Lemma PSD_Matrix_Inverse

Suppose A is a PSD matrix and D is a diagonal matrix with positive entries. Then for any vector x, we have

$$\|D^{-1}x\| \geq \|(A+D)^{-1}x\|$$

Proof

Notation: For matrix B, define $B^2 = BB$.

It suffices to show that for all x,

$$x^T \left(D^{-2} - (A+D)^{-2}\right) x \geq 0$$

That is, we are interested in showing that $D^{-2} - (A+D)^{-2}$ is PSD. This can be shown by noting that

$$(A+D)^2 \succeq D^2 \implies D^{-2} \succeq (A+D)^{-2}$$