ECE 4094

Project A

Progress Report

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1. Introduction

When modelling large datasets, it is useful to evaluate the covariance matrix. As part of the calculations, the logdet of the matrix needs to be computed. As the matrix gets very large, this logdet computation becomes exceedingly time and space inefficeint. In [1], a method to compute an esimate for the logdet of a Large, Sparse, Positive Semi-Definite Matrix (LSPSDM) was proposed which uses a Chebyshev Polynomial function and a Monte-Carlo approach. This method computes the logdet to a very high degree of accuracy, at a much faster rate than the exact method, and uses far less space.

A flaw in this method is that it requires knowledge of the condition number to perform certain elements of the computation. James Saunderson has proposed a new method, which uses a rational function estimator instead of a Chebyshev Polynomial to eliminate the need for the condition number. The primary goals of this project are to implement the method shown in [1] and the method proposed by James Saunderson, and compare the effectiveness of the two methods.

2. Objectives

Objective Name	Brief Description	Completed	Realistic before Relevant Due Date
Generate LSPSDM LSPSDM using the method in [1].	This is diagonally dominant, which is a special case of of the data we will be working with. It is easier to generate than a non-diagonally dominant matrix.	Yes	Yes
Implement method in [1]	The paper [1] introduces a new algorithm for estimating the log determinant of a LSPSDM. It uses a combination of Monte-Carlo methods and Polynomial funciton estimation for the log function	Yes	Yes
Experiment 1 from [1]	Run code many times over datasets of varying sizes. Use algorithm form paper and cholesky decomposition and produce graphs similar to paper.	Yes	Yes
More advanced analaysis	More advanced tests for runtime, space consumption, and other tests.	No	No
Implement new method	James Saunderson has developed the outline for a new method for estimating the log determinant. This method uses a rational function estimator for the log function and as a result is not reliant on knowing the condition number of the matrix.	In Progress	Yes
Progress Report	See introduction to this document	Yes	Yes
Design Specifications	A document detailing the design choices made in this project and the reasons for said choices.	Yes	Yes
Some simple comparisons of method in [1] and new method	Runtime, space, relative accuracy compared to exact method.	No	Yes
Paralellise Rational function method	Use MATLAB's inbuild parallel toolbox to paralellise the new algorithm	No (S1 2018)	Yes
Some simple comparisons of method in [1] and parallelised new method	Runtime, space, relative accuracy compared to exact method.	No (S1 2018)	Yes
Some advanced comparisons of method in [1] and parallelised new method	More advanced tests for runtime, space consumption, and other tests to compare the two methods	No (S1 2018)	Yes
Real world dataset analysis	Find real world data that can be converted to relevant format and be analysed by parallel new method for final results	No (S1 2018)	Yes (Optional)
More advanced dataset	Generate random dataset that is not diagonally dominant	No (S1 2018)	Yes (Optional)

GUI	Make a GUI that allowes user to get a visual indicator of the performance of the code	No (S1 2018)	Yes (Optional)
Final Report	A detailed report summerising the entire project	No (S1 2018)	Yes
Poster	A poster to present at the Spark Night	No (S1 2018)	Yes
Video	A short video summerising the project,	No (S1 2018)	Yes

3. Progress to date

The initial part of project was to research various parts of the theory behind the project. While I was familiar with much of it beforehand, the refresher was very useful, and some fo the fields were new to me as well. Some of the fields I looked into are:

- Positive Semi-Definite Matrix:
  - A matrix where all the eigenvalues are non-negative.
- Sparse Matrix
  - A matrix where the overwhelming majority of the entries are 0.
- Matrix functions
  - Each of the polynomial functions has an analogue Matrix function.
- Cholesky decomposition
  - An exact method for computing the logdet of a matrox
- Chebyshev polynomials
  - A series of orthogonal polynomials used in [1] to estimate the logdet
- Gershgorin circle theorem
  - A method used to bound the eigenvalues of a square matrix.
- Gaussian-Legendre quadrature
  - A method of estimating the integral of a function. Uses a series of predefined weights and nodes to estimate the integral.

After ensuring that I had a solid grasp of the theory I began writing code to generate a dataset that could be used in testing and experimentation. The dataset needed to be in the form of a Large $(10^7 \times 10^7)$ LPSDM stored in sparse form. The primary MATLAB functions used in this section are kron, rand, randi and sparse.

Once I could reliable generate a LPSDM I began writing code to replicate the algorithm devised in [1]. This algorithm iterativley updates a guess for the logdet. It requires knowledge of the condition number of the matrix, which is approximated using Gershgorin Circle Theorem. The codition number is then fed into an equation which is then used to generate a chebyshev approximation for the log function used in the algorithm.

With the algorithm from [1] working I began working on replcating the first experiment performed in said paper. The experiment produces the following results;

- Runtime vs Matrix size graph (1e3 to 1e7)
- Relative accuracy compared to Cholesky decompostion, an exact method (1e3 to 3e4)
- Runtime comparison for cholesky decomposition vs algorithm

- Comparison to method used by Zhang & Leithead, 2007 using n = 1000

The first three experiments have been performed, and I am in the process of setting up the fourth one. In addition, I have began implementing the new method. I started by implementing the gauss-legendre quadrature rule on a matrix of size 1x1. I then used that code to generalize the method to a large matrix. I am currently in the process of debugging that code. After the debugging is complete, I will perform some comparisons on time and space efficeincy of the two methods, as well as comparing their relative accuracy.

4. Work to be completed

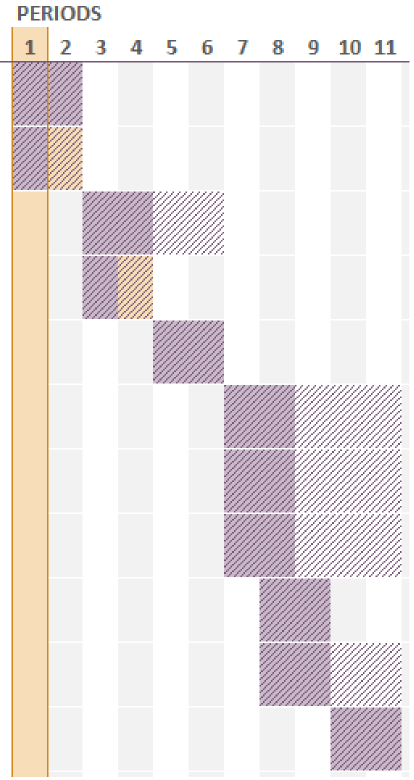
4.1 Table

Tasks and expected time allotment		
Task	**Expected hours**	**Start week**
Complete tasks from first half of project	20	Feb 12 2018
Some more advanced alalysis of method in [1]	15	1
Some simple comparisons of method in [1] and new method	5	1
Parallelise new method	20	3
Some simple comparisons of method in [1] and parallelised new method	5	3
Some advanced comparisons of method in [1] and parallelised new	15	5
Poster	15	7
Final Report	40	7
Video	10	7
Real world dataset analysis (*)	20	8
More advanced dataset (*)	20	8
GUI (*)	25	10

* optional goal for late stages of project, listed in order of priority

4.2 Gantt Chart

ACTIVITY	PLAN START	PLAN DURATION	ACTUAL START	ACTUAL DURATION	PERCENT COMPLETE
Advanced Analysis of Chebyshev Method	1	2	2		0%
Simple Analysis of Chebyshev vs Gaussian	1	1	2		0%
Parallelise Gaussian	3	4	4		0%
Simple analysis of Chebyshev vs parallel Gaussian	3	1	4		0%
Advanced analysis of Chebyshev vs parallel Gausian	5	2	6		0%
Poster	7	5	8		0%
Final Report	7	5	8		0%
Video	7	5	8		0%
Analysis on real world dataset	8	2	9		0%
Generate more advanced dataset	8	4	9		0%
GUI	10	2	11		0%



5. References

[1] “Large-scale Log-determinant Computation through Stochastic Chebyshev Expansions”, I. Han, D. Mallioutov, J. Shin