Detecting gravitational-wave signals with stochastic sampling

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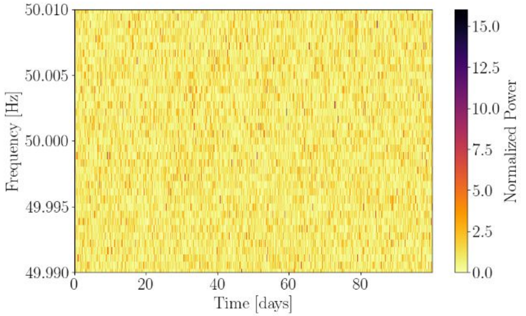
Overview

Gravitational-wave astronomy is an exciting field when it comes to data analysis:

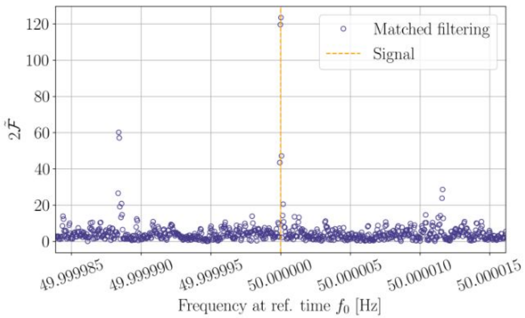
- Well-known models for the expected signals (Black-hole/Neutron star mergers).
- Complicated noise and detector response.

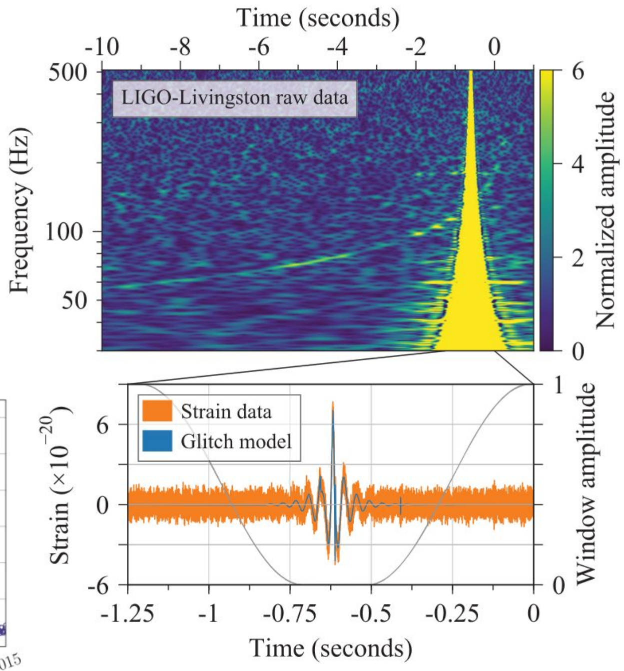
This talk: Follow-up of continuous gravitational-wave candidates.

1. What are continuous waves (CWs)? Why do we care?
2. How do we search for CW?
3. Follow-up of GW candidates using MCMCs
4. How to quantify the "success" of an MCMC follow-up
5. Setup and sensitivity optimisation



Search f_0





https://arxiv.org/pdf/1710.05832

Gravitational waves : a quick introduction

Gravitational waves : a quick introduction

GWs: traveling perturbations of the spacetime geometry predicted by General Relativity (A. Einstein, ca. 1915) and detected using LIGO [LIGO-Virgo Collaboration arxiv:1602.03837]

Source: “Time-dependent quadrupolar accelerations.”

Extremely *weak*, need extreme phenomena at astrophysical scales:

- Mergers of black holes or neutron stars.
- Supernovae explosions.
- Rapidly rotating non-axisymmetric neutron stars.

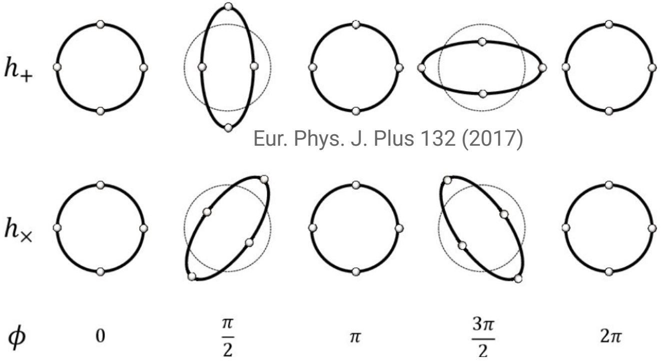
Complementary information to EM channels:

- How often do Black holes/Neutron stars merge?
- How spherical are neutron stars?
- What happens during a supernova explosion?
- Direct dark matter detection, exotic physics…

Observable effect: Relative displacement of test masses.

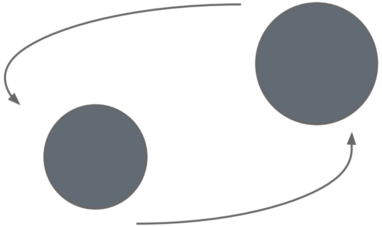
Interferometric detectors (LIGO, Virgo, KAGRA):

- Test masses==mirrors, use lasers to measure their position.



Eur. Phys. J. Plus 132 (2017)

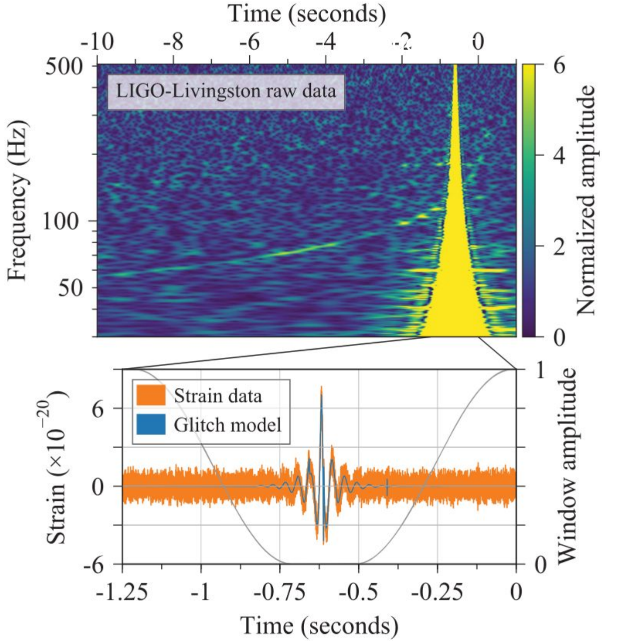
Gravitational waves : a quick introduction



Binary neutron star,
~40 Mpc away

Gravitational waves
travelling through
spacetime
h ~1e-21

LIGO/Virgo/KAGRA
detectors
(~3-4 km long)



Laser interferometry,
displacement ~1e-5 * proton radius

Continuous gravitational-wave signals (CWs)

Continuous gravitational-wave signals

So far (2025), detected GW signals come from merging binaries.

Neutron stars (NSs) *may* sustain surface deformations while they spin hundreds of time per second (see e.g. millisecond pulsars).

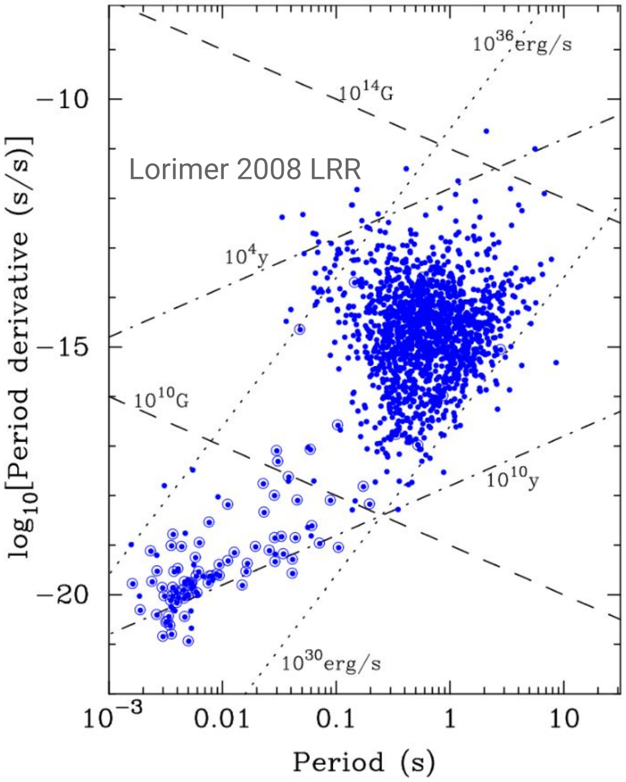
Non-axisymmetric deformation + rapid spin == GW emission!

Long-duration (continuous!) GW signal.

Implications:

- We are missing ~100 million NS, we may observe them using GW.
- NS interiors are *very* complicated and can be observed using GWs! (superconducting inner fluids? Rosby-mode oscillations? nuclear pasta? glitches?)

Deformation ~ NS ellipticity (ϵ). Theoretical models: log10(ϵ) < -6 [Gittins arxiv:2401.01670]



$$h_0 = \frac{4\pi^2 G}{c}\frac{I_z\epsilon}{d}[2f_{\rm rot}]^2 \simeq 4.2\cdot 10^{-26}\left(\frac{\epsilon}{10^{-6}}\right)\left(\frac{f_{\rm rot}}{100\ \mathrm{Hz}}\right)^2\left(\frac{d}{1\ \mathrm{kpc}}\right)^{-1}$$

at least ~5 orders of magnitude weaker than BNS. Need to integrate *long* data periods to detect.

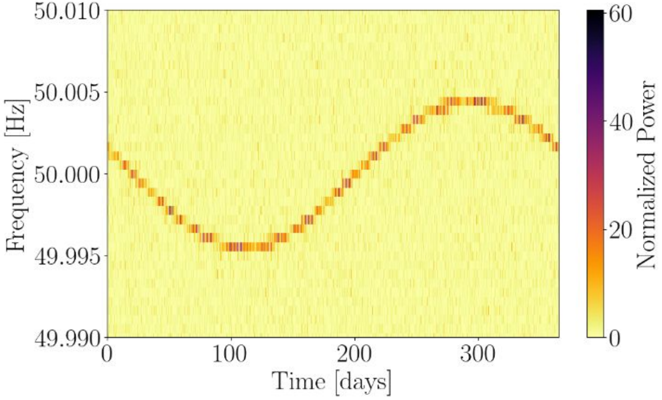
Continuous gravitational-wave signals

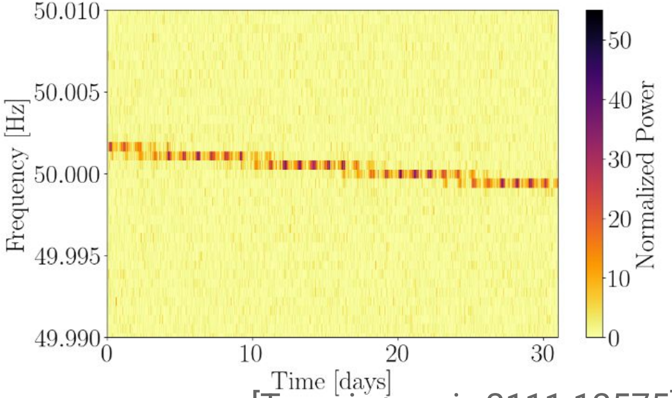
Morphology:
- Quasi-monochromatic signal.
- Frequency modulation due to Earth's motion around the Sun.
- Amplitude modulation due to detector's antenna pattern.

Search using *matched-filtering*:
1) Propose/select a specific set of signal parameters (*template*).
2) Subtract corresponding signal from data.
3) Check if result is consistent with noise
(i.e. compute detection statistic!)

$\boldsymbol{\lambda}$ = {Frequency, spindown, sky position,...}

Multiple ways:
- EM observations.
- Interesting sky regions.
- Optimize sensitivity.
- ...





[Tenorio+ arxiv:2111.12575]

Continuous gravitational-wave signals

Example search: Search in O4a LIGO data targeted at known pulsars.
LVK arxiv:2501.01495. ~ 6 months of integration time.

No detection: Set upper limits on ϵ for known pulsars.

Crab pulsar (for example):

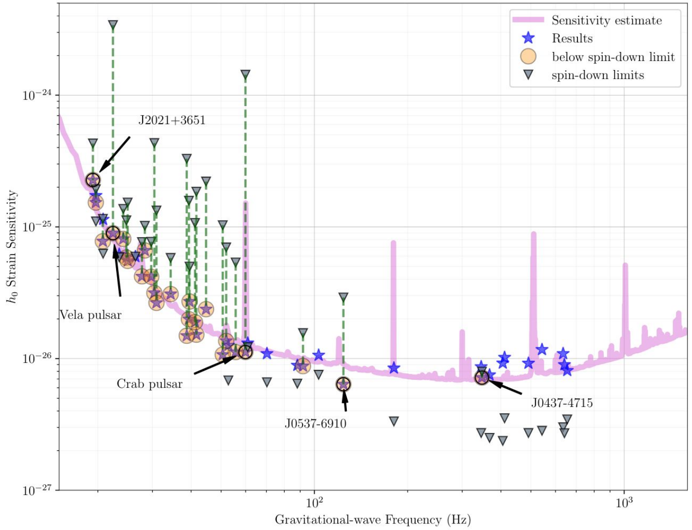
- Distance to the source ~ 2kpc .
- Less than 1% of the energy is loss as CW signals.
- ϵ < 6e-6 .
- NS Radius ~ 10km → "Mountain height" < 6cm

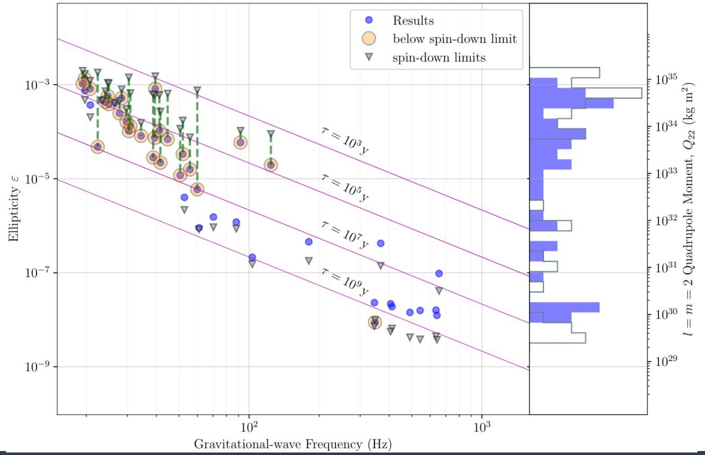
"Imagine a flat Milano - Lecco road with less than 6cm imperfections"

Summary:

- LIGO can set constraining upper limits on nearby NSs.
- Observed NS (pulsars) are *very* spherical.
- …?

Can we look for NS from unknown sources?
Would they behave like pulsars?
What if they are more extreme?





All-sky CW searches

All-sky CW searches

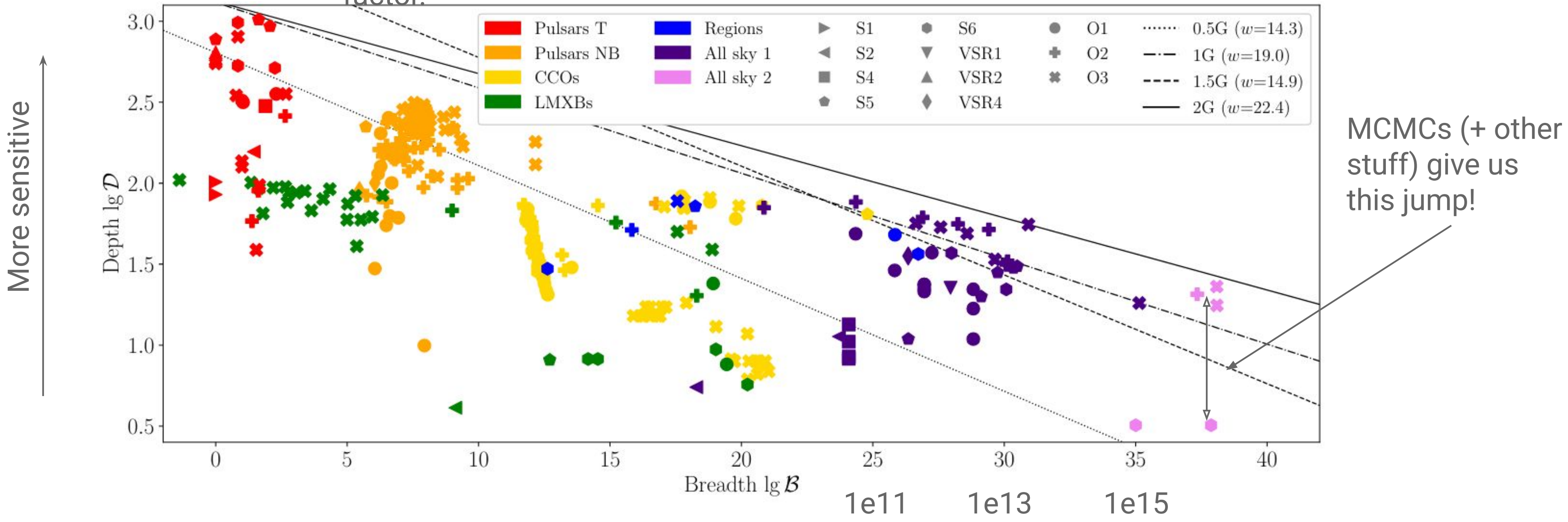
Wette arxiv:2305.07106

Historical set of CW searches
(2003 - 2023)

Breadth == Number of templates
Depth == (Noise amplitude) / (Signal amplitude)

$$\mathcal{D} = \frac{\sqrt{S_h/\mathrm{Hz}}}{h_0^C}$$

Clear *downward* trend:
Broader space → Broader priors → Bigger Occam factor.



All-sky CW searches

The less we *assume* about a source (bigger prior), the more complicate is to detect it (~ bigger Occam's factor).

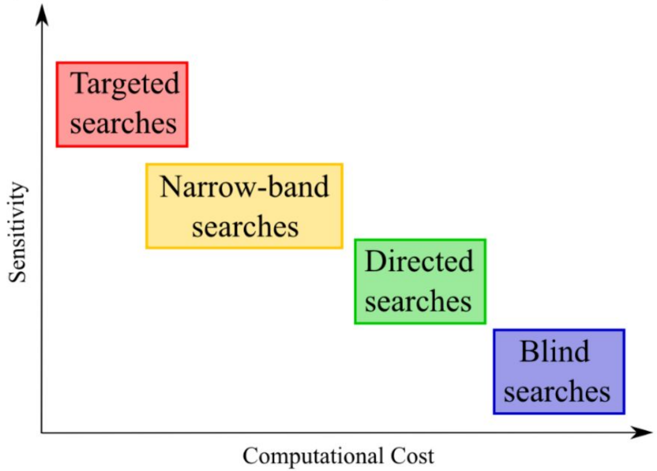
We can look for CW sources across the whole sky and LIGO sensitive band to look for *unknown* NSs.

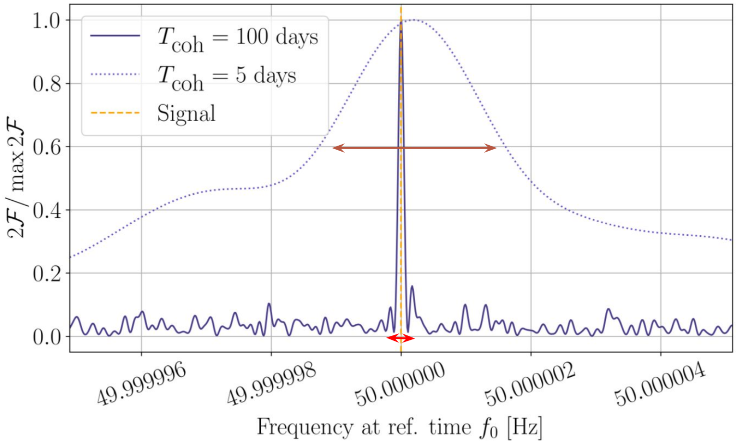
How many frequencies/sky positions/spindowns (*templates*) should we evaluate? We count them!

We can *simulate* a signal and study how much we can deviate from the true parameters before the detection statistic degrades too much (mismatch).

Different detection statistics will have different resolution requirements.

(Sieniawska & Bejger 2019)





All-sky CW searches

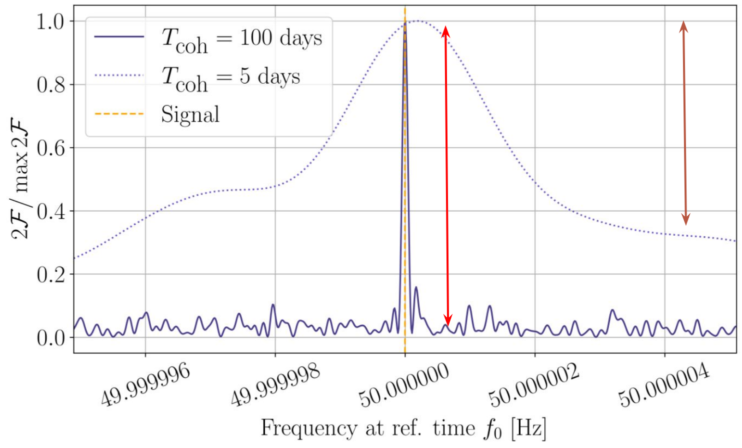
Cost of a detection statistic on 1 template ~1mus.
Typical budget ~ 1 million CPU hours ~ 1 CPU century.

Optimal statistic:

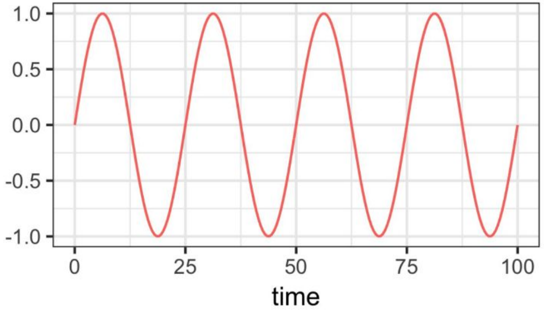
~1e20 templates == 3k millenia. → No.

Semicoherent statistic:

~1e15 templates == 0.3 century → Ok :)

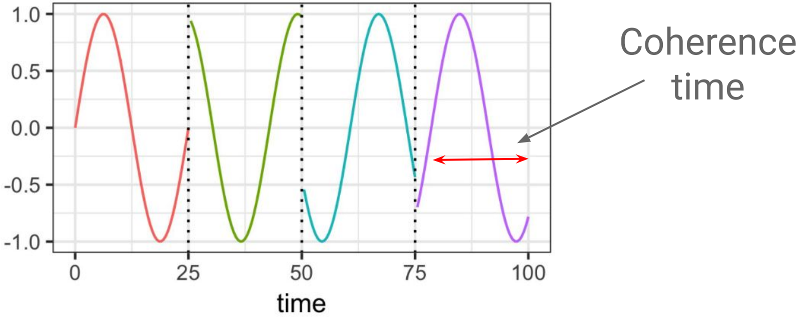


Coherent strategy



Very expensive, less false alarms.

Semicoherent strategy



Less expensive, more false alarms.

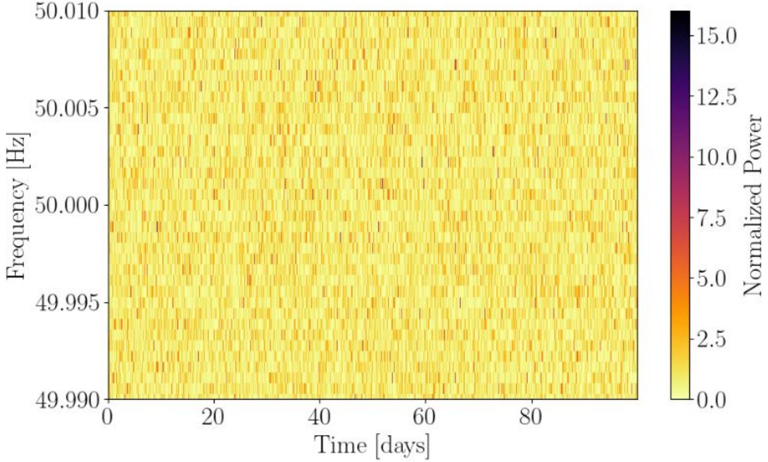
All-sky CW searches

Situation so far (i.e. summary!):

- Want to find CW signals from unknown sources.
- *Must* use sub-optimal methods to fit within budget.
- Initial Tcoh ~ 30 min.
- The search will produce *many* false alarms.

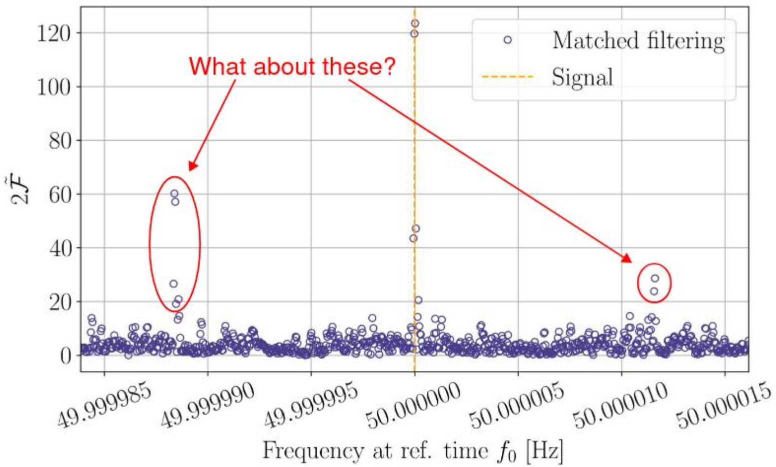
How do we deal with false alarms?

Increase coherence time in a *controlled* manner.
Place templates efficiently using probabilistic methods.



Search f_0 $\longrightarrow$

(+ sky position, spin down, …)



MCMC Follow-up of CW candidates

MCMC Follow-up of CW candidates

Asthon & Prix arxiv:1802.05450
https://github.com/PyFstat/PyFstat

Run a search → Identify interesting candidates → Run another "search" around with longer Tcoh.

Increase Tcoh → Need more templates to cover the region, increasing computing cost.

Is there a "cheap" way to run this search cheaply?

Simple idea: *Detection statistics* are related to *Posterior distributions* by an "unimportant" factor.

$$\underbrace{P(\boldsymbol{\lambda}|\boldsymbol{x}, \mathcal{H}_{\mathrm{S}})}_{\text{Signal parameters after looking at the data}} \propto \underbrace{B_{\mathrm{S/G}}(\boldsymbol{x}; \boldsymbol{\lambda})}_{\text{Detection statistic}} \underbrace{P(\boldsymbol{\lambda}|\mathcal{H}_{\mathrm{S}})}_{\text{Prior on signal parameters}}$$

$$B_{\mathrm{S/G}}(\boldsymbol{x}) \equiv \frac{P(\boldsymbol{x}|\mathcal{H}_{\mathrm{S}})}{P(\boldsymbol{x}|\mathcal{H}_{\mathrm{G}})}$$

$$B_{\mathrm{S/G}}(\boldsymbol{x}; \boldsymbol{\lambda}, N_{\mathrm{seg}}) \equiv \left(\frac{70}{\hat{\rho}^4_{\mathrm{max}}}\right)^{N_{\mathrm{seg}}} e^{\widehat{\mathcal{F}}}$$

Higher detection statistic → Higher posterior probability.

Can we use a Markov-Chain Monte Carlo to place templates "smartly"?

$$P(\theta|\boldsymbol{x}, \mathcal{H}) = \frac{P(\boldsymbol{x}|\theta, \mathcal{H}) P(\theta|\mathcal{H})}{P(\boldsymbol{x}|\mathcal{H})}$$

MCMC Follow-up of CW candidates

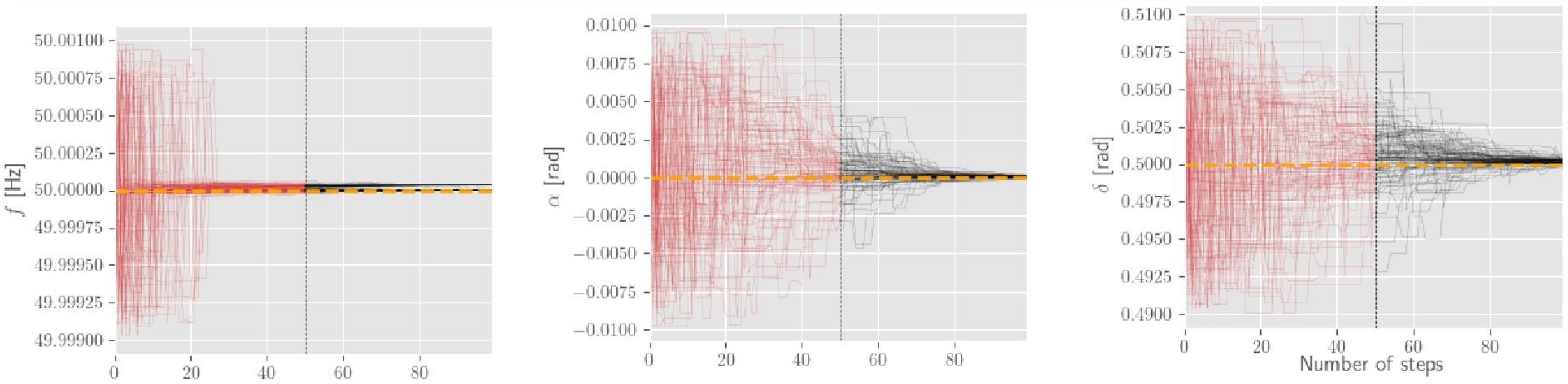
Tenorio+ arxiv:2105.13860

We use ptemcee [Vousden+ arxiv:1501.05823], a parallel-tempered ensemble MCMC.

Increase coherence time from 30min to 0.5 days (Detection statistic cost ~ 10ms).

Ensemble of ~100 walkers running for ~100 steps through 3 temperatures → ~30,000 templates.
(cf. initial ~1e15).

Seems to do a good job, right? → Standard situation: How do we quantify that?



MCMC Follow-up of CW candidates

Mirasola & Tenorio arxiv:2405.18934

Detection criterion: “Detection statistic high enough to be incompatible with noise”.

We don’t *need* convergence in terms of autocorrelation time, we only need the MCMC to *find* the peak.

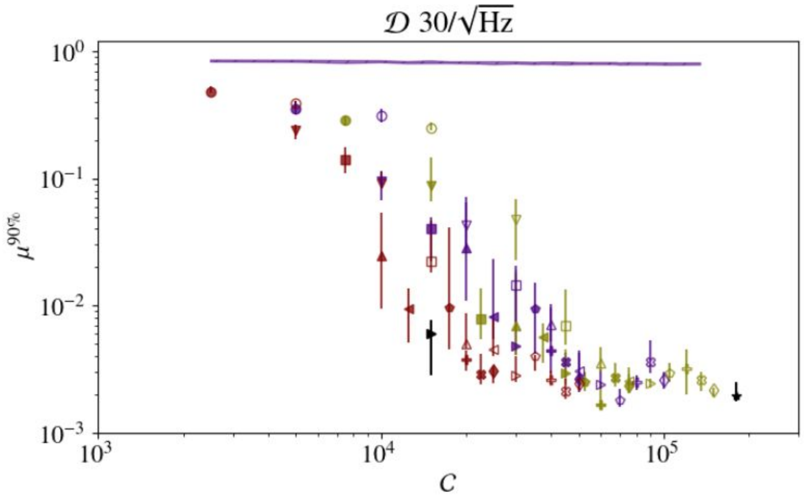
We quantify “how close” the MCMC gets to the peak using the *mismatch*:

$$B_{S/G}(\boldsymbol{x}) \equiv \frac{P(\boldsymbol{x}|\mathcal{H}_S)}{P(\boldsymbol{x}|\mathcal{H}_G)}$$

$$\mu(\Delta\lambda;\lambda_s) = \frac{2\hat{\mathcal{F}}(\lambda_s) - 2\hat{\mathcal{F}}(\lambda_s + \Delta\lambda)}{2\hat{\mathcal{F}}(\lambda_s) - 4N_{\text{seg}}}$$

We compute the resulting mismatch for each configuration using software-simulated signals.

These are compared to semianalytical estimates at a given false alarm and false dismissal (blue region), and then validated using simulations in real data.



MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

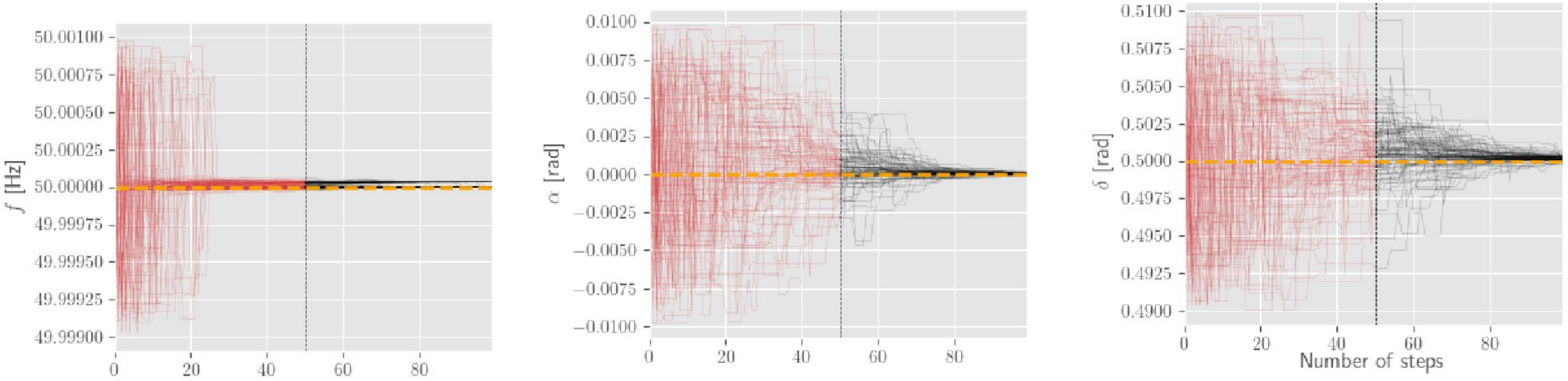
Right, now we know it has *finished* and we know the maximum detection statistic value.

What is the *result*? Was this a signal? Or was it only noise?

Two options:

1. Run another stage with longer coherence time and postpone the question.

2. Construct a Bayes factor to *decide* whether you have a signal or not. → We go directly to this one



MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

Assume we ran a few MCMC follow-ups increasing Tcoh.

Upon finishing an MCMC run, my *detection statistic* (2Fmax) is given by the *loudest* template:

{2Fmax[i], i=run 0, run 1, …}

If the data contains a signal, 2Fmax values will be *correlated*, as they are looking at *the same signal*.
If the data does not contain a signal, 2Fmax will be the *maximum* of a set of N random variables.

I want a Bayes factor that compares these two hypotheses:

$$\ln \mathcal{B}^{*}_{\mathrm{S/N}} = \ln \frac{\mathrm{P}(2\tilde{\mathcal{F}}^{*}|\mathcal{H}_{\mathrm{S}})}{\mathrm{P}(2\tilde{\mathcal{F}}^{*}|\mathcal{H}_{\mathrm{N}})}$$

Can I construct probability distributions for those two cases?

MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

Signal case: Detection statistic values are correlated.

Any pair of detection statistics depends on the signals SNR ϱ, so we can write the following integral:

$$\begin{aligned} \mathrm{P}(2\tilde{\mathcal{F}}|\mathcal{H}_{\mathrm{S}}) &= \int_0^\infty \mathrm{d}\rho^2 \; \mathrm{P}(2\tilde{\mathcal{F}}|\rho^2, 2\hat{\mathcal{F}}, N_{\mathrm{seg}}) \; \mathrm{P}(\rho^2|2\hat{\mathcal{F}}, N_{\mathrm{seg}}) \\ &\propto \int_0^\infty \mathrm{d}\rho^2 \; \mathrm{P}(2\tilde{\mathcal{F}}|\rho^2) \; \mathrm{P}(2\hat{\mathcal{F}}|\rho^2, N_{\mathrm{seg}}) \; \mathrm{P}(\rho^2) \;, \end{aligned}$$

If both det. stats. are high at the same ϱ value, the probability is high.

You'll have to believe me on this one.

tl;dr We can compute the probability under the signal hypothesis.

MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

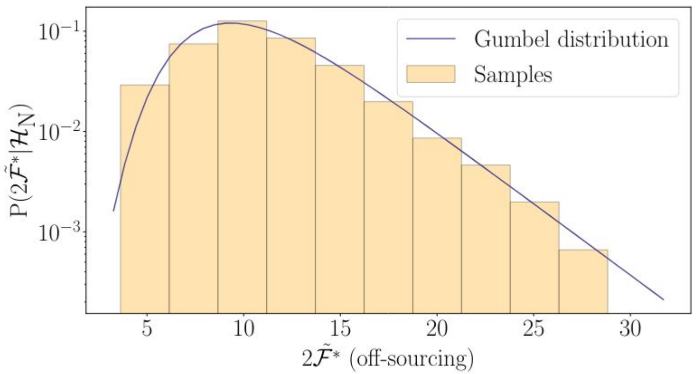
Noise case: Fisher-Tippett-Gnedenko Theorem

It's like the central limit theorem, but for min/max instead of for averages!
(checkout Extreme Value Theory).

The maximum of N variables, in this case, converges to a Gumbel distribution:

$$\mathrm{P}(2\tilde{\mathcal{F}}^* | \mathcal{H}_\mathrm{N}) = \frac{1}{\sigma_\mathrm{N}} \exp\left[-\left(\frac{2\tilde{\mathcal{F}}^* - \mu_\mathrm{N}}{\sigma_\mathrm{N}} \right) - e^{-\left(\frac{2\tilde{\mathcal{F}}^* - \mu_\mathrm{N}}{\sigma_\mathrm{N}} \right)} \right]$$

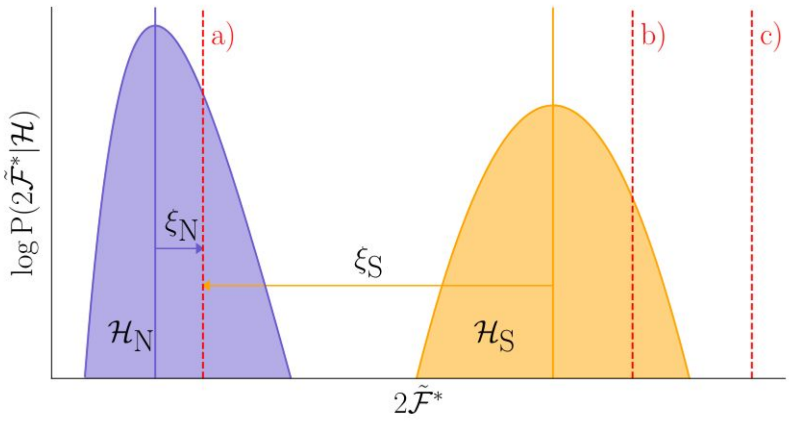
We can point away from the candidate in the sky and ru an MCMC to generate "independent" noise samples [Isi+arxiv.org:2010.12612] and fit the result using a Gumbel distribution [Tenorio+ arxiv:2111.12032].

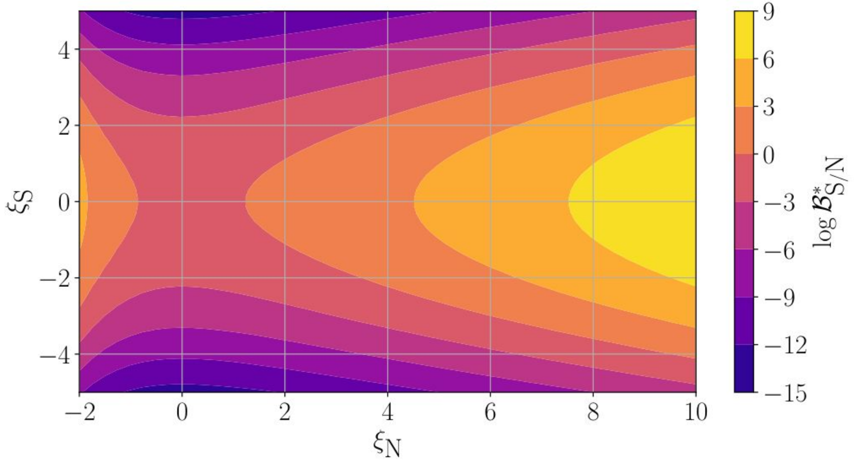


MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

Putting all together:





We end up with a sound Bayes factor to quantify the result of MCMC follow-ups!

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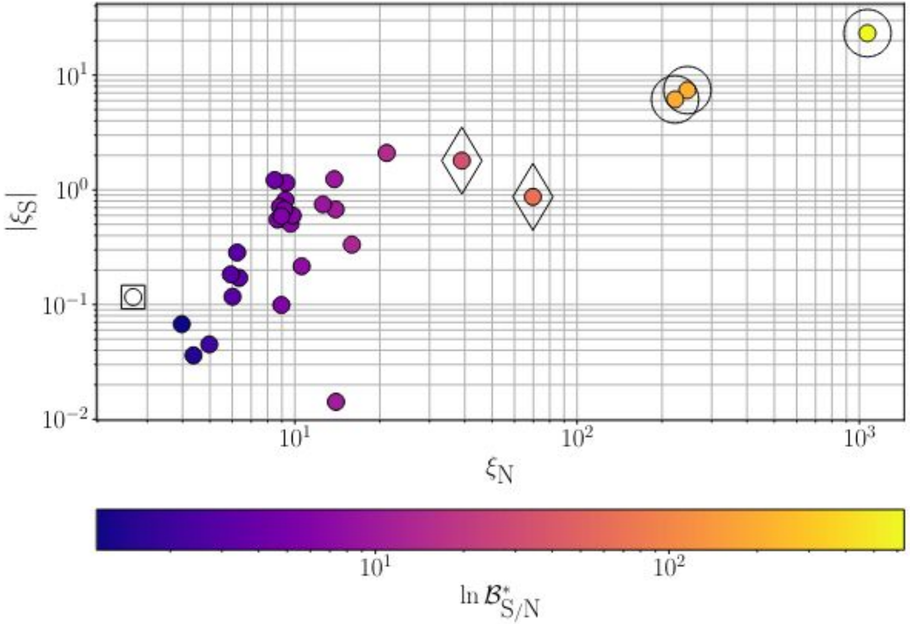
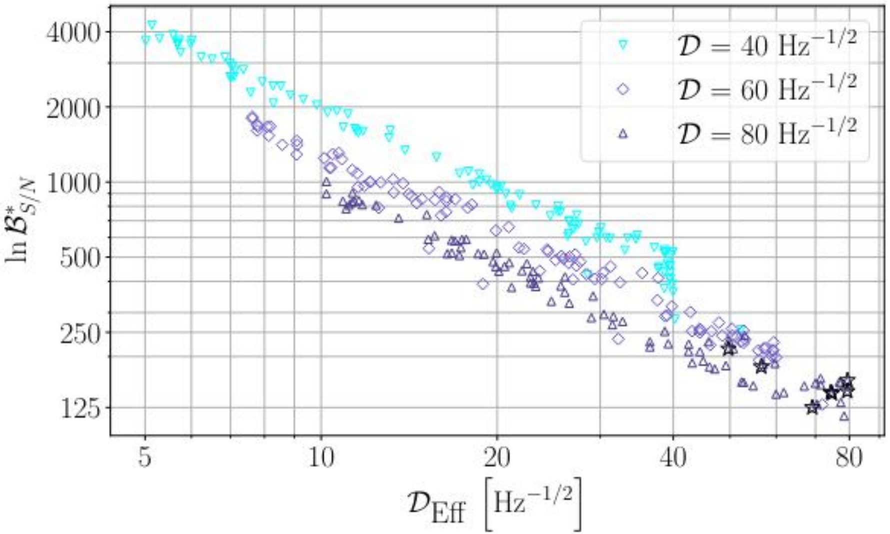
MCMC Follow-up of CW candidates

Tenorio+ arxiv:2105.13860

Pilot application: ~30 outliers produced in LIGO O2 searches.
Our Bayes factor disfavors an astrophysical origin for *all* of them.





Impact

Impact

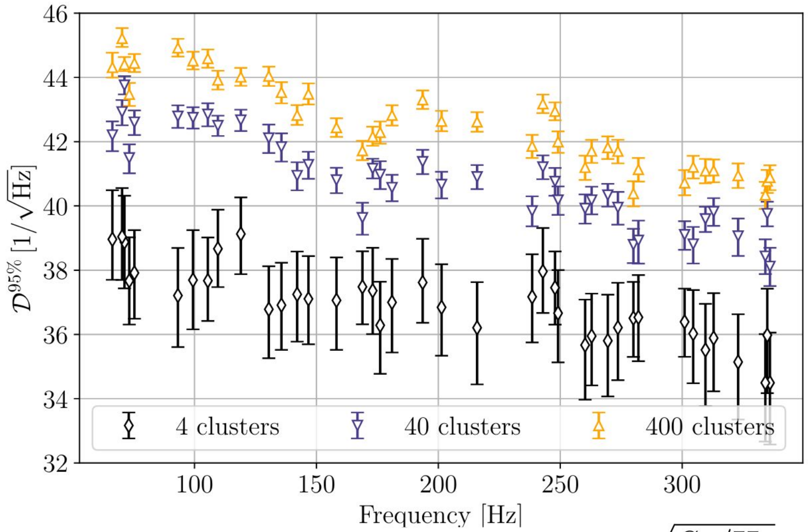
MCMC follow-up is a *systematic search-independent* approach to assess the astrophysical origin of a CW candidate.

First of its kind, this approach was used in O(10) LVK searches in O3 data, vastly simplifying their analyses.
Before, manual checks and ad-hoc vetoes where the norm.

Due to its high computing efficiency, it is now feasible to increase the *false alarm probability* of a search, as the MCMC follow-up will rapidly lower it again in a few stages.

More false alarms → More probability of detection!

Mirasola & Tenorio arxiv:2405.18934



$$\mathcal{D} = \frac{\sqrt{S_h/\mathrm{Hz}}}{h_0^C}$$

Specifically, ~(10-15)% sensitivity increase *at negligible computing cost* after calibrating the MCMC follow-up.

Increase of ~(30-50)% in surveyed volume!

Conclusion

Conclusion

Gravitational-wave astronomy is an exciting field when it comes to data analysis:
Complicated noise, intricate signal models, vast parameter spaces, …

We discussed the use of MCMC to follow-up CW candidates. In particular:
- How to decide whether the setup is “good”.
- How to decide if the candidate is “astrophysical” or not.

This involved the application of multiple probability results:
- Bayes factors and Bayesian probability in general.
- Extreme-value theory.
- Numerical simulations to estimate probabilities.

The resulting method has tangible impact:
- *Simpler* and more systematic CW follow-ups.
- Significant increase of the search sensitivity at a negligible computing cost.

This is just a small percentage of GW’s data-analysis potential, there’s many more avenues to explore! (low-latency/FPGA/GPU computing, machine-learning for detection and mitigation of non-Gaussianities, GenAI as a sampler… ask if interested!).