RICE

COMP 412
FALL 2010

Bottom-up Parsing, Part I

Comp 412

Reorganize this entire lecture (based on Fall 2010 experience)
Try giving it once and then reorder the points, so that the points are in a logical order.

Recap of Top-down Parsing

- Top-down parsers build syntax tree from root to leaves
- Left-recursion causes non-termination in top-down parsers
  - Transformation to eliminate left recursion
  - Transformation to eliminate common prefixes in right recursion
- FIRST, FIRST$^+$, & FOLLOW sets + LL(1) condition
  - LL(1) uses left-to-right scan of the input, leftmost derivation of the sentence, and 1 word lookahead
  - LL(1) condition means grammar works for predictive parsing
- Given an LL(1) grammar, we can
  - Build a recursive descent parser
  - Build a table-driven LL(1) parser
- LL(1) parser doesn't build the parse tree
  - Keeps lower fringe of partially complete tree on the stack

Parsing Techniques

Top-down parsers *(LL(1), recursive descent)*

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad "pick" $\Rightarrow$ may need to backtrack
- Some grammars are backtrack-free *(predictive parsing)*

Bottom-up parsers *(LR(1), operator precedence)*

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of grammars

Bottom-up Parsing (definitions)

The point of parsing is to construct a ***derivation***

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a sentence in $L(G)$
  - If γ contains 1 or more non-terminals, γ is a sentential form
- To get γ_i from γ_{i-1}, expand some NT $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - In a leftmost derivation, it would be the first NT $A \in \gamma_{i-1}$

A *left-sentential form* occurs in a *leftmost* derivation

A *right-sentential form* occurs in a *rightmost* derivation

Bottom-up parsers build a rightmost derivation in reverse

We saw this definition earlier

Bottom-up Parsing (definitions)

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol *S*

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

$$\xleftarrow{\hspace{20em}} \text{bottom-up}$$

To reduce γ_i to γ_{i-1} match some *rhs* β against γ_i then replace β with its corresponding *lhs*, *A*. *(assuming the production $A \rightarrow \beta$)*

In terms of the parse tree, it works from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of β with *A* shrinks the upper fringe, we call it a *reduction*.
- "Rightmost derivation in reverse" processes words *left to right*

The parse tree need not be built, it can be simulated

$$|\textit{parse tree nodes}| = |\textit{terminal symbols}| + |\textit{reductions}|$$

Finding Reductions

Consider the grammar

0	*Goal*	→	a *A* *B* e
1	*A*	→	*A* b c
2		|	b
3	*B*	→	d

And the input string abbcde

Sentential	*Next Reduction*	
Form	*Prod'n*	*Pos'n*
abbcde	2	2
a *A* bcde	1	4
a *A* de	3	3
a *A* *B* e	0	4
Goal	—	—

The trick is scanning the input and finding the next reduction

The mechanism for doing this must be efficient

"Position" specifies where the right end of β occurs in the current sentential form.

While the process of finding the next reduction appears to be almost oracular, it can be automated in an efficient way for a large class of grammars

Finding Reductions (Handles)

The parser must find a substring β of the tree's frontier that *matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation* ($\Rightarrow \beta \rightarrow A$ is in RRD)

Informally, we call this substring β a *handle*

Formally,

A *handle* of a right-sentential form γ is a pair $\langle A\rightarrow\beta,k\rangle$ where $A\rightarrow\beta \in P$ and k is the position in γ of β's rightmost symbol.

If $\langle A\rightarrow\beta,k\rangle$ is a handle, then replacing β at k with A produces the right sentential form from which γ is derived in the rightmost derivation.

Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

$\Rightarrow$ the parser doesn't need to scan (*much*) past the handle

Most students find handles mystifying; bear with me for a couple more slides.

Example

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr + Term*
2		|	*Expr - Term*
3		|	*Term*
4	*Term*	→	*Term * Factor*
5		|	*Term / Factor*
6		|	*Factor*
7	*Factor*	→	<u>number</u>
8		|	<u>id</u>
9		|	<u>(</u> *Expr* <u>)</u>

A simple left-recursive form of the classic expression grammar

Bottom up parsers handle either left-recursive or right-recursive grammars.

We will use left-recursive grammars for arithmetic because of our bias toward left-to-right evaluation in algebra.

Example

derivation

0	*Goal*	$\rightarrow$	*Expr*
1	*Expr*	$\rightarrow$	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	$\rightarrow$	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	$\rightarrow$	number
8		|	id
9		|	(*Expr*)

Prod'n	*Sentential Form*
—	*Goal*
0	*Expr*
2	*Expr* - *Term*
4	*Expr* - *Term* * *Factor*
8	*Expr* - *Term* * <id,y>
6	*Expr* - *Factor* * <id,y>
7	*Expr* - <num,2> * <id,y>
3	*Term* - <num,2> * <id,y>
6	*Factor* - <num,2> * <id,y>
8	<id,x> - <num,2> * <id,y>

A simple left-recursive form of the classic expression grammar

Rightmost derivation of x - 2 * y

Example

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	<u>number</u>
8		|	<u>id</u>
9		|	<u>(</u> *Expr* <u>)</u>

A simple left-recursive form of the classic expression grammar

Prod'n	*Sentential Form*	*Handle*
—	*Goal*	—
0	*Expr*	0,1
2	*Expr* - *Term*	2,3
4	*Expr* - *Term* * *Factor*	4,5
8	*Expr* - *Term* * <id,<u>y</u>>	8,5
6	*Expr* - *Factor* * <id,<u>y</u>>	6,3
7	*Expr* - <num,<u>2</u>> * <id,<u>y</u>>	7,3
3	*Term* - <num,<u>2</u>> * <id,<u>y</u>>	3,1
6	*Factor* - <num,<u>2</u>> * <id,<u>y</u>>	6,1
8	<id,<u>x</u>> - <num,<u>2</u>> * <id,<u>y</u>>	8,1

↑ parse

Handles for rightmost derivation of <u>x</u> <u>-</u> <u>2</u> <u>*</u> <u>y</u>

Bottom-up Parsing (Abstract View)

A bottom-up parser repeatedly finds a handle $A \rightarrow \beta$ in the current right-sentential form and replaces β with A.

To construct a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow w$$

Apply the following conceptual algorithm

```
for i ← $\boxed{n}$ to 1 by –1
    Find the handle $\langle A_i \rightarrow \beta_i , k_i \rangle$ in $\gamma_i$
    Replace $\beta_i$ with $A_i$ to generate $\gamma_{i-1}$
```

of course, *n* is unknown until the derivation is built

This takes *2n* steps

Some authors refer to this algorithm as a *handle-pruning parser.*

The idea is that the parser finds a *handle* on the upper fringe of the partially complete parse tree and *prunes* it out of the fringe.

The analogy is somewhat strained, so I will try to avoid using it.

More on Handles

Bottom-up reduce parsers find a rightmost derivation in reverse order

- Rightmost derivation ⇒ rightmost NT expanded at each step in the derivation
- Processed in reverse ⇒ parser proceeds left to right

These statements are somewhat counter-intuitive

More on Handles

Bottom-up parsers find a reverse rightmost derivation

- Process input left to right
  - Upper fringe of partially completed parse tree is (*NT* | *T*)* *T**
  - The handle always appears with its right end at the junction between (*NT* | *T*)* and *T** (*the hot spot for LR parsing*)
  - We can keep the prefix of the upper fringe of the partially completed parse tree on a stack
  - The stack makes the position information irrelevant
- Handles appear at the top of the stack
- All the information for the decision is at the hot spot
  - The next word in the input stream
  - The rightmost NT on the fringe & its immediate left neighbors
  - In an LR parser, additional information in the form of a “state”

Handles Are Unique

Theorem:

If G is unambiguous, then every right-sentential form has a unique handle.

Sketch of Proof:

1 G is unambiguous $\Rightarrow$ rightmost derivation is unique

2 $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}

3 $\Rightarrow$ a unique position k at which $A \rightarrow \beta$ is applied

4 $\Rightarrow$ a unique handle $\langle A \rightarrow \beta, k \rangle$

This all follows from the definitions

If we can find the handles, we can build a derivation!

The handle always appears with its right end at the stack top.
$\rightarrow$ How many right-hand sides must the parser consider?

Shift-reduce Parsing

To implement a bottom-up parser, we adopt the shift-reduce paradigm

A shift-reduce parser is a stack automaton with four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
    - Locate left end of handle within the stack
    - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

Accept & *Error* are simple
Shift is just a push and a call to the scanner
Reduce takes |*rhs*| pops & 1 push

But how does the parser know when to shift and when to reduce?
It shifts until it has a handle at the top of the stack.

Bottom-up Parser

A simple *shift-reduce parser:*

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle $A \to \beta$
        then      // reduce $\beta$ to A
            pop $|\beta|$ symbols off the stack
            push A onto the stack
        else if (token ≠ EOF)
            then // shift
                push token
                token ← next_token( )
        else      // need to shift, but out of input
            report an error
```

What happens on an error?

- It fails to find a handle
- Thus, it keeps shifting
- Eventually, it consumes all input

This parser reads all input before reporting an error, not a desirable property.

Error localization is an issue in the handle-finding process that affects the practicality of shift-reduce parsers…

We will fix this issue later.

Figure 3.7 in EAC

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	*none*	*shift*
$ id	- num * id		

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr + Term*
2		|	*Expr - Term*
3		|	*Term*
4	*Term*	→	*Term * Factor*
5		|	*Term / Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

Back to x - 2 * y

Stack	Input	Handle	*Action*
$	id - num * id	*none*	*shift*
$ id	- num * id	8,1	*reduce 8*
$ *Factor*	- num * id	6,1	*reduce 6*
$ *Term*	- num * id	3,1	*reduce 4*
$ *Expr*	- num * id		

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

1. Shift until the top of the stack is the right end of a handle
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Back to x - 2 * y

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$	id - num * id	*none*	*shift*
$ id	- num * id	8,1	*reduce 8*
$ *Factor*	- num * id	6,1	*reduce 6*
$ *Term*	- num * id	3,1	*reduce 4*
$ *Expr*	- num * id		

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

Expr is not a handle at this point because it does not occur at this point in the derivation.

While that statement sounds like oracular mysticism, we will see that the decision can be automated efficiently.

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

Back to x - 2 * y

Stack	Input	Handle	*Action*
$	id - num * id	*none*	*shift*
$ id	- num * id	8,1	*reduce 8*
$ *Factor*	- num * id	6,1	*reduce 6*
$ *Term*	- num * id	3,1	*reduce 3*
$ *Expr*	- num * id	*none*	*shift*
$ *Expr* -	num * id	*none*	*shift*
$ *Expr* - num	* id		

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	none	shift
$ id	- num * id	8,1	reduce 8
$ *Factor*	- num * id	6,1	reduce 6
$ *Term*	- num * id	3,1	reduce 3
$ *Expr*	- num * id	none	shift
$ *Expr* -	num * id	none	shift
$ *Expr* - num	* id	7,3	reduce 7
$ *Expr* - *Factor*	* id	6,3	reduce 6
$ *Expr* - *Term*	* id		

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

1. Shift until the top of the stack is the right end of a handle
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Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	none	shift
$ id	- num * id	8,1	reduce 8
$ *Factor*	- num * id	6,1	reduce 6
$ *Term*	- num * id	3,1	reduce 3
$ *Expr*	- num * id	none	shift
$ *Expr* -	num * id	none	shift
$ *Expr* - num	* id	7,3	reduce 7
$ *Expr* - *Factor*	* id	6,3	reduce 6
$ *Expr* - *Term*	* id	none	shift
$ *Expr* - *Term* *	id	none	shift
$ *Expr* - *Term* * id			

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	none	shift
$ id	- num * id	8,1	reduce 8
$ *Factor*	- num * id	6,1	reduce 6
$ *Term*	- num * id	3,1	reduce 3
$ *Expr*	- num * id	none	shift
$ *Expr* -	num * id	none	shift
$ *Expr* - num	* id	7,3	reduce 7
$ *Expr* - *Factor*	* id	6,3	reduce 6
$ *Expr* - *Term*	* id	none	shift
$ *Expr* - *Term* *	id	none	shift
$ *Expr* - *Term* * id		8,5	reduce 8
$ *Expr* - *Term* * *Factor*		4,5	reduce 4
$ *Expr* - *Term*		2,3	reduce 2
$ *Expr*		0,1	reduce 0
$ *Goal*		none	accept

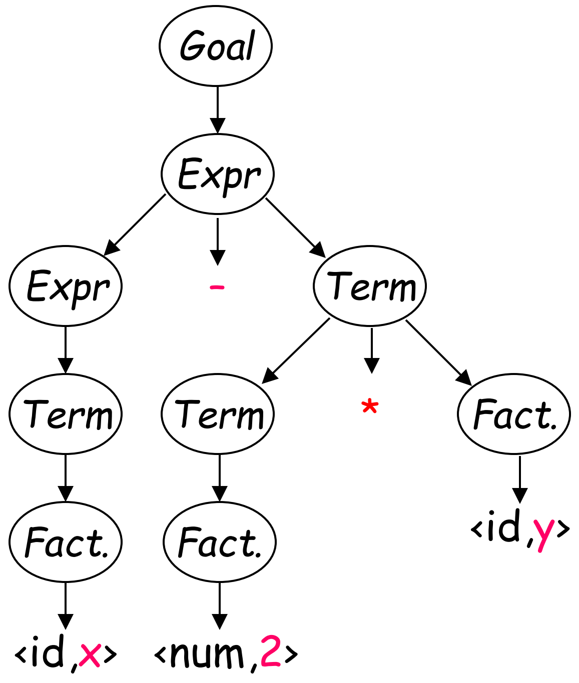
0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Expr* - *Term*
3		|	*Term*
4	*Term*	→	*Term* * *Factor*
5		|	*Term* / *Factor*
6		|	*Factor*
7	*Factor*	→	number
8		|	id
9		|	(*Expr*)

5 shifts +
9 reduces +
1 accept

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle and reduce

Back to x - 2 * y

Stack	Input	Action
$	id - num * id	shift
$ id	- num * id	reduce 8
$ *Factor*	- num * id	reduce 6
$ *Term*	- num * id	reduce 3
$ *Expr*	- num * id	shift
$ *Expr* -	num * id	shift
$ *Expr* - num	* id	reduce 7
$ *Expr* - *Factor*	* id	reduce 6
$ *Expr* - *Term*	* id	shift
$ *Expr* - *Term* *	id	shift
$ *Expr* - *Term* * id		reduce 8
$ *Expr* - *Term* * *Factor*		reduce 4
$ *Expr* - *Term*		reduce 2
$ *Expr*		reduce 0
$ *Goal*		accept



Corresponding Parse Tree

An Important Lesson about Handles

A handle must be a substring of a sentential form γ such that :

- It must match the right hand side β of some rule $A \rightarrow \beta$; and
- There must be some rightmost derivation from the goal symbol that produces the sentential form γ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - Answer: We use left context, encoded in the sentential form, left context encoded in a "parser state", and a lookahead at the next word in the input. (Formally, 1 word beyond the handle.)
  - Parser states are derived by reachability analysis on grammar
  - We build all of this knowledge into a handle-recognizing DFA

The additional left context is precisely the reason that LR(1) grammars express a superset of the languages that can be expressed as LL(1) grammars

LR(1) Parsers

- LR(1) parsers are table-driven, shift-reduce parsers that use a limited right context (1 token) for handle recognition
- The class of grammars that these parsers recognize is called the set of LR(1) grammars

Informal definition:

A grammar is LR(1) if, given a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

We can

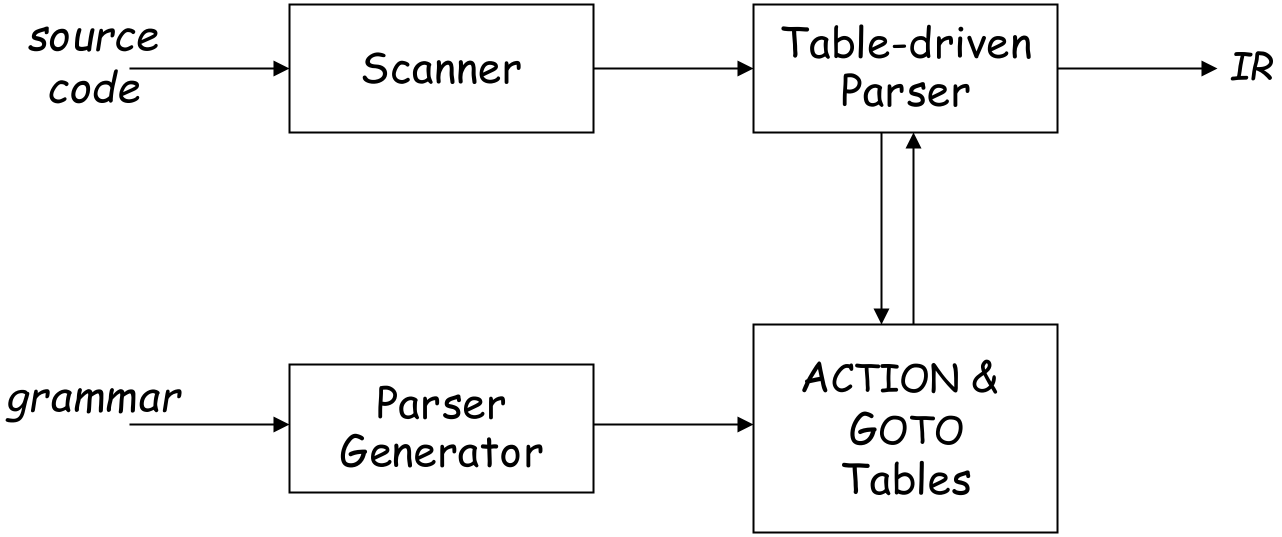
1. *isolate the handle of each right-sentential form* γ_i, and
2. *determine the production by which to reduce,*

by scanning γ_i from *left-to-right*, going at most *1* symbol beyond the right end of the handle of γ_i

LR(1) means left-to-right scan of the input, rightmost derivation (in reverse), and 1 word of lookahead.

LR(1) Parsers

A table-driven LR(1) parser looks like

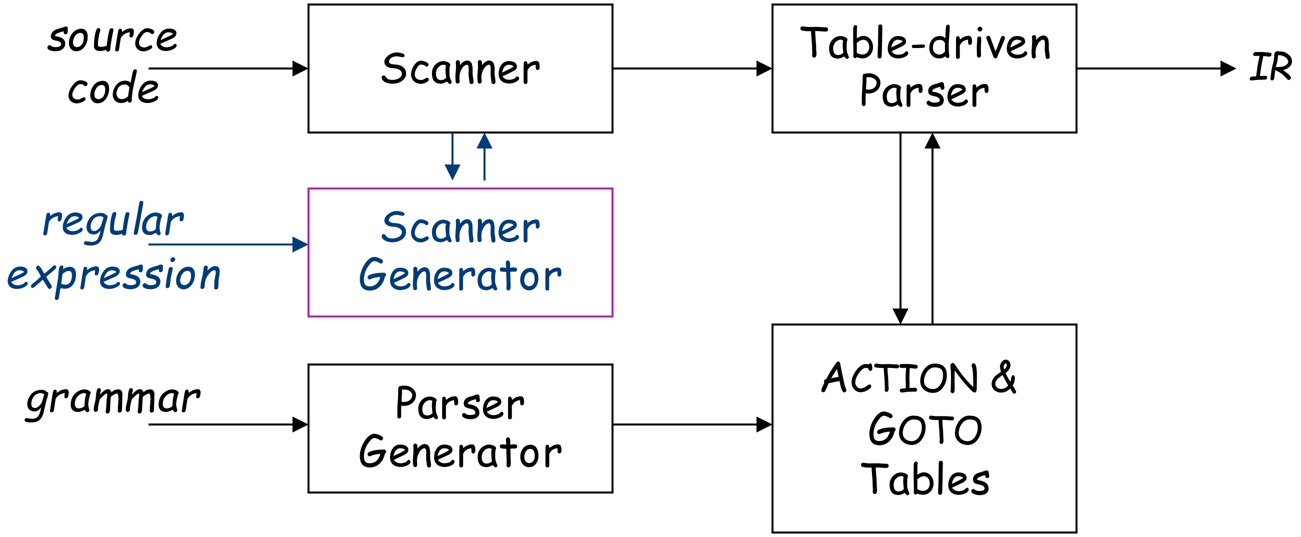


Tables *can* be built by hand

However, this is a perfect task to automate

LR(1) Parsers

A table-driven LR(1) parser looks like



Tables *can* be built by hand

However, this is a perfect task to automate

Just like automating construction of scanners …

Except that compiler writers use parser generators …

LR(1) Skeleton Parser

```
stack.push(INVALID);
stack.push($s_0$);                    // initial state
token = scanner.next_token();
loop forever {
        s = stack.top();
        if ( ACTION[s,token] == "reduce $A \to \beta$" ) then {
            stack.popnum(2*|$\beta$|);      // pop 2*|$\beta$| symbols
            s = stack.top();
            stack.push(A);              // push A
            stack.push(GOTO[s,A]);  // push next state
        }
        else if ( ACTION[s,token] == "shift $s_i$" ) then {
                stack.push(token); stack.push($s_i$);
                token ← scanner.next_token();
        }
        else if ( ACTION[s,token] == "accept"
                                & token == EOF )
                then break;
        else throw a syntax error;
}
report success;
```

The skeleton parser

- relies on a stack & a scanner
- uses two tables, called ACTION & GOTO

  ACTION: state x word → state

  GOTO: state x *NT* → state
- shifts |*words*| times
- reduces |derivation| times
- accepts at most once
- detects errors by failure of the other three cases
- follows basic scheme for shift-reduce parsing from last lecture

LR(1) Parsers *(parse tables)*

To make a parser for *L(G)*, need a set of tables

Remember, this is the left-recursive SheepNoise; EaC shows the right-recursive version.

The grammar

1	*Goal*	$\rightarrow$	*SheepNoise*
2	*SheepNoise*	$\rightarrow$	*SheepNoise* <u>baa</u>
3		|	<u>baa</u>

The tables

ACTION Table		
State	EOF	<u>baa</u>
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 1

The string baa

Stack	Input	Action
$ s_0	baa EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 1

The string <u>baa</u>

Stack	Input	Action
$ s_0	<u>baa</u> EOF	*shift 2*
$ s_0 <u>baa</u> s_2	EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* <u>baa</u>
3		|	<u>baa</u>

ACTION Table		
State	EOF	<u>baa</u>
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 1

The string baa

Stack	Input	Action
$ $s_0	baa EOF	*shift 2*
$ s_0 baa s_2	EOF	*reduce 3*
$ s_0 *SN* s_1	EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 1

The string baa

Stack	Input	Action
$ s_0	baa EOF	*shift 2*
$ s_0 baa s_2	EOF	*reduce 3*
$ s_0 *SN* s_1	EOF	*accept*

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

Notice that we never cleared the stack — the table construction moved *accept* earlier by one action

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string baa baa

Stack	Input	Action
$ s_0	baa baa EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string baa baa

Stack	Input	Action
$ s_0	baa baa EOF	*shift 2*
$ s_0 baa s_2	baa EOF	

1	*Goal*	$\rightarrow$	*SheepNoise*
2	*SheepNoise*	$\rightarrow$	*SheepNoise* baa
3		|	baa

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string <u>baa</u> <u>baa</u>

Stack	Input	Action
$ s_0	<u>baa</u> <u>baa</u> EOF	*shift 2*
$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	*reduce 3*
$ s_0 *SN* s_1	<u>baa</u> EOF	

1	*Goal*	$\rightarrow$	*SheepNoise*
2	*SheepNoise*	$\rightarrow$	*SheepNoise* <u>baa</u>
3		|	<u>baa</u>

Last example, we faced EOF and we accepted. With <u>baa</u>, we shift …

ACTION Table		
State	EOF	<u>baa</u>
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string baa baa

Stack	Input	Action
$ s_0	baa baa EOF	*shift 2*
$ s_0 baa s_2	baa EOF	*reduce 3*
$ s_0 *SN* s_1	baa EOF	*shift 3*
$ s_0 *SN* s_1 baa s_3	EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string baa baa

Stack	Input	Action
\$ s_0	baa baa EOF	*shift 2*
\$ s_0 baa s_2	baa EOF	*reduce 3*
\$ s_0 *SN* s_1	baa EOF	*shift 3*
\$ s_0 *SN* s_1 baa s_3	EOF	*reduce 2*
\$ s_0 *SN* s_1	EOF	

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* baa
3		|	baa

Now, we accept

ACTION Table		
State	EOF	baa
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

Example Parse 2

The string <u>baa</u> <u>baa</u>

Stack	Input	Action
\$ s_0	<u>baa</u> <u>baa</u> EOF	*shift 2*
\$ s_0 <u>baa</u> s_2	<u>baa</u> EOF	*reduce 3*
\$ s_0 *SN* s_1	<u>baa</u> EOF	*shift 3*
\$ s_0 *SN* s_1 <u>baa</u> s_3	EOF	*reduce 2*
\$ s_0 *SN* s_1	EOF	*accept*

1	*Goal*	→	*SheepNoise*
2	*SheepNoise*	→	*SheepNoise* <u>baa</u>
3		|	<u>baa</u>

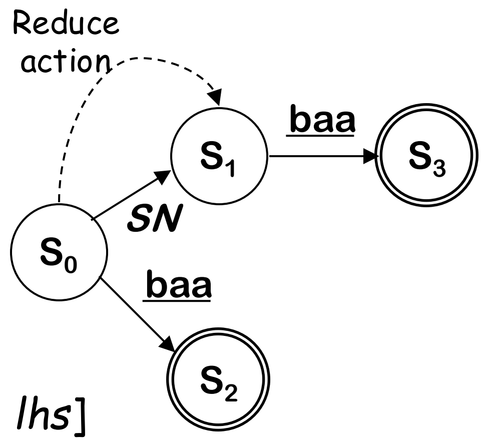
ACTION Table		
State	EOF	<u>baa</u>
0	—	*shift 2*
1	*accept*	*shift 3*
2	*reduce 3*	*reduce 3*
3	*reduce 2*	*reduce 2*

GOTO Table	
State	*SheepNoise*
0	1
1	0
2	0
3	0

LR(1) Parsers

How does this LR(1) stuff work?

- Unambiguous grammar $\Rightarrow$ unique rightmost derivation
- Keep upper fringe on a stack
  - All active handles include top of stack (TOS)
  - Shift inputs until TOS is right end of a handle
- Language of handles is regular (finite)
  - Build a handle-recognizing DFA
  - ACTION & GOTO tables encode the DFA
- To match subterm, invoke subterm DFA
  & leave old DFA's state on stack
- Final state in DFA $\Rightarrow$ a *reduce* action
  - New state is GOTO[state at TOS (after pop), *lhs*]
  - For *SN*, this takes the DFA to s_1



Control DFA for SN