Sample Solution for Assignment 3

Problem 1

One translation is the set of the formulas:

$$\begin{array}{rc}
1. & Mythical \supset \neg Mortal \\
2. & \neg Mythical \supset (Mortal \wedge Mammal) \\
3. & (\neg Mortal \vee Mammal) \supset Horned \\
4. & Horned \supset Magical
\end{array}$$

$Mythical$:
Can not be inferred from the clauses.
$Magical$:

5.	$\neg Magical$	Negation of the goal
6.	$\neg Horned \vee Magical$	from 4
7.	$\neg Horned$	from 5 and 6
8.	$(Mortal \vee Horned) \wedge (\neg Mammal \vee Horned)$	from 3
9.	$Mortal \wedge \neg Mammal$	from 7 and 8
10.	$(Mythical \vee Mortal) \wedge (Mythical \vee Mammal)$	from 2
11.	$Mortal \wedge Mythical$	from 9 and 10
12.	$Mortal \wedge \neg Mortal \implies [\,]$	from 1 and 11

Hence, the unicorn is Magical.
$Horned$:
Given the negation of the goal $\neg Horned$, repeat the step from 7 in the above question and then an empty set will be derived. Hence, the unicorn is Horned as well.

Problem 2

Vocabulary:
$Take(x, y)$: Student x take course y
$Fail(x, y)$: Student x fails in course y
$Like(x, y)$: Person x likes person y
$Vegetarian(x)$: Person x is a vegetarain
$Smart(x)$: Person x shave for person y in the town
$Student(x)$: Person x is a student
$DHF(x, y)$: Person x does homework for person y

a. Not all students take both History and Biology.
$\neg\forall x(Take(x, History) \wedge Take(x, Biology))$
b. Only one student failed History.
$\exists x(Fail(x, History) \wedge \forall y(Fail(y, History) \supset y = x))$
c. Every person who dislikes all vegetarians is smart.
$\forall x(\forall y(Vegetarian(y) \supset \neg Like(x, y)) \supset Smart(x))$
d. No person likes a smart vegetarian.
$\neg\exists x[\exists y(Like(x, y) \wedge Vegetarian(y) \wedge Smart(y))]$
e. There is a student who does homework for those and only those who do not do homework for themselves.

$\exists x\{Student(x) \wedge \forall y[DHF(x,y) \equiv \neg DHF(y,y)]\}$

Problem 3

1. $P(Sam)$
2. $G(Clyde)$
3. $L(Clyde, Oscar)$
4. $P(Oscar) \vee G(Oscar)$
5. $L(Oscar, Sam)$
6. $\neg G(x) \vee \neg P(y) \vee \neg L(x,y)$

The resolution refutation is:
7. $\neg G(Clyde) \vee \neg P(Oscar)$ **from 3 and 6**
8. $\neg P(Oscar)$ **from 2 and 7**
9. $\neg G(Oscar) \vee \neg P(Sam)$ **from 5 and 6**
10.$\neg G(Oscar)$ **from 1 and 9**
11. $P(Oscar)$ **from 10 and 4**
12. Nil **from 11 and 8**

Problem 4

Initially, $\Sigma_{cur} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, \}, \pi = \{\ \}$

- Iteration 1:
  $\Gamma = true$
  $\tau = \Gamma \supset HIRE$ (Note: for $\gamma_\alpha = n^+_\alpha / n_\alpha$, if both $n_\alpha = 0$ and $n^+_\alpha = 0$, we will have $\gamma_\alpha = 0$)

α	GPA	UST	HKU	CU	REC	EXP	γ
γ_α	4/7	1/3	2/4	1/4	4/8	**3/4**	EXP
γ_α	**3/3**	0/1	2/2	1/1	3/3	–	EXP $\wedge$ GPA

  $\tau = EXP \wedge GPA \supset HIRE$
  $\pi = \{EXP \wedge GPA \supset HIRE\}$
  $\Sigma_{cur} = \{e_2, e_4, e_6, e_7, e_8, e_9, e_{10}, e_{11}, \}$

- Iteration 2:
  $\Gamma = true$
  $\tau = \Gamma \supset HIRE$

α	GPA	UST	HKU	CU	REC	EXP	γ
γ_α	1/4	**1/3**	0/2	0/3	1/5	0/1	UST
γ_α	1/2	–	0/0	0/0	**1/1**	0/1	UST $\wedge$ REC

  $\tau = UST \wedge REC \supset HIRE$
  $\pi = \{EXP \wedge GPA \supset HIRE, UST \wedge REC \supset HIRE\}$
  $\Sigma_{cur} = \{e_2, e_6, e_7, e_8, e_9, e_{10}, e_{11}, \}$

Since all the positive instances are covered by the rules in, so the set of rules about when to hire an applicant learnt using GSCA is:
$EXP \wedge GPA \supset HIRE$ and $UST \wedge REC \supset HIRE$

Problem 5

In the following, let A denotes $Alarm$, J for $JohnCalls$, etc. You'll get fullmark as long as the formulas are correct, regardless if you have done the calculation.

$$\begin{aligned}
&P(A) = \sum_{B,E} P(A, B, E) = \sum_{B,E} P(A|B, E)P(B)P(E) = 0.0025 \\
&P(\neg A) = 1 - P(A) = 0.9975 \\
&P(M) = P(M|A)P(A) + P(M|\neg A)P(\neg A) = 0.012 \\
&P(J, M) = P(J, M, A) + P(J, M, \neg A) = P(J, M|A)P(A) + P(J, M|\neg A)P(\neg A) \\
&\qquad = P(J|A)P(M|A)P(A) + P(J|\neg A)P(M|\neg A)P(\neg A) = 0.002 \\
&P(J|M) = P(J, M)/P(M) = 0.17
\end{aligned}$$

Problem 6

1. Yes. All undirected paths between Test1 and Test2:

$$\begin{aligned}
&(Test1, Disease2, Test2), \\
&(Test1, Disease2, Symptom3, Disease3, Test3, Disease2, Test2), \\
&(Test1, Disease1, Symptom2, Disease2, Test2), \\
&(Test1, Disease1, Symptom2, Disease2, Symptom3, Disease3, Test3, Disease2, Test2).
\end{aligned}$$

   All of them going through Disease2 which has arrows coming in (type (3) in the definition of d-separation).

2. No. The path (Disease1,Test1,Disease2). The arrows going out of Test1 (type (2) in the definition of d-separation).

3. Yes. Same reason as above: in paths go through Disease2 according to type (3) in the definition of d-separation.

4. Consider all paths between D1 and D2:

$$\begin{aligned}
P_1 &: \quad (D1, T1, D2) \\
P_2 &: \quad (D1, S2, D2)
\end{aligned}$$

   So the condition for E is $T1 \in E$ and $S2 \notin E$.

5. Consider all paths between D1 and D3:

$$\begin{aligned}
P_1 &: \quad (D1, T1, D2, T3, D3) \\
P_2 &: \quad (D1, T1, D2, S3, D3) \\
P_3 &: \quad (D1, S2, D2, T3, D3) \\
P_4 &: \quad (D1, S2, D2, S3, D3)
\end{aligned}$$

   For P_1, the condition on E is

$$T1 \in E \vee T3 \in E \vee (D2 \notin E \wedge S2 \notin E \wedge S3 \notin E).$$

   For P_2:

$$T1 \in E \vee D2 \in E \vee S3 \notin E.$$

   For P_3:

$$S2 \notin E \vee D2 \in E \vee T3 \in E.$$

   For P_4:

$$S2 \notin E \vee D2 \in E \vee S3 \notin E.$$

So the condition on E is:

$$[T1 \in E \vee T3 \in E \vee (D2 \notin E \wedge S2 \notin E \wedge S3 \notin E)] \wedge$$
$$[T1 \in E \vee D2 \in E \vee S3 \notin E] \wedge$$
$$[S2 \notin E \vee D2 \in E \vee T3 \in E] \wedge$$
$$[S2 \notin E \vee D2 \in E \vee S3 \notin E]$$

For example $E = \emptyset$ will satisfy the above condition. So is $E = \{T1, D2, S2, S3\}$. Not sure if the condition can be much simplified.

Problem 7

The unique Nash equilibrium of this game would be **(Pol:expand, Fed:contract)**, i.e.(3,3) in the payoff matrix.

Problem 8

Formulate this auction as a game in normal form:

- A set of agents $N = \{1, 2\}$;
- The same set of actions for each agent $A_1 = A_2 = \{1, 2, 3, 4, 5, 6\}$;
- Utility functions

$$u_i(x_1, x_2) = \begin{cases} 6 - x_i & \text{if agent } i \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

Generate the payoff matrix as follows to find the Nash equilibria:

	1	2	3	4	5	6
1	$2.5, 2.5$	$0, 4$	$0, 3$	$0, 2$	$0, 1$	$0, 0$
2	$4, 0$	$2, 2$	$0, 3$	$0, 2$	$0, 1$	$0, 0$
3	$3, 0$	$3, 0$	$1.5, 1.5$	$0, 2$	$0, 1$	$0, 0$
4	$2, 0$	$2, 0$	$2, 0$	$1, 1$	$0, 1$	$0, 0$
5	$1, 0$	$1, 0$	$1, 0$	$1, 0$	$0.5, 0.5$	$0, 0$
6	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$

From the matrix, we can see that the Nash equilibria are $(4, 4), (5, 5)$ and $(6, 6)$.