

Implementation of Real Time Atmospheric Scattering

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1 Mathematical and Physical Background

Rayleigh scattering phase function [Preetham, 2003]

$$f_R(\theta) = \frac{3}{16\pi}(1 + \cos^2\theta) \quad (1)$$

Henye-Greenstein Approximation of the Mie scattering phase function: [Henyey and Greenstein, 1941, Preetham, 2003]

$$f_{HG}(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 - 2g \cos \theta + g^2)^{3/2}} \quad (2)$$

1.1 Extinction

Extinction in constant mediums can be calculated with the exponential function.

$$F_{ex}(s) = e^{-\beta_{ex}s} \quad (3)$$

where s is the length of the ray in the medium, and β_{ex} is the extinction coefficient. In the case of atmospheric scattering it is usually wave-length-dependent. The extinction coefficient is the sum of the absorption and out scattering.

$$\beta_{sc}(\theta) = \beta_{sc}f(\theta) \quad (4)$$

1.2 Optical Mass

Optical mass of a medium is given by [Preetham, 2003]

$$m = \int_0^s \rho(x)dx \quad (5)$$

It is the mass of the medium along a path of unit cross section with medium density $\rho(x)$

Optical length is optical mass divided by the density of earth's atmosphere at base height ρ_0

$$l = \frac{1}{\rho_0} \int_0^s \rho(x) dx \quad (6)$$

1.3 Rayleigh Scattering coefficient

The total β and angular $\beta(\theta)$ Rayleigh scattering coefficients are given by [Preetham, 2003]

$$\beta = \frac{8\pi^3(n^2 - 1)^2}{3N\lambda^4} \left(\frac{6 + 3p_n}{6 - 7p_n} \right) \quad (7)$$

$$\beta(\theta) = \frac{\pi^3(n^2 - 1)^2}{2N\lambda^4} \left(\frac{6 + 3p_n}{6 - 7p_n} \right) (1 + \cos^2 \theta) \quad (8)$$

where n is the refractive index of air ($n = 1.0003$), N is the number of molecules per unit volume ($N = 2.545 \times 10^{25}$) and p_n is the depolarization factor for air ($p_n = 0.0035$)

1.4 RGB Wavelengths

According to [Preetham, 2003]:

$$\lambda_{blue} = 400 \times 10^{-9} m \quad (9)$$

$$\lambda_{green} = 530 \times 10^{-9} m \quad (10)$$

$$\lambda_{red} = 700 \times 10^{-9} m \quad (11)$$

1.5 Mie Scattering coefficient

$$\beta = 0.434c\pi \left(\frac{2\pi}{\lambda} \right)^{v-2} K \quad (12)$$

where c is the concentration factor in the range of 6×10^{-17} and 25×10^{-17} , v is the *Junge exponent* ($v = 4$ for a standard sky model) and K varies from 0.656 to 0.69 depending on the wavelength λ .

2 In-Scattering

[Preetham, 2003]:

$$L_s = f * L_0 + L_{in} \quad (13)$$

Where f is the extinction coefficient, L_0 is the radiance at the end of the ray, L_{in} is the radiance scattered into the ray over the path s .

With a few simplifications (single scattering, constant density atmosphere) L_{in} can be formulated as

$$L_{in} = E^s \frac{\beta(\omega, \omega_s)}{\beta} (1 - e^{-\beta s}) \quad (14)$$

References

- [Henyey and Greenstein, 1941] Henyey, L. and Greenstein, J. (1941). Diffuse radiation in the galaxy. *The Astrophysical Journal*.
- [Preetham, 2003] Preetham, A. (2003). Modeling skylight and aerial perspective. *ATI Research, ACM SIGGRAPH*.