Framework of the Research

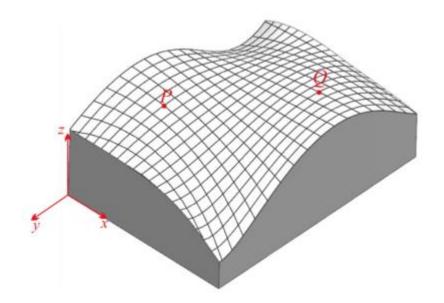
The framework diagram of the proposed algorithm in this study is shown in Fig, and the main steps are as follows:

Step 1 (Experiment Preparation): The experimental system is built, the sensing arrays are designed, the sensor coordinates are collected, A total of 16 PZT sensors, termed S1–S18, were glued to A piece of rail with epoxy resin to develop a rectangular sensing array.

Step 2 (Excitation Frequency Selection): A 50g weight is set in the center of the sensor network to test the signal amplitude under different excitation frequencies, and the excitation interval with the greatest change is noted and adopted in the experiment subsequently.

Step 3 (Signal Acquisition): It is planned to carry out experiments under three working conditions, the damage is in the center of the sensor network, the damage is at the intersection of the sensor path, and the damage is not on any sensor path. And the UGW response signals in the non-destructive and damaged structures are acquired as the inputs.

Step 4 (DF Modification): The effective path is optimized by scattering energy accumulation parameter E_0 , and threshold α . The modification function of group velocity with thickness is obtained experimentally, and the result is verified through numerical simulation. The MDF is calculated, and then it enters the next step.



$$D_{PQ} = \sqrt{(x_{p} - x_{q})^{2} + (y_{p} - y_{q})^{2}}.$$

$$t_{PQ} = \sum_{i} t_{i} = \frac{D_{PQ}}{N} \sum_{i} \frac{1}{v_{p}(f \cdot d_{i})}.$$

Step 5 (Weight Function Optimization): In practice, the ultrasonic energy of the sensing paths near the edge will be affected by the edges of rail significantly, in order to reduce this effect, the modified weight function in the traditional PDI algorithm is multiplied by a parameter η , which is obtained by calculating the effective detection area of the path.

$$E_{i} = \frac{\text{Energy}(c_{i} - b_{i})}{\text{Energy}(b_{i})}$$

$$E_{0i} = \frac{E_{i} - E_{\min}}{E_{\max} - E_{\min}} \ge \alpha.$$

$$\eta = \frac{S_{effective \ area}}{S_{total \ area}} = \frac{S_{effective \ area}}{ab\pi} = \frac{S_{effective \ area}}{a\sqrt{a^{2} - c^{2}\pi}} = \frac{S_{effective \ area}}{\frac{d}{2\beta}\sqrt{(\frac{d}{2\beta})^{2} - (\frac{d}{2})^{2}\pi}}$$

$$= \frac{S_{effective \ area}}{\frac{d^{2}}{4\beta^{2}}\pi\sqrt{1 - \beta^{2}}}$$

$$W_{i}(x, y) = \begin{cases} \eta\left(1 - \frac{1}{\beta} \cdot \tau_{d}\right), & \tau_{d} < \beta \\ 0, & \tau_{d} \ge \beta \end{cases}$$

Step 6 (Scaling Parameters Selection): The selection principles of the scaling parameters β are determined based on the path's probabilistic imaging pN. The imaging effects of different size of scaling parameters are compared and the appropriate parameter is selected as β .

Step 7 (Output and Assessment): The peak point of imaging is recognized as the damage position. Mean absolute error (MAE) and standard deviation (STD) are defined to assess algorithm performance.

Error =
$$\sqrt{(x_l - x_d)^2 + (y_l - y_d)^2}$$
.
MAE = $\frac{1}{m} \sum_{k=1}^{m} \text{Error}_k$
STD = $\sqrt{\frac{1}{m} \sum_{k=1}^{m} (\text{Error}_k - \overline{\text{Error}})^2}$