The Schrödinger equation is a second-order partial differential equation that describes the time evolution of a quantum system, represented by its wave function. It is given by:

$$i\partial_t u(x,t) = \Delta u(x,t) + V(x,t)u(x,t), \tag{1}$$

where Δ is the Laplacian operator, $V(x,t)$ is the potential, and $u(x,t)$ is the wave function. The weak form of the Schrödinger equation is given by:

$$i\int_\Omega (\partial_t u)\,\bar{v} = \int_\Omega \nabla u \cdot \nabla \bar{v} + \int_\Omega V(x,t)u\bar{v}, \tag{2}$$

where $v := v(x,t)$ is a test function. Discretizing the time domain and approximating the time derivative with finite differences, we get:

$$i\int_\Omega \left(\frac{u^{t+dt} - u^t}{dt}\right)\bar{v} = \int_\Omega \nabla u^{t+dt} \cdot \nabla \bar{v} + \int_\Omega V(x,t)u^t\bar{v} \tag{3}$$

Rearranging, we get:

$$i\int_\Omega u^{t+dt}\bar{v} - dt\int_\Omega \nabla u^{t+dt} \cdot \nabla \bar{v} = i\int_\Omega u^t\bar{v} + dt\int_\Omega V(x,t)u^t\bar{v}, \tag{4}$$

which can solved iteratively using a time-stepping method.