \{(S, I) : S = S^*, I > 0\}$.
Let M_2 *be the largest invariant subset of* M_1. *Suppose* (S^1, I^1) *is a point in* M_2. *Since* $M_2 \subseteq M_1$, *we must have* $S^1 = S^*$ *since* M_2 *is located on the vertical line* S^*. *Look at solutions through* (S^1, I^1). *It's in* M_2 $\forall$ *time t.*
So $S(t) = S^*$ $\forall$ t.
So $\left.\frac{dS}{dt}\right|_{(S^1,I^1)} = 0 = \Lambda - dS^1 - \beta S^1 I^1$ *(from a general SI model)*
But $S^1 = S^*$,
so $0 = \Lambda - dS^1 - \beta S^1 I^1 = \Lambda - dS^* - \beta S^* I^1$
$\implies I^1 = \frac{\Lambda - dS^*}{\beta S^*}$.
If we are to solve for I^*, *we will see that* $I^1 = I^*$.
$\therefore I^1 = I^*$.
$\therefore$ *the point* (S^1, I^1) *is the EEP.*
LaSalle's Invariance Principle says:

$$\lim_{t \to \infty} d(x(t), (S^*, I^*)) = 0$$

In words, we have shown that if we take a point (S^1, I^1) in the set M_2, which is the largest invariant subset of M_1, shown by the red line, then this point has to be the EEP. i.e the only time $V = 0$ is when we are at the EEP, and we know V is decreasing everywhere else since $V' \leq 0$. In more general terms, wherever the initial conditions start, we will spiral downwards towards the EEP and we will stop moving once we reach that point since $V' = 0$.

Relating this back to the MSEIR model studied, the intuitiveness is lost since we are higher than three dimensions. However, similar properties hold and a similar effect can be shown that $V' = 0$ only when we are at the EEP, and $V' < 0$ everywhere else, meaning that where the initial conditions begin, we will approach the EEP.
$\therefore$ the EEP is indeed globally stable when $\mathcal{R}_0 > 1$.

6. References

[1] cdc.gov/vaccines/vac-gen/immunity-types.htm
[2] Nelson G. Markley, Principles of Differential Equations

Communication with Dr. C. McCluskey from Wilfrid Laurier University aided in the completion of this paper.