Mathematische Beschreibung des RNN-Models

Sei X ein Vektor mit n Elementen, wobei n die Anzahl der Unicode Zeichen ist.

Sei Y ein Vektor mit n Elementen, wobei n die Anzahl der Klassifizierungskathegorien ist.

So ist Y durch

$$Y = \cancel{\sigma(} W_1 \cdot (X + H) + B_1 \cancel{)}$$

definiert.

Wobei σ durch

$$\sigma_i = \log\left(\frac{e^{x_i}}{\sum_{k=1}^{n} e^{x_k}}\right)$$

$$\Rightarrow \quad \sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \end{pmatrix}$$

und H durch

$$H = W_2 \cdot (X + H) + B_2$$

So kommen wir zu folgendem Optimierungsproblem:

$$J = \left(\sum_{i=1}^{N} \sigma_{y_{i,y}}\right) / n$$

Die Lösung des Optimierungsproblems erfolgt mit dem Gradientenverfahren:

$$\nabla W_1 = \frac{\partial J}{\partial W_1} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial W_1}$$

$$\nabla B_1 = \frac{\partial J}{\partial B_1} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial W_1}$$

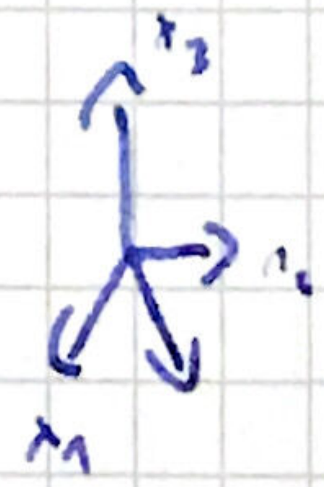
$$\nabla W_2 = \frac{\partial J}{\partial W_2} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial H} \cdot \frac{\partial H}{\partial W_2}$$

$$\nabla B_2 = \frac{\partial J}{\partial B_2} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial H} \cdot \frac{\partial H}{\partial B_2}$$

Daraus ergeben sich folgende rekursive Gleichungen

$$W_1 = W_1' - \alpha \cdot \nabla W_1$$

$$W_2 = W_2' - \alpha \cdot \nabla W_2$$



$$B_1 = B_1' - \alpha \cdot \nabla B_1$$

$$B_2 = B_2' - \alpha \cdot \nabla B_2$$

Notizen:

$$\sigma_i = \frac{e^{x_i}}{\sum_{k=1}^{K} e^{x_k}} \qquad u(x) = e^{x_i} \qquad v(x) = \sum_{k=1}^{K} e^{x_k}$$

$$\frac{d}{dx_i}\sigma_i' = \frac{e^{x_i}\cdot\sum_{k=1}^{K} e^{x_k} - e^{x_i}\cdot e^{x_i}}{\left(\sum_{k=1}^{K} e^{x_k}\right)^2}$$

$$= \frac{e^{x_i}\cdot\sum_{k=1}^{K} e^{x_k}}{\left(\sum_{k=1}^{K} e^{x_k}\right)^2} - \frac{e^{x_i}\cdot e^{x_i}}{\left(\sum_{k=1}^{K} e^{x_k}\right)^2}$$

$$\frac{d\sigma_i}{dx_i} = \frac{e^{x_i}}{\sum_{k=1}^{K} e^{x_k}} - \frac{(e^{x_i})^2}{\left(\sum_{k=1}^{K} e^{x_k}\right)^2}$$

$$\cancel{Y = \frac{e^{(W_1\cdot(x_1+H_1)+B_1)}}{\sum_{k=1}^{K} e}}$$

$$Y(i) = \frac{e^{(W_1^i\cdot(x_i+H)+B_1^i)}}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x_k+H)+B_1^k)}}$$

$$\nabla W_1 = \frac{\partial J}{\partial W_1} = \frac{\partial J}{\partial Y}\cdot\frac{\partial Y}{\partial W_1}$$

$$\cancel{= \frac{\partial J}{\partial Y}\cdot\left(\frac{e^{(W_1^i\cdot(x_i+H)+B_1^i)}}{\sum_{k=1}^{K} e^{(W_1}}\right.}$$

$$\sigma_i = \frac{e^{(W_1^i\cdot(x_i+H)+B_1^i)}}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}}$$

$$u(x) = e^{(W_1^i\cdot(x+H)+B_1^i)}$$

$$\frac{d}{dW_1^i} u'(x) = e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)$$

$$v(x) = \sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}$$

$$\frac{d}{dW_1^i} v'(x) = e^{(}$$

$$J = \sum_{i=1}^{N} Y(i)$$

$$\frac{dJ}{dY} =$$

$$(u(v(x)))' = u'(v(x))\cdot v'(x)$$

$$f(x) = W_1^i(x+H)+B_1^i$$

$$= W_1^i x + W_1^i H + B_1^i$$

$$\frac{df}{dH} = W_1^i$$

$$\frac{du}{dH_i} = e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot W_1^i$$

$$\frac{dv}{dH_i} =$$

$$\frac{\partial Y}{\partial B_1^i} = \frac{d\sigma_i}{d B_1^i}$$

$$= \frac{e^{(W_1^i \cdot (X+H) + B_1^i)} \cdot \sum_{k=1}^{K} e^{(W_1^k \cdot (X+H) + B_1^k)} - e^{(W_1^i \cdot (X+H) \cdot B_1^i)} \cdot e^{(W_1^i \cdot (X+H) \cdot B_1^i)}}{\left(\sum_{k=1}^{K} e^{(W_1^k \cdot (X+H) + B_1^k)} \right)^2}$$

$$= \frac{e^{(W_1^i \cdot (X+H) + B_1^i)}}{\sum_{k=1}^{K} e^{(W_1^k \cdot (X+H) + B_1^k)}} - \frac{\left(e^{(W_1^i \cdot (X+H) \cdot B_1^i)}\right)^2}{\left(\sum_{k=1}^{K} e^{(W_1^k \cdot (X+H) + B_1^k)} \right)^2}$$

$$\boxed{\frac{\partial Y}{\partial B_1^i} = \cancel{\frac{e^{(W_1^i (X+H) + B_1^i)}}{\sum_{k=1}^{K} e^{(W_1^k}}} = e^{(W_1^i \cdot (X+H) \cdot B_1^i)} \cdot \boxed{\lambda}}$$

$$\boxed{\frac{\partial y}{\partial W_1^i}}$$

$$\frac{\partial y}{\partial W_1^i} = \frac{d\sigma_i}{dW_1^i} = \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)\cdot\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}}{}$$

$$= \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)\cdot\cancel{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}} - e^{(W_1^i\cdot(x+H)\cdot B_1^i)}\cdot e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)}{\left(\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}\right)^{\cancel{2}}}$$

$$= \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}} - \frac{e^{(W_1^i\cdot(x+H)\cdot B_1^i)}\cdot e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)}{\left(\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}\right)^2}$$

$$= e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)\cdot\left(\frac{1}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}} - \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}}{\left(\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}\right)^2}\right)$$

$$\frac{\partial y}{\partial W_1^i} = e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)\cdot\left(\frac{1}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}} - \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}}{\left(\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}\right)^2}\right)$$

$$\boxed{\lambda = \left(\frac{1}{\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}} - \frac{e^{(W_1^i\cdot(x+H)+B_1^i)}}{\left(\sum_{k=1}^{K} e^{(W_1^k\cdot(x+H)+B_1^k)}\right)^2}\right)}$$

$$\boxed{\frac{\partial y}{\partial W_1^i} = e^{(W_1^i\cdot(x+H)+B_1^i)}\cdot(x+H)\cdot\boxed{\lambda}}$$

$$\frac{\partial Y}{\partial W_2^i} = \frac{\partial Y}{\partial H_i} \cdot \frac{\partial H_i}{\partial W_2^i}$$

$$\frac{\partial Y}{\partial H_i} = \frac{e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \sum_{k=1}^{K} e^{(W_1^k \cdot (x+H)+B_1^k)} - e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i}{\left(\sum_{k=1}^{K} e^{(W_1^k \cdot (x+H)+B_1^k)}\right)^2}$$

$$\boxed{\frac{\partial Y}{\partial H_i} = e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \boxed{\lambda}}$$

$$\frac{\partial H_i}{\partial W_2^i} = (x+H)$$

$$\Rightarrow \boxed{\frac{\partial Y}{\partial W_2^i} = e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \boxed{\lambda} \cdot (x+H)}$$

$$\frac{\partial Y}{\partial B_2^i} = \frac{\partial Y}{\partial H_i} \cdot \frac{\partial H_i}{\partial B_2^i}$$

$$\frac{\partial H_i}{\partial B_2^i} = 1$$

$$\Rightarrow \boxed{\frac{\partial Y}{\partial B_2^i} = e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \boxed{\lambda}}$$

$$\nabla W_1^i = \frac{\partial J}{\partial W_1^i} = \boxed{\frac{\partial J}{\partial Y} \cdot e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot (x+H) \cdot \underline{\lambda}}$$

$$\nabla B_1^i = \frac{\partial J}{\partial B_1^i} = \boxed{\frac{\partial J}{\partial Y} \cdot e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot \underline{\lambda}}$$

$$\nabla W_2^i = \frac{\partial J}{\partial W_2^i} = \frac{\partial J}{\partial Y} \cdot \frac{\partial Y}{\partial H_i} \cdot \frac{\partial H_i}{\partial W_2^i} = \boxed{\frac{\partial J}{\partial Y} \cdot e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \underline{\lambda} \cdot (x+H)}$$

$$\nabla B_2^i = \frac{\partial J}{\partial B_2^i} = \frac{\partial J}{\partial Y} \cdot \frac{\partial Y}{\partial H_i} \cdot \frac{\partial H_i}{\partial B_2^i} = \boxed{\frac{\partial J}{\partial Y} \cdot e^{(W_1^i \cdot (x+H)+B_1^i)} \cdot W_1^i \cdot \underline{\lambda}}$$

Wobei λ durch

$$\boxed{\lambda = \frac{1}{\sum_{k=1}^{K} e^{(W_1^k \cdot (x+H)+B_1^k)}} - \frac{e^{(W_1^i \cdot (x+H)+B_1^i)}}{\left(\sum_{k=1}^{K} e^{(W_1^k \cdot (x+H)+B_1^k)}\right)^2}}$$

definiert ist.