Robot Learning Homework 1

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1 Introduction

The purpose of this experiment is to study the behavior of an Extended Kalman Filter (EKF) applied to two nonlinear systems, namely, a single and a double pendulum. The EKF is a variant of the Linear Kalman Filter that works by dynamically linearizing the system at each iteration. Since the EKF is not optimal, in contrast to the linear Kalman Filter, the result may not be so accurate. In particular it performs better with system with small linearities. In the followin it is proposed first, a mathematical description of the two systems and their filters, and then a comparison of the performance of the respective EFK is presented.

2 Mathematical Models

2.1 Discretized Systems

Given that nowadays controllers are implemented using digital systems, such as microcontrollers, the model of the systems has to be rendered discrete. In particular we can make use of the forward Euler method to approximate the continuous dynamics in the discrete domain. Given the following nonlinear autonomous dynamical system

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases}$$

it can be linearized as

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \cdot \mathbf{f}(\mathbf{x}_{k-1})$$

where Δt is the sampling time. This representation allows us to express the jacobian of the discretized system, with respect to $\mathbf{x}_{k-1}$ as follows

$$\mathbf{A} = \frac{\partial \mathbf{x}_k}{\partial \mathbf{x}_{k-1}} = \mathbf{I}_n + \Delta t \left. \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_{k-1}}$$

2.2 Single Pendulum

Regarding the unactuated single pendulum system we use as state variables the angular position $x_1 = \theta$ and the angular velocity $x_2 = \dot{\theta}$, and read as output θ. The system presents a process noise $\boldsymbol{\epsilon}$ with variance $\mathbf{Q}$ and zero mean, and a measurement noise δ with variance R and zero mean. The resulting system can be described by the following state-space model

$$\begin{cases} \dot{x_1} = x_2 + \epsilon_1 \\ \dot{x_2} = -\frac{g}{l}\sin(x_1) + \epsilon_2 \\ y = x_1 + \delta_1 \end{cases}$$

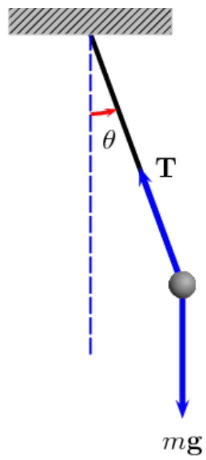

Figure 1: Unactuated Single Pendulum System

In order to develop the EKF we first need to derive the jacobian matrices of the discretized system

$$\mathbf{A} = \frac{\partial \mathbf{x}_k}{\partial \mathbf{x}_{k-1}} = \mathbf{I}_2 + \Delta t \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos(x_{1_{k-1}}) & 0 \end{bmatrix}$$

$$\mathbf{C} = \frac{\partial y}{\partial \mathbf{x}} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

2.3 Double Pendulum

The double pendulum system considered here consists of two masses connected by ropes, which we consider to be massless and inextensible. The two masses are, respectively, m_1 and m_2, and the two ropes have lenghts l_1 and l_2

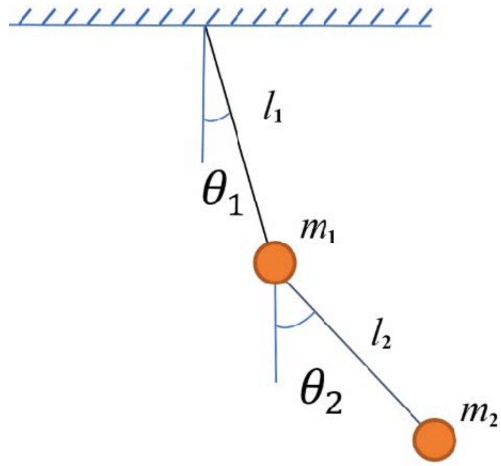

Figure 2: Unactuated Double Pendulum System

The position of the two masses can be expressed in function of $\theta_1, \theta_2, l_1, l_2$ as

$$\begin{cases} x_{m_1} = l_1 \sin(\theta_1) \\ y_{m_1} = -l_1 \cos(\theta_1) \\ x_{m_2} = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) \\ y_{m_2} = -l_1 \cos(\theta_1) - l_2 \cos(\theta_2) \end{cases}$$

Given the complexity of the system, to derive its space state model, we can first write the Lagrangian $\mathcal{L} = K - U$ of the system

$$\begin{aligned} \mathcal{L} \quad = \quad & \tfrac{1}{2} m_1 l_1^2 \dot{\theta_1}^2 + \tfrac{1}{2} m_2 l_1^2 \dot{\theta_1}^2 + \tfrac{1}{2} m_2 l_2^2 \dot{\theta_2}^2 \\ & + m_2 l_1 l_2 \dot{\theta_1} \dot{\theta_2} \cos(\theta_1 - \theta_2) + m_1 g l_1 \cos(\theta_1) \\ & + m_2 g l_1 \cos(\theta_1) + m_2 g l_2 \cos(\theta_2) \end{aligned}$$

We can now apply the Lagrange equation to describe the dynamics of the system

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i} = 0$$

which result in the following system of second order differential equations

$$\begin{cases} (m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) \\ + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + (m_1 + m_2) g \sin(\theta_1) = 0 \\ \\ l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \\ + g \sin(\theta_2) = 0 \end{cases}$$

We can now choose as our state vector $\mathbf{x} = [\ \theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2\ ]^{\mathrm{T}}$, and as output vector $\mathbf{y} = [\ \theta_1 \quad \theta_2\ ]^{\mathrm{T}}$. The state is perturbed by a process noise $\boldsymbol{\epsilon}$ with variance $\mathbf{Q}$ and zero mean, and the output is perturbed by a measurement noise with variance $\mathbf{R}$ and zero mean. By expliciting $\ddot{\theta}_1$ and $\ddot{\theta}_2$ from the previous system, we obtain continuous-time state-space model for the double pendulum system, which we can discretize and the derive the jacobian matrices $\mathbf{A} \in \mathbb{R}^{4,4}$ and $\mathbf{C} \in \mathbb{R}^{2,4}$.

3 ROS Integration

The single pendulum system and its EKF can be implemented in ROS as follows: one node takes the role of the real systems, and generates the real data corresponding to the system's state. Such data is then packed into a message of type `StateData.msg`, which comprise θ and $\dot{\theta}$. The message is sent to the second node via the `real_data` topic. The second node takes the role of a sensors, reading the state measurements and outputting θ, after having added it the noise. The data is packed in a message of type `OutputData.msg` and sent through the `sensor_data` topic. Finally, the last node, takes the role of the EKF, which reads the noisy measurements of the sensor, performs the EKF algorithm, and outputs the filtered state in a message of type `StateData.msg` via the `ekf_data` topic.

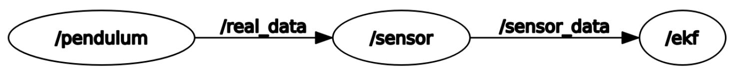

Figure 3: ROS Nodes and Topics for the Single Pendulum

The double pendulum system on ROS works equivalently, but the messages sent (`StateData.msg` and `OutputData.msg`) contain, respectively, $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)$ and (θ_1, θ_2).

4 Experimental Results

4.1 Single Pendulum

By running the simulation for $t = 400$ iterations, using a sampling time $\Delta t = 0.01s$, and having the covariance matrix of the process $\mathbf{Q} = 0.00001 \cdot \mathbf{I}_2$, we can observe the following EKF behavior for $R = 0.05$, $R = 0.5$ and $R = 5$

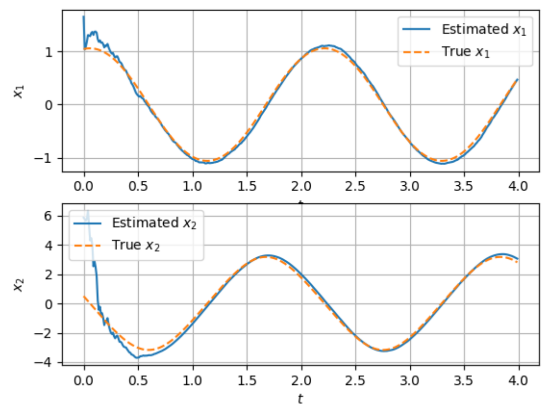

Figure 4: EKF Performance for $R = 0.05$

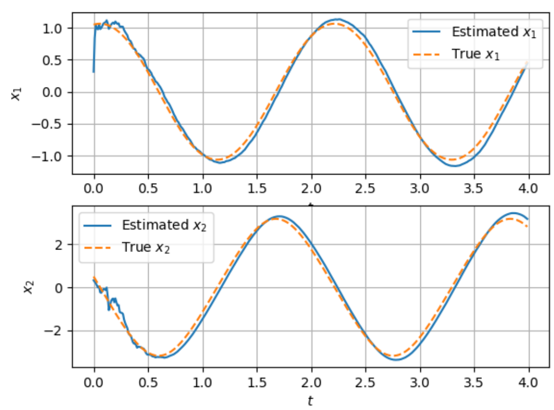

Figure 5: EKF Performance for $R = 0.05$

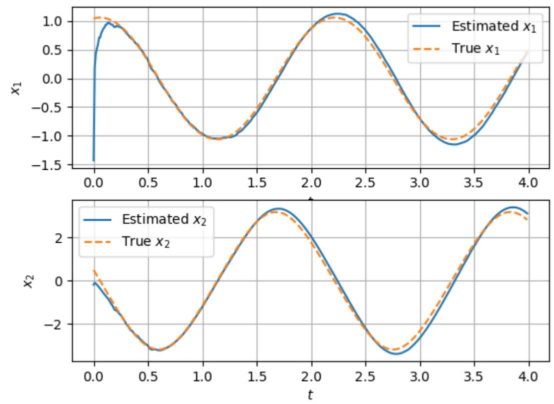

Figure 6: EKF Performance for $R = 0.05$

4.2 Double Pendulum

By running the simulation for $t = 400$ iterations, using a sampling time $\Delta t = 0.01s$ and system variables $m_1 = 1$, $m_2 = 2$, $l_1 = 1$, $l_2 = 2$, and having the covariance matrix of the process $\mathbf{Q} = 0.00001 \cdot \mathbf{I}_4$, we can observe the following EKF behavior for $\mathbf{R} = 0.05 \cdot \mathbf{I}_2$

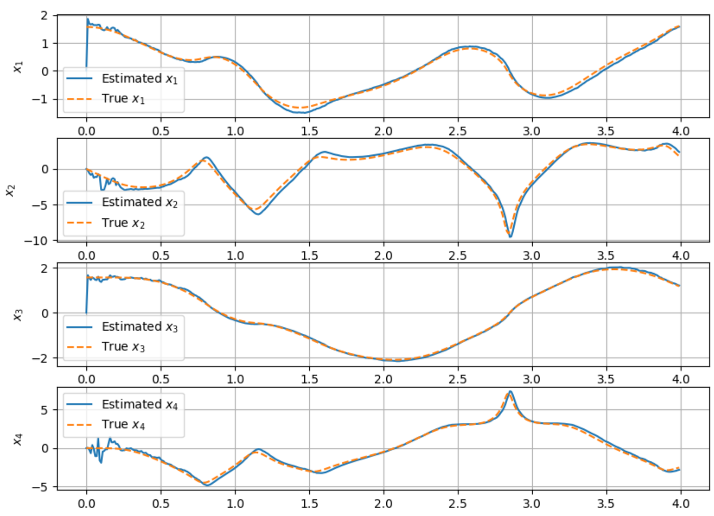

Figure 7: Double Pendulum EKF

5 Conclusion

As it is possible to see, the EKF of the single pendulum performs well, even in the presence of a greater uncertainty. This is due to the fact that the system's non-linearity is not so pronounced, and thus the linearization performed yields acceptable results.

On the contrary, if we examine the EKF results for the double pendulum, we can see that the filter does not perform as well, in particular with respect to the x_2 and x_4 states. The performance is significantly compromised by the strong non-linearity of the system under examination.

In conclusion we can say that the EKF can produce acceptable results if the system transitions from k to $k+1$ are smooth, i.e. if the non-linearities are not very pronounced.