

### Lemma PSD\_Matrix\_Inverse

Suppose  $A$  is a PSD matrix and  $D$  is a diagonal matrix with positive entries. Then for any vector  $x$ , we have

$$\|D^{-1}x\| \geq \|(A + D)^{-1}x\|$$

#### Proof

Notation: For matrix  $B$ , define  $B^2 = BB$ .

It suffices to show that for all  $x$ ,

$$x^T (D^{-2} - (A + D)^{-2}) x \geq 0$$

That is, we are interested in showing that  $D^{-2} - (A + D)^{-2}$  is PSD. This can be shown by noting that

$$(A + D)^2 \succeq D^2 \implies D^{-2} \succeq (A + D)^{-2}$$

### Lemma: Lipschitz Definition Equivalence

The following two conditions are equivalent:

1. For all  $u > 0$  and any  $\lambda^{(1)}, \lambda^{(2)}$  that satisfy

$$\|\lambda^{(1)} - \lambda^{(2)}\| \leq Cu$$

then

$$\|\hat{g}(\cdot|\lambda^{(1)}) - \hat{g}(\cdot|\lambda^{(2)})\|_D \leq u$$

1.  $\hat{g}(\cdot|\lambda)$  is  $1/C$ -Lipschitz in  $\lambda$ :

$$\|\hat{g}(\cdot|\lambda^{(1)}) - \hat{g}(\cdot|\lambda^{(2)})\|_D \leq \frac{1}{C} \|\lambda^{(1)} - \lambda^{(2)}\|$$

#### Proof

It is clear that Condition 2 implies Condition 1.

To show Condition 1 implies Condition 2, suppose for any  $\lambda^{(1)}, \lambda^{(2)}$ , we have

$$\|\lambda^{(1)} - \lambda^{(2)}\| = d = C \frac{d}{C}$$

Then

$$\|\hat{g}(\cdot|\lambda^{(1)}) - \hat{g}(\cdot|\lambda^{(2)})\|_D \leq \frac{d}{C} = \frac{1}{C} \|\lambda^{(1)} - \lambda^{(2)}\|$$