Model-Based Software Design, A.Y. 2022/23

Laboratory 2 Report

Components of the working group (max 2 people)

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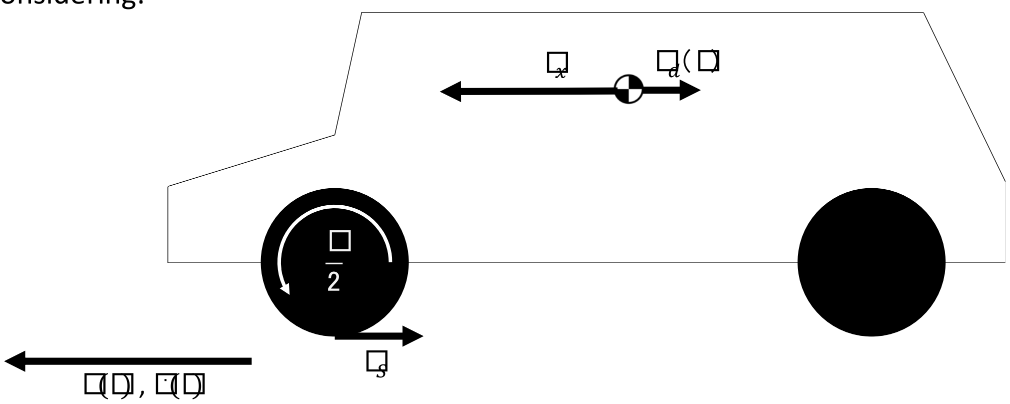
External interfaces of the plant

Name	Direction	Type
`Requested_Torque_Nm`	Input	CAN
`Vehicle_Speed_km_h`	Output	CAN
`Automatic_Transmission_Selector`	Input (from the driver to the controller)	CAN {P, R, N, D, P}
`Selected mode/errors`	Output (to the driver)	CAN

Equations of the plant

The plant considered in this model is the so-called *Vehicle Longitudinal Dynamics*.
Considering:



- $\dot{v}(t)$ the vehicle acceleration, expressed in $[\mathrm{m/s^2}]$
- $v(t)$ the vehicle longitudinal speed, expressed in $[\mathrm{m/s}]$
- m the vehicle mass, expressed in $[\mathrm{kg}]$
- $F_x(t)$ the longitudinal force applied to the vehicle center of gravity, expressed in $[\mathrm{N}]$
- $F_s(t)$ the longitudinal force applied to the wheel on the terrain, expressed in $[\mathrm{N}]$
- $F_d(v)$ the longitudinal force applied to the vehicle center of gravity due to the frictions with air and terrain, expressed in $[\mathrm{N}]$
- I the moment of inertia of each one of the wheels, expressed in $[\mathrm{kg \cdot m^2}]$
- r the radius of the wheel, expressed in $[\mathrm{m}]$
- $\omega(t)$ is the angular speed of the wheel, expressed in $[\mathrm{rad/s}]$
- $\omega_e(t)$ is the angular speed of the engine/electrical motor, expressed in $[\mathrm{rad/s}]$
- $\omega_{e_{RPM}}(t) = \frac{\omega_e(t) \cdot 60}{2\pi}$ is the angular speed of the engine/ electrical motor, expressed in $[\mathrm{revolutions\ per\ minutes}]$
- $\dot{\omega}(t)$ is the angular acceleration of the wheel, expressed in $[\mathrm{rad/s^2}]$
- S is the frontal surface of the car, expressed in $[\mathrm{m^2}]$
- $c_x = 0.3$ is the automobile drag coefficient
- ρ is the average density of air at sea level in standard conditions → $$\rho = 1.25\ \mathrm{kg/m^3}$$
- i_g is the gearbox reduction ratio
- i_f is the final drive reduction ratio
- $i_t = i_g \cdot i_f$ is the total power train reduction ratio.

An extremely simplified model can be obtained as follow:

$$\dot{v}(t) = \frac{F_x(t) - F_d(v)}{m} \qquad (1)$$

where $\dot{v}(t)$ is the vehicle acceleration, m is its mass, $F_x(t)$ is the longitudinal force applied to its center of gravity by the effects of the torque applied on the wheels, and $F_d(v)$ is the sum of the friction forces on the vehicle due to wheel-terrain and vehicle-air interactions.

Considering that the torque is equally split between the two wheels (valid only on straight tracks)

$$T(t) - 2 \cdot F_s(t) \cdot r = 2I \cdot \dot{\omega}(t) \quad (2)$$

the absence of slipping:

$$\begin{cases} 2 \cdot F_s(t) \cdot r = F_x(t) \\ v(t) = 2\,\pi \cdot r \cdot \ \omega(t) \end{cases}$$

and considering the moment of inertia of the wheels $I = 0$, we can define the following equation, given that $T(t) = 2 \cdot F_s(t) \cdot r \rightarrow F_s(t) = \frac{T(t)}{2 \cdot r}$.

The drag force that limits the maximum speed of the vehicle is equal to:

$$F_d\big(v(t)\big) = X_{air} \cdot \big(v(t)\big)^2 + X_{tyres} \cdot v(t) \quad (3)$$

where:

$$X_{air} = \frac{1}{2} \cdot S \cdot c_x \cdot \rho \quad (4)$$

and, as usually modeled:

$$X_{tyres}\Big|_{X_{tyres}\left(50\frac{\mathrm{km}}{\mathrm{h}}\right) = X_{air}\left(50\frac{\mathrm{km}}{\mathrm{h}}\right)} \rightarrow X_{tyres} = \frac{X_{air} \cdot 50}{3.6} \quad (5)$$

By substituting the (2) equation in (1), and by integrating both sides, we obtain:

$$v(t) = \frac{1}{m}\int_0^t F_x(t) - F_d\big(v(t)\big)\,dt = \frac{1}{m}\int_0^t 2 \cdot F_s(t) - F_d\big(v(t)\big)\,dt =$$

$$= \frac{1}{m}\int_0^t \frac{T(t)}{r} - F_d\big(v(t)\big)\,dt \quad (6)$$

and, by substituting (3) in (6):

$$v(t) = \frac{1}{m}\int_0^t \frac{T(t)}{r} - X_{air} \cdot \big(v(t)\big)^2 - X_{tyres} \cdot v(t)\,dt \quad (7)$$

Remember that the integrator block of Simulink requires an initial condition corresponding to the vehicle's longitudinal speed at the beginning of the simulation, $v(0)$. A possible configuration of the integration block is shown in Figure 2.

During the model development, put all the needed gain to obtain as an output of the physical model a speed expressed in km/h.
To simulate the slope θ of the terrain, it is possible to add the gravity force $F_g(\theta)$ as follows:

$$v(t) = \frac{1}{m}\int_0^t \frac{T(t)}{r} - X_{air} \cdot \big(v(t)\big)^2 - X_{tyres} \cdot v(t)dt + m \cdot g \cdot \sin{(\theta)} \quad (8)$$

With $g = 9.81$ the gravity acceleration on Earth.

Reasonable values for an electric compact car can be:

- $m = 1600$ kg
- $r = 0.3$ m
- The torque T (at the wheel) can vary in the range $[-60; 960]$ Nm
- $S = 3.5\ m^2$
- $c_x = 0.3$

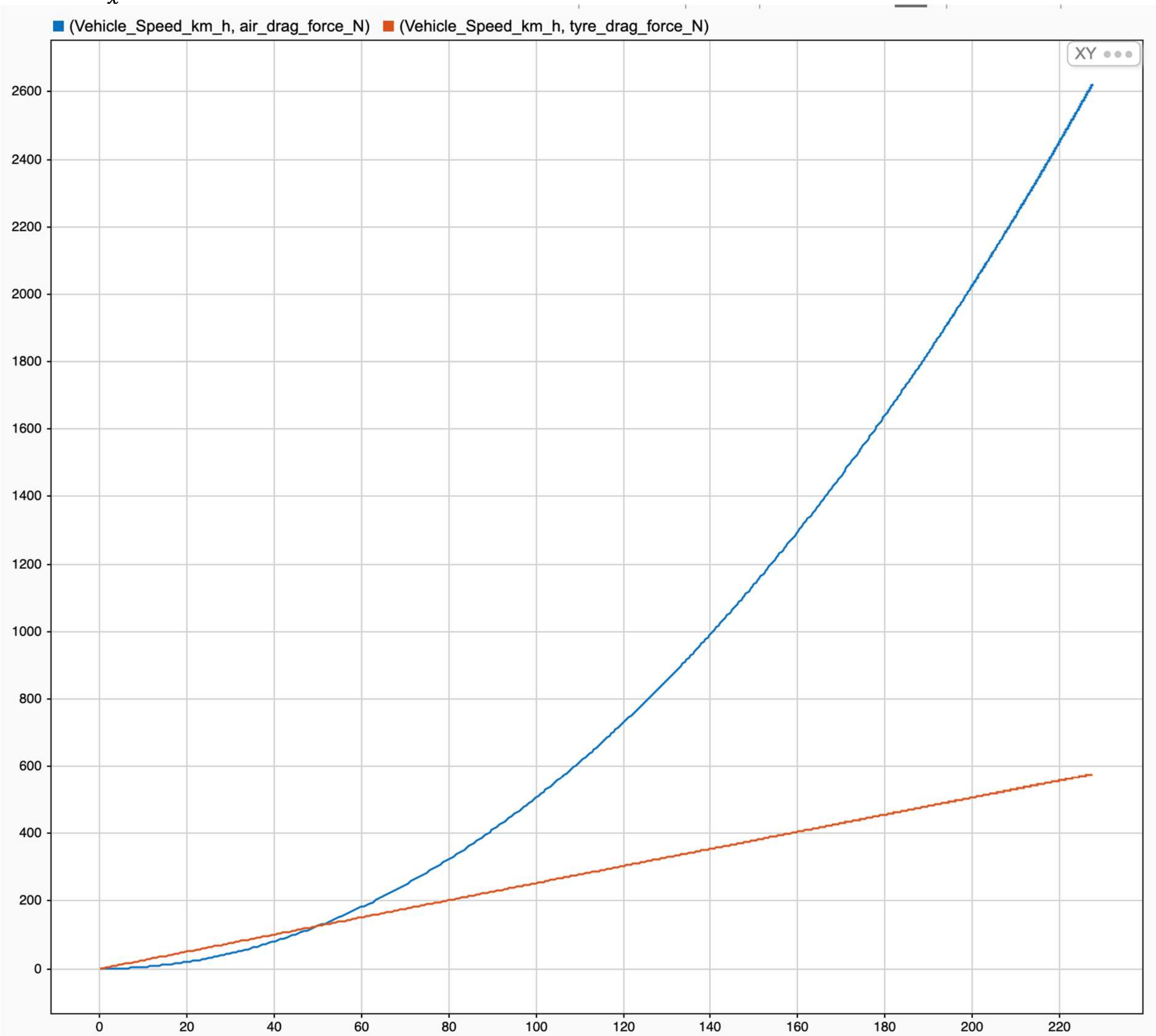


Figure 1 Graph showing drag forces of tires (in orange) and air (in blue) at various speeds. It is possible to observe that, as imposed in equation (5), $X_{tyre} = X_{air}$ at 50 km/h. Below this speed, the tire drag is dominant, after that, the air drag is dominant. Moreover, it is possible to see the top speed of the car (around 230 km/h) when $F_x = \frac{T}{r} = X_{tyre} + X_{air}$, with $F_x = 3200\ N$.

With those values, the top speed on level ground reachable by the car is about 230 km/h, where the drag forces equal the traction force (3200 N).
Considering the reverse direction, the maximum speed reachable with a limitation of -60 Nm is about 45 km/h.

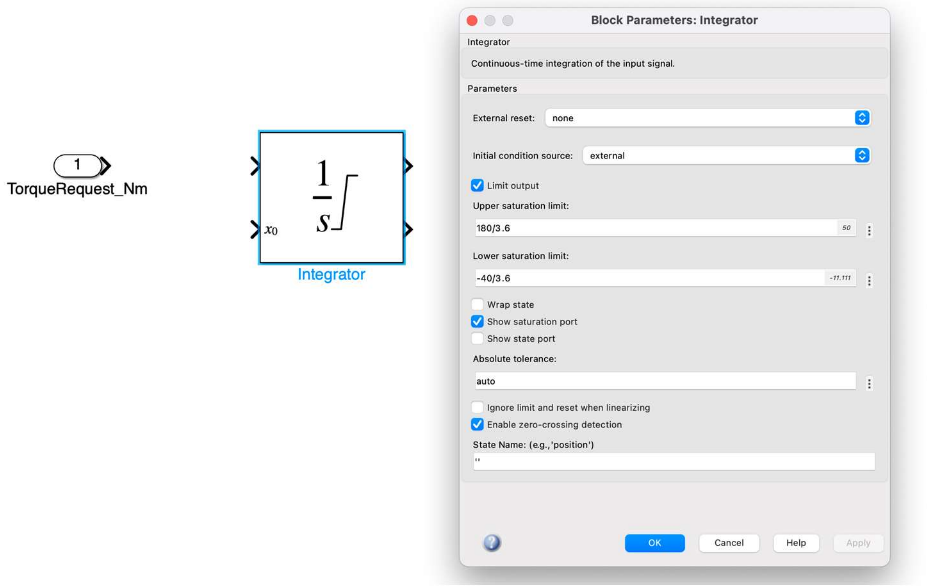


Figure 2 Settings window for the Integrator block of Simulink

Use these values (with a certain tolerance, for example, 10 %) to saturate the integrator block.

To make the model more realistic, it is possible to compute the torque request at the engine/motor. A typical ratio value for transmission of an electric car with a single gear can be around $i_t = 12$[1].

All the initialization parameters of the model are automatically loaded model by a callback of the function **init_fn** as shown in Figure 3.

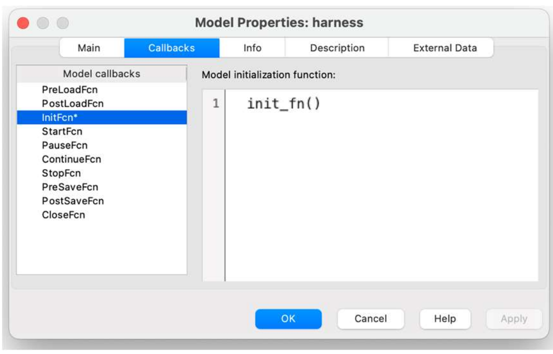


Figure 3 init_fn callback configuration in the harness model properties.

[1] Usually, the first gear of a car has a transmission ratio about $i_g = 3$, hence the engine makes 3 complete rotations very single rotation of the pinion gear of the differential. In the differential (final drive), the pinion gear makes about $i_f = 4$ revolutions for every revolution of the axle shafts. From here, the typical ratio of $i_t = 12$ between the torque at the wheel and the torque at the engine/electrical motor.

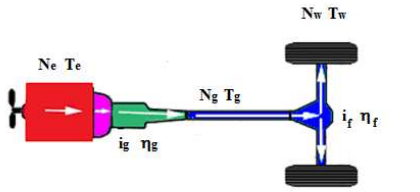
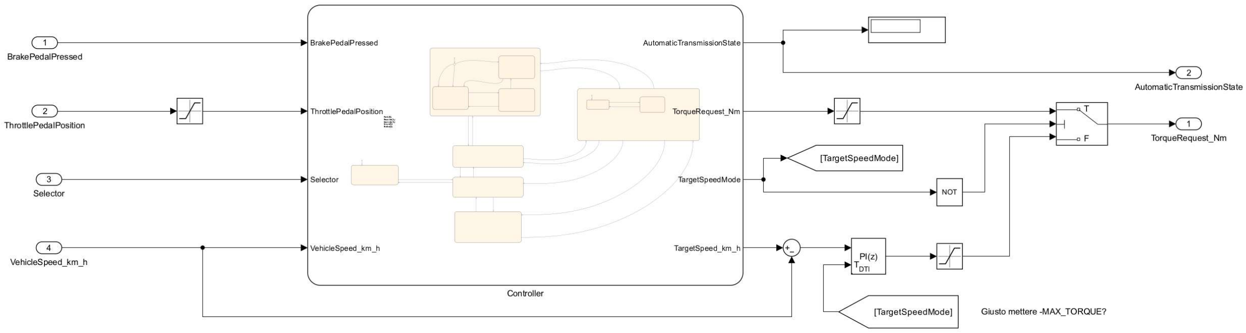
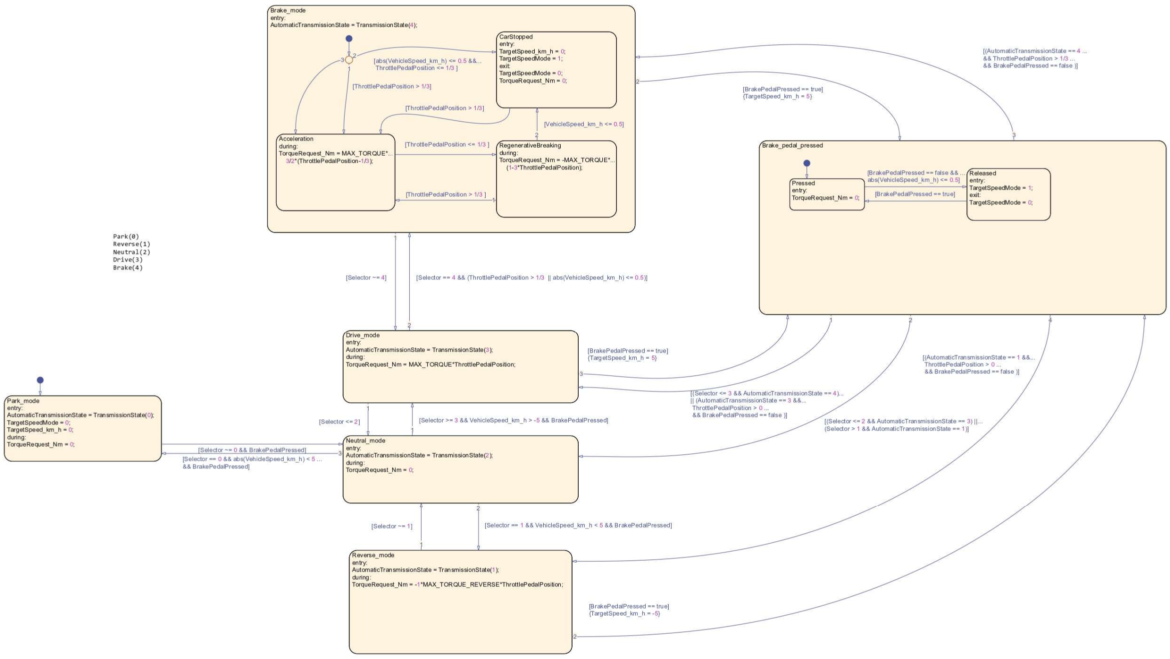


Figure from http://www.thecartech.com/subjects/auto_eng/Gearbox_ratios3.htm

Description of the whole system

Draw the I/O block diagram of the plant and of the controller, showing how they interact to each other.





Controller SW Unit specifications

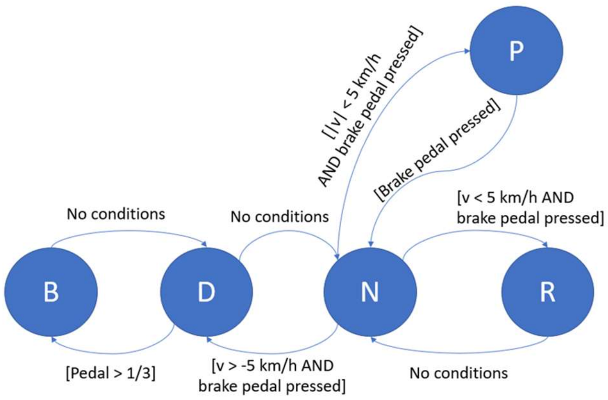
Provide a brief description of the Controller functionalities and its interfaces.

Interfaces

Name	Unit*	Type	Data Type	Dimension	Min	Max
BrakePedalPressed	N/A	CAN	bool	1x1	-	-
ThrottlePedalPosition	N/A	Analog	Single**	1x1	0	1
Automatic Transmission SelectorState	N/A	CAN {P, R, N, D, P}	Enum	1x1	0	4
VehicleSpeed_km_h	km/h	CAN	Single**	1x1	-60	240
Automatic TransmissionState (output)	N/A	CAN {P, R, N, D, P}	Enum	1x1	0	4
TorqueRequest_Nm (output)	Nm	CAN	Single**	1x1	-80	80

** Single precision floating point number

Draw the Finite State Machine (FSM) representing the controller logic:



the conditions selector == target state have been omitted

Comment on the design choices of the FSM, which are not trivial to be understood just by analyzing the controller logic.

- *It is possible to go from the Brake_pedal_pressed state to the Drive state even if the brake is still pressed if you change the position of the switch, if the brake is not released it will instantly return to the Brake_pedal_pressed state after updating the AutomaticTransmissionState.*
- *When TargetSpeedMode switches to true, the requested torque no longer depends on the pedal position but is calculated to reach the speed set in TargetSpeed_km_h, the integrative action in addition to the proportional action gives greater stability to the controller and guarantees its correct operation even in the event of sloping road.*