Implementation of Real Time Atmospheric Scattering

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1 Mathematical and Physical Background

Rayleigh scattering phase function [Preetham, 2003]

$$f_R(\theta) = \frac{3}{16\pi}(1 + cos^2\theta) \tag{1}$$

Henyey-Greenstein Approximation of the Mie scattering phase function: [Henyey and Greenstein, 194
Preetham, 2003]

$$f_{HG}(\theta) = \frac{1}{4\pi}\frac{1 - g^2}{(1 - 2g\cos\theta + g^2)^{3/2}} \tag{2}$$

1.1 Extinction

Extinction in constant mediums can be calculated with the exponential function.

$$F_{ex}(s) = e^{-\beta_{ex}s} \tag{3}$$

where s is the length of the ray in the medium, and β_{ex} is the extinction coefficient. In the case of atmospheric scattering it is usually wave-length-dependent. The extinction coefficient is the sum of the absorption and out scattering.

$$\beta_{sc}(\theta) = \beta_{sc}f(\theta) \tag{4}$$

1.2 Optical Mass

Optical mass of a medium is given by [Preetham, 2003]

$$m = \int_0^s \rho(x)dx \tag{5}$$

It is the mass of the medium along a path of unit cross section with medium density $\rho(x)$

Optical length is optical mass divided by the density of earth's atmosphere at base height ρ_0

$$l = \frac{1}{\rho_0} \int_0^s \rho(x) dx \tag{6}$$

1.3 Rayleigh Scattering coefficient

The total β and angular $\beta(\theta)$ Rayleigh scattering coefficients are given by [Preetham, 2003]

$$\beta = \frac{8\pi^3(n^2-1)^2}{3N\lambda^4}\left(\frac{6+3p_n}{6-7p_n}\right) \tag{7}$$

$$\beta(\theta) = \frac{\pi^3(n^2-1)^2}{2N\lambda^4}\left(\frac{6+3p_n}{6-7p_n}\right)(1+\cos^2\theta) \tag{8}$$

where n is the refractive index of air ($n = 1.0003$), N is the number of molecules per unit volume ($N = 2.545x10^{25}$) and p_n is the depolarization factor for air ($p_n = 0.0035$)

1.4 RGB Wavelengths

According to [Preetham, 2003]:

$$\lambda_{blue} = 400 \times 10^{-9} m \tag{9}$$

$$\lambda_{green} = 530 \times 10^{-9} m \tag{10}$$

$$\lambda_{red} = 700 \times 10^{-9} m \tag{11}$$

1.5 Mie Scattering coefficient

$$\beta = 0.434 c\pi \left(\frac{2\pi}{\lambda}\right)^{v-2} K \tag{12}$$

where c is the concentration factor in the range of 6×10^{-17} and 25×10^{-17}, v is the *Junge exponent* ($v = 4$ for a standard sky model) and K varies from 0.656 to 0.69 depending on the wavelength λ.

2 In-Scattering

[Preetham, 2003]:

$$L_s = f * L_0 + L_{in} \tag{13}$$

Where f is the extinction coefficient, L_0 is the radiance at the end of the ray, L_{in} is the radiance scattered into the ray over the path s.

With a few simplifications (single scattering, constant density atmosphere) L_{in} can be formulated as

$$L_{in} = E^s \frac{\beta(\omega, \omega_s)}{\beta}(1 - e^{-\beta s}) \tag{14}$$

References

[Henyey and Greenstein, 1941] Henyey, L. and Greenstein, J. (1941). Diffuse radiation in the galaxy. *The Astrophysical Journal*.

[Preetham, 2003] Preetham, A. (2003). Modeling skylight and aerial perspective. *ATI Research, ACM SIGGRAPH*.