Large-amplitude shocklets with trapped hot-electrons in space plasmas

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(Dated: December 14, 2022)

Abstract

Large-amplitude stationary electron-acoustic (EA) waves are studied, which are developed into the shocklets with the passage of time in an unmagnetized non-isothermal plasma. The latter comprises two groups of electrons; namely the hot and cool electrons that are distinctly characterized by different electron densities and temperatures. On EA timescales, the cool electrons behave as fluid, while non-isothermal hot electrons obey the vortex-like trapped distribution in the background of immobile ions. The nonlinear fluid equations are solved together both analytically and numerically within the framework of diagonalization matrix technique. Various parameters such as the free hot-to-trapped hot electron temperature ratio (β), the cool electron density ratio (α), and the cool-to-hot electron temperature ratio (σ) are numerically analyzed for dayside auroral zone plasma, showing a significant modification of solitary and shocklet structures. The plasma model is also applied to the electron diffusion region at the earth magnetopause, revealing the potential excitations depending significantly on the trapping parameter. The findings of these are important for understanding the nonlinear properties and steepening effects of the EA waves, especially in auroral zone and electron diffusion regions of earth's magnetosphere, where different types of electron populations are observed.

I. INTRODUCTION

Numerous spacecraft/satellite missions have revealed interesting observations on electron-acoustic (EA) structures in different regions of earth's magnetosphere. For instance, the Fast Auroral SnapshoT (FAST) [1, 2] has focused on the auroral regions at altitude less than 4000km while Geotail [3, 4] (Polar[5]) showed the existence of nonlinear structures in the tail (auroral and polar) region of magnetosphere at an altitude approximately $8R_E$ to $220R_E$ ($\sim 2R_E$ to $9R_E$), where R_E being the radius of the earth. These observations have primarily identified the bursts of broadband electrostatic noise (BEN) in auroral and other regions of magnetosphere, where the magnitude of electric fields varies from μV/m to 100 mV/m [3, 4, 6, 7]. In particular, the 3D structures involving the bursts of BEN [1, 2] have been determined in the dayside auroral zone with magnetic field effect. Later, the EA fluctuations were observed in the earth bow-shock, in neutron-star magnetosphere, in laser-produced plasmas, etc. Nonlinear propagation and shock structure formation have attracted significant interests of the researchers and consequently, the two-electron component (two groups of electrons) plasmas have extensively been studied for the long time and considered as hot topic due to its potential implications in the lab [8–10] and space [11–13] environments.

Two populations of electrons are characterized by different temperatures and different number densities, significantly modifying nonlinear features of the EA waves. The latter propagate through a two-electron component plasma on a cool-electron timescale, where cool electrons act as fluid while thermal pressure of hot electrons impart the restoring force to generate these waves in a neutralized background of static ions. In such a situation, the ion-plasma frequency is taken much smaller than the EA frequency. However, utilizing the inequality limits, i.e., $v_{Tc} \ll v_p \ll v_{Th}$, one may derive the *EA phase speed* as given by $v_p = c_{ea}/\left(1 + k^2\lambda_{Dh}^2\right)^{1/2}$, here v_{Tc}, v_{Th}, ω_{pc} and λ_{Dh} refer to the cool-electron thermal speed, hot-electron thermal speed, cool-electron plasma frequency, and hot-electron Debye length. It is worth mentioning that EA waves propagate into the plasma with a constant $v_p \simeq c_{ea}$ in the long wavelength limit (i.e., $k\lambda_{Dh} \ll 1$) and give rise to simple oscillations (i.e., $\omega \simeq \omega_{pc}$) in the short wavelength limit $k\lambda_{Dh} \gg 1$. It is also found that *electron-acoustic speed* $c_{ea}\left[= \omega_{pc}\lambda_{Dh} \equiv (k_B T_h/\alpha m_e)^{1/2}\right]$ of these waves significantly relies on the hot-to-cool electron density ratio $\alpha\,(= n_{h0}/n_{c0})$ in two-electron component plasmas.

Firstly, a theoretical model [14] was developed to investigate nonlinear properties of the

EA waves in a collisionless unmagnetized plasma, containing two distinct populations of electrons with static positive ions. It was shown that solitary EA waves only exist when an equilibrium density ratio is greater than the unity ($\alpha \gg 1$) and in the opposite limit $\alpha \ll 1$, the waves are strongly damped. By using the framework of kinetic model, Gary and Tokar [15] derived some necessary conditions for the existence of EA waves and identified the damping of the wave mode as long as its frequency lies in between the ion and electron plasma frequencies. The phenomenon of wave-particle interaction has also suggested that EA mode damps strongly for its phase speed resonating with the electron thermal speed and thus the propagation is more or less restricted to specific parametric regimes and values. Later, the research work on nonlinear EA waves was continued by many authors [7, 16, 17] in unmagnetized plasmas and particularly, Yu, Shukla and Ong [18] successfully discussed nonlinear coupling theory for the electrostatic EA and electromagnetic waves, revealing unique instabilities in the two-electron component plasmas. Interestingly, Berthomier et al. [17] solved the fluid equations by adopting the pseudopotential theory and obtained an energy-integral equation for the large-amplitude EA waves in unmagnetized plasmas. They expressed their findings in terms of potential pulses to investigate the beam temperature and beam electron density effects.

It is well-established that for hot electrons, the standard Maxwellian (isothermal) distribution is not always true in space plasma environments and therefore a non-isothermal approach can be required to model hot electrons by the vortex-like or trapped electron distribution [19] which is a more adequate tool to investigate collective modes and instabilities in the two-electron component plasmas. One of the main reasons for considering the trapped density distribution is the formation of phase space holes that are mostly caused by the trapping of hot electrons in the wave potential. In this context, Mamun and Shukla [20] highlighted the role of trapped hot electrons, affecting strongly the propagation characteristics of the weakly and fully nonlinear EA waves with applications to dayside auroral zone region [7]. They observed a positive potential for the EA waves showing a hole in the cold electron number density and a hump in hot electron number density. Later, new nonlinear features of the EA waves were identified in four component unmagnetized [21] and magnetized [22] plasmas by taking into account the warm electrons as beam. Due to trapping and beam electrons, wave amplitudes and widths were significantly modified both in weakly and fully nonlinear wave analyses. The amplitude of energy soliton was decreased by increasing the beam parameter effect. Recently, nonplanar EA waves have been examined [23] by incorporating the hot electrons as trapped species. A cylindrical (spherical) KdV equation was derived to show a fractional power nonlinearity within the framework of reductive perturbation technique. They compared a new family of analytical solutions with numerical solution with 2D and 3D displays. Both the solutions were found in good agreement and employed to study new progressive wave solutions.

Recently, a considerable attention has been focused on the nonlinear evolution of waves and shocks in two-electron component plasma, where the plasma constituents and their distribution significantly matter for nonlinear propagation and wave processes. In this perspective, Shukla et al. [24] have adopted the MHD model to study the formation of nonstationary nonlinear magnetoacoustic shocklets in terms of velocity and magnetic field profiles with inertialess isothermal electrons. In addition, dust dynamical equations [25] have been employed alongwith the charge-neutrality condition to obtain the two characteristic wave equations, which have been analyzed numerically for dust-acoustic (DA) solitary and shocklet structure formation in multicomponent dusty plasmas. Modified wave amplitudes and phase speeds that account for isothermal electrons and ions, are found in agreement with experimental observations. Later, the work was extended to dense plasmas [26] containing degenerate relativistic electrons and classical ions, and discussed the solutions for shocklets with degenerate relativistic electrons. In recent years, the large-amplitude EA shocklets [27] have been studied in the Kappa-distributed plasmas, where hot electrons are taken as superthermal species. The fluid equations were solved together by using the *diagonalized-matrix* technique and a nonlinear evolution equation was obtained to admit the EA shocklet formation. Numerically, the excitation/deformation of shocklets have been examined and the wave overtaking/breaking occurs as long as the time progresses ($\tau > 0$). However, in the present model, we take into account the effect of non-isothermal hot electrons to show the evolution of EA solitary waves into the nonlinear nonstationary large-amplitude shocklets. In this respect, the hot electrons are assumed to follow the trapped/vortex-like distribution and cool electrons as fluid in a neutralizing background of positive ions. Numerical evaluation reveals that density and non-isothermality parameters strongly influence the profiles of EA solitary and shocklet structures in two-temperature electron plasmas. The model is also applied to electron diffusion region at earth's magnetopause.

The layout of manuscript is given as under. Section II describes model equations for nonlinear EA waves in a two-electron component plasma and derives inviscid Burgers' equation. Section III shows parametric investigations for the time evolution of positive potential and cool electron density with temperature ratio and hot-electron isothermality effects. Section IV concludes the main results.

II. MATHEMATICAL FORMALISM

We consider a two-electron component plasma, comprising the dynamical cool electrons and non-isothermal hot electrons following the trapped electron density distribution in a neutralizing background of static positive ions. At equilibrium, the plasma charge-neutrality holds the condition $Z_i n_{i0} = n_{c0} + n_{h0}$, where n_{s0} refers to equilibrium number density of the sth species, (viz., s equals c for cool electrons, h for non-isothermal (trapped) hot electrons and i for singly charged ions with charging state $Z_i = 1$). The nonlinear dynamics of EA waves in one-dimensional unmagnetized collisionless plasma are governed by the following hydrodynamic equations:

$$\left(\frac{\partial}{\partial \tau} + U_c \frac{\partial}{\partial X}\right) N_c + N_c \frac{\partial U_c}{\partial X} = 0, \tag{1}$$

$$\frac{\partial U_c}{\partial \tau} + U_c \frac{\partial U_c}{\partial X} - \alpha \left(\frac{\partial \Phi}{\partial X} - 3\sigma N_c \frac{\partial N_c}{\partial X}\right) = 0, \tag{2}$$

$$N_h + \frac{N_c}{\alpha} - \left(1 + \frac{1}{\alpha}\right) = 0, \tag{3}$$

and

$$N_h = I(\Phi) + \frac{2}{\sqrt{\pi\beta}} W(\sqrt{-\beta\Phi}) \quad \text{for } \beta < 0, \tag{4}$$

where N_c (N_h), U_c and Φ represent the normalized cool (hot) electron number density, the normalized cool electron fluid velocity and normalized electrostatic potential, the other parameters such as $\alpha\,(= n_{h0}/n_{c0})$, $\beta = T_{hf}/T_{ht}$, and $\sigma = T_c/T_h$ denote the hot-to-cool electron density ratio, the free hot to trapped hot electron temperature ratio, and cool to hot electron temperature ratio, respectively. The Dawson integral is given by $W(\sqrt{-\beta\Phi}) = \exp(\beta\Phi) \int_0^{\sqrt{-\beta\Phi}} \exp(y^2)\ dy \equiv \frac{\sqrt{\pi}}{2} \exp(\beta\Phi)\,\mathrm{erf}\, i(\sqrt{-\beta\Phi})$ and $I(\Phi) = \left\{1 - \mathrm{erf}(\sqrt{\Phi})\right\} \exp(\Phi)$. Equations (1)-(4) are scaled by using the normalized parameters [28] like $N_c = n_c/n_{c0}$, $N_h = n_h/n_{h0}$, $U_c = u_c/c_s$, $\Phi = e\varphi/k_B T_h$, $X = x/\lambda_{Dh}$ and $\tau = t\omega_{pc}$, where the electron-acoustic speed, the cool electron plasma frequency and the characteristic scalelength are, respectively, defined by $c_s = (k_B T_h/\alpha m_e)^{1/2}$, $\omega_{pc} = (4\pi n_{c0} e^2/m_e)^{1/2}$ and $\lambda_{Dh} = (k_B T_h/4\pi e^2 n_{h0})^{1/2}$. Note that charge-neutrality condition at equilibrium $[n_{i0}/n_{h0} - 1 = 1/\alpha]$ indicates that for positive $\alpha > 0$, the density ratio (i.e., $n_{i0}/n_{h0} > 1$) is always greater than unity.

A. Linear analysis of EA waves

To study the linear dispersion relation of EA waves in a two-electron component plasma, we adopt the linearization theory which expresses the dependent variables in terms of power series in ϵ upto the first order quantities, as $N_c = 1 + \epsilon N_{c1}$, $N_h = 1 + \epsilon N_{h1}$, $\Phi = \epsilon \Phi_1$ and $U_c = \epsilon U_{c1}$. After using these expansions into Eqs. (1)-(3), we obtain the linearized set of equations for EA waves

$$\frac{\partial N_{c1}}{\partial \tau} + \frac{\partial U_{c1}}{\partial X} = 0, \tag{5}$$

$$\frac{\partial U_{c1}}{\partial \tau} - \alpha \left(\frac{\partial \Phi_1}{\partial X} - 3\sigma \frac{\partial N_{c1}}{\partial X} \right) = 0, \tag{6}$$

and

$$N_{h1} + \frac{N_{c1}}{\alpha} - \left(1 + \frac{1}{\alpha} \right) = 0. \tag{7}$$

For small-amplitude waves, we impose a limit $\Phi \ll 1$ in Eq. (4) to yield the following:

$$N_h = 1 + \Phi - \frac{4(1-\beta)}{3\sqrt{\pi}} \Phi^{3/2} + \frac{1}{2}\Phi^2 + .. \tag{8}$$

Now, taking the derivative of (6) with respect to X, Eqs. (1)-(4) can be solved together by eliminating the electrostatic potential to obtain

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial X^2} - 3\sigma\alpha \frac{\partial^2}{\partial X^2} \right) N_{c1} = 0 \tag{9}$$

By seeking a plane wave solution $[\propto e^{i(KX - \omega\tau)}]$ for the perturbed quantity, we finally obtain the dispersion relation for IA waves in a two-electron component plasma, as

$$\frac{\omega}{K} = (1 + 3\sigma\alpha)^{1/2}, \tag{10}$$

where $\omega(K)$ is the normalized wave frequency (wave number). It is important to note that electron trapping does not affect the linear dispersion of EA waves but have seen to modify the weakly nonlinear EA waves [20] through fractional nonlinearity. For a cold plasma, we take $\sigma = T_c/T_h \equiv 0$ in Eq. (10) and consequently, the normalized phase speed becomes unity. However, the restoration of dimensions leads the phase speed to the EA speed, $\omega/k = c_s \equiv (k_B T_h/\alpha m_e)^{1/2}$.

B. Nonlinear analysis of EA waves

For large-amplitude EA waves, we substitute (4) into (3) to easily express N_c in terms of Φ, as $N_c = 1 + \alpha - \alpha\left\{I(\Phi) + \frac{2}{\sqrt{\pi\beta}}W(\sqrt{-\beta\Phi})\right\}$. As a consequence, Eqs. (1) and (2) are accordingly simplified to

$$\frac{\partial \Phi}{\partial \tau} + U_c\frac{\partial \Phi}{\partial X} - \chi\left(\Phi\right)\frac{\partial U_c}{\partial X} = 0, \tag{11}$$

and

$$\frac{\partial U_c}{\partial \tau} + U_c\frac{\partial U_c}{\partial X} - \delta(\Phi)\frac{\partial \Phi}{\partial X} = 0, \tag{12}$$

with new nonlinear coefficients

$$\delta(\Phi) = \alpha\left\{1 + \frac{3\sigma N_c^2}{\chi\left(\Phi\right)}\right\}, \tag{13}$$

and

$$\chi\left(\Phi\right) = \frac{1 + \alpha - \alpha\left\{I(\Phi) + \frac{2}{\sqrt{\pi\beta}}W(\sqrt{-\beta\Phi})\right\}}{\alpha\left\{\exp(\Phi) - \operatorname{erf}(\sqrt{\Phi})\exp(\Phi) - \frac{1}{\sqrt{\pi\Phi}}\right\} + \frac{2\alpha}{\sqrt{\pi\beta}}\left\{-\frac{\beta}{2\sqrt{-\beta\Phi}} + \frac{\sqrt{\pi\beta}}{2}\exp(\beta\Phi)\operatorname{erf}i(\sqrt{-\beta\Phi})\right\}}. \tag{14}$$

Equation (11) and (12) represent a set of coupled nonlinear normalized equations, which is usually solved by diagonalization matrix technique for large-amplitude shocklets. The latter describe the arbitrary amplitude EA shocks admitting nonstationary solutions [27] to show irregular steep fronts in two-electron component plasma. The development of EA shocks into the self-steepened profiles occurs with the course of time and is mainly caused by the nonlinearity effects [25]. Thus, Eqs. (11) and (12) can be expressed in the following matrix form

$$\frac{\partial}{\partial \tau}\begin{bmatrix} \Phi \\ U_c \end{bmatrix} + \begin{bmatrix} U_c & -\chi\left(\Phi\right) \\ -\delta(\Phi) & U_c \end{bmatrix}\frac{\partial}{\partial X}\begin{bmatrix} \Phi \\ U_c \end{bmatrix} = 0. \tag{15}$$

Let us now find out the eigen values by calculating $\det(A - \lambda_{\pm}I) = 0$, where A and I stand for the square matrix and unit matrix, respectively, and given by

$$A = \begin{bmatrix} U_c & -\chi\left(\Phi\right) \\ -\delta(\Phi) & U_c \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

As a consequence, the eigen values ($\lambda_{\pm}$) are eventually retrieved, as

$$\lambda_{\pm} \equiv U_c \pm \sqrt{\delta(\Phi)\chi(\Phi)}, \tag{16}$$

Now we solve the relation $AX = \lambda_{\pm}X$ [i.e., $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$] by taking into account the eigen values separately from Eq. (16) and determine the diagonalizing matrix C as

$$C = \begin{bmatrix} 1 & 1 \\ -\frac{1}{\sqrt{\frac{\delta(\Phi)}{\chi(\Phi)}}} & \frac{1}{\sqrt{\frac{\delta(\Phi)}{\chi(\Phi)}}} \end{bmatrix} \text{ and } D = C^{-1}AC = \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix} \tag{17}$$

with

$$C^{-1} = \frac{1}{2\sqrt{\frac{\delta(\Phi)}{\chi(\Phi)}}} \begin{bmatrix} \sqrt{\frac{\delta(\Phi)}{\chi(\Phi)}} & -1 \\ \sqrt{\frac{\delta(\Phi)}{\chi(\Phi)}} & 1 \end{bmatrix}. \tag{18}$$

Recall that the square matrix which is represented by A, is now diagonalized by means of a diagonalizing matrix C, whose columns are the eigenvectors of A. Multiplying (15) by C^{-1} from the left side and express its second term in terms of D, we readily arrive at

$$C^{-1}\frac{\partial}{\partial\tau}\begin{bmatrix} \Phi \\ U_c \end{bmatrix} + DC^{-1}\frac{\partial}{\partial X}\begin{bmatrix} \Phi \\ U_c \end{bmatrix} = 0, \tag{19}$$

or

$$\frac{\partial\Psi_{\pm}}{\partial\tau} + \lambda_{\pm}\frac{\partial\Psi_{\pm}}{\partial X} = 0, \tag{20}$$

In deriving Eq. (20), we have used new variables $\Psi_{\pm}$ which can be expressed in terms of cool-electron fluid speed through $\Psi_{\pm} = U_c \mp F(\Phi)$ with $F(\Phi) = \int_0^{\Phi} \{\delta(\Phi)/\chi(\Phi)\}^{1/2} d\Phi$. Generally, Ψ_+ (Ψ_-) represents a wave that propagates in the positive (negative) x-direction. To find a simplest wave solution of (20), we should make either Ψ_+ or Ψ_- equal to zero. Thus, setting $\Psi_- = 0$, we eventually derive the cool electron fluid speed as $U_c = -F(\Phi)$, which further leads to have the relation $\Psi_+ = 2U_c$. Consequently, one can express Eq. (20) in terms of U_c to give

$$\frac{\partial U_c}{\partial\tau} + \lambda_+(\Phi)\frac{\partial U_c}{\partial X} = 0. \tag{21}$$

Similarly, the eigen value becomes modified as $\lambda_+(\Phi) = -F(\Phi) + \sqrt{\delta(\Phi)\chi(\Phi)}$. Since it is well-know that U_c is a function of Φ, therefore Eq. (21) can be written in this form

$$\frac{\partial\Phi}{\partial\tau} + \lambda_+(\Phi)\frac{\partial\Phi}{\partial X} = 0, \tag{22}$$

This is a nonlinear equation, which supports the self steepening of positive potential Φ and its general solution is given in the form $\Phi = \Phi_0 [X - \lambda_+ (\Phi) \tau]$, where Φ_0 being a function of one variable, is determined by the initial condition for Φ at $t = 0$. Note that the solution holds only if Φ is differentiable, whereas $\lambda_+ (\Phi)$ indicates the effective nonlinear phase speed dependent on the normalized electrostatic potential Φ, containing all the plasma parameters. Due to nonlinearity effects, the pulse becomes self-steepened and developed into the shocklets with the course of time, while propagating in the two-electron component plasma.

III. NUMERICAL RESULTS AND DISCUSSION

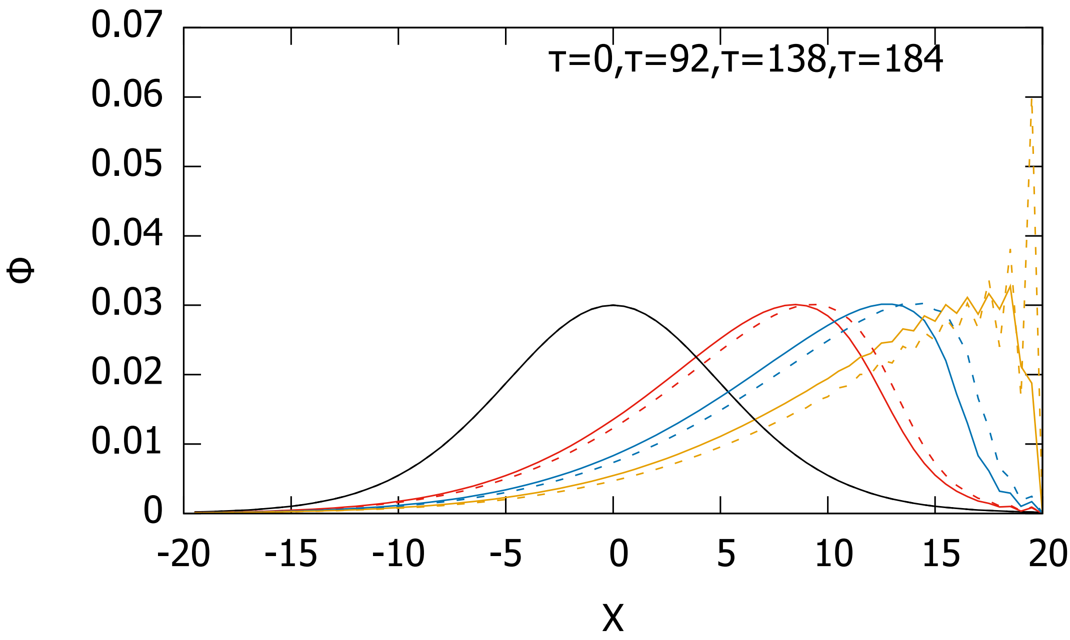
In this section, we present numerical evolution to discuss the formation of large-amplitude electron-acoustic (EA) waves with vortex-like trapped distributed hot electrons in a two-electron component plasma. For this purpose, we numerically solve Eqs. (21) and (22) and study electrostatic waves in the following regions of space plasmas.

A. Dayside auroral zone plasma

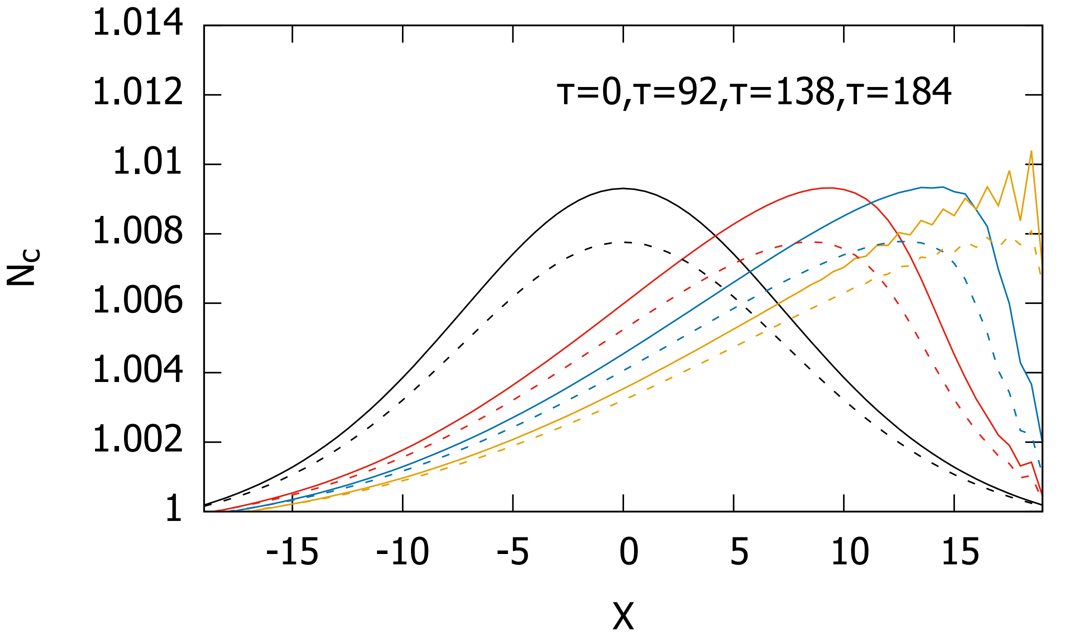
For numerical illustration, we consider typical values, which are the representative of the dayside auroral zone plasma, given by [20], $n_{h0} \sim 2.5\text{cm}^{-3}$; $n_{c0} \sim 0.5\text{cm}^{-3}$; $T_c \sim 5\text{eV}$; and $T_h \sim 250\text{eV}$. Based on these parameters, we can easily evaluate other quantities, such as the cold electron plasma frequency $\omega_{pc} \sim 3.9 \times 10^4\text{Hz}$, the cold electron thermal speed $v_{Tc} \sim 9.37 \times 10^8\text{cm/s}$, the hot-electron thermal speed $v_{Th} \sim 6.63 \times 10^9\text{cm/s}$, and the characteristic length $\lambda_{Dh} \sim 74340\text{cm}$, the electron-acoustic speed $c_s \sim 2.96 \times 10^9\text{cm/s}$, the hot-to-cool electron density ratio $\alpha = 5$ and the cool-to-hot electron temperature ratio $\sigma = 0.02$. Since the model investigates the stationary and non-stationary fully nonlinear excitations associated with EA waves in the form of electrostatic potential and cool electron density with trapped hot electrons, therefore, we fix the pulse amplitude [20] at $\Phi_{am} = 0.03$, which corresponds to an electric field amplitude $E_0 \simeq 100\text{mV/m}$. This model also infers about the trapping parameter (viz., the free hot-to-trapped hot electron temperature ratio β) that significantly affects the nonlinear steepening of the waves in two-electron component plasmas. It is worth to mention that various situations can be taken into account, for instance, $\beta = 1$, $\beta = 0$, and $\beta < 0$, which represent the Maxwellian, flat topped and vortex-like distributions, respectively. Specifically, in this model, we shall focus on a situation when $\beta < 0$ and investigate the propagation characteristics of nonlinear EA waves in a two-electron component plasma.

Figure 1 displays the temporal evolution of normalized (a) electrostatic potential $\Phi\,(= e\varphi/k_BT_h)$ and (b) cool electron density $N_c\,(= n_c/n_{c0})$ against the normalized position $X(= x/\lambda_{Dh})$ for varying the trapping parameter $\beta =-0.5$ (solid curves) -0.9 (dashed curves) with fixed $\alpha \left(= \frac{n_{h0}}{n_{c0}} \equiv 5\right)$ and $\sigma = \frac{T_c}{T_h} \equiv 0.02$. For stationary case, when time $\tau = 0$; the potential pulses exactly coincide with each other and non-stationary excitations appear when time (viz., $\tau = 92, 138, 184$) progresses, resulting in the enhancement of pulse amplitude with respect to β. One also observes that as times goes on the solitary pulses become self-steepened more and more and eventually lead to an oscillatory shocklet with increased self-steepness. It is observed that nonlinearity effect become stronger with the course of time started with a humb in the wave crest then developed to undular bore. Thus, increase of the free hot-to-tapped hot electron parameter (β) causes to increasing the nonlinearity effect in solitary and shock pulses. Moreover, the magnitudes owing to non-isothermal hot electrons and time evolution on nonlinear structures associated with the potential profiles are slightly smaller as compared to density profiles, where the variation of β parameter leads to the reduction of solitary and shocklet's pulse width and slightly increasing the pulse amplitude with time.

Figure 2 displays how the hot-to-cool electron density ratio $\alpha(= 4, 5)$ modifies the temporal evolution of (a) nonlinear EA waves associated with the potential $\Phi\,(= e\varphi/k_BT_h)$ and (b) cool electron density $N_c\,(= n_c/n_{c0})$ excitations with fixed $\beta = -0.5$ and $\sigma = 0.3$. As the value of alpha increases, the amplitude of solitary and shock waves almost remain constant and the pulse width insignificantly decreases. However, at time $\tau = 0$, solitary potentials does not coincide with each other in this plot. The impact of non-zero time $\tau = 92, 138, 184$ enhances the nonlinearity impact and resulting in the increase of self-steepening of waves. However, the profiles of cool electron density show an increase of amplitudes due to hot-to-cold-electron density variation while decreasing the pulse width. Figure 3 represents the temporal evolution of large-amplitude (a) EA waves associated with the potential (Φ) and (b) cool electron density (N_c) profiles for varying of the value of cool-to-hot electron temperature ratio (σ) with fixed α and β. At time $\tau = 0$, the positive solitary potentials exactly coincide with each other. We also note that temporal variation of potential and cool electron



(a)



(b)

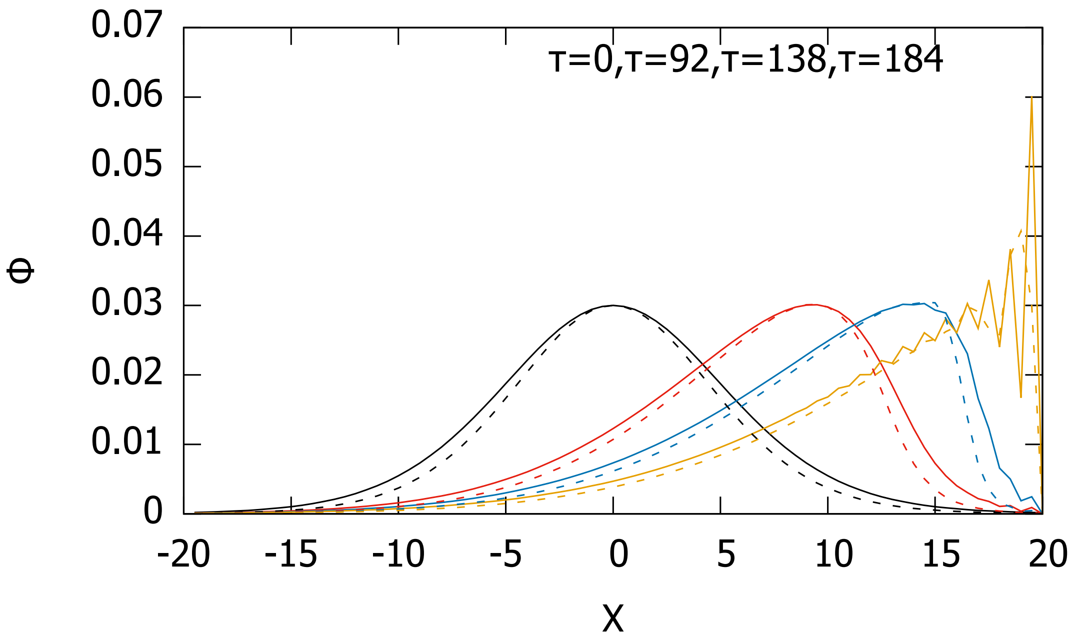
FIG. 1: The time evolution of the normalized (a) positive potential and (b) cold electron density excitations is shown against the normalized position for different values of $\beta = -0.5$ (solid curves), and $\beta = -0.9$ (dashed curves) with $\alpha = 5$ and $\sigma = 0$.

density profiles occurs as long as the value of σ increases, causing the nonlinearity effect to be decreased.

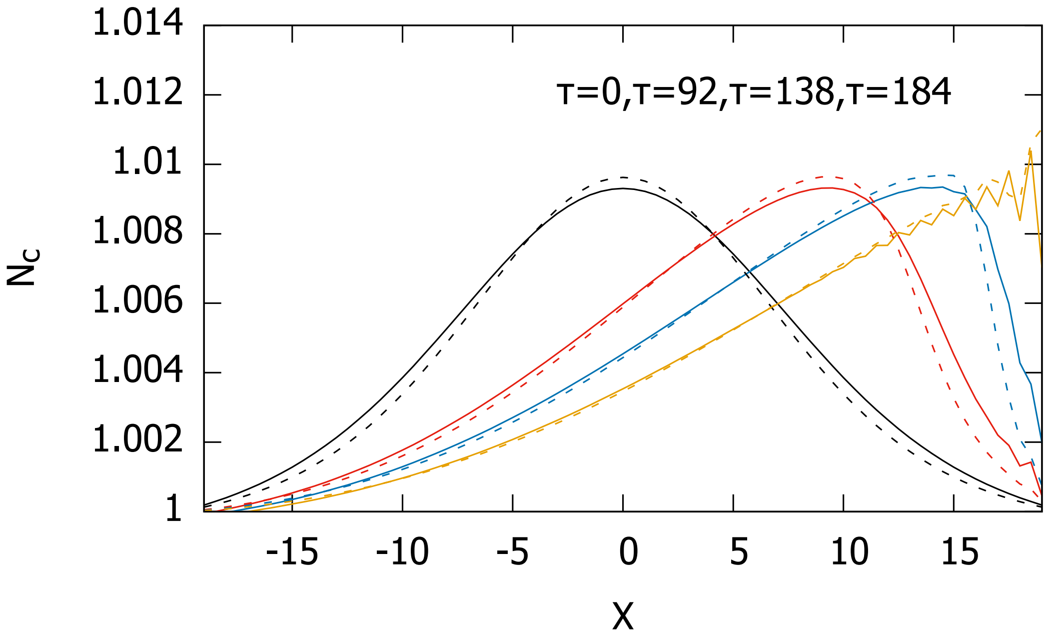
B. Electron diffusion region (EDR) at Earth's Magnetopause

For EDR plasma, we reconsider basic set of equations (1)-(4) and investigate the formation of large-amplitude EA shocklets at earth's magnetopause. For this purpose, the current model only replaces the cool electrons by the warm magnetosheath electrons ($\sim$ 100eV), which are merged with the inflowing magnetosphere plasma containing magnetospheric hot electrons ($\sim$ 1keV) with stationary ions. In EDR plasma at earth magnetopause, the warm electrons act as dynamical fluid, whereas hot electrons obeying the trapped (vortex-like) distribution to impart restoring force to the wave. Typical parameters consistent to the EDR plasma [29, 30] are chosen as the hot electron density $n_{h0} \sim 0.5\text{cm}^{-3}$, the warm electron density $n_{w0} \sim 1.5\text{cm}^{-3}$, the hot electron temperature $T_h \sim$ 1keV, and the warm electron temperature $T_c \sim$ 100eV. Upon using these parameters, we find the warm electron plasma frequency $\omega_{pw} \sim 6.9 \times 10^4 Hz$ and the warm electron thermal speed $v_{Tw} \sim 4.19 \times 10^9 cm/s$ with other quantities like $v_{Th} \sim 1.32 \times 10^{10}$cm/s, $\lambda_{Dh} \sim$ 332459cm and $c_s \sim 2.29 \times 10^9$cm/s with a hot-to-warm electron density ratio $\alpha \sim 0.333$. Considering the electrostatic potential amplitude to be $\Phi_{am} = 0.1$, for which the electric field amplitude in the EDR plasma then turns out as $E_0 \simeq$ 30mV/m. This lies within the valid range of electric field, e.g., from 1mV/m$-$100mV/m, as observed by MMS missions at the earth magnetopause. Both the small [31] and large [30, 32] amplitude electric fields were revealed in the MMS spacecraft observations for the EDR of earth's magnetosphere. In particular, Mozer et al [33] showed the existence of large amplitude bipolar electric field structures and accounted for trapped electrons in electron space holes at earth's magnetopause.

In Figure 4, the impact of trapping parameter (β) is shown on the profiles of electrostatic potential $\Phi\,(= e\varphi/k_B T_h)$ as a function of position $X(= x/\lambda_{Dh})$ for varying $\beta(=-0.5, -0.9)$ with fixed $\alpha(= \frac{n_{h0}}{n_{w0}}) \equiv 0.33$ and $\sigma(= \frac{T_w}{T_h}) \equiv 0.1$. It may be examined that the variation of trapping parameter alters significantly the nonlinear (positive) potential excitations both at time $\tau = 0$ as well as $\tau(= 92, 138, 184)$ and lead to the formation of stationary and nonstationary profiles, respectively. The temporal evolution thus develops into the shocklets having large self-steepening due to nonlinear effect. However, the pulse amplitude increases

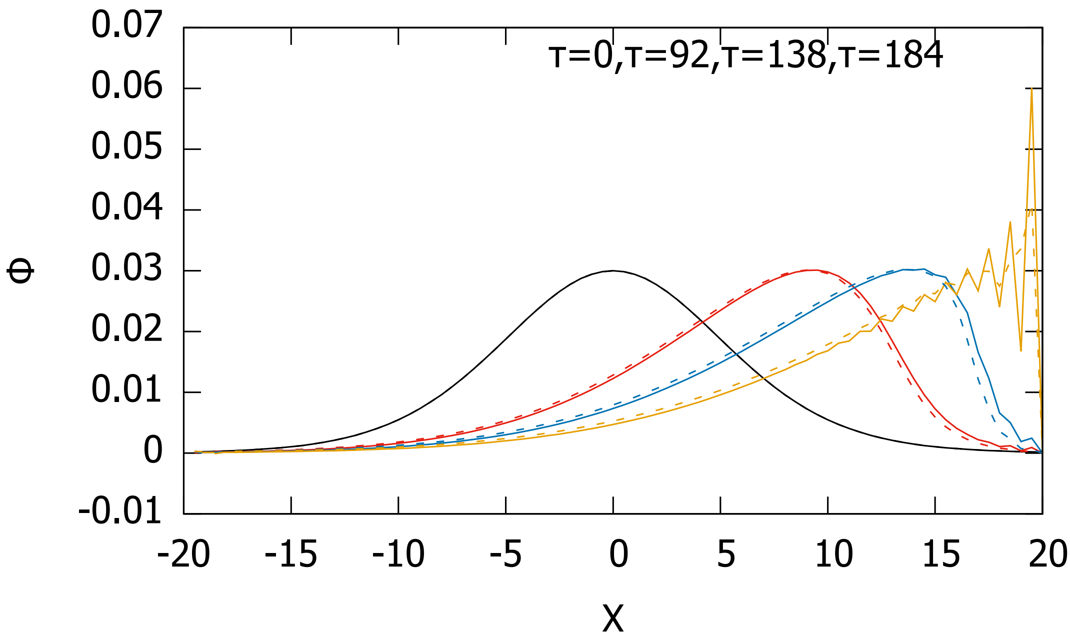


(a)

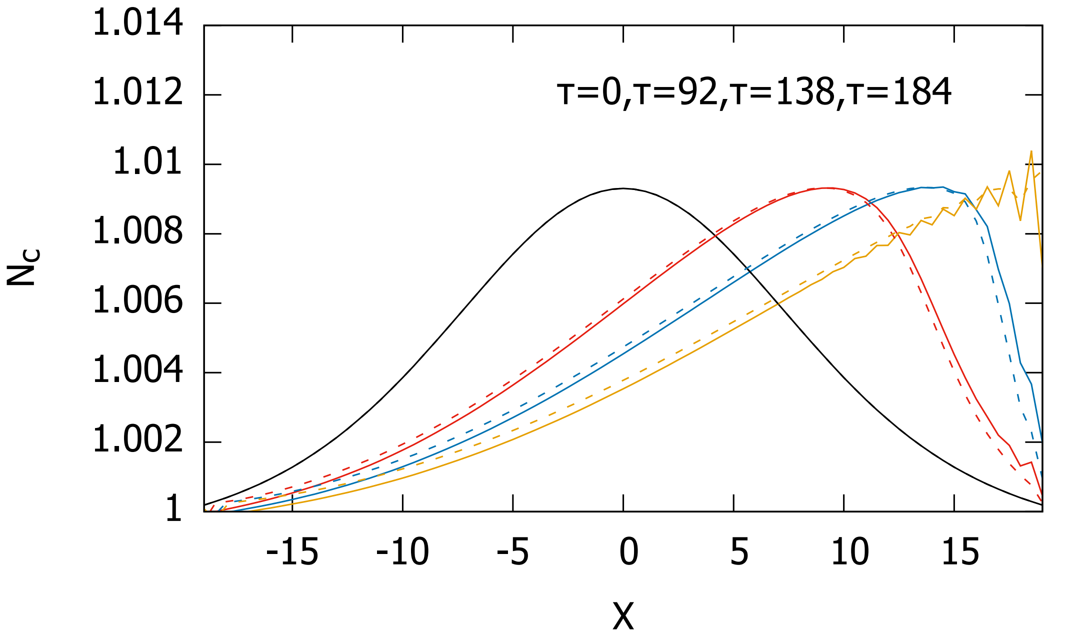


(b)

FIG. 2: The time evolution of normalized (a) positive potential and (b) cool electron density excitations is displayed against the normalized position for varying the values of $\alpha = 4$ (solid curves) and $\alpha = 5$ (dashed curves) with fixed $\beta = -0.5$ and $\sigma = 0.02$.



(a)



(b)

FIG. 3: The time evolution of normalized (a) positive potential and (b) cool electron density excitations is shown against the normalized position for different values of $\sigma = 0.02$ (solid curves) and $\sigma = 0.09$ (dashed curves) with $\alpha = 5$ and $\beta = -0.5$.

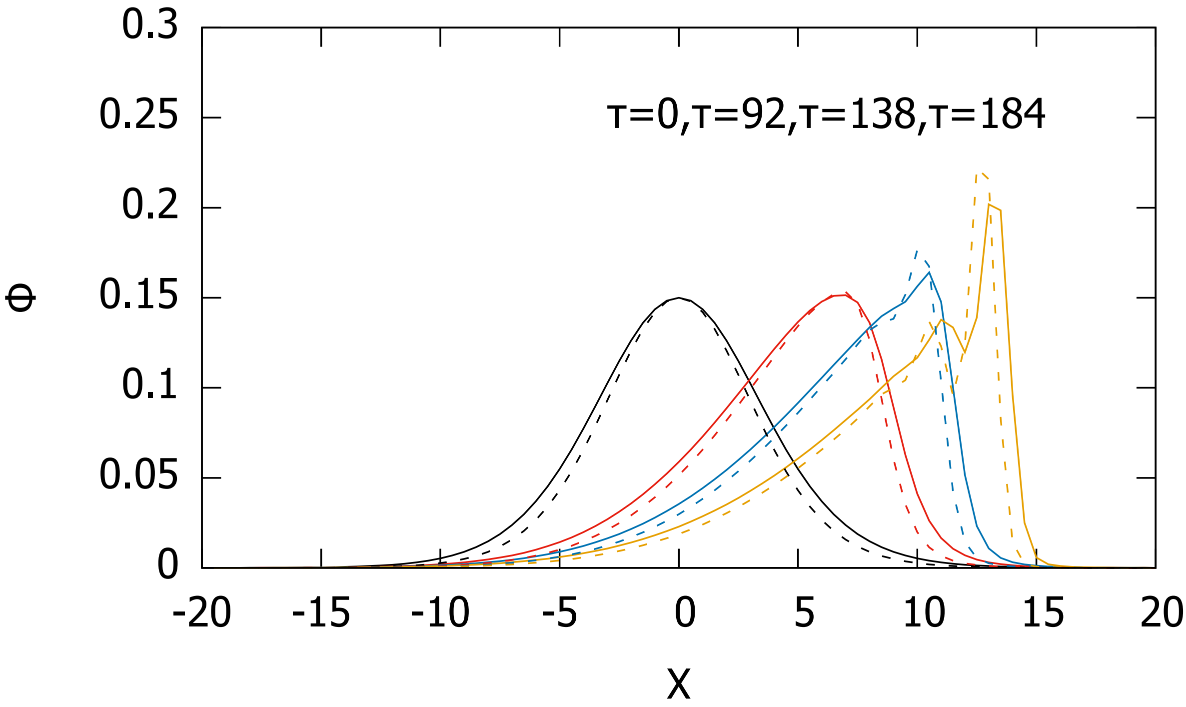


FIG. 4: The time evolution of normalized positive potential is shown against the normalized position for varying trapping parameter $\beta = -0.5$ (solid curves) and $\beta = -0.9$ (dashed curves) with fixed values of $\alpha \simeq 0.33$ and $\sigma = 0.1$.

and width decreases with respect to β. Figure 5 represents the temporal evolution of the normalized positive potential against the normalized position to showing the effect of warm-to-hot electron temperature ratio $\sigma = 0.1$, 0.3 with fixed $\alpha \simeq 0.33$ and $\beta = -0.5$. It is clear to observe that temperature ratio does not affect the stationary pulse but effectively modifies the formation of shocklets with large self-steepening, leading to the periodic oscillations.

IV. CONCLUSION

To conclude, we have investigated the formation of large-amplitude electron-acoustic (EA) waves and their development into shocklets with the course of time in unmagnetized non-isothermal plasmas. The latter is composed of two-electron populations of the electrons with different electron densities and temperatures. The cool electrons are assumed as fluid whereas non-isothermal hot electrons follow the vortex-like trapped distribution in a neutralizing background of static ions. In this regard, we have taken into account the

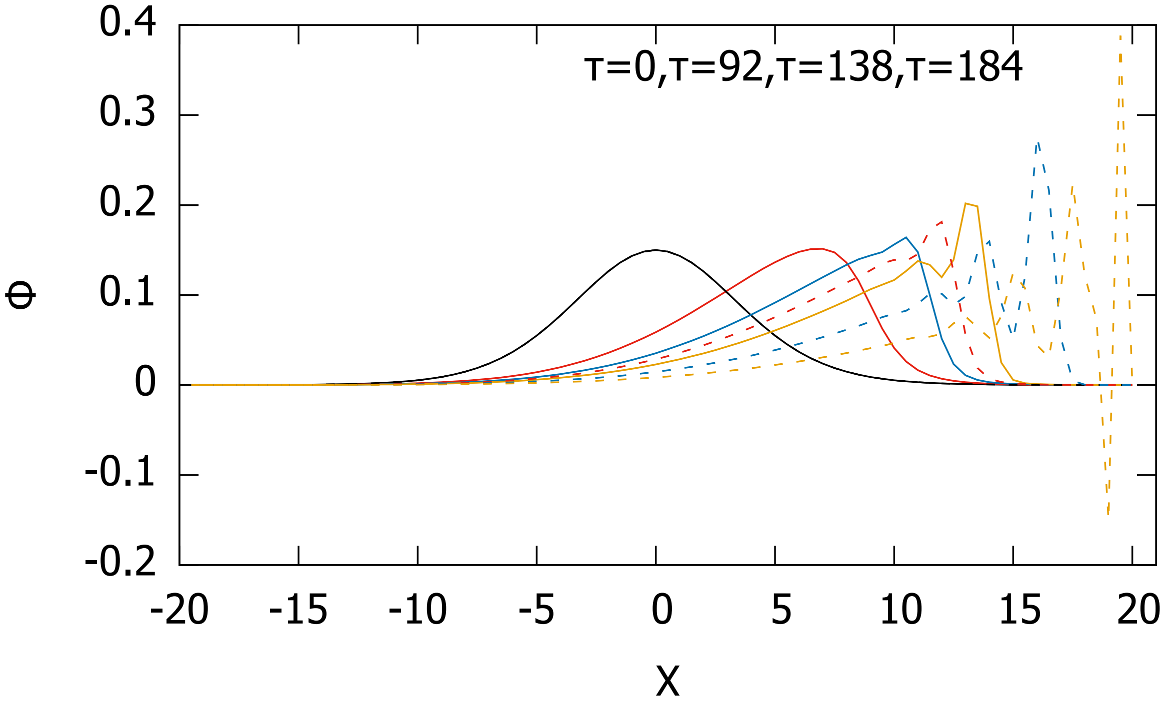


FIG. 5: The time evolution of normalized positive potential is displayed against the normalized position for changing $\sigma = 0.1$ (solid curves) and $\sigma = 0.3$ (dashed curves) with $\alpha \simeq 0.33$ and $\beta = -0.5$.

nonlinear fluid equations for cool electrons and solving them together both analytically and numerically within the framework of diagonalization matrix technique. The effects of the parameters such as the free hot-to-trapped hot electron parameter (β), the hot-to-cool electron density ratio (α), and the cool-to-hot electron temperature ratio (σ) are then examined on the excitations of solitary and oscillatory shocklets. Numerical results reveal that density and non-isothermality parameters strongly modify the nonlinear properties and steepening of the EA structures in two-electron component plasmas. The model is also applied to understand shocklet formation in the electron diffusion region plasma at earth magnetopause, where magnetospheric hot electrons are merged with warm magnetosheath electrons in a neutralizing background of ions.

Acknowledgment

The authors thank the University of Jeddah for providing technical and financial support. This work was funded by the University of Jeddah, Saudi Arabia under the grant No. UJ-21-DR 112.

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