Derivatives

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1 Test Functions

$f(x,y) = f(r)$
$r = \sqrt{x^2 + y^2}$

1.1 Square Root Function

$$f(r) = r \tag{1}$$

$$\textbf{Grad}\, f(r) = \begin{bmatrix} \frac{x}{r} \\ \frac{y}{r} \end{bmatrix} \tag{2}$$

$$\textbf{Div}\, f(r) = \frac{1}{r} \tag{3}$$

1.2 Sine Function

$$f(r) = sin(r) \tag{4}$$

$$\textbf{Grad}\, f(r) = \begin{bmatrix} \frac{xcos(r)}{r} \\ \frac{ycos(r)}{r} \end{bmatrix} \tag{5}$$

$$\textbf{Div}\, f(r) = \frac{cos(r) - sin(r)r}{r} \tag{6}$$

1.3 Cosine Function

$$f(r) = -(cos^2(x) + cos^2(y))^2 \tag{7}$$

$$\textbf{Grad}\, f(r) = \begin{bmatrix} 4\cos^3(x)\sin(x) + 4\cos(x)\sin(x)\cos^2(y) \\ 4\cos^3(y)\sin(y) + 2\cos^2(x)\sin(2y) \end{bmatrix} \tag{8}$$

$$\textbf{Div}\, f(r) = -(cos^2(x) + cos^2(y))^2 \tag{9}$$

2 Kernel Functions

2.1 Gaussian

$$W(r) = \frac{1}{\pi h^2} e^{-(\frac{r}{h})^2}, \text{ for } 0 \leq r \leq c \\ = 0, \quad \text{else} \tag{10}$$

$$\mathbf{Grad}\, W(r) = \begin{bmatrix} \frac{2x}{\pi h^4} e^{-(\frac{r}{h})^2} \\ \frac{2y}{\pi h^4} e^{-(\frac{r}{h})^2} \end{bmatrix} \tag{11}$$

$$\mathbf{Div}\, W(r) = \frac{-4(h^2 - x^2 - y^2)}{\pi h^6} e^{-(\frac{r}{h})^2} \tag{12}$$