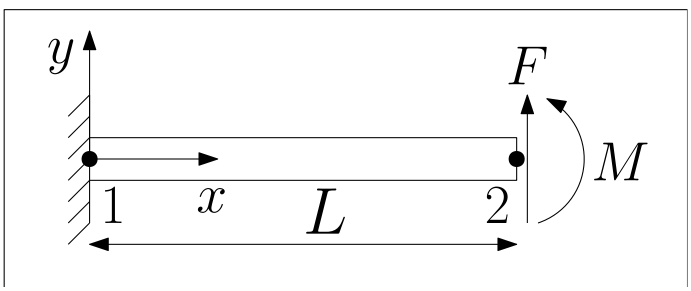
Stiffness matrix for a cantilever bending beam

Compute the stiffness matrix of the bending beam model in the figure above using two approaches:

1. by writing the strain complementary energy,
2. from the compliance matrix

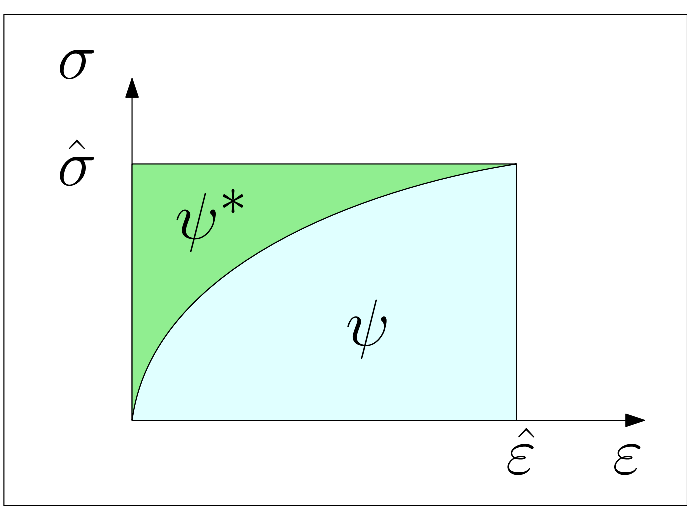
when the shear strain related to the shear load:

1. is neglected (Euler-Bernoulli's thin beam case);
2. is not neglected (Timoshenko's relatively thick beam case). The beam in the (x, y)-plane is clamped at the node 1 and is submitted to both a shear force and a torque at the node 2.



Refresher: complementary strain energy

Consider the following uniaxial stress-strain curve:



The strain energy density is the area underneath the stress-strain curve

$$\psi = \int_0^{\hat{\varepsilon}} \sigma \, d\varepsilon$$

The complementary strain energy density is the complementaty area to obtain a rectangular area

$$\psi^* = \int_0^{\hat{\sigma}} \varepsilon \, d\sigma.$$

In the linear case the two areas are equal, i.e. $\psi = \psi^*$.

The complementary energy expresses the energy in terms of stress rather than in terms of strains and it is suitable when one wants to apply e.g. Castigliano theorem.